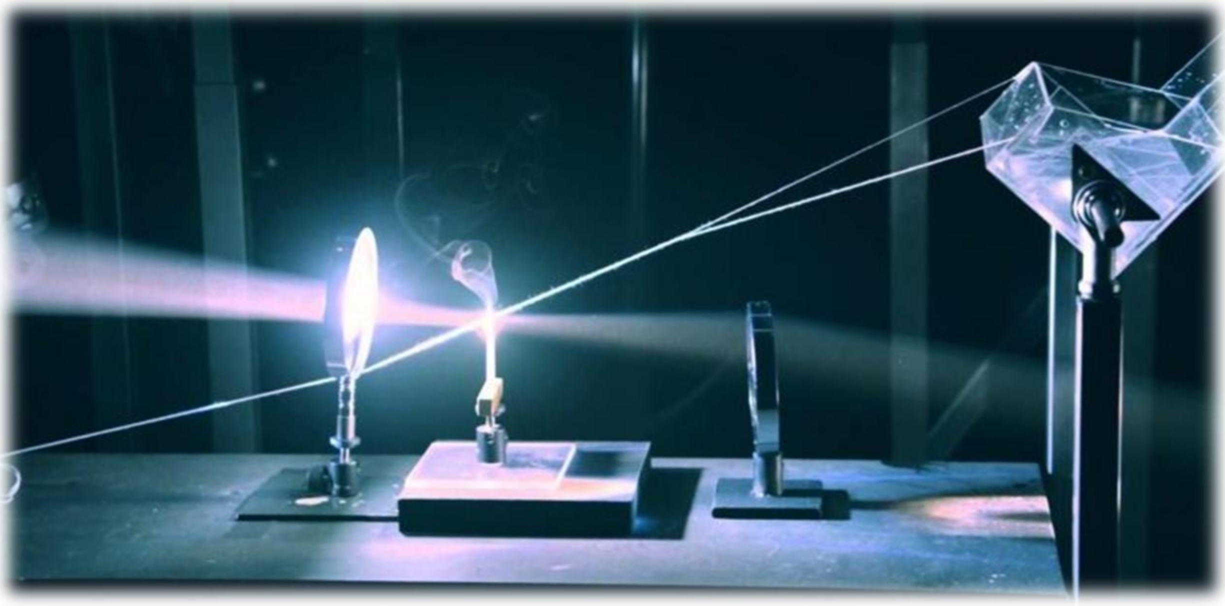
Engineering Physics



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MODULE 1. Magnetism

LESSON 1. Basic concept of Magnetism & its terminology

Introduction

Magnetism is a force of attraction or repulsion that acts at a distance. It is due to a magnetic field, which is caused by moving electrically charged particles. It is also inherent in magnetic objects such as a magnet.

A magnet is an object that exhibits a strong magnetic field and will attract materials like iron to it. Magnets have two poles, called the north (N) and south (S) poles. Two magnets will be attracted by their opposite poles, and each will repel the like pole of the other magnet. Magnetism has many uses in modern life.

Questions you may have include:

- What is a magnetic field?
- What is a magnetic force?
- What is the relationship between magnetism and electricity?

Magnetic field

A magnetic field consists of imaginary lines of flux coming from moving or spinning electrically charged particles. Examples include the spin of a proton and the motion of electrons through a wire in an electric circuit.

What a magnetic field actually consists of is somewhat of a mystery, but we do know it is a special property of space.

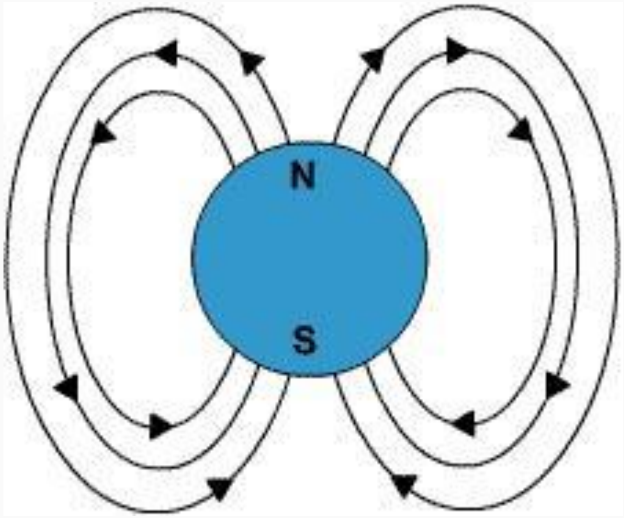


Fig.1(1) Magnetic field or lines of flux of moving charged particle

Names of poles

The lines of magnetic flux flow from one end of the object to the other. By convention, we call one end of a magnetic object the N or North-seeking pole and the other the S or South-seeking pole, as related to the Earth's North and South magnetic poles. The magnetic flux is defined as moving from N to S.

Note: The Earth does not follow the magnetic configuration in the above illustration. Instead, the lines of flux are opposite from a moving charged particle.

Magnets

Although individual particles such as electrons can have magnetic fields, larger objects such as a piece of iron can also have a magnetic field, as a sum of the fields of its particles. If a larger object exhibits a sufficiently great magnetic field, it is called a magnet.

Magnetic force

The magnetic field of an object can create a magnetic force on other objects with magnetic fields. That force is what we call magnetism.

When a magnetic field is applied to a moving electric charge, such as a moving proton or the electrical current in a wire, the force on the charge is called a Lorentz force.

Attraction

When two magnets or magnetic objects are close to each other, there is a force that attracts the poles together.

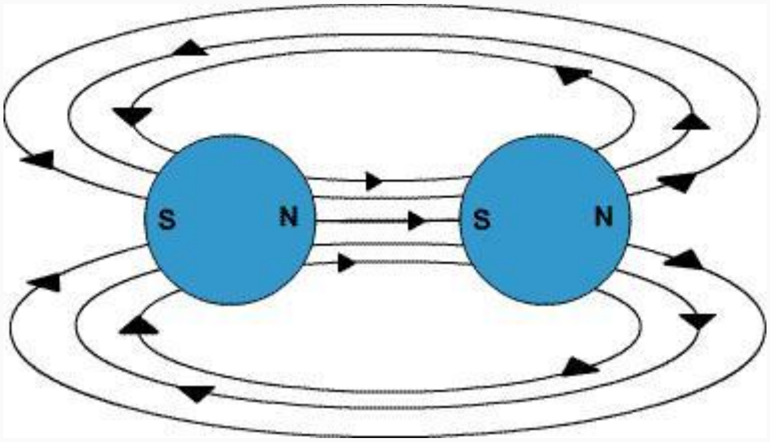


Fig.1(2) Force attracts N to S

Magnets also strongly attract ferromagnetic materials such as iron, nickel and cobalt.

Repulsion

When two magnetic objects have like poles facing each other, the magnetic force pushes them apart.

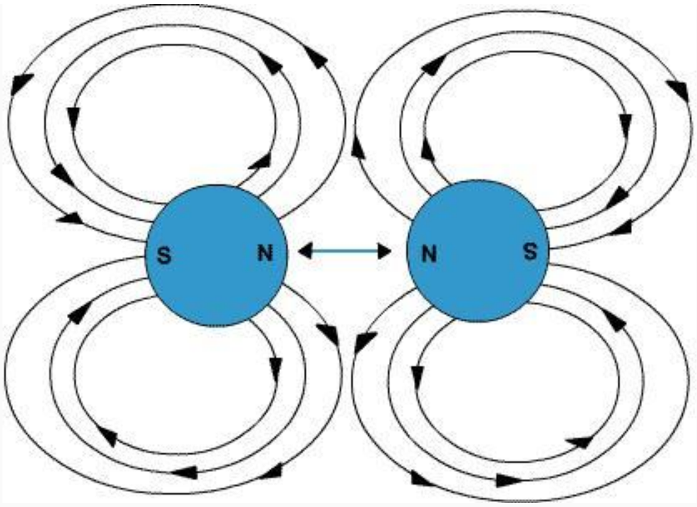


Fig.1(3) Force pushes magnetic objects apart

Magnetic and electric fields

The magnetic and electric fields are both similar and different. They are also inter-related.

Electric charges and magnetism similar

Just as the positive (+) and negative (−) electrical charges attract each other, the N and S poles of a magnet attract each other.

In electricity like charges repel, and in magnetism like poles repel.

Electric charges and magnetism different

The magnetic field is a dipole field. That means that every magnet must have two poles.

On the other hand, a positive (+) or negative (−) electrical charge can stand alone. Electrical charges are called monopoles, since they can exist without the opposite charge.

Magnetic Flex density (B)

If the total numbers of lines of forces comprising the magnetic field is called the magnetic flux , then the magnetic flux density is defined as the flux is passing per unit area within a material through a plane at right angles to the flux and is given by

$$B={\varphi \over A}{{Wb} \over {{m^2}}}$$

The magnetic flux density is also proportional to the intensity of the magnetic field H

Hence, $B \propto H = \mu H$

Where, μ is a constant known as permeability and depens on the magnetic nature of the medium. In the absence of any medium, $B \propto H = \mu H$ is given by

$$B = {\mu _o}H$$

Where, \[{\mu _o}\] is absolute permeability of free space and is equal to \[4\pi\times{10^{ - 7}}{H \over m}\]

The ratio \[{\mu\over{{\mu _o}}}\] is called a relative permeability \[{\mu _r}\], and is dimensionless quantity.

Magnetism is a class of physical phenomena that includes forces exerted by magnets on other magnets.

\[{\mu _r}\] = \[{\mu\over{{\mu _o}}}\]

Hence \[\mu\] = \[{\mu _o}\] \[{\mu _r}\]

Hence B = \[{\mu _o}\] \[{\mu _r}\]H

Intensity of magnetic field

The intensity (strength) of magnetic field at any point within a field is measured by the magnitude of a force experienced by a unit N pole placed at that point.

Suppose a positive pole of strength + m Wb experiences a force of Newton at any point in the magnetic field, and then the intensity of the field at that point is

\[H={F\over m}{N\over{W{b^{ - 1}}}}\]

The intensity of magnetic field has a definite direction at every point acts along the lines which are called the magnetic lines of force.

Intensities of magnetization (H)

Adding and subtracting \[{\mu _o}\]H from B = \[{\mu _o}\]\[{\mu _r}\]H , We get

B = \[{\mu _o}\]\[{\mu _r}\]H + \[{\mu _o}\]H - \[{\mu _o}\]H

Hence B = \[{\mu _o}\]H + \[{\mu _o}\]H(\[{\mu _r}\] - 1) the first term is magnetic flux density in free space and the second term gives the indication of increase in flux density due to the magnetic material.

The quantity H (\[{\mu _r}\] - 1) = M is called the intensity of magnetization and equal to the magnetic moment per unit volume.

\[H\left({{\mu _r} - 1} \right)=M\]

Magnetic Susceptibility

The ratio between the intensity of magnetization M to the intensity of magnetic field is known as magnetic susceptibility X_m

Hence \[{\chi _m}={M \over H}=\left({{\mu _r} - 1}\right)\]

LESSON 2. Classification of Magnetic Material

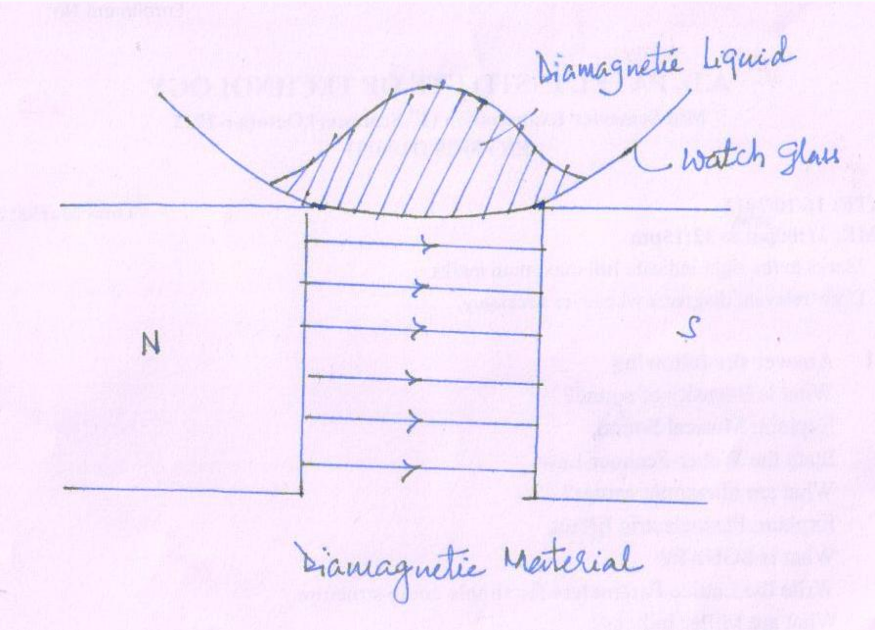
Diamagnetism

Diamagnetism is the property of an object or material that causes it to create a magnetic field in opposition to an externally applied magnetic field.

It is a quantum mechanical effect that occurs in all materials; where it is the only contribution to the magnetism the material is called a diamagnet.

Unlike a ferromagnetic, a diamagnet is not a permanent magnet.

The diamagnetic substances when subjected to a non-uniform magnetic field move from stronger to weaker part of the field. E.g. when a diamagnetic liquid like water or alcohol in a watch glass is placed over two different poles so apart, the liquid concentrates at the center as the field is minimum(weak) at the center



When a bar of diamagnetic material is placed in an external magnetic field, the magnetic induction becomes less than magnetizing field H

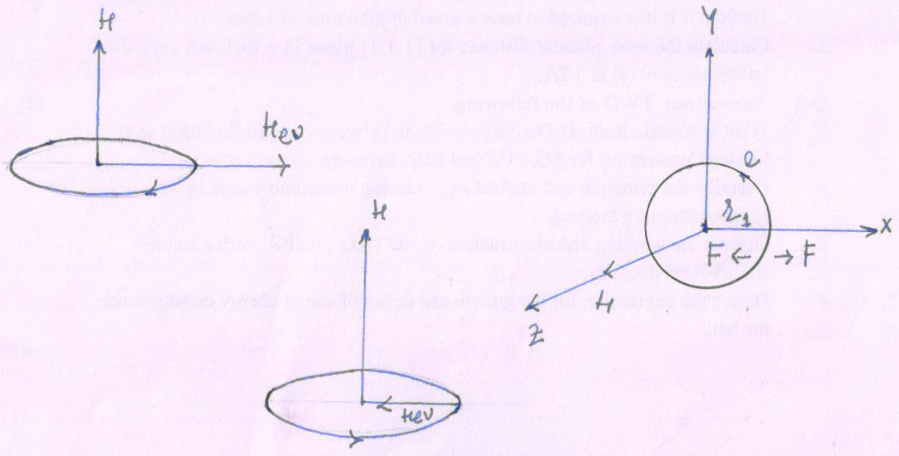
The permeability μ_r of diamagnetic material is less than one but positive.

The diamagnetic susceptibility values X_D are small negatives. The susceptibility values are unaffected by changes in temperature.

Langevin's theory of Diamagnetism

Suppose the electron is moving around the nucleus in an atom.

Let m - mass, e - charge, r_1 - radius of the orbit, V - liner velocity and W_O- angular velocity respectively.



Electron moving around the nucleus of orbital radius r_1

The revolving electron moves in a circular orbit due to the electrostatic force between the electron and the nucleus, which is balanced by centrifugal force.

F = $\frac{mv^2}{r_1} = m\omega_o^2 r_1$; $v = r_1\omega_o$; …….(1)

Let H be the intensity of the applied magnetic field which acts perpendicular to the plane of the orbit, say, along z-axis.

Now force,

F = Hev……………(2) act on the electron due to the magnetic field.

The direction of the force is given by Fleming's left hand rule.

When the electron is moving in clock wise direction, the force is directed outwards along the radius and when the electron is moving in anticlockwise direction, this force is directed inwards. The effect of applied field is only a precessorial motion of the orbit without any change in its form. It is called Larmor precession.

Due to this motion, the angular velocity of the electron is changed from say ω_o to ω.

Then

$m\omega^2 r_1 = F = m\omega_o^2 r_1 \pm F$

($m\omega^2 - m\omega_o^2) r_1$ = change of radial force = Hev

($\omega^2 - \omega_o^2) m r_1$ = Hev

$\left({\omega-{\omega _o}}\right)$ = $\frac{He{r_{1}}\omega}{m{r_1}\left({\omega+{\omega _o}}\right)}$ ……………..(3)

Let $\omega - \omega_o = \Delta\omega$ and $\omega + \omega_o = 2\omega$

$\Delta\omega$ = $\frac{He\omega}{2m\omega c}$………….. (4)

C is velocity of light introduced to convert electrostatic unit of electronic charge into electromagnetic unit.

This change of angular velocity (frequency) is termed Larmor Frequency. This change in frequency of the electronic motion produces a net current

$di=- e{\frac{\omega}{2\pi}}$………(5)

Using equn (4) in equn (5)

di=$\frac{-{e^2}H}{4\pi mc}$ ……………..(6)

Due to this additional current, the additional magnetic moment produced is

$d\mu=\frac{-{e^2}Hr_1^2\pi}{4m{c^2}}$…….(7)

The negative sign indicates that the direction of the induced moment is opposite to the direction of the applied field, which is the intrinsic property of diamagnetism. If there are z electronic orbits in an atom (z -atomic number) the total induced magnetic moment in each atom is

$$\mu = \frac{-e^2 H_z}{4mc^2}\sum r_1^2 \text{ ……………..(8)}$$

See From Fig.1(5)

$r_1^2={x^2} + {y^2}$

r_1^2 is the mean square of the perpendicular distance of the electron from the field axis through the nucleus. If is the mean value of all possible orbital radii, then

r_1^2 = ${x^2} + {y^2} + {z^2}$…………(9)

For a spherically symmetrical charge distributions

${x^2} = {y^2} = {z^2}$

So $r_1^2 = 2{x^2}= \frac{2{r^2}}{3}$ ……(10)

Using equation (10) in equation (8) we get ,

$$\mu = \frac{-e^2 H_z}{6mc^2}\sum r^2 \text{ …………..(11)}$$

For solid containing N atoms per unit volume, the total magnetic moment per unit volume or magnetization is

$$\mu = \frac{-Ne^2H_z}{6mc^2}\sum r^2 \text{ ....(12)}$$

The diamagnetic susceptibility is the ratio of induced magnetization to the magnetic field

$$\chi_{\text{dia}} = \frac{\text{M}}{\text{H}} = \frac{-Ne^2H_z}{6mc^2}\sum r^2 \text{ ....(13)}$$

This is classical Langevin's expression for diamagnetic susceptibility.

The diamagnetic susceptibility is independent of the field strength and temperature, which was later proved to be correct by Curie's experimental observation.

Paramagnetism

It may be defined as a type of magnetism in which the material gets weakly magnetized in the same direction as that of the applied magnetic field.

The materials which exhibit paramagnetism are alkali metals, transition metals, and rare earth elements.

The origin of paramagnetism arises from the presence of permanent magnetic moments in the atoms or molecules.

The permanent magnetic moments in a single atom are created due to

The moving of an electron is in a circular or elliptical orbit around the nucleus.

The motion of an electron is about its own axis (spin).

It has been observed that the magnetic moments created due to orbital motion of electrons disappear due to the effect of the electric field of the neighboring charges. But magnetic moments due to the spin motion of the electrons remain un affected by this field.

It will be interesting to add that the spin magnetic moments are composed of two groups.

One of the groups contains electrons with clockwise spin and the other with anticlockwise spin.

In the absence of an electric field, these magnetic moments are randomly distributed, or in other words they have no mutual interactions among them.

This is called paramagnetism.

In presence of magnetic field, the moments have a tendency to turn towards the direction of the field.

If no opposing forces act, a complete alignment of the atomic moments will be produced and the sample as a whole acquires a very large magnetization in the direction of the field.

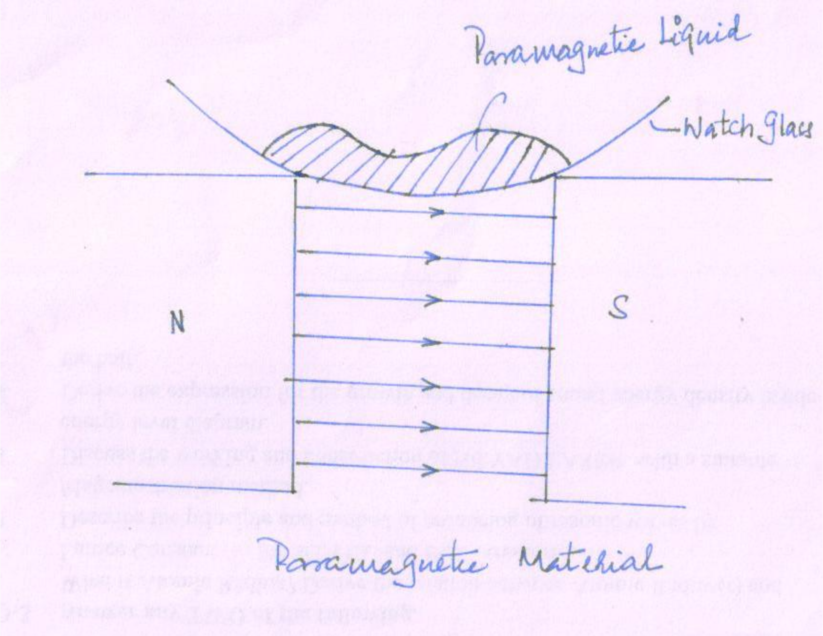
However, thermal agitation of the atoms opposes this tendency and tends to randomize the atomic the atomic dipole moments.

The results are only a partial alignment in the field direction and hence paramagnetic materials exhibits weak inherent magnetization and a small positive susceptibility.

An increase in thermal energy will increase the randomizing effect on the alignment of magnetic dipoles along the field direction and hence decrease the susceptibility.

Properties of Paramgnetism

- Para magnetic material when placed in a magntic field acquires a feeble magnetization in the same sense as the applied field. Platinum, Aluminum, Copper, Sulphate and Nickle.solutions are the example of paramgentism.
- These substances when exposed to a non-uniform magnetic field experience an attractive force toward the stronger part of the field. For example, if we place a paramagnetic liquid in a watch glass and place it over a non-uniform magnetic field, the liquid moves toward the poles as the field is strongest near the poles.



- When paramagnetic materials are placed in uniform magnetic field, the magnetic induction becomes more than the magnetizing field and thus the relative permeability is slightly greater than unity.
- The paramagnetic susceptibility is however small but positive.
- The susceptibility is inversely proportional to the operating temperature. Thus with increase in temperature the X value decreases and becomes negative at elevated high temperatures to exhibits diamagnetic properties.

Langevin’s theory of Paramgetism

In absence of external magnetic field, the dipoles are oriented along different directions, such that the resultant sum of all projections of magnetic moments on any reference line is zero.

When the external field is applied, the state of magnetization depends upon the following two factors:

The applied magnetic field- it align the magnetic axes of molecules along the field direction.

The thermal agitation- It is disorganizes the orderly state produced by the applied field.

For the equilibrium state, the following two kinetic theory principles are used

(1) The number of molecules having orientation, inclined at an angle \[\theta\] with the given line of reference is directly proportional to the \[\sin \theta d\theta\]

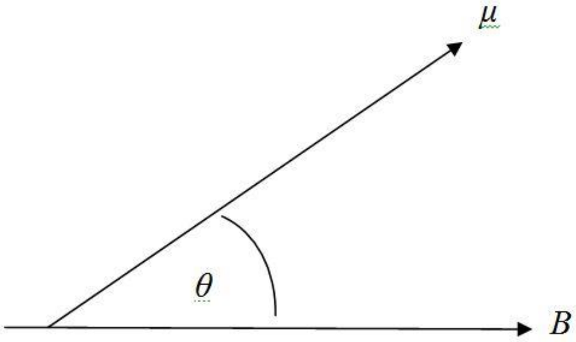
(2) As per of equpartition of energy, the number of molecules whose potential energy is proportional to, energy is proportional to,

w is magnetostatic energy,

k is Boltzmann factor and

T is absolute temperature.

Let us consider a molecular magnetic moment of μ , inclined at an angle \[\theta\] with respect to the external magnetic field of induction B



The magnitude of magneto static energy

$$w = -\mu B\cos\theta \;\ldots\ldots\ldots\ldots(1)$$

The –ve sign suggest the work is done against the thermal agitation misalignment.

Let dN molecules obey the two kinetic theory principle, out of total number of N in a unit volume of the gas

$$dN \;\propto exp\left(\frac{\mu B\cos\theta}{k}\right)\sin\theta d\theta$$

$$dN = \; C\; exp\left(\frac{\mu B\cos\theta}{k}\right)\sin\theta d\theta \;\ldots\ldots\ldots\ldots\ldots(2)$$

C is proportionality constant

Putting $\frac{\mu B}{KT} = a$ and integrating equation (2) between θ to π we have

$$N = C\;\int_0^{\pi}\exp(\,a\cos\theta)\sin\theta\,d\theta$$

$\cos\theta = x$ and differentiate w.r.t θ , $\sin\theta\,d\theta = -dx$

$$N = \int_1^{-1} C\exp(ax)\,dx$$

Hence, $C = \frac{Na}{e^{a} - e^{-a}} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$

The intensity of magnetization may be derived as follow

We know that the resolved component of magnetic moment of each molecule along the field direction is μ cos \[\theta\] . Thus the resultant magnetic moment due to molecules is dN μ cos \[\theta\] . Hence the magnetic moment of all N molecules in unit volume which is equal to I is given by

$$I = \int_o^{\pi}\mu\cos\theta\;dN \;\ldots\ldots\ldots\ldots(4)$$

Form equation (2)

$$I = \int_0^\pi \mu \cos\theta c exp(a\cos\theta)\sin\theta d\theta \; \ldots\ldots.(5)$$

Let $\cos\theta = x$ and differentiate w.r.t. $\theta - \sin\theta d\theta = dx$

$$I = c\mu \int_{-1}^{1} x e^{ax}\, dx$$

Integrating by part, we get after applying boundary condition

$$I = \mu C\left[\frac{e^{a}+e^{-a}}{a} - \frac{e^{a}-e^{-a}}{a^2}\right] \ldots\ldots\ldots\ldots\ldots(6)$$

Use the value of c from equation (3) in equation (6)

$$I = \frac{\mu Na}{e^{a}-e^{-a}}\left[\frac{e^{a}+e^{-a}}{a} - \frac{e^{a}-e^{-a}}{a^2}\right]$$

$$I = \mu N\left[\frac{e^{a}+e^{-a}}{e^{a}-e^{-a}} - \frac{1}{a}\right]$$

$$I = \mu N\left[\cot a - \frac{1}{a}\right] = \mu N\, L(a) \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

$L(a) = \left(\cot a - \frac{1}{a}\right)$ is known as Langevin function

When $\frac{\mu B}{KT} \gg 1$, then $\frac{\mu B}{KT} = a$ is very large, In this case either T must be small or B must be large we find

$$|L(a)|_{a\to\infty} = \left|\cot a - \frac{1}{a}\right|_{a\to\infty} = \left|\frac{e^{a}+e^{-a}}{e^{a}-e^{-a}} - \frac{1}{a}\right|_{a\to\infty}$$

Multiple numerator and denominator by e^{-a} we get,

$$|L(a)|_{a\to\infty} = \left|\frac{1+e^{-2a}}{1-e^{-2a}} - \frac{1}{a}\right|_{a\to\infty} = 1 \ldots\ldots\ldots\ldots(8)$$

Equation (7) becomes

$I = \mu N = I_s$ ……………..(9)

I_s is the saturation value of magnetization.

Thus at lower temperature of higher external magnetic field conditions, the Langevin function tends to be unity, thereby the magnetization shows the saturation value which is in accordance with the theory.

When $\mu B \ll kT$ i.e temperature is large and B is very small, we can show the Langevin function as

$|L(a)|_{a \to \infty} = \frac{a}{3}$ ……………..(10)

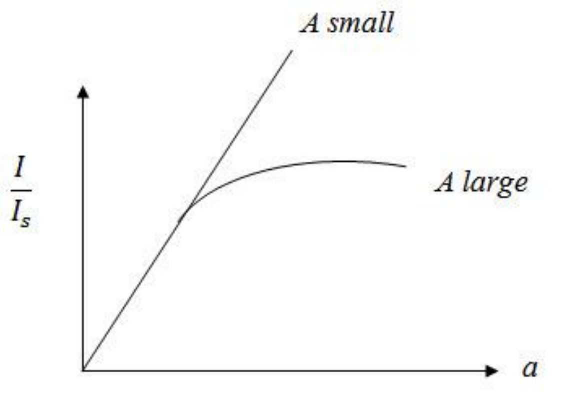
From equation (7) and (9)

$I = \mu N\, L(a) = I_s L(a)$

Or

$\frac{I}{I_s} = L(a) = \left[\cot a - \frac{1}{a}\right]$ ……………..(11)

From graph



a is small and linear, thus

$I = \mu N \frac{a}{3}$

But $\frac{\mu B}{KT} = a$ we get

$$I = \frac{\mu^2 \mu_0 HN}{3kT} \text{ Where, } B = \mu_o H \quad \ldots\ldots\ldots\ldots (12)$$

The paramagnetic susceptibility

$$\chi_p = \frac{I}{H} = \frac{\mu^2 \mu_0 N}{3kT} \ldots\ldots\ldots\ldots (13)$$

Hence

$$\chi_p = \frac{C}{T} \quad \ldots\ldots\ldots\ldots (14)$$

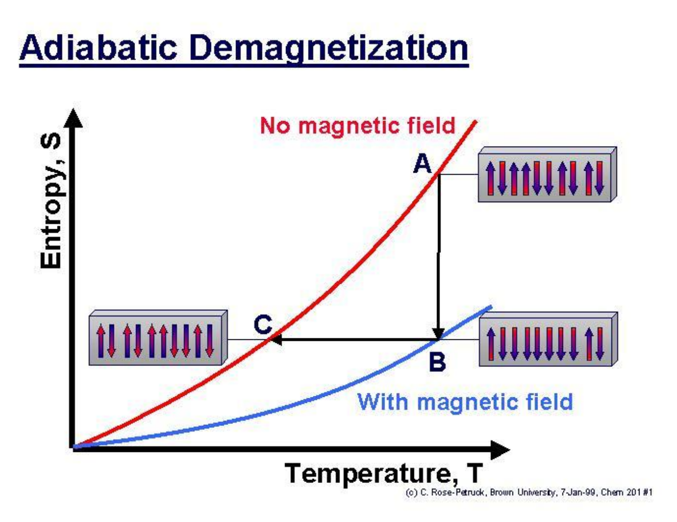
Where $C = \frac{\mu^2 \mu_0 N}{3kT}$ ……………. (15) is called Curie constant

LESSON 3. Adiabatic Demagetization & its properties

Adiabatic Demagnetization: Magnetic Cooling

Yet another method of cooling objects, magnetic cooling, exploits the relationship between the effects of the magnetic field strength of an applied field and the entropy of an object.

One particular method of magnetic cooling is Adiabatic Demagnetization, which capitalizes on the paramagnetic properties of some materials to cool those materials (usually in gaseous form) down into the millikelvin -- or colder -- range. This method can also be used to cool solid objects, but the most drastic cooling in the fractions of a Kelvin range are generally accomplished for low-density gases that have already been greatly cooled (around 3-4 K)



Adiabatic Demagnetization diagram

What are Paramagnetic Materials?

When an outside magnetic field is applied to these materials, a magnetic field that is parallel to the applied field is induced. The amount of particles that align with the applied field depends on the field strength: if there is higher field strength, more particles are aligned with the field.

The Process of Adiabatic Demagnetization

- First, the sample to be cooled (typically a gas) is allowed to touch a cold reservoir (which has a constant temperature of around 3-4 K, and is often liquid Helium), and a magnetic field is induced in the region of the sample.

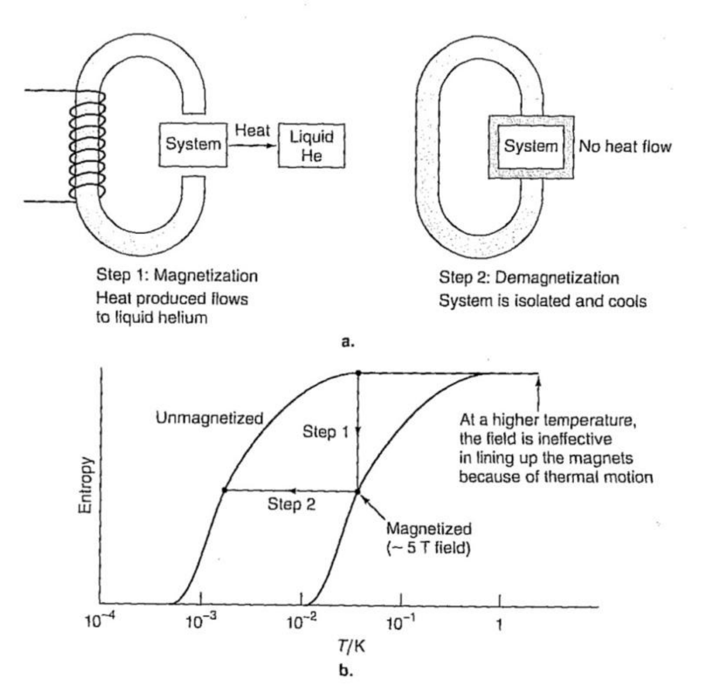


Fig.2(2) Process of adiabatic demagnetization

- Once the sample is in thermal equilibrium with the cold reservoir, the magnetic field strength is increased. This causes the entropy of the sample to decrease, because the system becomes more ordered as the particles align with the magnetic field. The temperature of the sample is still the same as that of the cold reservoir at this point.
- Then the sample is isolated from the cold reservoir, and the magnetic field strength is reduced. The entropy of the sample remains the same, but its temperature drops in reaction to the reduction in the magnetic field strength. If the sample was already at a fairly low temperature, this temperature decrease can be ten-fold or greater.
- This process can be repeated, permitting the sample to be cooled to very low temperatures.

Limitations of the Process of Adiabatic Demagnetization

If the sample material is an electronic paramagnet, the lowest temperatures that can be attained using these methods are on the order of 1 milliKelvin.

If the sample is a nuclear paramagnet, the lowest achievable temperatures are much lower (because the interactions between the dipoles are much weaker).

Weiss Molecular Field Theory and Ferromagnetism Or

Weiss theory of Ferromagnetism Or

Mean Field Theory of Ferromagnetism Or

Molecular Theory of Ferromagnetism

It has been observed that the ratio of the magnetic moment to the angular momentum for the spinning electron is twice for an orbital electron. In case of ferromagnetic materials this ratio is almost same as that for a spinning electron. Hence, the spinning electrons make a major contribution to magnetic properties of ferromagnetic.

According to Weis, the effective field strength may be regarded as the vector sum of external applied field strength and the internal molecular field strength B_i

\[{B_e}=B + {B_i}=B + \lambda I\]…… (1)

Consider gram molecule of the substance whose density is and molecular weight and and being the gram molecular moment and its saturation value respectively then

And

\[{I_s} = {{{\sigma _s}\rho } \over M}\]……….. (3)

Since I and I_s refer to the unit volume

\[{I \over {{I_s}}}\] = \[{\sigma\over{{\sigma _s}{\rm{}}}}\]………… (4)

As the domains are assumed to obey the general theory of paramgnetism, then from Langevin's theory of paramgnetism, we have

\[{I \over {{I_s}}}=L\left(a \right)=\left[{\cot a -{1\over a}}\right]\]…………. (5)

Equating (4) and (5)

\[{I \over {{I_s}}}\] = \[{\sigma\over{{\sigma _s}{\rm{}}}}\] = \[L\left(a\right)=\left[{\cot a-{1\over a}}\right]\]………….. (6)

Where \[{{\mu {B_e}}\over {KT}}\] = a = \[{{\mu \left({B + \lambda I} \right){\rm{}}} \over {KT}}\]…….. (7)

When the external field is zero B = 0

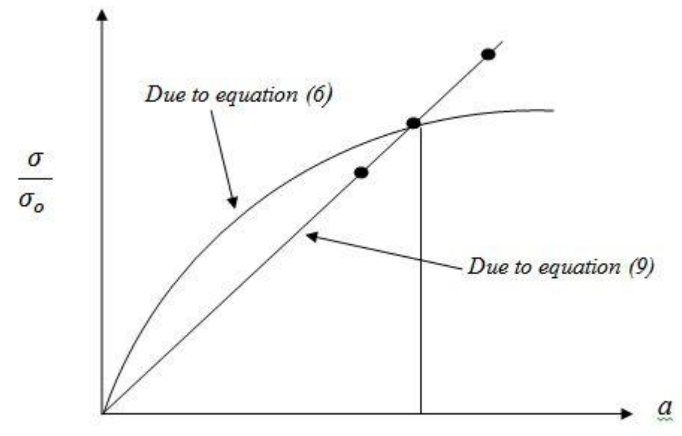
\[{B_e}=\lambda I={{\lambda \sigma \rho } \over M}\]…………. (8)

Putting equation (8) in equation (7)

\[{\mu\over{KT}}{{\lambda\sigma\rho}\over M}\] = a = \[{{{\sigma _o}} \over N}\] \[{{\lambda \sigma \rho } \over {MKT}}={{{\sigma _o}} \over N}{{\left({B + \lambda I} \right){\rm{}}} \over {KT}}\]

Hence

\[{\sigma\over{{\sigma _o}}}={{RMT} \over {\lambda \rho {\sigma _o}^2}}a\]……… (9)



Equation (9) and (7) simultaneously determine the condition for spontaneous magnetization.

LESSON 4. Theory of Magnetism

Curie-Weiss Law- Weiss theory of paramgnetism

We know that the curie's law

\[{\chi _m}={C \over T}\]

It has been observed that many paramagnetic do not obey the Curie law, they modified the equation is called Curie's-Law

\[{\chi _m}={C \over T}\] …………….(1)

Where \[\theta\] is constant with dimension of temperature and have small values of the order of 10K or less for most paramagnetics.

It was Weiss who improved upon Langvien theory with considerable success by his new concept on an internal molecular field within the gas produced at any point by the neighboutring molecular magnets.

If this internal molecular field is represented by H_m , Weiss assume that H_m =\[\beta I\]

I is intensity of magnetization and a constant is called molecular field constant.

Effective magnetic field

\[{H_e}=H + {H_m}\]…………..(2)

\[{H_e}=H + \beta I\]……………(3)

\[I={{{\mu ^2}{\mu _o}HN} \over {3kT}}\]

Or

\[I=\mu NL\left(a \right)={I_s}L\left(a \right)\]

\[{I \over {{I_s}}}={{{\mu _o}\mu \left({H + \beta I} \right){\rm{}}} \over {3kT}}\]…………….(4)

Taking one gram molecular weight of the material and expressing the gram molar magnetic moment and its saturation value given by σ_s

So,

\[{I\over{{I_s}}}\] =\[{{\sigma\over{{\sigma _s}{\rm{}}}}}\]= \[{{{\mu _o}\mu \left({H + \beta I} \right){\rm{}}} \over {3kT}}\]…………..(5)

Let ρ is a density and is molecular weight of the sample

$$I = \frac{\sigma}{\frac{M}{\rho}} \;\ldots\ldots\ldots\ldots\ldots (6)$$

And

$$I_s = {{\sigma_s \rho} \over M}$$ ....(7)

$$\mu = {{\sigma_s} \over N}$$ ....(8) N Avogadro's number

Using equation (5) and (8)

We get,

$${\sigma \over {\sigma_c}} = {{\mu_o \sigma_s} \over {3NkT}}\left(H + \beta I \right)$$ ....(9)

Where R is molecular gas constant = NK

Using equation (9) and (7)

$$\sigma = {{\mu_o \sigma_s^2} \over {3RT}}\left(H + \beta {{\sigma \rho} \over M} \right)$$ ....(10)

The molecular susceptibility

$$\chi_m = {\sigma \over H}$$

Using equation (10)

$$\chi_m \left(T - {{\mu_o \sigma_s^2 \beta \rho} \over {3Rm}} \right) = C_m$$

Or

$$\chi_m \left(T - \theta \right) = C_m$$

$\theta = {{\mu_o \sigma_s^2 \beta \rho} \over {3Rm}}$ is called Curie point and $C_m = {{\mu_o \sigma_s^2} \over {3R}}$ is called Curie molecular constant.

So

$$\chi_m = {{C_m} \over {\left(T - \theta \right)}}$$ ....(11)

This expression is called Curie-Weiss law.

Ferromagnetism

The term ferromagnetism may be defined as a type of magnetism in which a material gets magnetized to a very large extent in the presence of an external field. The direction in which the material gets magnetized is the same as that of the magnetic field.

Properties

- Ferromagnetic materials acquire a high degree of magnetization in the same sense as the applied field some classical examples of ferromagnetic are iron, cobalt and nickel.
- These materials attract the line of forces strongly and hence permeability values are very high.

- The value of susceptibility of ferromagnetic is also very large and positive.
- When ferromagnetic material is placed in an external magnetic field, such that the length of the material is parallel to the field direction, a small change in the dimensions occurs. This effect is called magnetostriction.
- As the temperature, increases the value of susceptibility decreases. Above a certain temperature, ferromagnetic materials become ordinary paramagnetic and this temperature is called the curie temperature.

Weiss Molecular Field Theory and Ferromagnetism Or

Weiss theory of Ferromagnetism Or

Mean Field Theory of Ferromagnetism Or

Molecular Theory of Ferromagnetism

It has been observed that the ratio of the magnetic moment to the angular momentum   for the spinning electron is twice for an orbital electron. In case of ferromagnetic materials this ratio is almost same as that for a spinning electron. Hence, the spinning electrons make a major contribution to magnetic properties of ferromagnetic.

According to Weis, the effective field strength may be regarded as the vector sum of external applied field strength and the internal molecular field strength B_i

$$B_e = B + B_i = B + \lambda I \text{……} \quad (1)$$

Consider gram molecule of the substance whose density is and molecular weight and   and being the gram molecular moment and its saturation value respectively then

$$I = \frac{\sigma}{\frac{M}{\rho}} \text{…………..} \quad (2)$$

And

$$I_s = \frac{\sigma_s \rho}{M} \text{……….} \quad (3)$$

Since I and I_s refer to the unit volume

$$\frac{I}{I_s} = \frac{\sigma}{\sigma_s} \text{…………} \quad (4)$$

As the domains are assumed to obey the general theory of paramgnetism, then from Langevin's theory of paramgnetism, we have

$$\frac{I}{I_s} = L(a) = \left[\cot a - \frac{1}{a}\right] \text{…………} \quad (5)$$

Equating (4) and (5)

$$\frac{I}{I_s} = \frac{\sigma}{\sigma_s} = L(a) = \left[\cot a - \frac{1}{a}\right] \text{…………..} \quad (6)$$

Where \[{{\mu {B_e}}\over {KT}}\] = a = \[{{\mu \left({B + \lambda I} \right){\rm{}}} \over {KT}}\]…….. (7)

When the external field is zero B = 0

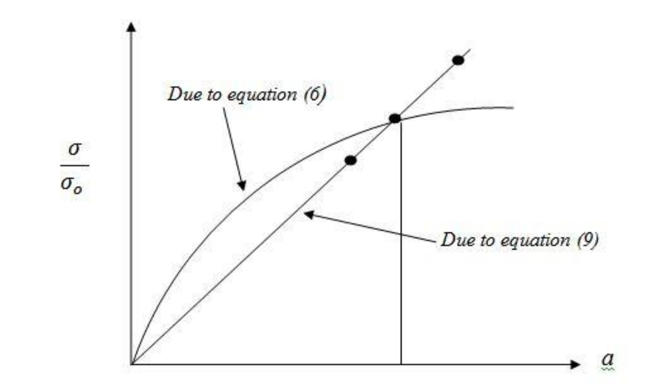
\[{B_e}=\lambda I={{\lambda \sigma \rho } \over M}\]…………. (8)

Putting equation (8) in equation (7)

\[{\mu\over{KT}}{{\lambda\sigma\rho}\over M}\] = a = \[{{{\sigma _o}} \over N}\] \[{{\lambda \sigma \rho } \over {MKT}}={{{\sigma _o}} \over N}{{\left({B + \lambda I} \right){\rm{}}} \over {KT}}\]

Hence

\[{\sigma\over{{\sigma _o}}}={{RMT} \over {\lambda \rho {\sigma _o}^2}}a\]……… (9)



Equation (9) and (7) simultaneously determine the condition for spontaneous magnetization.

MODULE 2. Particle Physics

LESSON 5. Basic concept of Wave and Particle

Wave Particle Duality of radiation

For the wave and particle duality we must to know,

What is a particle?

What is wave?

The concept of particle is easy to understand because

(i) It has mass

(ii) It is located at definite point

(iii) It moves from one to another place.

(iv) It gives energy when stopped.

The concept of wave is quite difficult compare to particle because

(i) A wave is spread out over a relatively large region of space

(ii) It is can't be said to be location.

(iii) It is hard to think of mass being associated with it.

For a simplicity,

The wave is specified by its

(i) Frequency

(ii) Wavelength

(iii) Phase or wave velocity

(iv) Amplitude

(v) Intensity

Basic of above concept, It is difficult to accept the conflicting ideas that, the radiation has dual nature; the radiation is a wave, which is spread out over the space. So, the radiation sometimes behaves like a wave and as a particle.

How is it?

Radiation includes

(i) Visible light

(ii) Infrared

(iii) Ultra-violet ray and X-Ray are behaved as a wave in the phenomena of interference and diffraction.

Because in these phenomena. Two waves are simultaneously coinciding at the same position at same time. Actually, it is difficult for two particles to occupy the same position at same time. So the radiation behaves like wave.

The Black Body Radiation, Photoelectric Effect and Compton Effect are established that there is a interaction of radiation energy with matter. Here radiation interact with matter in the form of photons so radiation behaves liked a particle. Thus, the wave and particle nature is not exhibits simultaneously.

De-Broglie Concept of matter wave

From phenomena of Diffraction, Interference and Polarization,

It is noted that light is one kind of wave motion. Further, some new phenomena such as Photo-electric Effect and Compton Effect are not explained on the basis of wave-theory. But these are explained on the basis of quantum theory.

In quantum theory, the light or photons are asset with literature on mass, velocity and momentum.

But photon theory was applied to the phenomena of interference and diffraction. It is not able to explain.

So light has dual nature, it posses both particle and wave properties.

- In some cases, the waves are predominant and some other cases the particles are predominant.
- The dual properties are depending upon the condition under which the particular phenomena occur.
- It is note that wave and particle never expected to appear together.

De-Broglie extended the wave-particle parallelism of optics to all fundamental things existing independently from other things.

Fundamental existing things like

- Electron
- Protons
- Neutrons
- Atoms and

- Molecules

He suggested that matter has also dual nature i.e. wave and particle like radiation. There is detail connection between waves and written work in caseof particle also.

A moving particle is always associated with the wave and it is controlled by wave same as photon is controlled by wave.

On the concept of wave-particle duality, he noted that if the radiation (light) can act as wave sometime and act as particle at other time.

Then the material (matter) particle (electron and neutron) should act as wave at some other times. Hence, the De-Broglie hypothesis

A moving particle is associated with wave which is known as De-Broglie wave.

The wavelength of matter wave is

\[\lambda\] = \[{h \over {mv}}\] = \[{h \over P}\]

m Mass of material particle

v Velocity of material particle

P Momentum of material particle

LESSON 6. Theory of Wave and particle

De-Broglie wavelength

The expression of the De-Broglie wave associated with a material particle can be derived on comparison of radiation.

Suppose, according to Plank theory of radiation the energy of photon (quantum) is

$E = hv$

But $v = {c \over \lambda }$ c - velocity of light

$E = {{hc} \over \lambda }$…… (1) λ - wavelength of light

But according to energy and mass relation $E = mc^2$…… (2)

Equating (1) and (2)

$mc^2 = {{hc} \over \lambda }$

Hence $\lambda = {h \over {mc}}$……. (3)

We know that c- velocity of light (photon) so

$mc = P$ ……. (4) Associated with photon (radiation)

Now suppose a material particle of mass –m and material particle is moving with velocity v

Hence, the momentum of particle $P = mv$

The wavelength (λ) is associated with particle (material particle)

$\lambda = {h \over {mc}} = {h \over P}$…….(5)

Let the kinetic energy of moving material particle is

$E = {{m{v^2}} \over 2}$

$E = {{m{v^2}} \over 2} \times {m \over m}$

$E = {{{m^2}{v^2}} \over {2m}}$

$E = {{{P^2}} \over {2m}}$ where, $P = mv$

$E = {{m{v^2}} \over 2} = {{{m^2}{v^2}} \over {2m}} = {{{P^2}} \over {2m}}$

P = $\sqrt {2mE}$…….(6)

De-Broglie wavelength

λ = ${h \over {\sqrt {2mE} }}$……(7)

De-Broglie wavelength associated with electron

Suppose

Electron is at rest

It has having mass- m_0

Charge of electron- e

Now electron associated by potential voltage and obtained the velocity v from its rest position.

KE = ${{{m_0}{v^2}} \over 2}$ = eV

v^2 = ${{2eV} \over {{m_0}}}$

v = $\sqrt {{{2eV} \over {{m_0}}}}$

De-Broglie wavelength is associated with electron

λ = ${h \over {{m_0}v}}$

λ = ${h \over {\sqrt {2{m_0}eV} }}$

h - Plank's constant- 6.625×10^{-34} Joule.sec

m_0 - Rest mass of electron- 9.11×10^{-31} kg

E - Charge of electron- 1.62×10^{-19}C

V - 100 Volts

λ = 1.226 Å ……(8)

The wavelength is associated with an electron accelerated to 100 volts is

1.226 Å

Properties of matter wave

- Lighter in the particle, greater is the wavelength associated with it
- λ = ${h \over {mv}}$ = ${h \over P}$
- Smaller is the velocity of the particle, greater is the wavelength associated with it.

- Suppose,v = 0, then \[\lambda\] = ∞ ; wave becomes indeterminate
- Suppose v = ∞ then \[\lambda\] = 0; wave becomes a zero. These conditions suggest, the matter waves are generated by the motion of particles.
- These waves (\[\lambda\] = ∞ and \[\lambda\] = 0) are produced whether the particles are charged particles or the particles are un-changed.
- These waves are not electromagnetic waves but they are a new kind of waves (electromagnetic waves are produced only by motion of charged particle). The velocity of material particle is not a constant while the velocity of electromagnetic wave is constant.
- The velocity of matter waves is greater than the velocity of light. This can be provide as under

The wave velocity ω is v × \[\lambda\]

So, ω = \[{{{c^2}} \over v}\]
As a particle velocity v can't exceed c hence ω is greater than velocity of light.

- The wave and particle aspect of moving bodies can never appear together in the same experiment. What we can say is that waves have particle like properties and particle have wave like properties and the concepts are inseparably linked. Matter wave representation is only a symbolic representation.
- The wave nature of matter introduced an uncertainty in the location of the position of the particle because a wave can't be said exactly at this point or exactly at that point. However, where the wave is large(strong) there is a good chance of finding the particle while, where the wave is small there is very small(weak) chance of finding the particle

LESSON 7. Heisenberg Uncertainty Principle

Heisenberg Uncertainty Principle

Heisenberg explained a principle which is aim toward importance of dual nature of the matter.

According to classical mechanics

- Suppose a particle is moving, so at any instant has a Fixed Position in space and definite momentum can be measured if the initial value are known.

But according to wave mechanics,

- A particle is considered as a wave packets.
- Now suppose, wave packet (concept of \[{\rm{\lambda }}\]) is small, the position of particle may be fixed but velocity or momentum of particle become indefinite.
- In other way, wave packet is large, the velocity or momentum may be fixing and position of particle is indefinite.
- Thus, certain (definite) position of the particle has uncertainty in momentum or velocity and certain momentum and velocity has uncertain in position of particle.

From these probabilities it is impossible to know

- Where within the wave packet the particle is?
- What is its exact momentum or velocity?

According to Heisenberg uncertainty principle

It is impossible to specify precisely and simultaneously the value of both of physical variables that describe the behavior of an atomic system.

Qualitative principle states

The order of magnitude of the product of the uncertainties in the knowledge of two variables must be at least Plank's Constant

Let pair of two variables (physical variables) as position of particle and momentum of particle.

$\Delta x \, \Delta p = h$

Δx - Uncertainty in determining the position of particle.

Δp - Uncertainty in determining the momentum of particle

$\Delta E \, \Delta t = h$

ΔE - Uncertainty in determining the energy

Δt - Uncertainty in determining in determining the time

$\Delta j \, \Delta\theta = h$

Δj - Uncertainty in determining angular momentum

$\Delta\theta$ - Uncertainty in determining angle

Experimental Justification of uncertainty Principle

(1) Determining of the position of a particle by microscope

For the measured of position of a particle (electron) in the range of microscope the resolving power of the microscope can be measure the smallest distance between the two points,

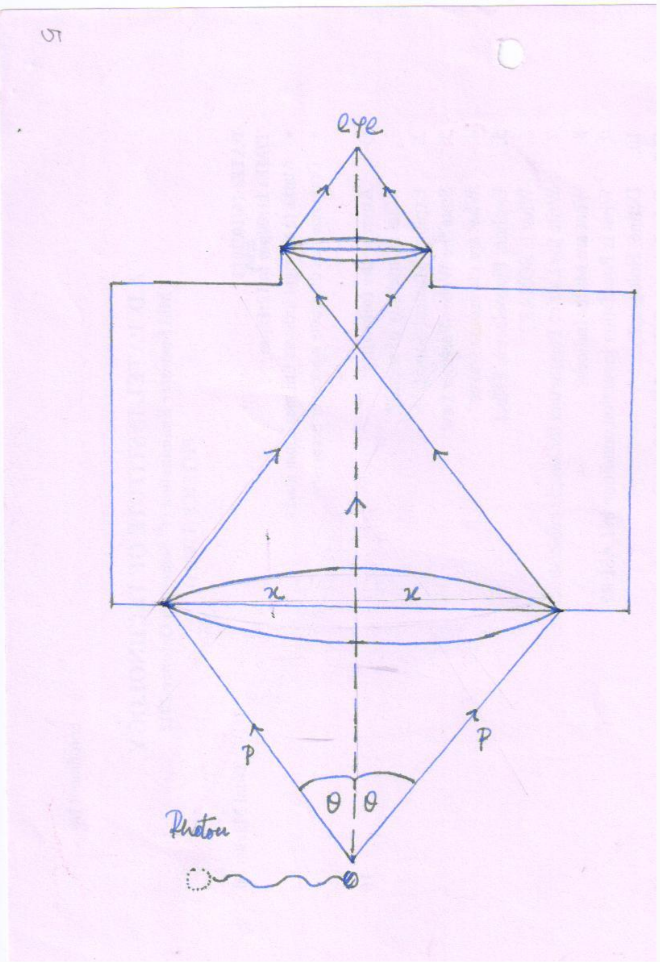
$\Delta x={\lambda\over{2\sin \theta }}$……..(1)

Δx - Uncertainty in determining the position of particle

λ - Wavelength of light used

θ - Semi vertical angle of the cone of light

To observed the particle (position of electron), it is necessary a photon strikes the electron (particle) and scattered inside the microscope.



When a photon of initial momentum $\rho = {h \over \lambda }$ after scattering (as a momentum is conserved in the collision) a particle enters in the field of view of microscope. A particle may be anywhere within angle 2θ

Therefore, the momentum(x-component) in microscopic region is $p \sin \theta$ and $-p \sin \theta$

Hence,

$\Delta p = p \sin \theta - (-p \sin \theta)$

$\Delta p = 2p \sin \theta$

But , $p = {h \over \lambda }$

$\Delta p = {{2h\sin\theta}\over\lambda}$……..(2)

From (1) and (2)

$\Delta x \, \Delta p = h$

Product of uncertainties in position and momentum is of order of Plank's Constant.

(2) Diffraction by single slit

Suppose a narrow beam of electrons passes through a single narrow slit (Δy) and produces a diffraction pattern on the screen.

Hence diffraction equation

$$d\sin \theta=n\lambda$$

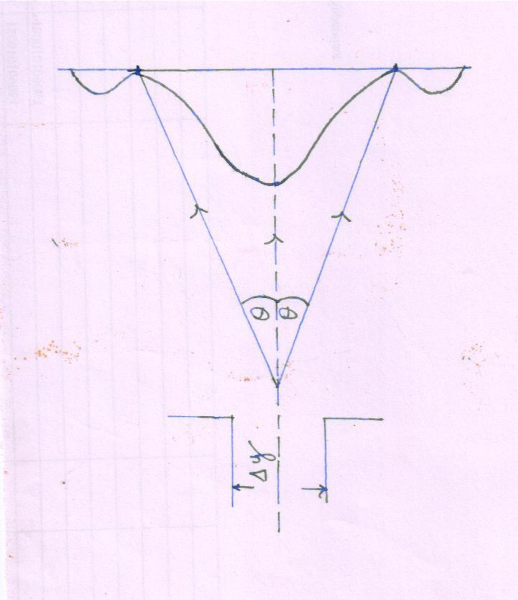
For 1st order

$$d\sin\theta=\lambda$$

d - Width of the slit = Δy

$\Delta y \sin \theta = \lambda$

Regarding diffraction pattern of on screen, all electrons have passed through the slit. But we can’t say that exact position of a particle within slit



Hence the uncertainty can be determine the position of the electron equal to the width Δy.

$\Delta y = {\lambda \over{\sin \theta }}$

Here, initially electrons are moving along -axis and they have no component of momentum along -axis. After diffraction at the slit electrons are deviated from their initial path to form diffraction pattern and have a component . The y - component of momentum may lie between p sin θ and -p sinθ.

The uncertainties in y – component of momentum = Δp = 2p sinθ

But, $p = {h \over \lambda }$

$\Delta p = 2p \sin\theta = {{2h\sin\theta}\over\lambda }$

$\Delta y \, \Delta p = {\lambda\over{\sin \theta }} \times {{2h\sin\theta }\over\lambda }$

$\Delta y \, \Delta p = 2h$

Or

$\Delta y \, \Delta p \cong h$

It shows the product of uncertainties in position and momentum is of order of Plank’s constant.

LESSON 8. Mathematical Derivation for Uncertainty Priciple

(1) Determining of the position of a particle by microscope

For the measured of position of a particle (electron) in the range of microscope the resolving power of the microscope can be measure the smallest distance between the two points,

$\Delta x={\lambda\over{2\sin \theta }}$……..(1)

Δx - Uncertainty in determining the position of particle

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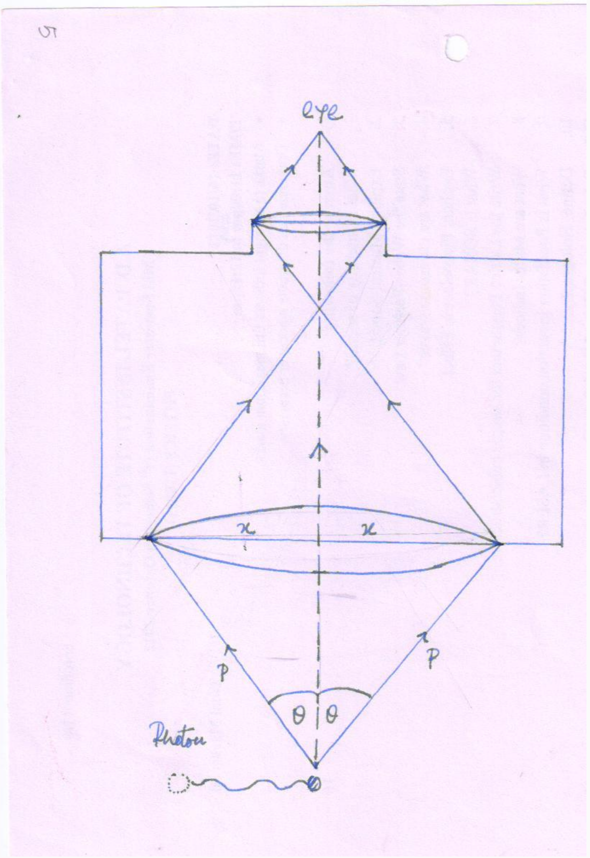

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$\Delta p = p \sin\theta - (-p \sin\theta)$

$\Delta p = 2p \sin\theta$

But , p = ${h \over \lambda}$

$\Delta p = {{2h\sin\theta}\over\lambda}$……..(2)

From (1) and (2)

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Product of uncertainties in position and momentum is of order of Plank's Constant.

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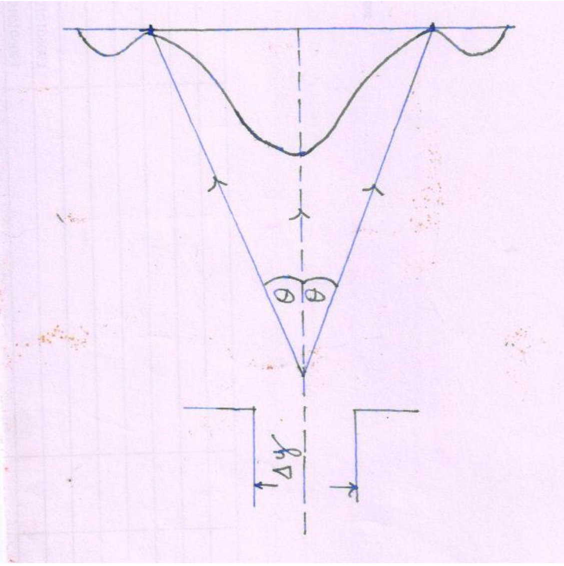


Hence the uncertainty can be determine the position of the electron equal to the width Δy.

Δy =\[{\lambda \over{\sin \theta }}\]

Here, initially electrons are moving along -axis and they have no component of momentum along -axis. After diffraction at the slit electrons are deviated from their initial path to form diffraction pattern and have a component . The y - component of momentum may lie between p sin \[\theta\] and -p sin\[\theta\].

The uncertainties in y – component of momentum = Δp = 2p sin\[\theta\]

But, p = \[{h \over \lambda }\]

Δp = 2p sin\[\theta\] = \[{{2h\sin\theta}\over\lambda }\]

Δy Δp = \[{\lambda\over{\sin \theta }}\] X \[{{2h\sin\theta }\over\lambda }\]

$\Delta y \, \Delta p = 2h$

Or

$\Delta y \, \Delta p \cong h$

It shows the product of uncertainties in position and momentum is of order of Plank's constant.

LESSON 9. Wave Function and its significance

Physical interpretation Wave Function

- It is now interesting as well as befitting to define of interprets the significance of wave function \[\Psi\] in terms of observable properties associated with the particle or the system.

- It should be defined in such wav that any meaningful question about the result of an experiment performed upon the system can be answered if the wave function is known.

- In the beginning it was considered that the wave function is merely an auxiliary mathematical quantity employed to facilitate computation relative to the experimental result.

- The first and simple interpretation of \[\Psi\] was given by Schrodinger himself in terms of charge density.

- We know that in any electromagnetic wave system if A is the amplitude of the wave, then the energy density i.e energy density per unit volume is equal to A^2 , so that the number of photons per unit volume i.e photon density is equal to A^2 , so that the number of photons per unit volume i.e photon density equal to \[\frac{{{A^2}}}{{hv}}\] or the photon density in proportion la to A^2 as hv is constant.

- If the \[\Psi\] is amplitude of matter wave at any point in space then the particle density at that point may be taken as proportion to \[{\Psi ^2}\] . Thus \[{\Psi ^2}\] is a measure of particle density.

- When this is multiplied be the charge of the particle, the charge density is obtained. In this way \[{\Psi ^2}\] is a measure of charge density.

- It is observed that in some cases \[\Psi\] is appreciably different from zero within some finite region known as wave packets.

- It is natural to ask

- Where is the particle in relation to wave packet? To explain it, Max Born suggested a new idea about the physical significance of \[\Psi\] which is generally accepted now a day. According to him \[\Psi {\Psi ^2}\] = \[{\left| \Psi\right|^2}\] gives the probabilities of finding the particle in the state \[\Psi\] i.e \[{\Psi ^2}\] is measure of probability density. The probability of finding a particle volume \[dV = \] \[dxdydz\] is given by \[{\left| \Psi\right|^2}dxdydz\]

- For the total probability of finding the particle somewhere is, of course, unit i.e particle is certainly to be found somewhere in space

$\iiint$ \[{\left| \Psi\right|^2}dxdydz\] = 1

\[\Psi\] Satisfying above requirement is said to be normalized.

Properties of wave function \[\Psi\]

- \[\Psi\] Contains all the measurable information about the particle.
- The wave function is complex with real and imaginary parts. The complex conjugate of \[\Psi\] is denoted by \[{\Psi ^*}\]

Thus, if

\[\Psi\] = \[X + iY\]

\[{\Psi ^*}\] = \[X - iY\]

However,   is always positive and real as \[\Psi {\Psi ^*}={X^2} + {Y^2}\] because \[{i^2}=1\]

- \[\Psi {\Psi ^*}\] Summed up overall space = 1 the wave function must be normalized.
- \[\Psi\] is continuous , i.e. its partial derivatives are \[\frac{{\partial \Psi }}{{\partial x}}\] \[\frac{{\partial \Psi }}{{\partial y}}\] \[\frac{{\partial \Psi }}{{\partial z}}\] must be also continuous everywhere.
- \[\Psi\] allows energy calculation via the Schrodinger Wave Equation
- \[\Psi\] Establishes the probability distribution in three dimensions.
- \[\Psi\] Permits calculations of the most probable value of a given variable.
- \[\Psi\] For a free particle is in sine wave, implying a precisely determined momentum and a totally uncertain position.
- The wave function must be finite everywhere. The particle exists somewhere in the space, therefore, the integral \[\Psi {\Psi ^*}\] overall space must be finite.

LESSON 10. Derivation for Schrodinger Wave Equation

Equation of motion of matter waves

(1) Time Independent Schrodinger Wave Equation

According to be Broglie theory, a particle of mass m is always associated with a wave whose wavelength is given by

λ = $\frac{h}{{mv}}$. If the particle has wave properties, it is expected that there should be some sort of wave equation which describes the behavior of the particle.

Suppose, a system of stationary waves associated with a particle.

x, y, z be the coordinates of the particle and Ψ the wave displacement for the de Broglie waves at any time .

The classical differential equation of wave motion is given by

$\frac{{{\partial ^2}\Psi }}{{\partial {t^2}}}$ = ${v^2}\left({\frac{{{\partial ^2}\Psi }}{{\partial {x^2}}} + \frac{{{\partial ^2}\Psi }}{{\partial {y^2}}} + \frac{{{\partial ^2}\Psi }}{{\partial {z^2}}}} \right)$ = ${v^2}{\nabla ^2}\Psi$ …….. (1)

Where, ${\nabla ^2}$ = $\frac{{{\partial ^2}}}{{\partial x}} + \frac{{{\partial ^2}}}{{\partial {y^2}}} + \frac{{{\partial ^2}}}{{\partial {z^2}}}$

${\nabla ^2}$ Is being Lasplacian operator and is wave velocity.

The solution of equation (1) gives Ψ as a periodic displacement in term of time i.e.

$\Psi \left({x,y,z,t} \right) = {\Psi _o}\left({x,y,z} \right){e^{ - i\omega t}}$ ……..(2)

Where ${\Psi _o}\left({x,y,z} \right)$ is a function of x, y, z and gives the amplitude at the point considered.

Equation (2) can also be expressed as

$\overrightarrow {\Psi \left({r,t} \right)}$ = $\overrightarrow {{\Psi _o}\left(r \right)} {e^{ - i\omega t}}$ ……….(3)

Differentiating equation (3) twice we get

$\frac{{{\partial ^2}\Psi }}{{\partial {t^2}}}$ = $- {\omega ^2}{\Psi _o}\left(r \right){e^{ - i\omega t}}$

$\frac{{{\partial ^2}\Psi }}{{\partial {t^2}}}$ = $- {\omega ^2}\Psi$

Substituting the value of $\frac{{{\partial ^2}\Psi }}{{\partial {t^2}}}$ in equation (1) we have

\[\frac{{{\partial ^2}\Psi }}{{\partial {x^2}}} + \frac{{{\partial ^2}\Psi }}{{\partial {y^2}}} +
\frac{{{\partial ^2}\Psi }}{{\partial {z^2}}}\] +
\[\frac{{{\omega ^2}}}{{{v^2}}}\Psi\] = 0 …….(4)

But, \[\omega= 2\pi v = 2\pi \frac{v}{\lambda }\]

Because \[\frac{\omega }{v}=\frac{{2\pi }}{\lambda }\]

Substituting the value of \[\frac{\omega }{v}\] in equation (4) we have

\[\frac{{{\partial ^2}\Psi }}{{\partial {x^2}}} + \frac{{{\partial ^2}\Psi }}{{\partial {y^2}}} +
\frac{{{\partial ^2}\Psi }}{{\partial {z^2}}}\] +
\[\frac{{4{\pi ^2}}}{{{\lambda ^2}}}\Psi\] = 0

\[{\nabla ^2}\Psi\] + \[\frac{{4{\pi ^2}}}{{{\lambda ^2}}}\Psi\] = 0 ……. (5)

Now from de Broglie relation \[\lambda\] = \[\frac{h}{{mv}}\]

\[{\nabla ^2}\Psi\] + \[\frac{{4{\pi ^2}}}{{{h^2}}}{m^2}{v^2}\Psi\] = 0 ……. (6)

If E and V be the total and potential energies of the particle respectively, then its kinetic energy \[\frac{{m{v^2}}}{2}\] is given by

\[\frac{{m{v^2}}}{2}\] = \[E - V\]

\[m{v^2}\] = \[2m\left({E - V{\text{}}} \right)\] ……... (7)

From equation (6) and (7) we have

\[{\nabla ^2}\Psi\] + \[\frac{{4{\pi ^2}}}{{{h^2}}}2m\left({E - V{\text{}}} \right)\Psi\] = 0

Or

\[{\nabla ^2}\Psi\] + \[\frac{{8{\pi ^2}m}}{{{h^2}}}\left({E - V{\text{}}} \right)\Psi\] = 0 ……… (8)

It is called Schrodinger time independent wave equation

Substituting \[\hbar \] = \[\frac{h}{{2\pi }}\] in equation (8), the Schrodinger wave equation can be written as

\[{\nabla ^2}\Psi\] + \[\frac{{2m}}{{{\hbar ^2}}}\left({E - V{\text{}}} \right)\Psi\] = 0……… (9)

For free particle V = 0 , hence the Schrodinger wave equation for a particle can be expressed as

\[{\nabla ^2}\Psi\] + \[\frac{{2mE}}{{{\hbar ^2}}}\Psi\] = 0………. (10)

(2) Time Dependent Schrodinger Wave Equation

The Schrodinger time depending wave equation may be obtained from Schrodinger time independent wave equation by eliminating E .

Differentiating equation (3) with respect to t , we get

\[\frac{{\partial \Psi }}{{\partial t}}\] = \[- i\omega {\Psi _o}\overrightarrow {\left(r \right)} {e^{ - i\omega t}}\]

\[\frac{{\partial \Psi }}{{\partial t}}\] = \[- i2\pi v{\Psi _o}\overrightarrow {\left(r \right)} {e^{ - i\omega t}}\]

\[\frac{{\partial \Psi }}{{\partial t}}\] = \[- 2\pi i\] \[\frac{E}{h}\] \[\Psi\] = ………….(11)

Substituting the value of E and \[\Psi\] in Schrodinger time independent wave equation, we get

\[{\nabla ^2}\Psi\] + \[\frac{{2m}}{{{\hbar ^2}}}\left({i\hbar \frac{{\partial \Psi }}{{\partial t}} - V{\text{}}\Psi } \right)\] = 0

\[{\nabla ^2}\Psi\] = \[- \frac{{2m}}{{{\hbar ^2}}}\left({i\hbar \frac{{\partial \Psi }}{{\partial t}} - V{\text{}}\Psi } \right)\]

\[- \frac{{{\hbar ^2}}}{{2m}}\] \[{\nabla ^2}\Psi\] + \[V{\text{}}\Psi\] = \[i\hbar \frac{{\partial \Psi }}{{\partial t}}\] ……………. (12)

This equation is known as Schrodinger time independent wave equation

It can be written as

\[\left({ - \frac{{{\hbar ^2}}}{{2m}}{\text{}}{\nabla ^2}{\text{}} + {\text{}}V{\text{}}} \right)\] \[\psi\] = \[i\hbar \] \[\frac{\partial }{{\partial t}}\Psi\]

Or

\[H\Psi\] = \[E\Psi\] ………….. (13)

Where \[H\] = \[- \frac{{{\hbar ^2}}}{{2m}}{\text{}}{\nabla ^2}{\text{}} + {\text{}}V\] and \[E\] = \[i\hbar\] \[\frac{\partial }{{\partial t}}\]

The equation (13) describes the motion of a non relativistic material particle.

MODULE 3. Modern Physics

LESSON 11. Qualitative Explanation of Zeeman Effect

Qualitative explanation of Zeeman Effect

The Zeeman Effect, named after the Dutch physicist Pieter Zeeman, is the effect of splitting a spectral line into several components in the presence of a static magnetic field. It is analogous to the Stark effect, the splitting of a spectral line into several components in the presence of an electric field

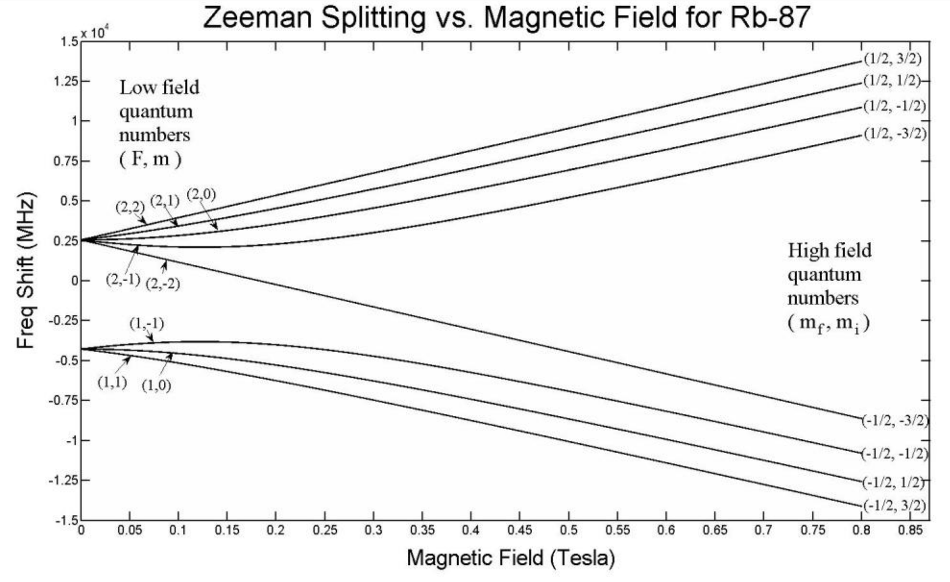


Fig Zeeman Splitting against Magnetic Field

Also similar to the Stark effect, transitions between different components have, in general, different intensities, with some being entirely forbidden (in the dipole approximation), as governed by the selection rules.

The distance between the Zeeman sub-levels is a function of the magnetic field; this effect can be used to measure the magnetic field

The Zeeman Effect is very important in applications such as nuclear magnetic resonance spectroscopy, electron spin resonance spectroscopy, Magnetic Resonance Imaging (MRI).

It may also be utilized to improve accuracy in atomic absorption spectroscopy. A theory about the magnetic sense of birds assumes that a protein in the retina is changed due to the Zeeman Effect.

When the spectral lines are absorption lines, the effect is called inverse Zeeman Effect.

Theoretical Background

The total Hamiltonian of an atom in a magnetic field is

$H = H_0 + V_M$,

where H_0 is the unperturbed Hamiltonian of the atom, and V_M is perturbation due to the magnetic field:

$$V_M = -\vec{\mu} \cdot \vec{B},$$

where $\vec{\mu}$ is the magnetic moment of the atom. The magnetic moment consists of the electronic and nuclear parts; however, the latter is many orders of magnitude smaller and will be neglected here. Therefore,

$$\vec{\mu} = -\mu_B g \vec{J}/\hbar,$$

where μ_B is the Bohr magneton, $\vec{J}$ is the total electronic angular momentum, and g is the Landé g-factor. The operator of the magnetic moment of an electron is a sum of the contributions of the orbital angular momentum $\vec{L}$ and the spin angular momentum $\vec{S}$, with each multiplied by the appropriate gyromagnetic ratio:

$$\vec{\mu} = -\mu_B (g_l \vec{L} + g_s \vec{S})/\hbar,$$

where $g_{l=1}$ and $g_s \approx 2.0023192$ (the latter is called the anomalous gyromagnetic ratio; the deviation of the value from 2 is due to Quantum Electrodynamics effects). In the case of the LS coupling, one can sum over all electrons in the atom:

$$g\vec{J} = \left\langle \sum_i (g_l \vec{l_i} + g_s \vec{s_i}) \right\rangle = \left\langle (g_l \vec{L} + g_s \vec{S}) \right\rangle,$$

Where $\vec{L}$ and $\vec{S}$ are the total orbital momentum and spin of the atom, and averaging is done over a state with a given value of the total angular momentum.

If the interaction term V_M is small (less than the fine structure), it can be treated as a perturbation; this is the Zeeman Effect proper. In the Paschen-Back effect, described below, V_M exceeds the LS coupling significantly (but is still small compared to H_0). In ultra strong magnetic fields, the magnetic-field interaction may exceed H_0, in which case the atom can no longer exist in its normal meaning, and one talks about Landau levels instead. There are, of course, intermediate cases which are more complex than these limit cases.

If the spin-orbit interactions dominate over the effect of the external magnetic field $\vec{L}$ and $\vec{S}$ are not separately conserved, only the total angular momentum is. The spin and orbital angular momentum vectors can be thought of as processing about the (fixed) total angular momentum vector. The (time-)"averaged" spin vector is then the projection of the spin onto the direction of :

$$\vec{S}_{avg} = \frac{(\vec{S} \cdot \vec{J})}{J^2} \vec{J}$$

and for the (time-)"averaged" orbital vector:

$$\vec{L}_{avg} = \frac{(\vec{L}\cdot\vec{J})}{J^2}\vec{J}.$$

Thus,

$$\langle V_M \rangle = \frac{\mu_B}{\hbar}\vec{J}(g_L\frac{\vec{L}\cdot\vec{J}}{J^2} + g_S\frac{\vec{S}\cdot\vec{J}}{J^2})\cdot\vec{B}.$$

Using $\vec{L}$ = $\vec{J}$ - $\vec{S}$ and squaring both sides, we get

$$\vec{S}\cdot\vec{J} = \frac{1}{2}(J^2 + S^2 - L^2) = \frac{\hbar^2}{2}[j(j+1) - l(l+1) + s(s+1)],$$

and: using $\vec{S}$ = $\vec{J}$ - $\vec{L}$ and squaring both sides, we get

$$\vec{L}\cdot\vec{J} = \frac{1}{2}(J^2 - S^2 + L^2) = \frac{\hbar^2}{2}[j(j+1) + l(l+1) - s(s+1)].$$

Combining everything and taking $J_z = \hbar m_j$, we obtain the magnetic potential energy of the atom in the applied external magnetic field,

$$\begin{aligned} V_M &= \mu_B B m_j \left[g_L \frac{j(j+1) + l(l+1) - s(s+1)}{2j(j+1)} + g_S \frac{j(j+1) - l(l+1) + s(s+1)}{2j(j+1)} \right] \\ &= \mu_B B m_j \left[1 + (g_S - 1)\frac{j(j+1) - l(l+1) + s(s+1)}{2j(j+1)} \right], \\ &= \mu_B B m_j g_j \end{aligned}$$

where the quantity in square brackets is the Landé g-factor gJ of the atom ($g_L = 1$ and $g_S \approx 2$) and mj is the z-component of the total angular momentum. For a single electron above filled shells s = 1 and j = 1 ± s, the Landé g-factor can be simplified into:

$$g_j = 1 \pm \frac{g_S - 1}{2l + 1}$$

The Lyman alpha transition in hydrogen in the presence of the spin-orbit interaction involves the transitions

$2P_{1/2} \rightarrow 1S_{1/2}$ and $2P_{3/2} \rightarrow 1S_{1/2}$.

In the presence of an external magnetic field, the weak-field Zeeman effect splits the 1S1/2 and 2P1/2 levels into 2 states each (mj =1/2,-1/2) and the 2P3/2 level into 4 states (mj = 3/2,1/2,-1/2,-3/2).

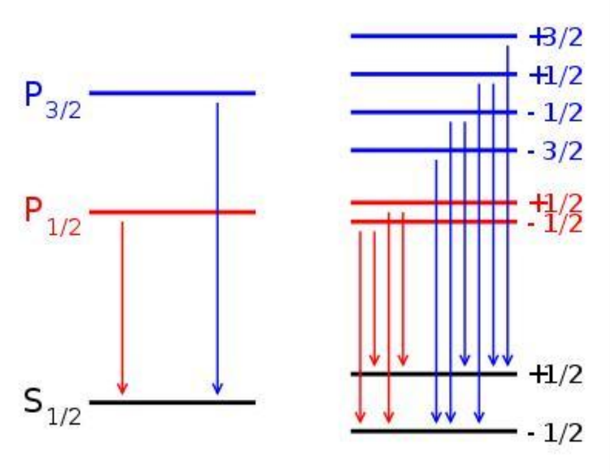


Fig. Zeeman Splits

The Landé g-factors for the three levels are:

$g_{J} = 2$ For (j=1/2, l=0)

$g_{J} = 2/3$ For (j=1/2, l=1)

$g_{J} = 4/3$ For (j=3/2, l=1).

Note in particular that the size of the energy splitting is different for the different orbital, because the gJ values are different. On the left, fine structure splitting is depicted. This splitting occurs even in the absence of a magnetic field, as it is due to spin-orbit coupling. Depicted on the right is the additional Zeeman splitting, which occurs in the presence of magnetic fields.

LESSON 12. Qualitative Explanation of Stark Effect

Stark effect

The Stark effect is the shifting and splitting of spectral lines of atoms and molecules due to presence of an external static electric field.

The amount of splitting and or shifting is called the Stark splitting or Stark shift. In general one distinguishes first- and second-order Stark effects.

The first-order effect is linear in the applied electric field, while the second-order effect is quadratic in the field.

The Stark effect is responsible for the pressure broadening (Stark broadening) of spectral lines by charged particles. When the split/shifted lines appear in absorption, the effect is called the inverse Stark effect.

The Stark effect is the electric analogue of the Zeeman Effect where a spectral line is split into several components due to the presence of a magnetic field.

The Stark effect can be explained with fully quantum mechanical approaches, but it has also been a fertile testing ground for semi classical methods.

Overview

An electric field pointing from left to right, for example, tends to pull nuclei to the right and electrons to the left.

In another way of viewing it, if an electronic state has its electron disproportionately to the left, its energy is lowered, while if it has the electron disproportionately to the right, its energy is raised.

Other things equal, the effect of the electric field is greater for outer electron shells, because the electron is more distant from the nucleus, so it travels farther left and farther right.

The Stark effect can lead to splitting of degenerate energy levels. For example, in the Bohr model, an electron has the same energy whether it is in the 2s state or any of the 2p states. However, in an electric field, there will be hybrid orbital (also called quantum superposition) of the 2s and 2p states where the electron tends to be to the left, which will acquire a lower energy, and other hybrid orbital where the electron tends to be to the right, which will acquire a higher energy. Therefore, the formerly degenerate energy levels will split into slightly lower and slightly higher energy levels.

Classical electrostatics

The Stark effect originates from the interaction between a charge distribution (atom or molecule) and an external electric field. Before turning to quantum mechanics we describe the interaction classically and consider a continuous charge distribution $\rho(r)$. If this charge distribution is non-polarizable its interaction energy with an external electrostatic potential $V(r)$ is

$$E_{\text{int}} = \int \rho(\mathbf{r})V(\mathbf{r})d\mathbf{r}^3$$

If the electric field is of macroscopic origin and the charge distribution is microscopic, it is reasonable to assume that the electric field is uniform over the charge distribution. That is, V is given by a two-term Taylor expansion,

$$V(\mathbf{r}) = V(\mathbf{0}) - \sum_{i=1}^{3} r_i F_i$$, with the electric field: , $$F_i \equiv -\left(\frac{\partial V}{\partial r_i}\right)\Big|_{\mathbf{0}}$$

where we took the origin 0 somewhere within ρ. Setting V(0) as the zero energy, the interaction becomes

$$E_{\text{int}} = -\sum_{i=1}^{3} F_i \int \rho(\mathbf{r}) r_i d\mathbf{r} \equiv -\sum_{i=1}^{3} F_i \mu_i = -\mathbf{F} \cdot \boldsymbol{\mu}$$

Here we have introduced the dipole moment μ of ρ as an integral over the charge distribution. In case ρ consists of N point charges qj this definition becomes a sum

$$\boldsymbol{\mu} \equiv \sum_{j=1}^{N} q_j \mathbf{r}_j$$

Perturbation theory

Turning now to quantum mechanics we see an atom or a molecule as a collection of point charges (electrons and nuclei), so that the second definition of the dipole applies. The interaction of atom or molecule with a uniform external field is described by the operator

$$V_{\text{int}} = -\mathbf{F} \cdot \boldsymbol{\mu}.$$

This operator is used as a perturbation in first- and second-order perturbation theory to account for the first- and second-order Stark effect.

First order

Let the unperturbed atom or molecule is in a g-fold degenerate state with ortho- normal 0th -order state functions. $\psi_1^0, \ldots, \psi_g^0$ (Non-degeneracy is the special case g = 1). According to perturbation theory the first-order energies are the Eigen values of the g x g matrix with general element

$$(\mathbf{V}_{\text{int}})_{kl} = \langle \psi_k^0 | V_{\text{int}} | \psi_l^0 \rangle = -\mathbf{F} \cdot \langle \psi_k^0 | \boldsymbol{\mu} | \psi_l^0 \rangle, \qquad k, l = 1, \ldots, g.$$

If g = 1 (as is often the case for electronic states of molecules) the first-order energy becomes proportional to the expectation (average) value of the dipole operator,

$$E^{(1)} = -\mathbf{F} \cdot \langle \psi_1^0 | \boldsymbol{\mu} | \psi_1^0 \rangle = -\mathbf{F} \cdot \langle \boldsymbol{\mu} \rangle.$$

Because a dipole moment is a polar vector, the diagonal elements of the perturbation matrix Vint vanish for systems with an inversion center (such as atoms). Molecules with an inversion center in a non-degenerate electronic state do not have a (permanent) dipole and hence do not show a linear Stark effect.

In order to obtain a non-zero matrix Vint for systems with an inversion center it is necessary that some of the unperturbed functions ψ_i^0 have opposite parity (obtain plus and minus under inversion), because only functions of opposite parity give non-vanishing matrix elements. Degenerate 0th -order states of opposite parity occur for excited hydrogen-like (one-electron) atoms. Such atoms have the principal quantum number n among their quantum numbers. The excited state of hydrogen-like atoms with principal quantum number n is n2-fold degenerate and

$$n^2 = \sum_{\ell=0}^{n-1} (2\ell + 1),$$

Here ℓ is the angular momentum quantum number

For instance, the excited n = 4 state contains the following ℓ states,

$$16 = 1 + 3 + 5 + 7 \quad \Longrightarrow \quad n = 4 \text{ contains } s \oplus p \oplus d \oplus f.$$

The one-electron states with even are even under parity, while those with odd ℓ are odd under parity. Hence, hydrogen-like atom with n>1 shows first-order Stark effect.

The first-order Stark effect occurs in rotational transitions of symmetric top molecules (but not for linear and asymmetric molecules). In first approximation a molecule may be seen as a rigid rotor. A symmetric top rigid rotor has the unperturbed Eigen states

$$|JKM\rangle = (D^J_{MK})^* \quad \text{with} \quad M, K = -J, -J+1, \dots, J$$

With 2(2J+1)-fold degenerate energy for |K| > 0 and (2J+1)-fold degenerate energy for K=0. Here DJMK is an element of the Wigner D-matrix. The first-order perturbation matrix on basis of the unperturbed rigid rotor function is non-zero and can be diagonalized. This gives shifts and splitting in the rotational spectrum. Quantitative analysis of these Stark shift yields the permanent electric dipole moment of the symmetric top molecule.

Second order

As stated, the quadratic Stark effect is described by second-order perturbation theory. The 0th -order problems

$$H^{(0)}\psi_k^0 = E_k^{(0)}\psi_k^0, \quad k = 0, 1, \dots, \quad E_0^{(0)} < E_1^{(0)} \le E_2^{(0)}, \dots$$

are assumed to be solved. It is usual to assume that the 0th -order state to be perturbed is non-degenerate. If we take the ground state as the non-degenerate state under consideration (for hydrogen-like atoms: n = 1), perturbation theory gives

$$E^{(2)} = \sum_{k>0} \frac{\langle \psi_0^0 | V_{\text{int}} | \psi_k^0 \rangle \langle \psi_k^0 | V_{\text{int}} | \psi_0^0 \rangle}{E_0^{(0)} - E_k^{(0)}} = -\frac{1}{2} \sum_{i,j=1}^{3} F_i \alpha_{ij} F_j$$

With the components of the polarizability tensor α defined by

$$\alpha_{ij} \equiv -2 \sum\nolimits_{k>0} \frac{\langle \psi_0^0 | \mu_i \, | \psi_k^0 \rangle}{E_0^{(0)} - E_k^{(0)}}$$

The energy E(2) gives the quadratic Stark effect.

Because of their spherical symmetry the polarizability tensor of atoms is isotropic,

$$\alpha_{ij} = \alpha_0 \, \delta_{1D456j} = E^{(2)} = -\frac{1}{2} \alpha_0 F^2$$

This is the quadratic Stark shift for atoms. For many molecules this expression is not too bad an approximation, because molecular tensors are often reasonably isotropic.

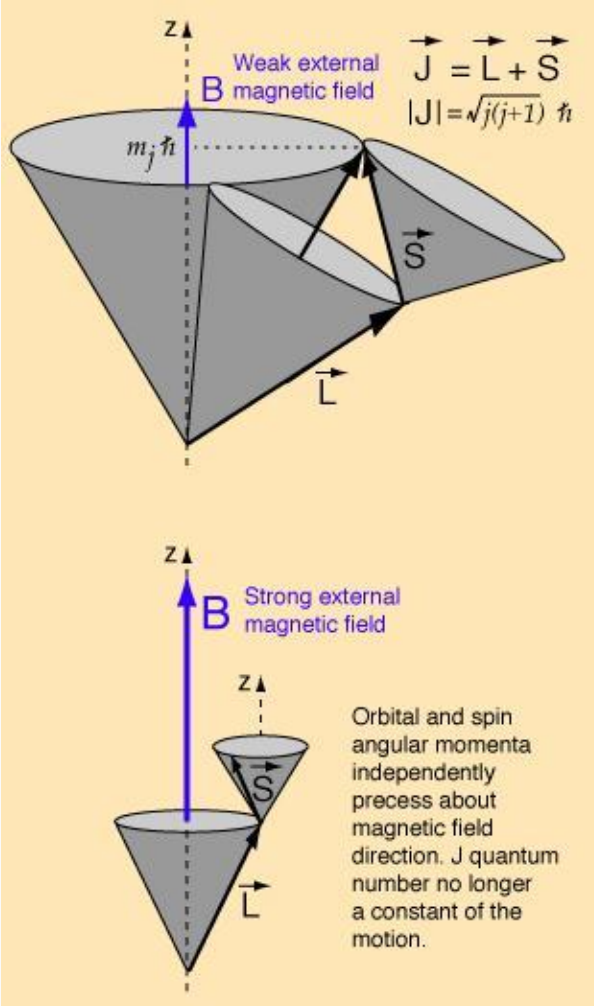
Quantum-confined Stark effect

In a semiconductor heterostructure, where a small band gap material is sandwiched between two layers of a larger band gap material, the Stark effect can be dramatically enhanced by bound excitons. This is because the electron and hole which form the exciton are pulled in opposite directions by the applied electric field, but they remain confined in the smaller band gap material, so the exciton is not merely pulled apart by the field. The quantum-confined Stark effect is widely used for semiconductor-based optical modulators, particularly for optical fiber communications.

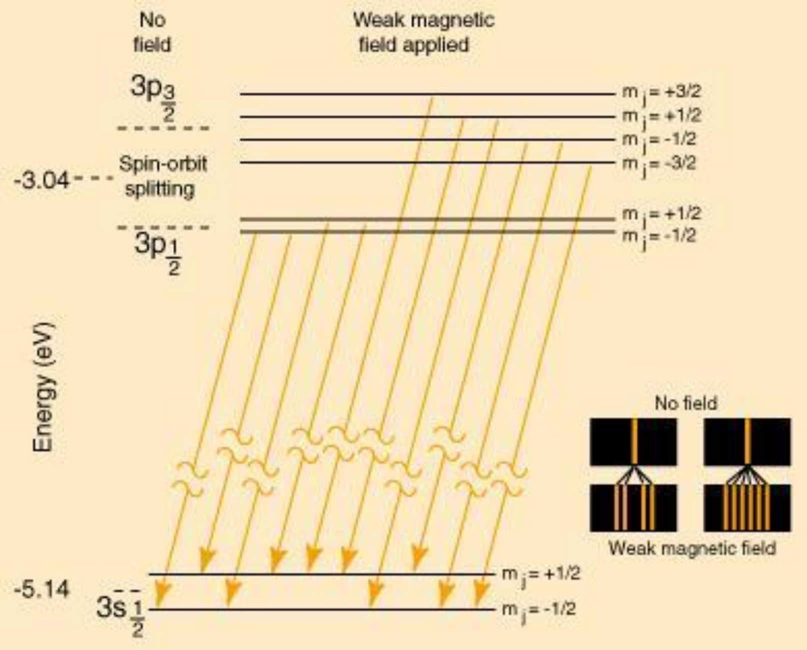
LESSON 13. Paschan Back Effect & Introduction to Raman Spectroscopy

Paschan Back Effect

In the presence of an external magnetic field, the energy levels of atoms are split. This splitting is described well by the Zeeman effect if the splitting is small compared to the energy difference between the unperturbed levels, i.e., for sufficiently weak magnetic fields. This can be visualized with the help of a vector model of total angular momentum. If the magnetic field is large enough, it disrupts the coupling between the orbital and spin angular momentum, resulting in a different pattern of splitting. This effect is called the Paschen-Back effect



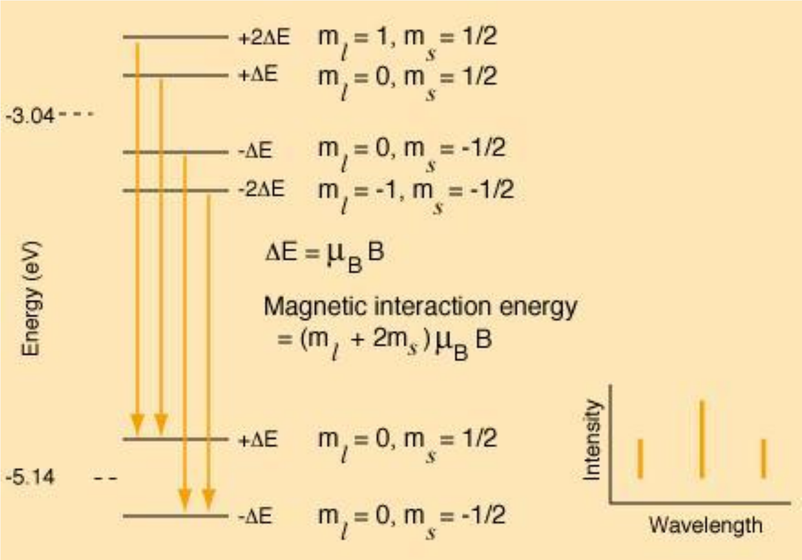
In the weak field case the vector model at left implies that the coupling of the orbital angular momentum L to the spin angular momentum S is stronger than their coupling to the external field. In this case where spin-orbit coupling is dominant, they can be visualized as combining to form a total angular momentum J which then precesses about the magnetic field direction



In the strong-field case, S and L couple more strongly to the external magnetic field than to each other, and can be visualized as independently precessing about the external field direction.

For reference, the sodium Zeeman Effect is reproduced below to show the nature of the magnetic interaction for weak external magnetic fields.

The following is a model of the changes in the pattern if the magnetic field were strong enough to decouple L and S. The resulting spectrum would be a triplet with the center line twice the intensity of the outer lines.



To create this pattern, the projections of L and S in the z-direction have been treated independently and the ms multiplied by the spin g-factor. The energy shift is expressed as a multiple of the Bohr magnetrons mB. The selection rules explain why the transitions shown are allowed and others not.

Sodium was used as the basis of the model for convenience, but the fields required to create Paschen-Back conditions for sodium are unrealistically high. Lithium, on the other hand, has a spin-orbit splitting of only 0.00004 eV compared to 0.0021 eV for sodium. Such small energy values are sometimes expressed in "wave numbers", or 1/l in cm-1. In these units the lithium separation is about 0.3 cm-1and the sodium separation is about 17 cm-1 .The Paschen-Back conditions are met in some lithium spectra observed on the Sun, so this effect does have astronomical significance.

Raman Spectroscopy

Introduction

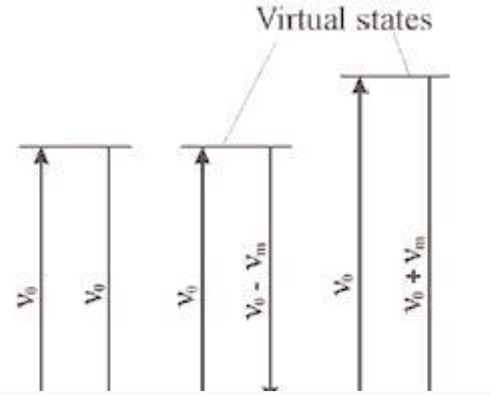
Raman spectroscopy is a spectroscopic technique based on inelastic scattering of monochromatic light, usually from a laser source. Inelastic scattering means that the frequency of photons in monochromatic light changes upon interaction with a sample. Photons of the laser light are absorbed by the sample and then reemitted. Frequency of the reemitted photons is shifted up or down in comparison with original monochromatic frequency, which is called the Raman Effect. This shift provides information about vibrational, rotational and other low frequency transitions in molecules. Raman spectroscopy can be used to study solid, liquid and gaseous samples.

Origins of Raman

The Raman Effect is based on molecular deformations in electric field E which is determined by molecular polarizability α. The laser beam can be considered as an oscillating electromagnetic wave with electrical vector E. Upon interaction with the sample it induces electric dipole moment P = αE which deforms molecules. Because of periodical deformation, molecules start vibrating with characteristic frequency υm.

Amplitude of vibration is called a nuclear displacement.

In other words, monochromatic laser light with frequency υ0 excites molecules and transforms them into oscillating dipoles. Such oscillating dipoles emit light of three different frequencies



when:

- A molecule with no Raman-active modes absorbs a photon with the frequency υ0. The excited molecule returns back to the same basic vibrational state and emits light with the same frequency υ0 as an excitation source. This type if interaction is called an elastic Rayleigh scattering.

- A photon with frequency υ0 is absorbed by Raman-active molecule which at the time of interaction is in the basic vibrational state. Part of the photon’s energy is transferred to the Raman-active mode with frequency υm and the resulting frequency of scattered light is reduced to υ0 - υm. This Raman frequency is called Stokes frequency, or just “Stokes”.

- A photon with frequency υ0 is absorbed by a Raman-active molecule, which, at the time of interaction, is already in the excited vibrational state. Excessive energy of excited Raman active mode is released, molecule returns to the basic vibrational state and the resulting frequency of scattered light goes up to υ0 + υm. This Raman frequency is called Anti- Stokes frequency, or just “Anti-Stokes”.

About 99.999% of all incident photons in spontaneous Raman undergo elastic Rayleigh scattering. This type of signal is useless for practical purposes of molecular characterization. Only about 0.001% of the incident light produces inelastic. Raman signal with frequencies υ0 ± υm. Spontaneous Raman scattering is very weak and special measures should be taken to distinguish it from the predominant Rayleigh scattering. Instruments such as notch filters, tunable filters, laser stop apertures, double and triple spectrometric systems are used to reduce Rayleigh scattering and obtain high-quality Raman spectra.

LESSON 14. Practical view of Raman Spectroscopy

Instrumentation

A Raman system typically consists of four major components:

- Excitation source (Laser).
- Sample illumination system and light collection optics.
- Wavelength selector (Filter or Spectrophotometer).
- Detector (Photodiode array, CCD or PMT).

A sample is normally illuminated with a laser beam in the ultraviolet (UV), visible (Vis) or near infrared (NIR) range. Scattered light is collected with a lens and is sent through interference filter or spectrophotometer to obtain Raman spectrum of a sample. Since spontaneous Raman scattering is very weak the main difficulty of Raman spectroscopy is separating it from the intense Rayleigh scattering. More precisely, the major problem here is not the Rayleigh scattering itself, but the fact that the intensity of stray light from the Rayleigh scattering may greatly exceed the intensity of the useful Raman signal in the close proximity to the laser wavelength. In many cases the problem is resolved by simply cutting off the spectral range close to the laser line where the stray light has the most prominent effect. People use commercially available interference (notch) filters which cut-off spectral range of ± 80-120 cm-1 from the laser line. This method is efficient in stray light elimination but it does not allow detection of low-frequency Raman modes in the range below 100 cm-1. Stray light is generated in the spectrometer mainly upon light dispersion on gratings and strongly depends on grating quality.

Raman spectrometers typically use holographic gratings which normally have much less manufacturing defects in their structure then the ruled once. Stray light produced by holographic gratings is about an order of magnitude less intense then from ruled gratings of the same groove density. Using multiple dispersion stages is another way of stray light reduction. Double and triple spectrometers allow taking Raman spectra without use of notch filters. In such systems Raman-active modes with frequencies as low as 3-5 cm-1 can be efficiently detected.

In earlier times people primarily used single-point detectors such as photon-counting Photomultiplier Tubes (PMT). However, a single Raman spectrum obtained with a PMT detector in wave number scanning mode was taking substantial period of time, slowing down any research or industrial activity based on Raman analytical technique. Nowadays, more and more often researchers use multi-channel detectors like Photodiode Arrays (PDA) or, more commonly, a Charge-Coupled Devices (CCD) to detect the Raman scattered light. Sensitivity and performance of modern CCD detectors are rapidly improving. In many cases CCD is becoming the detector of choice for Raman spectroscopy.

Ways to improve Raman signal intensity

Raman signal is normally quite weak and people are constantly improving Raman spectroscopy techniques. Many different ways of sample preparation, sample illumination or scattered light detection were invented to enhance intensity of Raman signal. Here we will examine some of them.

Stimulated Raman

It was found that if the sample was irradiated with a very strong laser pulse, new "non-linear" phenomena were observed in Raman signal. In comparison with continuous wave (CW) lasers with electric field of about only 104 V ·cm-1 pulsed lasers with electric field of about 109 V ·cm-1 transform a much larger portion of incident light into useful Raman scattering and substantially improve signal-to-noise ratio.

Stimulated Raman scattering is an example of "non-linear" Raman spectroscopy. Very strong laser pulse with electric field strength > 109 V ·cm-1 transforms up to 50% of all laser pulse energy into coherent beam at Stokes frequency υ0 – υm.

The Stokes beam is unidirectional with the incident laser beam. Only the mode um which is the strongest in the regular Raman spectrum is greatly amplified. All other, weaker Raman active modes are not present. The Stokes frequency is so strong it acts as a secondary excitation source and generates the second Stokes line with frequency υ0 - 2υm. The second Stokes line generates the third one with the frequency υ0 - 3υm etc. Stimulated Raman technique enjoys 4-5 orders of magnitude enhancement of Raman signal as compared to the spontaneous Raman scattering.

CARS

Coherent Anti-Stokes Raman, CARS, is another type of "non-linear" Raman spectroscopy. It stands for Coherent Anti-Stokes Raman Spectroscopy. Instead of the traditional one laser, two very strong collinear lasers irradiate a sample. Frequency of the first laser is usually constant, while the frequency of the second one can be tuned in a way that the frequency difference between the two lasers equals exactly the frequency of some Raman-active mode of interest. This particular mode will be the only extremely strong mode in the Raman signal.

With CARS we can obtain only one strong Raman peak of interest. In this case a monochromator is not really required. A wideband interference filter and a detector behind the filter would do the job. Below is more specific description with a little bit of math to understand CARS more in depth.

Two laser beams with frequencies υ1 and υ2 (υ1 > υ2) interact coherently, and because of the wave mixing, produce strong scattered light of frequency 2υ1 - υ2

If the frequency difference between two lasers υ1 - υ2 is equal to the frequency um of a Raman-active rotational, vibrational or any other mode then a strong light of frequency υ1 + υm is emitted. In other words, to obtain strong Raman signal the second laser frequency should be tuned in a way that υ2 = υ1 - υm . Then the frequency of strong scattered light will be 2υ1 - υ2 = 2υ1- (υ1 - υm) = υ1 + υm, which is higher than the excitation frequency u1 and therefore considered to be Anti-Stokes frequency. Coherent Anti-Stokes Raman Spectroscopy derives its name from the fact that it uses two Coherent laser beams and the resulting signal has Anti-Stokes frequency.

Resonance Raman (RR)

Many substances, especially colored ones, may absorb laser beam energy and generate strong fluorescence which contaminates Raman spectrum. This is one of the central problems in Raman spectroscopy especially when UV lasers are used. However, it was found that under certain conditions some types of colored molecules can produce strong Raman scattering instead of fluorescence. This effect was called Resonance Raman. The Resonance Raman effect takes place when the excitation laser frequency is chosen in a way that it crosses frequencies of electronic excited states and resonates with them. Intensity of Raman bands which originate from electronic transitions between those states are enhanced 3-5 orders of magnitude. Not all the bands of spontaneous Raman spectrum are enhanced. The so-called chromophoric group, which is responsible for the molecule's coloration, experiences the highest level of enhancement. The reason is the chromophoric group normally has the highest level of light absorption.

The highest intensity of Resonance Raman signal is obtained when laser frequency equals to the first or the second electronic excited state

Therefore, tunable lasers are the most appropriate choice for the RR technique. Although,

Even when the frequency of the laser does not exactly match the desired electronic excited states an impressive enhancement of Raman signal occurs.

SERS and SERRS

Surface-Enhanced Raman Spectroscopy utilizes the following effect. Raman signal from molecules adsorbed on certain metal surfaces can be 5-6 orders of magnitude stronger then the Raman signal from the same molecules in bulk volume. The exact reason for such dramatic improvement is still under discussion. However, since intensity of Raman signal is proportional to the square of electric dipole moment $P = \alpha E$, there are two possible reasons - the enhancement of polarizability α, and the enhancement of electrical field E. The first enhancement of polarizability a may occur because of a charge-transfer effect or chemical bond formation between metal surface and molecules under observation. This is a so-called chemical enhancement. The second one takes into account interaction of the laser beam with irregularities on the metal surface such as metal micro-particles or roughness profile. It is believed that laser light excites conduction electrons at the metal surface leading to a surface plasma resonance and strong enhancement of electric field E. It is also called electromagnetic enhancement. In all cases choice of appropriate surface substrate is very important. The most popular and universal substrates used for

SERS are electrochemically etched silver electrodes as well as silver and gold colloids with average particle size below 20 nm.

One disadvantage of SERS is the difficulty of spectra interpretation. The signal enhancement is so dramatic that Raman bands that are very weak and unnoticeable in spontaneous Raman spectra can appear in SERS. Some trace contaminants can also contribute additional peaks. On the other hand, because of chemical interactions with the metal surface, certain peaks which are strong in conventional Raman might not be present in SERS at all. Non-linear character of signal intensity as a function of concentration complicates things even further. Very careful consideration of all physical and chemical factors should be done while interpreting SERS spectra which makes it extremely difficult for practical use. The SERRS technique, Surface-Enhanced Resonance Spectroscopy, was developed because of such complications. It utilizes both Surface-Enhancement effect and Raman Resonance effect so the resulting enhancement in Raman signal intensity can be as high as 1014. The main advantage of SERRS is its spectra resemble very much regular Resonance Raman spectra, which makes it much easier to interpret.

LESSON 15. Concept of Bloch Function & Bloch Electron

Charged Particles

Electric charges, positive or negative, occur in multiples of the electronic charge. The electron is one of the fundamental particles constituting the atom. The charge of an electron is negative and is denoted by e.

The magnitude of e is 1.6 X 10^{-19} coulomb . The mass of an electron changes with its velocity in accordance with the theory of relativity. An electron moving with a velocity v has the mass

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \;\ldots\ldots\ldots\ldots (1)$$

where c is the velocity of light in free space (c = 3 \[\times {10^8}{{met} \over {sec}}\]).

If v<< c, \[m \approx {m_o}\], called the rest mass of the electron.

The rest mass of the electron has the value $m_o = 9.11 \times 10^{-31}$ kg .

Equation (1) shows that the mass m increases with the velocity v and approaches infinity as v → c . An electron, starting from rest and accelerated through a potential difference of 6kv , acquires a velocity of 0.15c . At this high speed, the mass of the electron increases only by 1 per cent. Therefore, the change of the mass of an electron with velocity can be neglected for accelerating potentials less than 6kv .

The radius of an electron is about 10^{-15}m and that of an atom is 10^{-10}m . The radii are so small that electrons and atoms are ordinarily taken as point masses.

Electron Volt- A Unit of Energy

For energies involved in electron devices, ‘joule’ is too large a unit. Such small energies are conveniently measured in electron volt, abbreviated eV. The electron volt is the kinetic energy gained by an electron, initially at rest, in moving through a potential difference of 1 volt. Since

e = 1.6 X 10^{-19} coulomb

1eV = 1.6 X 10^{-19} coulomb X 1 volt = 1.6 X 10^{-19} joule

Atomic Energy Levels

An atom of an element is generally made up of electrons, protons, and neutrons. The only exception is the hydrogen atom which possesses one electron and one proton, but no neutron. While an electron is negatively charged, the proton is a positively charged particle. The charge of a proton is numerically equal to that of an electron, but the mass of a proton is 1837 times that of an electron. A neutron is a neutral particle having a mass nearly equal to the proton mass. Because the protons and neutrons carry practically the entire mass of the atom, they remain almost immobile in a region, called the atomic nucleus. The electrons revolve round the nucleus in definite orbits which are circular or elliptical. The motion is analogous to that of planets around the sun. The atom is electrically neutral because the number of orbital electrons is equal to the number of protons in the nucleus. The atom of one element differs from that of another due to the differing numbers of protons, neutrons, and electrons in the atom. In the Bohr atomic model, the electrons are assumed to move about the nucleus in certain discrete circular orbits without radiating any energy. In any orbit, the angular momentum of the electron is equal to an integral multiple of ${h \over {2\pi }}$, where h is Planck's constant (h = 6.62 X 10^{-34} J.s). The integral number n has values 1, 2, 3 etc. for different orbits. The higher the value of has the larger the radius of the orbit.

Velocity of Bloch Electron

- A Bloch wave or Bloch state, named after Swiss physicist Felix Bloch, is the wave-function of a particle (usually, an electron) placed in a periodic potential. Bloch's theorem states that the energy Eigen function for such a system may be written as the product of a plane wave envelope function and a periodic function (periodic Bloch function) un K(r) that has the same periodicity as the potential, giving:

$${\psi _{nk}}\left(r \right)={e^{ikr}}{u_{nk}}\left(r \right)$$

- The corresponding energy Eigen values are $\epsilon_n(k) = \epsilon_n(k+K)$, , periodic with periodicity K of a reciprocal lattice vector.
- The energies associated with the index n vary continuously with wave vector and form an energy band identified by band index n .

The Eigen values for given n are periodic in K; all distinct values of $\epsilon_n(k)$ occur for K -values within the first Brillouin zone of the reciprocal lattice.

Statement of Bloch's Function

Suppose that V(x) denotes the potential energy of an electron in a in a linear lattice of lattice constant a and that V(x) = V(x + a) i.e the period of the potential is also a .

The wave functions of the electron in this potemtial are then obtained the Schrodinger equation

$${{{\partial ^2}\Psi } \over {\partial {x^2}}}={{2m} \over {{\hbar ^2}}}\left[{E - V\left(x \right)} \right]\Psi=0 \qquad \text{…………..(1)}$$

With reference to the solution of this equation there is an important theorem which predict that it has solution of the form

$$\Psi \left(x\right)={e^{ \pm iKx}}{u_K}\left(x \right)$$

$${u_K}\left(x \right)={u_K}\left({x + a} \right) \qquad \text{…………..(2)}$$

The solutions are plane waves $\Psi \left(x \right)={e^{ \pm iKx}}$ modulated by function $u_K(x)$ which depends in general on the wave vector K and have the periodically of the potential.

This theorem is known as the Bloch Theorem and the functions (2) as the Bloch solution.

Suppose g(x) and f(x) are two teal and independent solution of the Schrodinger equation (1)

Then most general solution is

$$\Psi \left(x \right)=Af\left(x \right) + Bg\left(x \right)$$ ………… (3)

Where A and B are arbitrary constant

Since V(x) = V(x + a) and only g(x) and f(x), but also g(x + a) and f(x + a) are the solution.

Now differential equation of the 2nd order has only two independent solutions, these must be expressible as a linear combination of the independent ones.

We must have relation

$$f\left({x + a} \right)={\alpha _1}f\left(x \right) + {\alpha _2}g\left(x \right)$$

$$g\left({x + a}\right)={\beta_1}f\left(x\right)+{\beta_2}g\left(x\right)$$………(4)

a1, a2 , and ${\beta_2}$ are real functions of energy E

According to these solution, the general solution,

$$\Psi \left({x + a} \right)=Af\left({x + a} \right) + Bg\left({x + a} \right)$$

Or

$$\Psi \left({x + a} \right)=\left({A{\alpha _1} + B{\beta _1}} \right)f\left(x \right) + \left({A{\alpha _2} + B{\beta _2}} \right)g\left(x \right)$$………..(5)

Now if we choose A and B such that

$$A{\alpha _1} + B{\beta _1}=\lambda A$$

$${\alpha _2} + B{\beta _2}=\lambda B$$…….(6)

λ is a constant then the functions $\Psi \left(x \right)$ will have the property

$$\Psi \left({x + a} \right) = \lambda \Psi \left(x \right)$$………(7)

The equation (6) has non-vanishing solution for A and B only if the determinant of their coefficient

$$\left|{\matrix{{{\alpha _1}-\lambda }&{{\beta _1}}\cr{{\alpha _2}}&{{\beta _2}-\lambda }\cr}}\right|= 0$$

$${\lambda^2}-\left({{\alpha_1}+{\beta_2}}\right)\lambda+{\alpha _1}{\beta _2}-{\alpha _2}{\beta _1} = 0$$ ………..(8)

We can show that

${\alpha_1}{\beta_2}-{\alpha_2}{\beta_1}=1$ in the following manner.

Since f(x) and g(x) are two real independent solution of equation (1)

\[{{{\partial^2}g\left(x\right)}\over{\partial{x^2}}}={{2m}\over{{\hbar^2}}}\left[{E-V\left(x\right)}\right]g\left(x\right)=0\]

\[{{{\partial^2}f\left(x\right)}\over{\partial{x^2}}}={{2m}\over{{\hbar^2}}}\left[{E-V\left(x\right)}\right]f\left(x\right)=0\]

Multiplying the former by f(x) and latter by g(x) and subtracting

\[f\left(x\right){{dg\left(x\right)}\over{dx}}-g\left(x\right){{df\left(x\right)}\over {dx}}=constant\]

LHS of this equation is called Wronskian W(x), of the solutions and constant in this case.

Further, from equation (4) one can derive

\[W\left({x+a}\right)=\left({{\alpha_1}{\beta_2}-{\alpha_2}{\beta_1}}\right)W\left(x\right)\]……….(9)

The equation (8) becomes

\[{\lambda^2}-\left({{\alpha_1}+{\beta_2}}\right)\lambda+1=0\]…………(10)

\[{\alpha_1}+{\beta_2}\] is real function of energy

In general there will be two functions

\[{\Psi _1}\left(x \right)\] and \[{\Psi _2}\left(x \right)\] which have the property (7)

Noting the product \[{\lambda _{1}}{\lambda _{2}}=1\]we may taken them as complex conjugates

\[{\lambda _{1}}={e^{iKa}}\] and \[{\lambda _{2}}={e^{iKa}}\] …………(11)

K is real

The corresponding functions $\Psi_1(x)$ and $\Psi_2(x)$ then have the property

\[{\Psi_1}\left({x + a}\right)={e^{iKa}}{\Psi_1}\left(x\right)\]

\[{\Psi_2}\left({x + a}\right)={e^{iKa}}{\Psi_2}\left(x\right)\]…………(12)

This is the property of Bloch Function is seen .

Put x + a = a in equation (2)

$$\Psi(x+a) = e^{\pm iK(x+a)}u_K(x+a) = \Psi(x)e^{\pm iKa}$$

Hence

$\Psi \left({x + a}\right)=\lambda\Psi\left(x\right)$ where, $\lambda={e^{\pm iKa}}$……….(13)

We see that (12) and (13) are same. Thus, we can write the solution of (1) and (2) and the Bloch Theorem is proved. The wave functions in this case are labeled by κ and are called the Bloch Wave functions.

LESSON 16. Bands in Solid

Bands in Solids

Introduction

Electronics (coined from the word 'electron') is the branch of science and engineering dealing with the theory and use of a class of devices in which electrons are transported through a vacuum, gas or semiconductor. The motion of electrons in such devices, called electron devices, is usually controlled by electric fields. Diodes, triodes, transistors etc. are examples of electron devices. Electronics can be broadly classified into two branches: physical electronics and electronic engineering. Physical electronics treats the motion of electrons in a vacuum, gas or semiconductor. The design, fabrication, and application of electron devices form the subject matter of electronic engineering. Electronics has inroads into many fields of science and technology, and has profoundly influenced the life of modern man. Modern computers and communication systems are intimately linked with advances in electronics. We list below some areas of application of electronics; the list is, however, far from complete.

(i) In home, we use a variety of electronic systems: radios, TVs, VCRs, CD players, telephone answering machines, PCs, microwave ovens, pocket calculators, digital watches, etc.

(ii) Round-the-globe communication via microwave or fiber-optic links and satellites, and access to internets has been possible with advances in electronics. This has shaped the global village.

(iii) Sophisticated electronic instruments are used for researches in various fields of science and engineering.

(iv) The commercial and industrial sectors rely on electronic communication, information processing, and control systems.

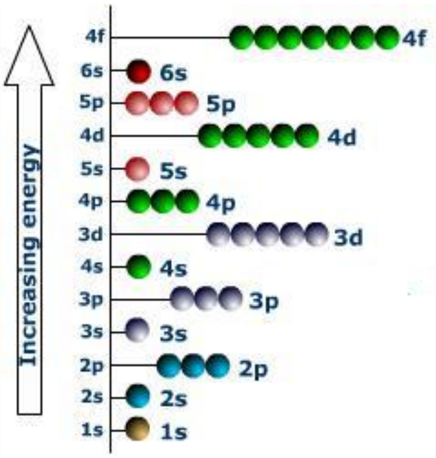
(v) Electronic radar systems are employed for a safe flight from one airport to another. A modern aircraft is equipped with electronic sensors and computers.

(vi) Electronic communication and radars play a vital role in meteorology, defense, and military services.

(vii) Modern medical practice relies on precise diagnostics and monitoring electronic systems. For better times, and at times for worse, electronics has shaped our lives. Our living standards have improved, but we are also at the peril of deadly weapons that would not have appeared without the progress in electronics.

Energy Bands in Solids

The allowable discrete values of n show that all energies are not permitted for the electrons. The electrons can have only certain discrete energies corresponding to the different integral values of n. In other words, the electron energy is quantized. The allowable energies are represented by horizontal lines in a diagram, called the energy level diagram of the atom.



When an electron jumps from a higher energy level E_1 to a lower energy level E_h , an electromagnetic radiation of frequency is emitted, where

\[v={\rm{}}{{{E_h}-{\rm{}}{E_l}}\over h}\] ………….(2)

h is Being Planck's constant.

On the contrary, on absorbing a photon of energy hv, an electron initially at the energy state E_1 can move to the energy state hv .

An electron normally occupies the lowest energy state, called the ground level, in the atom. The other higher lying energy levels are called the excited levels of the atom. By absorbing more and more energies, an electron can move into excited states which are farther and farther away from the nucleus. If the energy is sufficiently high, the electron can overcome the attraction of the nucleus and gets detached from the atom. The energy required for this to occur is known as the ionization potential. The energy level corresponding to in n = ∞ **See Above Fig.** represents the ionization level. As the electrons are electrostatically attracted by the positively charged nucleus, the allowed energies for the electrons are negative. The ionization level represents the zero level of energy. The energies become more and more negative with decreasing values of n .

The wavelengths emitted from the atom due to electronic transitions from higher energy states to lower ones give the spectral lines characterizing the atom. To explain the details of the spectral lines of some elements, improvements over the simple Bohr model have been made. Quantum mechanical treatments have introduced four quantum numbers, designated by n, l , m_l and m_s . The quantum numbers can take the following values:

n, = 1, 2, 3...

l = 0, 1, 2,... (n – 1)

m_l = 0, ± 1, ± 2,...,± l

m_s = ±1/2

The quantum number n, called the principal quantum number, primarily determines the energy of the orbital electrons. The quantum number l measures the angular momentum of the electron, and is referred to as the orbital angular momentum quantum number. The number ml gives the splitting of the energies for a given n and l in a magnetic field, and is called the magnetic quantum number. The quantity ms, known as the spin quantum number, shows that the spin of the electron about its own axis is quantized.

The state of an electron in the atom is uniquely specified by the four quantum numbers. This is a result of Pauli's exclusion principle. The principle states that no two electrons in an electronic system can occupy the same quantum state described by the same set of four quantum numbers n , l , m_l and m_s.
Electronic Shells

The specific value of the principal quantum number n determines an electronic shell. All the electrons of a given atom having the same value of n belong to the same electronic shell. L diagram of the letters K, L, M, N, ... denote the shells for n = 1, 2, 3, 4, ... , respectively. The different values of l for a given n define the subshells for the shell. The subshells are represented by s, p, d, f, ... corresponding to l = 0, 1, 2, 3, ... , respectively.

For n= 1, there are two states corresponding to l = 0, m_l= 0, and ms = ± 1/2. These two states are known as 1 states. For n = 2, we have two 2s states corresponding to l= 0, = 0, and m_s ± 1/2 . In addition, there are six states for n = 2, l= 1, m_l = –1, 0, + 1, and m_s = ± 1/2.

These are 2p states. The total number of electrons in the K shell (n = 1) is thus 2, and that in the L shell (n = 2) is 2 + 6 = 8. Similarly, it can be shown that a maximum of 10 electrons can be accommodated in a d subshell, a maximum of 14 electrons in an f subshell, and so on. The number of protons in the nucleus is the atomic number Z. Since the atom is electrically neutral, the number of orbiting electrons is also Z. For sodium, Z = 11. The electronic configuration of the sodium atom is $1s^2\,2s^2\,2p^2\,3s^1$, where the superscripts denote the number of electrons in a particular subshell. Clearly, the sodium atom has one electron in the outermost subshell.

Energy Bands in Crystals

A crystal is a solid consisting of a regular and repetitive arrangement of atoms or molecules (strictly speaking, ions) in space. If the positions of the atoms in the crystal are represented by points, called lattice points, we get a crystal lattice. The distance between the atoms in a crystal is fixed and is termed the lattice constant of the crystal.

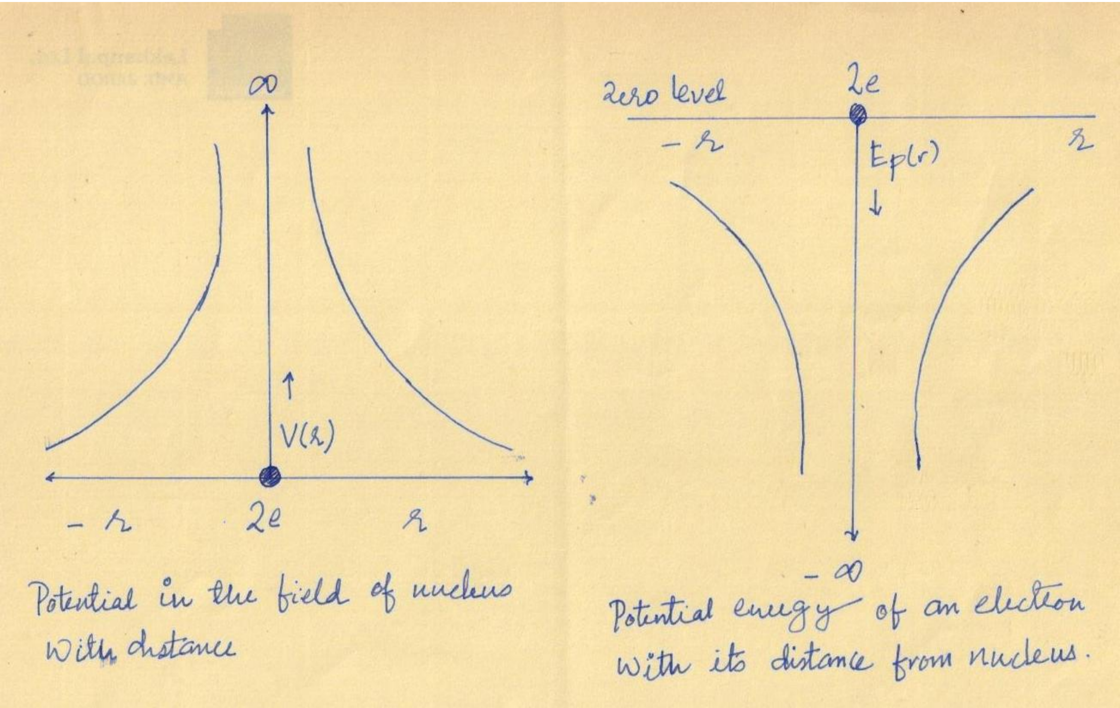
To discuss the behavior of electrons in crystal, we consider an isolated atom of the crystal. If Z is the atomic number, the atomic nucleus has a positive charge Ze. At a distance r from the nucleus, the electrostatic potential due to the nuclear charge is (in SI units)

$$V_{r}={{Ze}\over {4\pi {\varepsilon _o}r}} \quad \ldots\ldots\ldots\ldots(3)$$

Where is the permittivity of free space. Since an electron carries a negative charge, the potential energy of an electron at a distance from the nucleus is

$$E_p\left(r\right)=eV\left(r\right)={{Z{e^2}}\over{4\pi{\varepsilon_o}r}} \quad \ldots\ldots\ldots\ldots(4)$$

is positive while is negative. Both and are zero at an infinite distance from the nucleus.

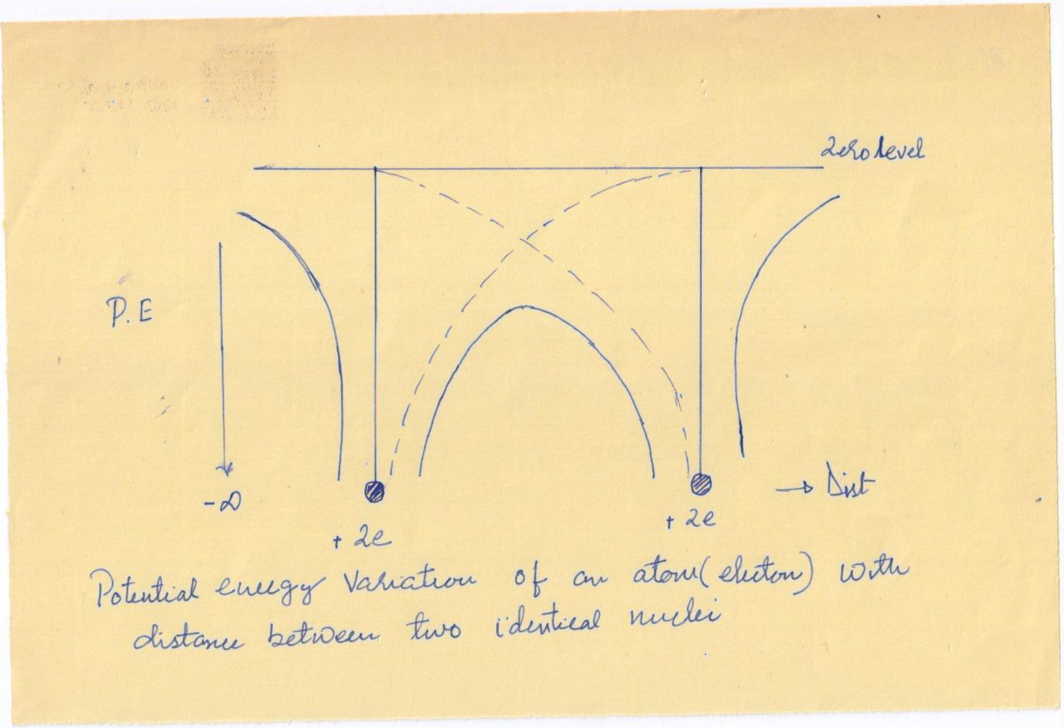


Potential in the field of nucleus with distance

Potential energy of an electron with its distance from nucleus.

Fig show the variation of V_r and $E_p(r)$, respectively with r .

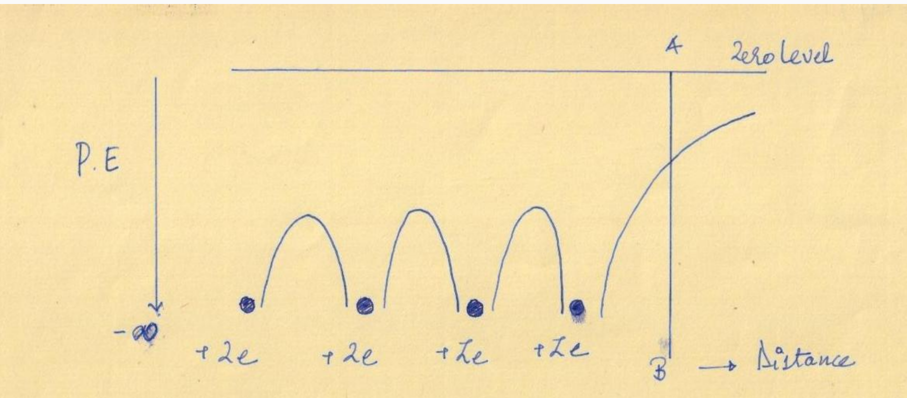
We now consider two identical atoms placed close together. The net potential energy of an electron is obtained as the sum of the potential energies due to the two individual nuclei. In the region between the two nuclei, the net potential energy is clearly smaller than the potential energy for an isolated nucleus.



Potential energy Variation of an atom(electron) with distance between two identical nuclei

The potential energy along a line through a row of equispaced atomic nuclei, as

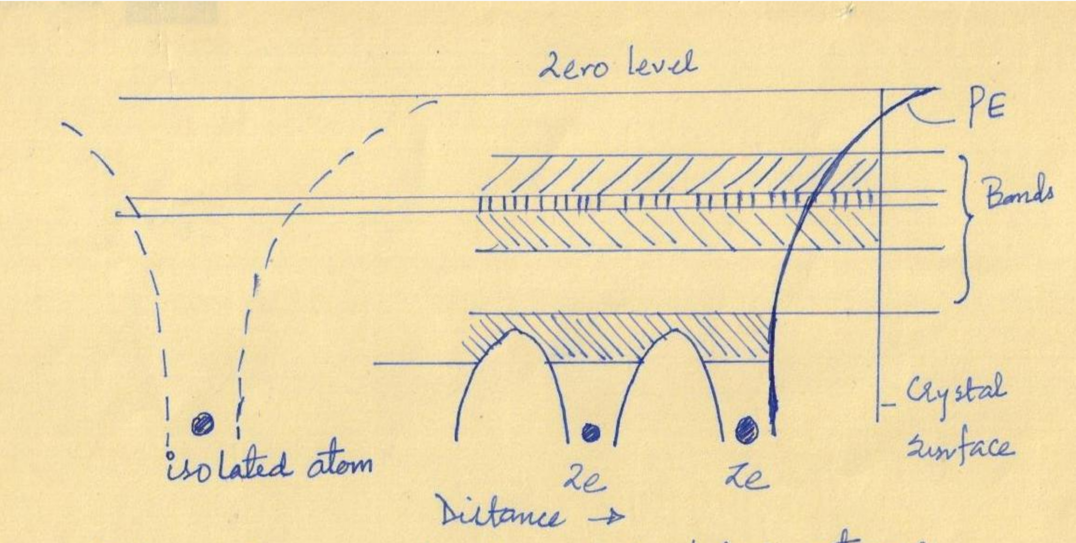
in a crystal, is diagrammatically shown in



Potential Energy of an electron along a rows of atoms in a crystal

The potential energy between the nuclei is found to consist of a series of humps. At the boundary AB of the solid, the potential energy increases and approaches zero at infinity, there being no atoms on the other side of the boundary to bring the curve down.

The total energy of an electron in an atom, kinetic plus potential, is negative and have discrete values. These discrete energy levels in an isolated atom are shown by horizontal lines in



Spliting of energy levels of isolated atoms into energy bands as these atoms are brought close to-gether to form a crystal.

When a number of atoms are brought close together to form a crystal, each atom will exert an electric force on its neighbors. As a result of this interatomic coupling, the crystal forms a single electronic system obeying Pauli's exclusion principle. Therefore, each energy level of the isolated atom splits into as many energy levels as there are atoms in the crystal, so that Pauli's exclusion principle is satisfied. The separation between the split-off energy levels is very small. This large number of discrete and closely spaced energy levels forms an energy band. Energy bands are represented schematically by shaded regions in **See Above Fig**.The width of an energy band is determined by the parent energy level of the isolated atom and the atomic spacing in the crystal. The lower energy levels are not greatly affected by the interaction among the neighboring atoms, and hence form narrow bands. The higher energy levels are greatly affected by the interatomic interactions and produce wide bands. The interatomic spacing, although fixed for a given crystal, is different for different crystals. The width of an energy band thus depends on the type of the crystal, and is larger for a crystal with a small interatomic spacing. The width of a band is independent of the number of atoms in the crystal, but the number of energy levels in a band is equal to the number of atoms in the solid. Consequently, as the number of atoms in the crystal increases, the separation between the energy levels in a band decreases. As the crystal contains a large number of atoms (), the spacing between the discrete levels in a band is so small that the band can be treated as continuous.

The lower energy bands are normally completely filled by the electrons since the electrons always tend to occupy the lowest available energy states. The higher energy bands may be completely empty or may be partly filled by the electrons. Pauli's exclusion principle restricts the number of electrons that a band can accommodate. A partly filled band appears when a partly filled energy level produces an energy band or when a totally filled band and a totally empty band overlap.

As the allowed energy levels of a single atom expand into energy bands in a crystal, the electrons in a crystal cannot have energies in the region between two successive bands. In other words, the energy bands are separated by gaps of forbidden energy.

The average energy of the electrons in the highest occupied band is usually much less than the zero level marked in See **See Above Fig.**. The rise of the potential energy near the surface of the crystal, as shown in **See Above Fig.** serves as a barrier preventing the electrons from escaping from the crystal. If sufficient energy is imparted to the electrons by external means, they can overcome the surface potential energy barrier, and come out of the crystal surface.

LESSON 17. Effective Mass and Mathematical Derivation

Effective Mass

- We know that an electron has a well defined mass, and it obey Newtonian Mechanics when accelerated by an electric field E.

- Now in the situation of an electron moving inside a crystal i.e. in initial state k, where it is not free to move, the question arises as to what the mass of an electron inside crystal is under accelerated motion and what is the effect of an electric field?

- It is found that the mass of and electron in a crystal appears, in general, different from the free electron mass, and is usually referred to as the effective mass.

- The velocity of the electron in a one-dimensional lattice is described by its group velocity given by

$v={{2\pi } \over h}{{dE} \over {dk}}$……………….(1)

and acceleration is given by

$$a={{dv} \over {dt}}=\left({{{2\pi } \over h}} \right){d \over {dt}}\left({{{dE} \over {dk}}} \right)$$

$a=\left({{{2\pi } \over h}} \right)\left({{{{d^2}E} \over {d{k^2}}}} \right){{dk} \over {dt}}$………………..(2)

Now E and k relation,

We can obtain $ {{{d^2}E} \over {d{k^2}}}$. Now under the influence of an applied electric field E we have to fiend the value of ${{dk} \over {dt}}$.

Consider that an electron is subjected to an external electric field E for a time dt

If the velocity of the electron is v and the distance travelled in time dt is v dt

$dE=eEvdt$……………….(4)

Substituting the value of equation (1) in equation (4)

$dE=eE{{2\pi } \over h}{{dE} \over {dk}}dt$………..(5)

${{dk} \over {dt}}$ = ${{2\pi eE} \over h}$................(6)

Substitute of equation (6) in equation (2)

$$a=\left({{{2\pi } \over h}} \right)\left({{{{d^2}E} \over {d{k^2}}}} \right){{2\pi eE} \over h}$$

Hence

$$a=\left({{4{\pi ^2}} \over {{h^2}}} \right)eE\left({{{d^2}E} \over {d{k^2}}} \right)$$……….(7)

Comparing equation (7) with that for a free, classical particle

F = ma, $F=m{{dv} \over {dt}}$ and F = eE

Hence

ma = $m{{dv} \over {dt}}$ = eE

$a={{eE} \over m}$ ……….(8)

From eqn(7) and (8)

$${{eE} \over m} = \left({{4{\pi ^2}} \over {{h^2}}} \right)eE\left({{{d^2}E} \over {d{k^2}}} \right)$$

Therefore the effective mass is given by

$$m^* = \frac{h^2}{4\pi^2\left(\frac{d^2E}{dk^2}\right)}$$

Hence,

It indicates the effective mass of an electron moving through the crystal lattice is determine by ${{{d^2}E} \over {d{k^2}}}$

For free electron m* = m because

$E={{{\hbar ^2}{k^2}} \over {2m}}$ and

${{{d^2}E} \over {d{k^2}}} = {{{\hbar ^2}} \over m}$

This result is expected, but for an electron moving in a periodic potential, E dose not vary with κ in the above manner, and so m* ≠ m. Thus we conclude that all the results of the free electron theory are correct provided m in each case is replaced by a suitable m* .

Therefore, $E={{{\hbar ^2}{k^2}} \over {2{m^*}}}$ is known as effective mass approximation.

LESSON 18. Band Theory of Solid

Band theory of solids

In an atom, the electrons in inner shells are tightly bound to the nucleus while the electrons in the outermost shell (i.e the valance electron) are loosely bound to the nucleus.

During the formation of a solid, a large number of atoms are brought very close together; the energy levels of these valence electrons are affected most.

The energies of inner shell electrons are not affected much.

The band formed by a series of energy levels containing the valence electrons is called the **Valence Band (VB)**.

- It is highest occupied energy band.
- It may be completely filled or partially filled with electrons.

The next higher permitted energy band is called the **Conduction Band (CB)**.

- It may also be defined as the lowest unfilled permitted energy band.
- It may be empty or partially filled with electrons.
- The electrons can move freely in the conduction band and hence the electrons in conduction band are called conduction electrons.

The energy gap between the **VB** and **CB** is called the **Forbidden Energy Gap or Forbidden Band**.

- This band is formed by a series of non-permitted energy levels above the top of valence band and below the bottom of the conduction band.
- It is denoted by E_g and is the amount of energy to be supplied to the electron in **VB** to get excited into the **CB**.
- When an electron gains sufficient energy, it is ejected from the valence band. Because of this, a covalent bond is broken and a vacancy for electron, called Hole, is generated.
- It is supposed to behave as a positive charge.
- This Hole can travel to the adjacent atom by acquiring an electron from an atom.
- When an electron is captured by a Hole, the covalent bond is again re-established.
- Thus conduction electrons are found in and above freely in the conduction band. The Holes exist in and move in the valence band.

Distinction between Metal, Insulator and Semiconductor

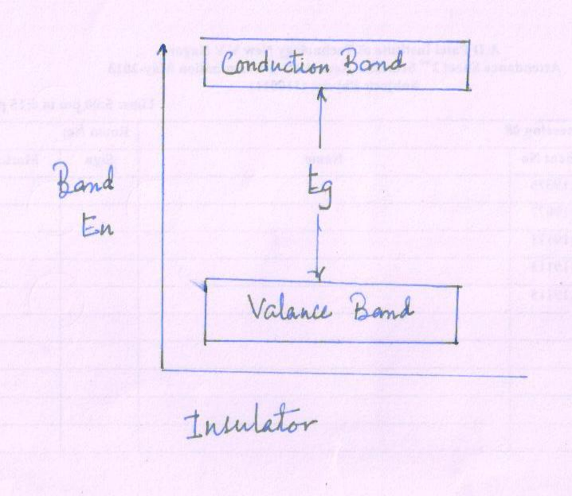
- There are many energy bands in solids as there are energy levels in parent atom.
- The electrical conduction properties of solids are concerned, only the valance band and conduction band energies of the electron are considered.
- Completely filled bands and completely empty bands do not contribute to electrical conduction.
- The electrons in the innermost filled shell do not take part in conduction process.
- The electrons in conduction band are free and move easily under an electric field.
- The electrons in valance band are attached to the lattice and are not free to move
- If they acquire sufficient energy to cross the forbidden gap E_g, they occupy the conduction band states are available for conduction.
- On the basis of the band theory, solids are classified into three categories;
    - Insulator
    - Semiconductor
    - Conductor

Insulator

These are those materials in which the valence electrons are bound very tightly to their parent atoms.

Hence very large electric field is needed to remove them from attraction of nuclei.

The band picture if insulator is shown in **Fig.**



Features of Insulator

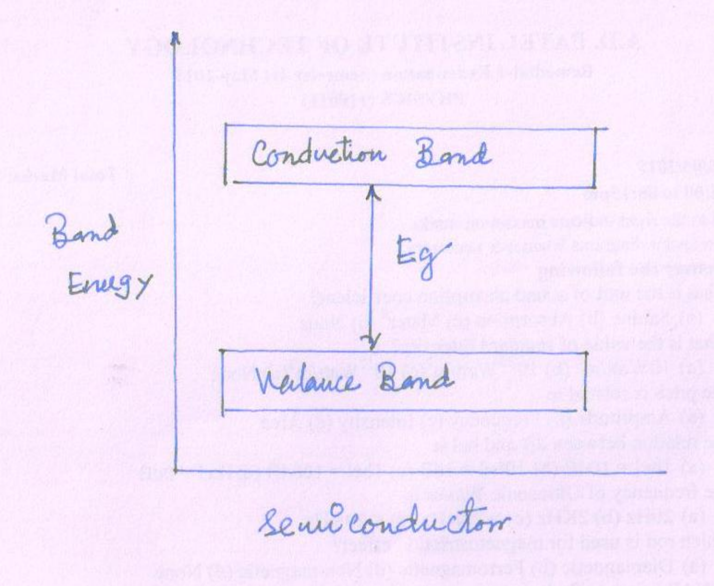
- A full valance band
- An empty conduction band
- A large band gap E_g; = 5-10 eV
- Very high resistivity of order of 10^9 Ω cm
- Some samples at high temperature and under sufficient electric field, some electrons may move from valance band to conduction band. Hence insulator shows small conductivity at high temperature.

e.g. diamond and glass

Semiconductors

A semiconducting material has electrical properties between those of insulators and good conductors.

The band diagram of semiconductor is shown in **Fig.**



Features of Semiconductors

- Almost empty conduction band and filled valance band with a very narrow energy gap $E_g = 1$ eV
- At 0K, the valance band is completely filled and the conduction band is empty.
- As temperature increases the electron in valance band acquire enough energy to cross the small energy gap and move to CB. Thus the conductivity of semiconductors increases with temperature.

- The electron moving to CB leave behind positive holes in VB . Hence, semiconductor current is the sum of electron and hole current flowing in opposite direction.

- The resistivity varies from to 10^{-12} to 10^{9} Ω cm

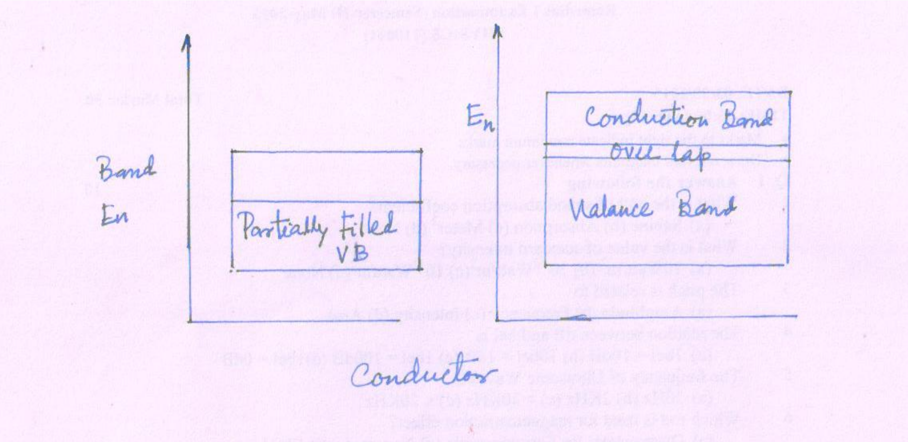
e.g. Ge = E_g = 0.7 eV and Si = E_g = 1.1 eV

Conductors

- The conductors have a large number of free electrons available for electrical conduction.

- Conductors may be defined as solids characterized by a single energy band called conduction band, which is partially filled at any temperature.

- Alternatively, a conductor as a solid, in which the conduction and valance bands overlap and there is no energy gap between the two bands



- As there is no forbidden gap, there is no structure to establish holes.
- The total current is simply due to the flow of electrons.

e.g. Lithium, Beryllium, Sodium

LESSON 19. Classification of Solids & Type of Semiconductor

Type of Semiconductor

It has been already discussed; a semiconductor is material whose electrical properties lie between those of insulator and conductor.

A semiconductor has almost filled valence band and almost empty conduction band with a very small energy gap between the two.

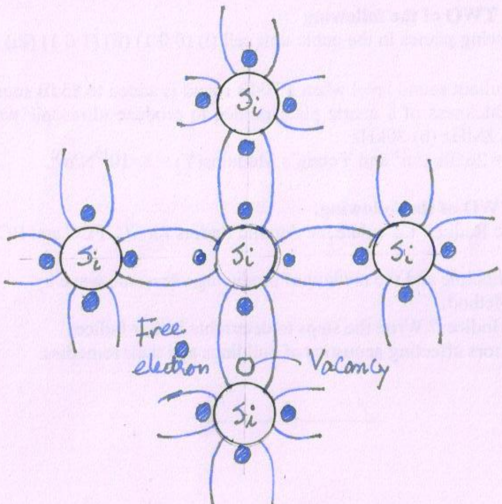
The semiconductors are of two types:

(i) Pure or Intrinsic Semiconductor

(ii) Doped or extrinsic semiconductor

Pure or Intrinsic Semiconductors

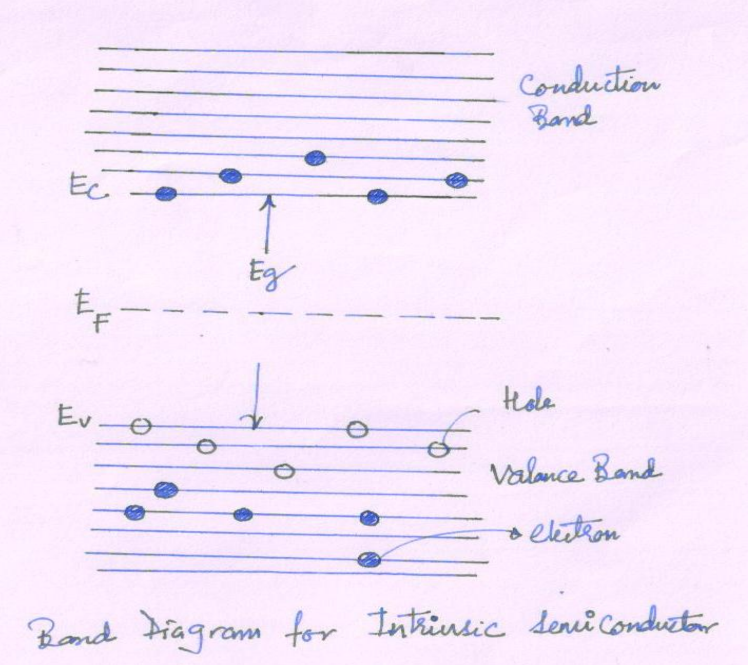
- Semiconductor like Ge and Si has crystalline structure. Their atoms are arranged in an ordered array known as the crystal lattice.
- These materials are tetravalent in nature; with four valence electrons in the outermost shell.
- The neighboring atoms form covalent bonds by sharing four electrons with each other so as achieved stable structure.
- A two dimensional structure of Ge crystal lattice is shown in **Fig.**



Intrinsic Semiconductor

- in which the circles represent atom cores consisting of the nuclei and inner 28 electrons.
- Each pair of lines represents a covalent bond.

- The dots represent the valence electrons.
- The intrinsic semiconductor is the material in its very pure form.
- The band diagram of pure semiconductor is shown in **Fig.**



Band Diagram for Intrinsic Semiconductor

- At temperature 0K all the electrons are paired and are contained in the completely filled valence band.
- No electron is free for conduction band. Hence the conduction band is completely empty.
- The energy gap between the two bands is small Eg= 0.72 eV for Ge
- At 0K the energies of this magnitude are usually not acquired by the applied field. Hence the valence band remains full, the conduction band empty and the material behaves as insulator.
- At room temperature, nearly 300K, some of the valence electrons acquire thermal energy greater than Eg and hence they can jump to the higher conduction band.
- They now act as free electrons and can move under the influence of small applied field.
- The absence of electron in the valence band is the Hole.
- Under the influence of electric field, an electron form the neighboring covalent bond may fill this vacancy i.e. hole moves in a direction opposite to electron motion in valence band.
- The Hole moves in the same direction as would a positive charge carrier. Hence, in semiconductors there are two types of charge carrier. The total current is the sum of the electrons and hole current.
- In the intrinsic semiconductor, electrons and holes are generated in pair.

- At any given temperature, the number of electrons in conduction band and number of holes in valance band are same.
- With the rise in temperature, more and more electron hole pairs are formed and more charge carriers are available for conduction. Hence the conductivity of intrinsic semiconductors increases with the rise in temperature.
- The intrinsic semiconductors have low conductivity.

Doped or Extrinsic Semiconductors

Most of the engineering applications of semiconductors involves doped semiconductors with specify impurities rather than the intrinsic material.

Doping is the process of adding a controlled quantity of impurity to intrinsic semiconductors, so as to increase its conductivity. A semiconductor doped with impurity atoms is called an extrinsic semiconductor.

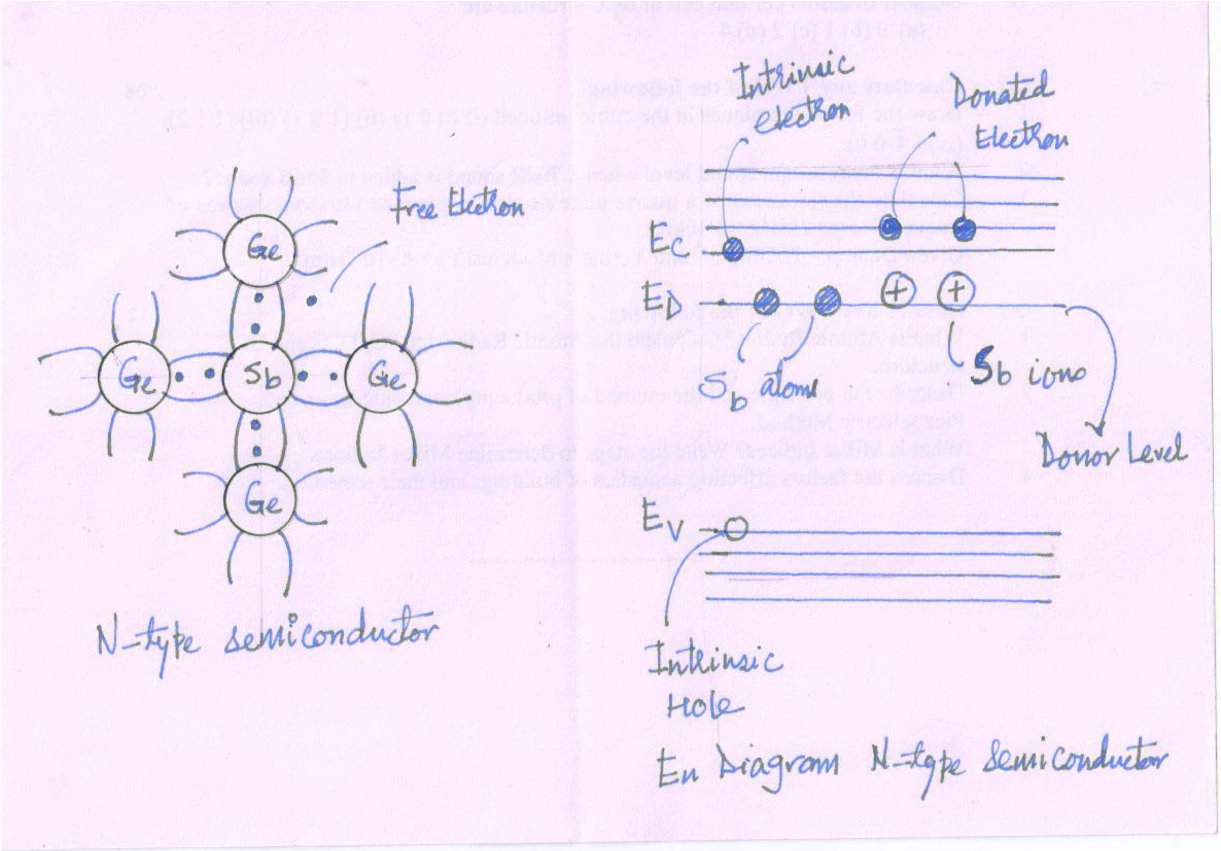
The impurity is added to the melt of Ge or Si , then the crystal is grown in which the impurities are incorporated.

The impurity atoms occupy lattice positions which were occupied by germanium atoms in pure metal.

Two types of extrinsic semiconductors are produced depending upon the group impurity atom.

N -Type of semiconductor

- Consider a crystal of Ge in which a small fraction of Ge atoms is replaced by Sb atoms. The Sb atom is pentavalent and it occupies normal Ge lattice site. It forms four covalent bonds with four neighboring Ge atoms. **Fig.**



N-type semiconductor

En Diagram N-type Semiconductor

- The fifth electron, not used in bonding, is loosely bound. With supply of little energy, it can be made free leaving behind a positively charged immobile ion.

- Thus the impurity atoms donate free electrons to the crystal thereby increasing the conductivity of material. Hence they are called the donor impurities.

- The conductivity is due to the negatively charged electrons. Hence, the material is called N-Type semiconductor.

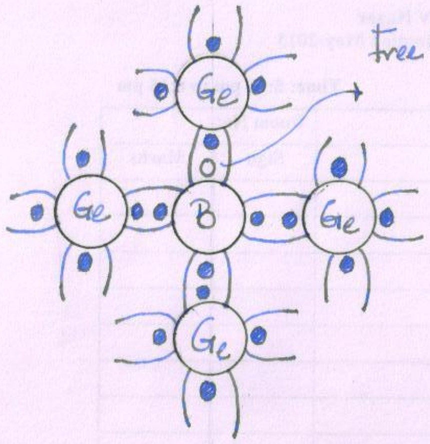
- The electron and hole pairs are generated in Ge due to thermal energy.

- For N-Type of semiconductor, the concentration of free electron is far greater than concentration of holes. The addition of donor impurity generates new energy levels in the band picture. The donor energy levels of neutral donor atoms lie very close to the lower edge of conduction band.

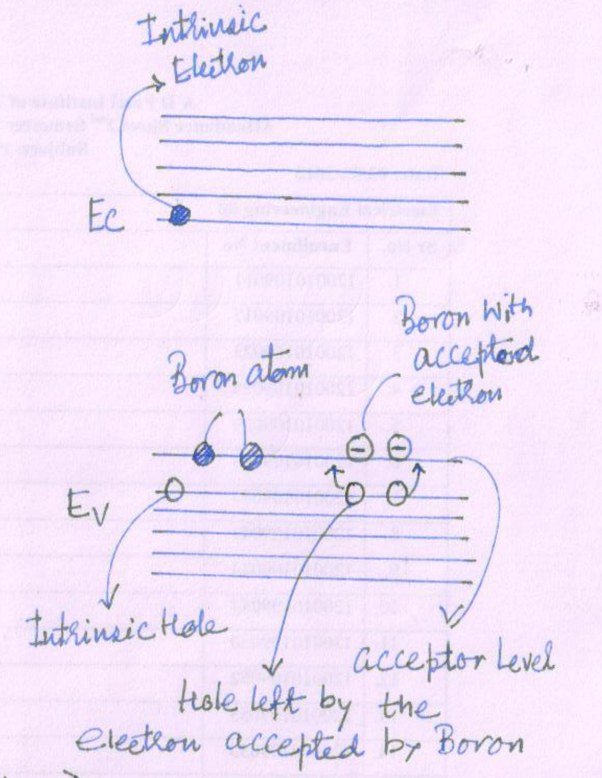
- With the supply of a little energy the neutral donor atom loses the fifth electron for conduction and it gets positively charged.

P-Type of semiconductors

- If the intrinsic semiconducting material, says Germanium, is doped by atoms of trivalent element like Boron, then the impurity atoms occupy places in the lattice which were previously occupied by Ge in pure crystal. **Fig.10(4)**



P-type Semiconductor



En Diagram - P-type Semiconductor

- Boron, is being trivalent, is one electron short of being able to complete the stable structure. The absence of electron in one of these bond is hole. With a small amount of energy, it can accept an electron from the neighboring germanium atom and the vacancy shifts there.

- The impurity atoms thus supply holes which are ready to accept electrons. Hence the impurity is known as acceptor impurity. The hole concentration is much more than electron concentration (thermally generated). The conductivity is due to the positively charged holes. Hence the semiconductor is called P-Type of semiconductor.

- The addition of impurity introduces additional acceptor energy level, in the band diagram, slightly above the top of the conduction band.

- With the supply of little energy, these vacancies can be occupied by electrons in valence band and thus increases the hole in valence band.

- It should be remembered that even though the conduction in P-Type and N-Type materials is due to holes and electrons respectively. It does not mean that P-Type material has a positive charge and N-Type material has a negative charge.

- The extrinsic semiconductors are electrically neutral at any given temperature.

LESSON 20. Law of Mass Action

Law of Mass action

We know that the concentration of electrons in conduction band is

\[{n_o}=2{\left({{{2\pi m_e^*{k_B}T} \over {{h^2}}}} \right)^{{3 \over 2}}}e{^{\left({{{{E_f} - {E_g}} \over {{K_B}T}}} \right)}}\] …………………(1)

Similarly, hole concentration in valance band is

\[{n_h} = 2{\left({{{2\pi m_h^*{k_B}T} \over {{h^2}}}} \right)^{{3 \over 2}}}e{^{\left({{{ - {E_f}} \over {{K_B}T}}} \right)}}\] ………………….(2)

Multiplying equation (1) and (2) we have the useful equation

\[{n_e}{n_h}\] = 4 \[{\left({{{2\pi m_e^*{k_B}T} \over {{h^2}}}} \right)^{{3 \over 2}}}e{^{\left({{{{E_f} - {E_g}} \over {{K_B}T}}} \right)}}\] \[{\left({{{2\pi m_h^*{k_B}T} \over {{h^2}}}} \right)^{{3 \over 2}}}e{^{\left({{{ - {E_f}} \over {{K_B}T}}} \right)}}\]………….. (3)

This is very useful relation since for the given semiconductor (i.e given effective masses and energy gap) at a given temperature the product of electron and hole concentration is constant.

We have assume that the semiconductor is intrinsic; the only assumption which has been is that the distance of the Fermi level from the edges of both the bands should be large in concentration with K_BT . Due to this consideration, the results of equation (3) will also apply to extrinsic semiconductor. Thus introducing an impurity in an intrinsic semiconductor to increase n , say will decrease ρ as the product must remain constant. Thus the equation (3) governs the relative concentrations of electrons and holes in a given semiconductor and is sometime called the law of mass action.

LESSON 21. Basic Concept of Superconductor

Superconductivity

It is a state of matter exhibited at very low temperatures where the resistivity of the material drops to zero and become perfect diamagnetic.

Super conducting state is influenced by temperature, current and magnetic field.

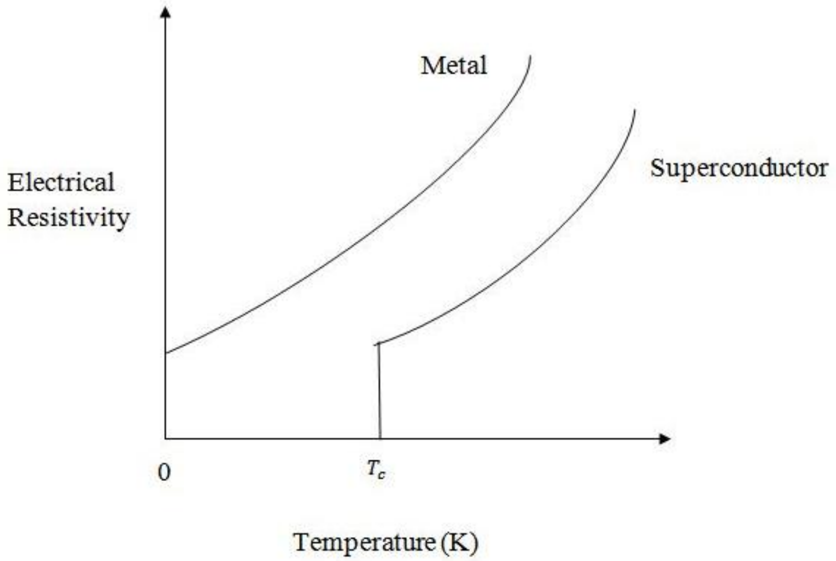
There exist critical values for these three parameters, above which the material passes into normal state.

It is designated for use in the field of medicine and particle physics. It is being tried for use in transportation and transmission of power.

Recently, the high temperature ceramic superconductors raised the hopes of using superconductors in making more efficient and smaller electrical and electronic devices at normal temperature.

Phenomena

- Metal are good conductors of electricity as they contain large number of free electrons. As a result low resistance offered by them to the flow current is attributed to the scattering of the free electrons by vibrating ions of the lattice. As temperature increases, the amplitude of the lattice vibrations increases and causes more scattering of electrons leading to more resistance.
- Even at 0 K (-273), metals offers finite resistance is called residual resistance. It is attributed to scattering of electrons by impurities and crystal defects present in the material.
- The variation of resistivity of normal metals with temperature is shown in **Fig.**



- It indicates the existence of residual resistance (ρr).
- H Onne was explained the behavior of metals at very low temperature.
- The electrical resistance of highly purified H_g dropped suddenly to zero at a temperature of 4.2K.
- This transition is reversible, when the metal heated above the transition temperature of 4.2K, the H_g regained its resistivity.
- H Onne named the phenomenon as superconductivity.
- Superconductivity was a strictly low temperature phenomena; some ceramic were found to exhibit superconductivity at high temperature of about 120K.
- It was discovered for Lead, Zink, Aluminum and other metals and number of alloys.
- Superconductor is the phenomenon in which the electrical resistance of materials suddenly disappears below a certain temperature.
- The materials that exhibit superconductivity and which are in the superconducting state are called superconductors.

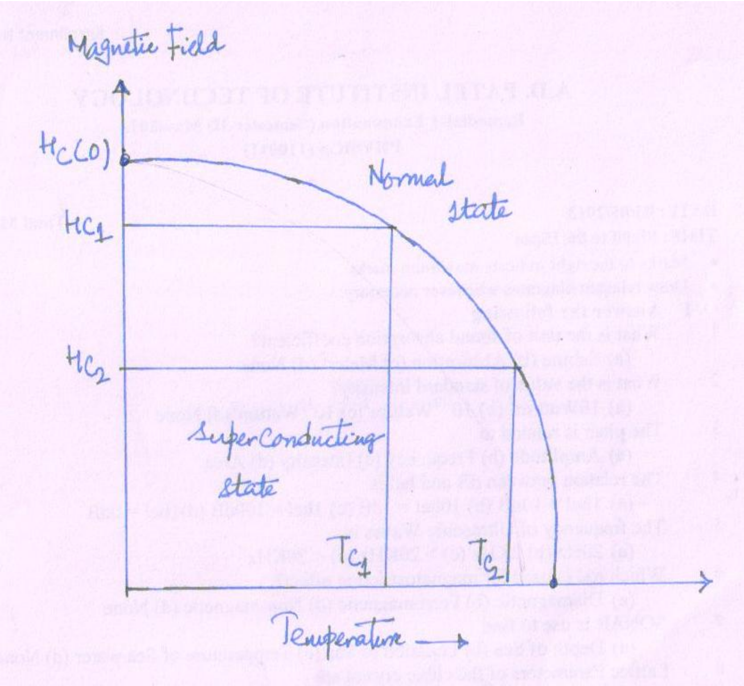
Transition Temperature

The temperature at which a normal material suddenly changes into a superconductor is called transition temperature T_c .

It is also called as critical temperature.

Critical Magnetic Field

- Superconducting state depends on the strength of the magnetic field in which the material is placed **Fig.**



- It is vanishes if a sufficiently strong magnetic field is applied.
- The minimum magnetic field, which is essential to regain the normal resistivity, is called critical magnetic field H_c .
- When the applied magnetic field exceeds the critical value H_c , the superconducting state is destroyed and the material goes into normal state.
- The value of H_c varies with the temperatures.
- It shows the dependence of H_c on temperature in a typical semiconductor.
- At temperature below T_c in absence of magnetic field, the material is in super conducting state.
- When magnetic field is applied and as its strength reaches the critical value material is in super conducting state.
- When magnetic field is applied and as its strength reaches the critical value H_c, the superconductivity in the material will be disappears.
- At any temperature $T < T_c$, the material remains superconducting until a corresponding critical magnetic field H_c is applied.
- When the magnetic field is exceed than critical magnetic field H_c, the material goes into normal state.
- The relation between magnetic field and temperature for the superconducting phenomena is

$$H_c\left(T \right) = H_c\left(0 \right) \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$H_c\left(T \right)$ - Critical magnetic field at temperature K

$H_c\left(0 \right)$ - Critical magnetic field at 0K

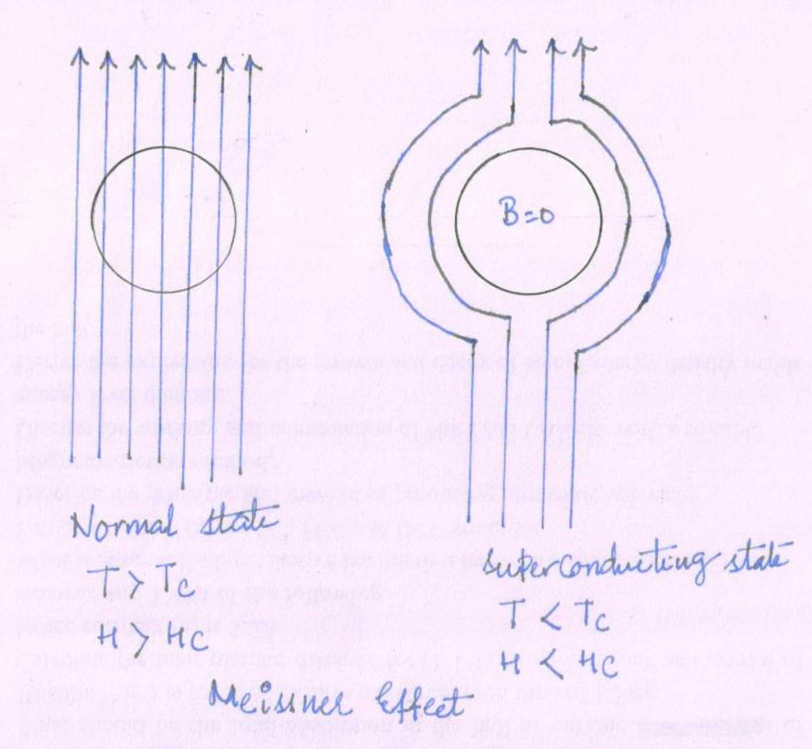
T - Temperature in K

T_c - Critical temperature at magnetic zero field

Meissner Effect

In 1933, W Hans Meissner found that when superconductors are cooled below the critical temperature in presence of a magnetic field, the magnetic flux is expelled from the interior of the specimen and a super conductor becomes a perfect diamagnetic in nature.

This phenomena is known as Meissner Effect **Fig.**



Meissner found that as the temperature of the specimen is lowered to T_c, the magnetic flex is suddenly and completely expelled from it.

The flux expulsion continues for $T < T_c$.

The effect is reversible, when the temperature is raised from below T_c, the flux suddenly penetrates the specimen at $T = T_c$ and material returns to the normal state.

The magnetic induction inside the sample is

$B = {\mu _o}\left({H + M} \right)$

H Applied magnetic field

M Magnetization produced in the sample

At $T < T_c$, $B = 0$

$0 = {\mu _o}\left({H + M} \right)$

$H = -M$

The susceptibility of the material

${M \over H} = \chi = -1$

The sample is diamagnetic and the state in which magnetization cancels the external magnetic field completely is referred to as perfect diamagnetic.

LESSON 22. Isotop Effect & Type of superconductor

Isotope Effect

In 1950, C A Reynolds and E M Maxwell found that the critical temperature decreases with increasing isotopic mass M

The variation is given by

$T_c \propto {M^{ - \alpha }}$

α is constant and is approximately 0.5.

The phenomenon of decrease of critical temperature with increasing atomic mass is called isotope effect.

Since the mean square of amplitude of atomic vibrations is proportional to $\sqrt M$ at low temperature.

The value of α can be expressed as

$\alpha = {{\partial \left({\ln{T_c}} \right)} \over {\partial \left({lnM} \right)}}$

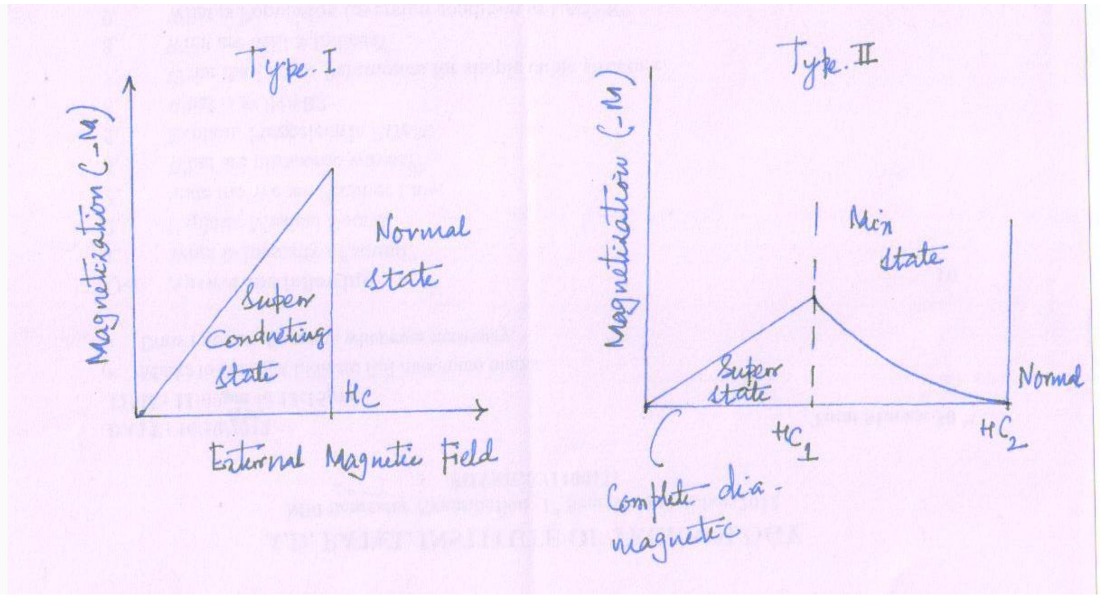
- A change in the isotopic mass does not change the electronic structure.
- Dependence of transition temperature on the isotopic mass suggests involvement of electron-lattice interaction.

Type I and II superconductor

Superconductors are divided into two types depending on the way of transition from superconducting to normal state proceed when the externally applied magnetic field exceeds.

Type-I

- The transition from superconducting state to normal state is the presence of magnetic field occurs sharply at the critical value of H_c. **Fig.13(1)**



- It is perfectly diamagnetic below H_c and completely expels the magnetic field from the interior of the superconducting phase.

- Up to the critical field strength, the magnetization of the material grows in proportion to the external field and then suddenly drops to zero at the transition to the normal state.

- It has only one critical field

- The magnetic field can penetrate only the surface layer and current can flow only in this layer.
- It is poor carriers of electric current.

- The critical field H_c is relatively low; it would be generate magnetic fields about 100 to 2000Gauss.

- It is not much used in production of high magnetic field.

- It has low critical field.

- It shows complete Meissner effect. Aluminum, Lead, Indium and Tin

Type – II

- It is discovered by Schubnikov in 1930.

- It is characterized by two critical fields H_{c1} and H_{c2}. **Fig.13(2)**

- It is hard superconductor.

- The transition from superconducting state to normal state occurs gradually as the magnetic field is increased from H_{c1} to H_{c2} .

- The magnetization of material grows in proportion to the external magnetic field up to the lower critical value H_{c1}.

- The external magnetic flux is expelled from the interior of the material till then.
- At H_{c1} the magnetic field lines begin penetrating the material.
- As a magnetic field increases, the magnetic flux through the material increases.
- At upper critical H_{c2}, the magnetization vanishes completely and the external field has completely penetrate and destroyed the superconductivity.
- Between H_{c1} and H_{c2}, the material is in a magnetically mixed state but electrically it is a superconductor.
- H_{c2} can be as high as 20-50 \[{{Wb} \over {{m^2}}}\] and the retention of superconductivity in such high magnetic fields make type II materials very useful in application of creating very high magnetic fields.
- Transition metals and alloys Niobeum, Silicon, Venedium

LESSON 23. Application of Superconductor

AC Josephson's and DC Josephson's

- In 1962, Brian Josephson suggested that cooper pair could tunnel through an insulating layer, which separates two super conductors.
- The superconductor- Insulator - Superconductor layer constitutes the Josephson junction.
- The insulating layer is of the order of 1 thickness.

He predicted that tunneling current can occur without any resistance, giving rise to direct current when the voltage applied across the junction is zero and an alternating current when the applied voltage is a dc voltage.

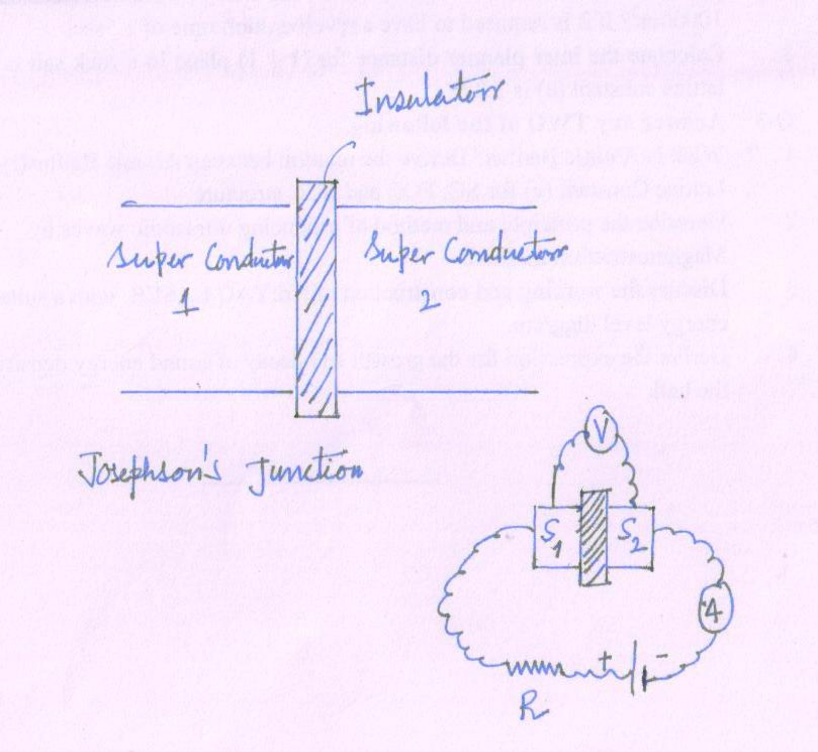


Fig.14(1)

DC Josephson Effect

- Two super conductors separated by thick insulator layer, say 10 thick, and are essentially two independent superconductors without any joint properties.

- When insulator layer is thin, say 1 nm thick, they become a system of coupled conductors. The cooper pairs tunnel through the barrier (insulating layer) as a single unit.
- The cooper pairs in a superconductor can be represented by a wave function, which is same for all pairs.
- The cooper pairs tunnel from one side of the junction to other side easily.
- The effect of the insulating layer is that it introduces a phase difference between the wave function of the pair on the other side. Because of this, supper current appears across the junction even through the applied voltage is zero. This is called dc Josephson Effect.

It is noticed by

$$I_s = I_c\sin \varphi_0$$

φ_0 is the phase difference between the wave function describing cooper pairs on the both side.

I_c is critical current at zero voltage; depending upon the thickness, width of the insulator and temperature.

AC Josephson Effect

Suppose a dc voltage is applied across the junction, it introduces a addition phase on cooper pairs during tunneling.

So, a strictly new phenomenon will be observed.

The dc voltage generates an alternating current

$I = I_c\sin \left(\varphi_0 + \Delta \varphi \right)$

Because of the dc voltage, the energy of the cooper pairs on both sides of the barrier differ by $2eV$.

Using quantum mechanical calculation;

$\Delta \varphi = 2\pi t\frac{2eV}{h}$

$I = I_c\sin \left(\varphi_0 + 2\pi t\frac{2eV}{h} \right)$

It is representing an alternating current which has frequency, $v = \frac{2eV}{h}$

Thus, a dc voltage is applied across the junction; an ac current is produced.

SQUID

A superconducting quantum interference device (SQUID) is a device used to measure extremely weak magnetic flux.

- It is basically a sensitive magnetometer, which can measure very weak magnetic fields of the order of $10^{-13}\frac{Wb}{m^2}$

- The sensitivity of a SQUID to magnetic fields can be increased by using a flux transformer.
- The heart of a SQUID is a superconducting ring, which contains one or more Josephson Junctions.

There are main two types of SQUID;

(i)DC SQUID:

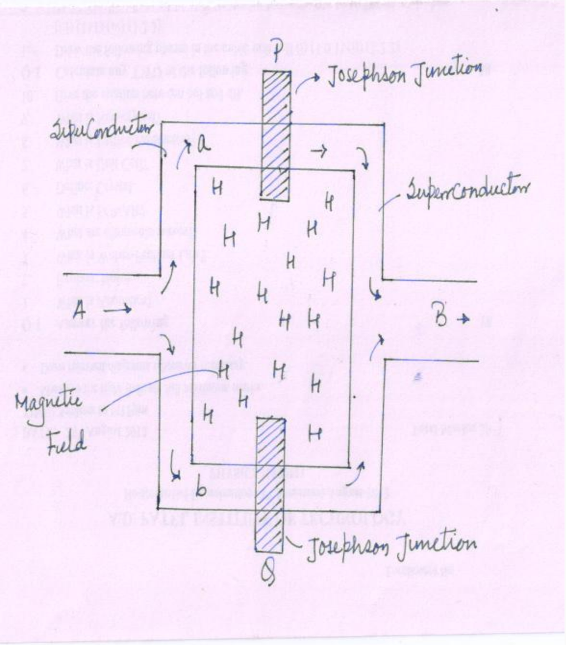
- It is made of two Josephson Junctions in parallel and relies on the interference of the current from each.
- It is more difficult and expensive to produce, but it is more sensitive.

(ii) AC or RF SQUID:

- It is made of one Josephson junction.

Fabrication

- SQUIDs are usually fabricated from Lead or pure Niobium.
- The Lead is taken in the form of allow with 10% Gold or Indium.
- A thin Niobium layer deposited on to it acts as the base electrode of the SQUID and tunnel barrier is oxidized onto this Niobium surface.
- The schematic of a two-junctions SQUID (DC) is shown in **Fig.14(2) and Fig.14(3)**



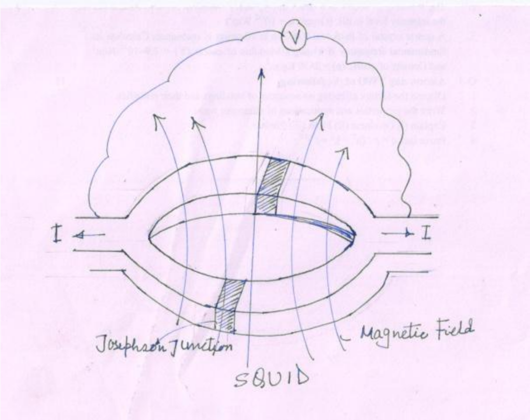


Fig.14(2) **Fig.14(3)**

It consists of two Joseph junctions arranged in parallel so that electrons tunneling through the junctions demonstrate quantum interference.

- The DC superconducting is applied to the SQUID. This current, called as bias current, enters into device through arm A.
- It is divided into two paths “a” and “b” and again recombines into one path. It leaves through the arm B.
- Here, P and Q are two Josephson junctions with different thickness of the insulators P and Q. The I_1 and I_2 are tunneling current through junctions P and Q respectively.
- In superconductor, the wave function experiences a phase shift at the junction P and Q.
- Suppose phase difference between points A and B taken on the path through junction P be δ_a and the phase difference between points A and B taken on the path through junction Q be δ_b .
- Hence, in absence of magnetic field these two phases are equal i.e phase difference is zero. $\delta_b - \delta_a = 0$
- Now, the magnetic field is applied perpendicular to the loop,

The flux passes through the loop, and changes the quantum mechanical phase difference across each of the two junctions. The wave functions at the two Josephson junctions interfere with each other. The super current flows along the paths a and b interfere. Hence, the device is called SQUID.

In super conductor, the waves are de Broglie wave of cooper pair and phase difference is caused by the applied magnetic field. So, according the Josephson’s theory the phase difference between re collected current is directly proportional to the magnetic flux (Φ_0) through ring

Total current $I_L = 2 (I_0 \sin \delta_0) \cos \frac{\Phi}{\hbar c}$

It indicates a progressive increase or decrease of the magnetic flux causes the current to oscillate between a maximum and a minimum value. Maxima in the current occur whenever the magnetic flux increases by one flux.

In practice, instead of the current, we measure the voltage across the SQUID which also oscillates with the changing magnetic field. Thus, the SQUID is a flux to voltage transducer which is converting a tiny change in magnetic flux into voltage. The flux (ϕ) is related to the magnetic field *B*. Hence, ϕ = BA

is Area of the ring.

Application of SQUID

- SQUIDs are use to measure very small magnetic field ($10^{-13}\frac{Wb}{m^2}$). Hence it acts as a very sensitive magnetometer.
- In geology, it is used to measure rock magnetism and conditional drift.

- It is also used in non-destructive test, testing of corrosion of aluminum sheet riveted together in aircraft.
- The extreme sensitivity of SQUIDs makes them ideal for studies in biology. Magnetoencephalography (MEG), for example, uses measurements from an array of SQUIDs to make inferences about neural activity inside brains
- In the clinical environment SQUIDs are used in cardiology for magnetic field imaging (MFI), which detects the magnetic field of the heart for diagnosis and risk stratification
- Probably the most common commercial use of SQUIDs is in magnetic property measurement systems (MPMS)
- SQUIDs are being used as detectors to perform magnetic resonance imaging (MRI).
- A potential military application exists for use in anti-submarine warfare as a magnetic anomaly detector (MAD) fitted to maritime patrol aircraft.

LESSON 24. High Temperature SuperConductor (HTS)

Introduction to High T_c superconductors

Basic concept of High Temperature Superconductors (HTS)

- High-temperature superconductors are materials that behave as superconductors at unusually high temperatures.
- Symbolically, it is denoted by high- T_c or HTS.
- The first high- T_c superconductor was discovered in 1986 by the researchers Karl Müller and Johannes Bednorz.
- The “ordinary” or “metallic” superconductors usually have transition temperatures (temperatures below which they exhibit the superconductivity phenomena) below 30 K (-243.2°C).
- The HTS have been observed with transition temperatures as high as 138 K (-135°C).
- The compounds of Copper and Oxygen (so-called "cuprates") were believed to have HTS properties, and the term high-temperature superconductor was used interchangeably with cuprites superconductor for compounds such as Bismuth Strontium Calcium Copper Oxide (BSCCO) and Yttrium Barium Copper Oxide (YBCO).
- However, several iron-based compounds are now known to be superconducting at high temperatures.

History

The phenomenon of superconductivity was discovered by Kamerlingh Onnes in 1911, in metallic Mercury below 4 K (-269.15°C). For seventy-five years after that, researchers attempted to observe superconductivity at higher and higher temperatures.

In 970, superconductivity was observed in certain metal oxides at temperatures as high as 13 K (-260.2°C), which were much higher than those for elemental metals.

In 1987, K Alex Müller and J. Georg Bednorz were exploring a new class of ceramics for superconductivity. Bednorz encountered a Barium-doped compound of Lanthanum and Copper oxide whose resistance dropped down to zero at a temperature around 35 K (-238.2°C).

The superconductor with the highest transition temperature that has been confirmed by more than one independent research groups is mercury barium calcium copper oxide ($HgBa_2Ca_2Cu_3O_8$) at around 133 K.

Crystal structures of high-temperature ceramic superconductors

- The structure of high-T_c Copper oxide or cuprate superconductors are often closely related to perovskite structure [A perovskite structure is any material with the same type of crystal structure as Calcium Titanium Oxide ($CaTiO_3$)].
- One of the properties of the crystal structure of oxide superconductors is an alternating multi-layer of CuO_2 planes with superconductivity taking place between these layers.
- This structure causes a large anisotropy in normal conducting and superconducting properties, since electrical currents are carried by holes induced in the oxygen sites of the CuO_2 sheets.

$YBaCuO$ Superconductors **Fig 15(1)**

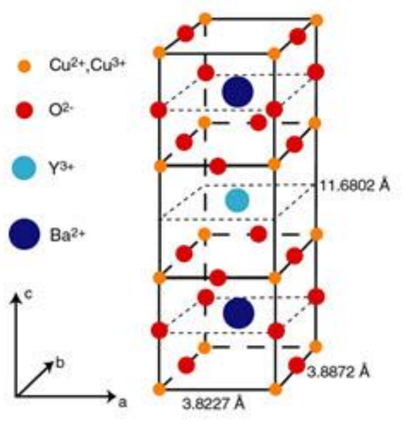


Fig.15 (1)

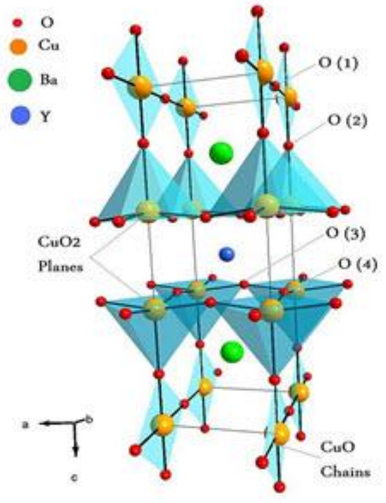


Fig.15 (1)

The first superconductor found with ${T_c}$ > 77 K (liquid nitrogen boiling point) is Yttrium Barium Copper Oxide ($YB{a_2}C{u_3}{O_{7 - x}}$)

The proportions of the 3 different metals in the $YB{a_2}C{u_3}{O_{7}}$ superconductor are in the mole ratio of 1:2:3 for yttrium to barium to copper, respectively. So it is referred to as the 123 superconductor.

The unit cell of $YB{a_2}C{u_3}{O_{7}}$ consists of three pseudocubic elementary perovskite unit cells.

Each perovskite unit cell contains a Y or Ba atom at the center: Ba in the bottom unit cell, Y in the middle one, and Ba in the top unit cell.

Thus, Y and Ba a are stacked in the sequence [Ba - Y - Ba] along the c-axis.

All corner sites of the unit cell are occupied by Cu, which has two different coordination, Cu(1) and Cu(2) , with respect to oxygen.

There are four possible crystallographic sites for oxygen: O(1) , O(2) , O(3) and O(4) . The coordination polyhedral of Y and Ba with respect to oxygen is different.

The tripling of the perovskite unit cell leads to nine oxygen atoms, whereas $YB{a_2}C{u_3}{O_7}$ has seven oxygen atoms and, therefore, is referred to as an oxygen-deficient perovskite structure. The structure has a stacking of different layers: $\left({CuO} \right)\left({BaO} \right)\left({Cu{O_2}} \right)\left(Y \right)\left({Cu{O_2}} \right)\left({BaO} \right)\left({CuO} \right)$.

One of the key features of the unit cell of $YB{a_2}C{u_3}{O_{7 - x}}$ is the presence of two layers of $Cu{O_2}$. The role of the Y plane is to serve as a space between two $Cu{O_2}$ planes.

In $YBCO$, the Cu - O chains are known to play an important role for superconductivity.

${T_c}$ is maximal near 92 K when x $\approx$ 0.15 and the structure is orthorhombic.

Superconductivity disappears at x $\approx$ 0.6, where the structural transformation of $YBCO$ occurs from orthorhombic to tetragonal.

${B_i}$ - , Tl - and ${H_g}$ - based high- ${T_c}$ superconductors

- The crystal structure of ${B_i}$ - , Tl - and ${H_g}$ - based high- ${T_c}$ superconductors are very similar. Like $YBCO$,
- It is perovskite-type feature and the presence of $Cu{O_2}$ layers also exists in these superconductors.
- However, unlike $YBCO$, - Cu - O chains are not present in these superconductors. The superconductor has an orthorhombic structure, whereas the other high- superconductors have a tetragonal structure.

The $B{i_2}$-Sr -Ca -Cu -O -system has three superconducting phases forming a homologous series as $B{i_2}S{r_2}C{a_{n - 1}}C{u_n}{O_{4 + 2n + x}}$ (*n* = 1, 2 and 3)

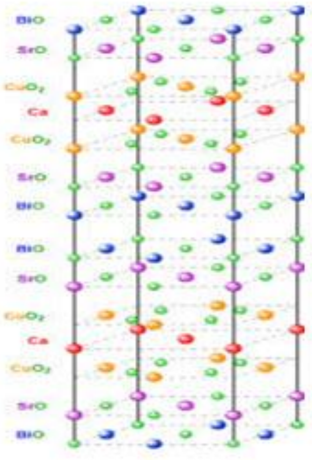


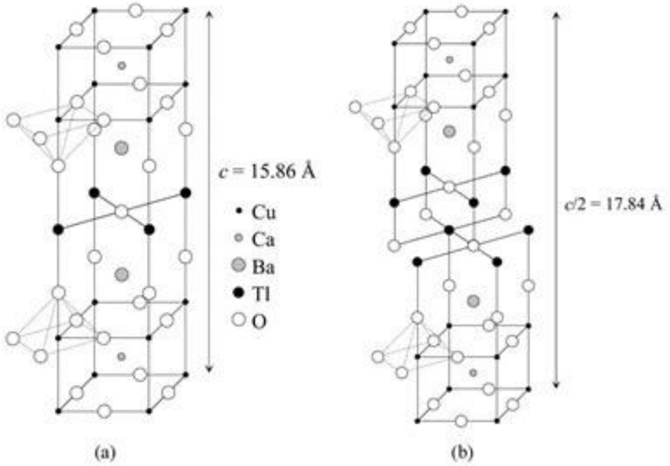
Fig.15

These three phases are \[Bi\]-2201, \[Bi\]-2212 and \[Bi\]-2223, having transition temperatures of 20, 85 and 110 K, respectively, where the numbering system represent number of atoms for \[Bi\], \[Si\] , \[Cu\] Ca and Cu respectively.

The two phases have a tetragonal structure which consists of two sheared crystallographic unit cells. The unit cell of these phases has double \[Bi - O\] planes which are stacked in a way that the Bi atom of one plane sits below the oxygen atom of the next consecutive plane. The Ca atom forms a layer within the interior of the \[Cu{O_2}\] layers in both \[Bi\]-2212 and \[Bi\]-2223; there is no \[Ca\] layer in the \[Bi\]-2201 phase. The three phases differ with each other in the number of \[Cu{O_2}\] planes; \[Bi\]-2201, \[Bi\]-2212 and \[Bi\]-2223 phases have one, two and three \[Cu{O_2}\] planes, respectively. The c axis of these phases increases with the number of \[Cu{O_2}\] planes (see table below). The coordination of the \[Cu\] atom is different in the three phases. The \[Cu\] atom forms an octahedral coordination with respect to oxygen atoms in the 2201 phase, whereas in 2212, the \[Cu\] atom is surrounded by five oxygen atoms in a pyramidal arrangement. In the 2223 structure, \[Cu\] have two co-ordinations with respect to oxygen: one \[Cu\] tom is bonded with four oxygen atoms in square planar configuration and another \[Cu\] atom is coordinated with five oxygen atoms in a pyramidal arrangement.

\[Tl - Ba - Ca - Cu - O\] Superconductor:

- The first series of the \[TL\] -based superconductor containing one \[TL\]–\[O\] layer has the general formula \[TlB{a_2}C{a_{n - 1}}C{u_n}{O_{2n + 3}}\] whereas the second series containing two \[TL\] –\[O\] layers has a formula of \[T{l_2}B{a_2}C{a_{n - 1}}C{u_n}{O_{2n + 4}}\] with *n* = 1, 2 and 3



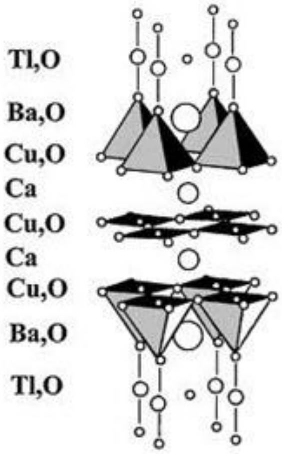


Fig.15 (3)

- In the structure of $T{l_2}B{a_2}Cu{O_6}$ (TL-2201), there is one $Cu{O_2}$ layer with the stacking sequence (TL–O) (TL–O) (Ba–O) (Cu–O) (Ba–O) (TL–O)(TL–O).

- In $T{l_2}B{a_2}CaC{u_2}{O_8}$ (TL-2212), there are two Cu–O layers with a Ca layer in between. Similar to the $T{l_2}B{a_2}Cu{O_6}$ structure, TL–O layers are present outside the Ba–O layers.

- In $T{l_2}B{a_2}C{a_2}C{u_3}{O_{10}}$ (TL-2223), there are three $Cu{O_2}$ layers enclosing Ca layers between each of these.

- In TL -based superconductors, ${T_c}$ is found to increase with the increase in $Cu{O_2}$ layers. However, the value of ${T_c}$ decreases after four layers in $TlB{a_2}C{a_{n - 1}}C{u_n}{O_{2n + 3}}$ and in the $TlB{a_2}C{a_{n - 1}}C{u_n}{O_{2n + 4}}$ compound, it decreases after three $Cu{O_2}$ layers.

$Hg - Ba - Ca - Cu - O$ Superconductor:

- The crystal structure of \[HgB{a_2}Cu{O_4}\] (\[Hg\] -1201), \[HgB{a_2}CaC{u_2}{O_6}\] (\[Hg\] -1212) and \[HgB{a_2}C{a_2}C{u_3}{O_8}\] (\[Hg\] -1223) is similar to that of \[TL\]-1201, \[TL\]-1212 and \[TL\]-1223, with in place of \[TL\] .

- It is noteworthy that the \[{T_c}\] of the Hg compound (\[Hg\]-1201) containing one \[Cu{O_2}\] layer is much larger as compared to the one- \[Cu{O_2}\] -layer compound of thallium (\[TL\]-1201).

- In the \[Hg\] -based superconductor, \[{T_c}\] is also found to increase as the \[Cu{O_2}\] layer increases.

- For \[Hg\]-1201, \[Hg\]-1212 and \[Hg\]-1223, the values of \[{T_c}\] are 94, 128 and the record value at ambient pressure 134 K, respectively.

Table-1 High temperature superconductors

Formula	Notation	Tc (K)	No. of Cu-O planes in unit cell	Crystal structure
\[YB{a_2}C{u_3}{O_7}\]	123	92	2	Orthorhombic
\[B{i_2}S{r_2}Cu{O_6}\]	Bi-2201	20	1	Tetragonal
\[B{i_2}S{r_2}CaC{u_2}{O_8}\]	Bi-2212	85	2	Tetragonal
\[B{i_2}S{r_2}C{a_2}C{u_3}{O_6}\]	Bi-2223	110	3	Tetragonal
\[T{l_2}B{a_2}Ca{O_6}\]	Tl-2201	80	1	Tetragonal
\[T{l_2}B{a_2}CaC{u_2}{O_8}\]	Tl-2212	108	2	Tetragonal
\[T{l_2}B{a_2}C{a_2}C{u_3}{O_{10}}\]	Tl-2223	125	3	Tetragonal
\[TlB{a_2}C{a_3}C{u_4}{O_{11}}\]	Tl-1234	122	4	Tetragonal
\[HgB{a_2}Cu{O_4}\]	Hg-1201	94	1	Tetragonal
\[HgB{a_2}CaC{u_2}{O_6}\]	Hg-1212	128	2	Tetragonal

$HgB{a_2}C{a_2}C{u_3}{O_8}$	Hg-1223	134	3	Tetragonal

Properties of High Temperature Superconductors.

- HTS are brittle in nature
- The properties of the normal state of these materials are highly anisotropic.
- The Hall Coefficient is positive indicating the charge carriers are hole.
- Their behavior can't be explained by BCS theory.
- The isotopic effect is almost absent in these materials.
- The magnetic properties of these materials are highly anisotropic.
- The effect of pressure is different on different materials e.g the application of pressure increases the critical temperature of $LBCO$ compounds but decreases the critical temperature of $YBCO$

Application of HTSC

Commercial quantities of HTSC wire based on $BSCCO$ are now available and applications include transformers, fault current limiters, power storage, motors and fusion reactors etc.

HTSC cables-

- Lower voltage at the same power rating
- Low impedance power guiding
- Lower life cycle costs
- No soil heating
- No electromagnetic spray fields.

Electrical Machines-

- High quality of generated power
- Improved voltage stability
- More reactive power is available at a given rating
- Low total harmonic distortion
- Low maintenance
- Size and Weight reduction

- High efficiency and low operating cost

Fault current limiters-

Over 100times faster response time

Time adjust response function

Full power protection cycles

Magnets-

- Higher operational temperature range up to 77 K
- Compact and high weight
- Higher magnetic field of the order of 100 Tesla
- Greater stability and thermal efficiency
- Less complex cryogenic system
- Higher signal to noise ratio

Transformers-

- HTS turns power transformer into compact, highly efficient and environment friendly performers.

RF and Microwave Filters-

Medical Diagnosis-

MODULE 6. Optics

LESSON 25. Basic concept of LASER

LASER

No other scientific discovery of the 20th century has been demonstrated with so many exciting applications as laser acronym for

Light Amplification by Stimulated Emission of Radiation.

The basic concepts of laser were first given by an American scientist- Charles Hard Townes and Soviet scientists- Alexander Mikhailovich Prokhorov and Nikolai Gennediyevich Basov

- Laser is a powerful source of light having extraordinary properties which are not found in the normal light sources like tungsten lamps, mercury lamps, etc. The unique property of laser is that its light waves travel very long distances with every little divergence.
- In case of a conventional source of light, the light is emitted in a jumble of separate waves that cancel each other at random

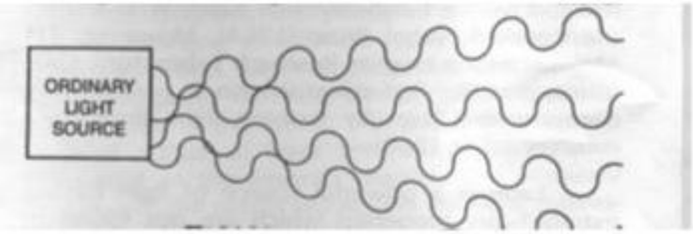


Fig.16 (1)

Fig.16(1) and hence can travel very short distances only.

- On the other hand, if a small stones are thrown into a pool one by one at the same place and also at constant intervals of time, the waves thus generated strengthen each other and travel long distances. In this case, the waves are said to travel coherently. In laser, the light waves are exactly in step with each other and thus have a fixed phase relationship

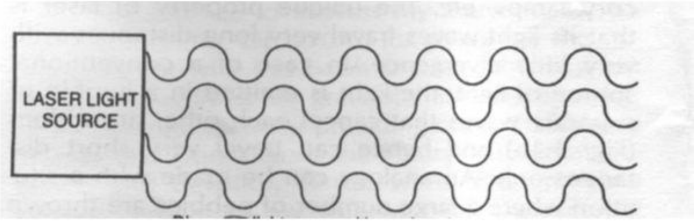


Fig.16 (2)

It is this coherency that makes all the difference to make the laser light so narrow, so powerful and so easy to focus on a given object. The light with such qualities is not found in nature.

Basic Concept of the LASER

Absorption

- Let the atom is in the lower energy state (E_1).
- Now, a photon of energy (E_1 - E_2) is incident on the atom at lower energy state.
- Photon gives its energy to the atom and disappeared.
- Then we can say that atom absorbed the incident photon.
- Due to sufficient absorption of energy by the atom, the atom jumps to the exited state (E_2) from lower energy state (E_1).

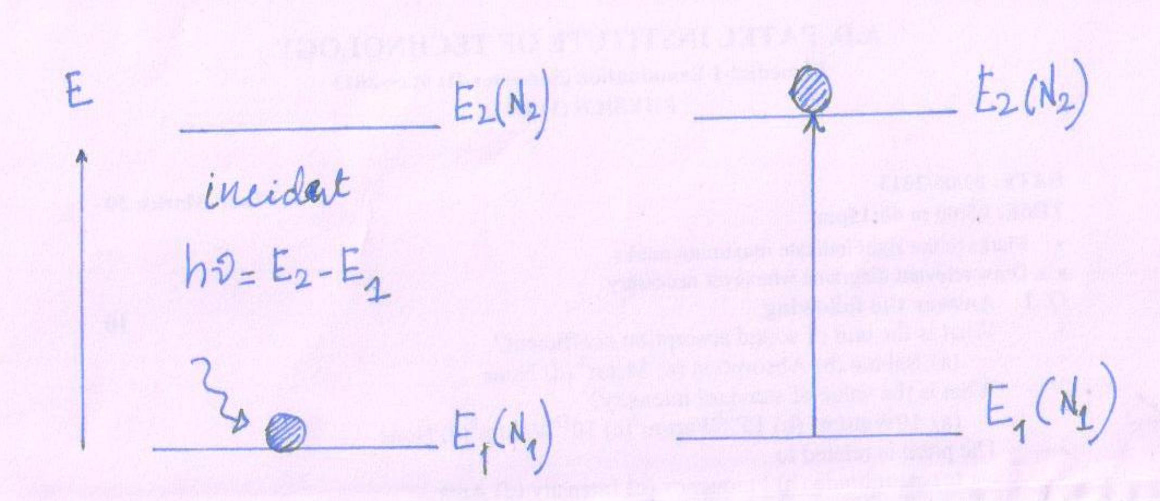


Fig16(3)

This is called absorption process and corresponding transition is called Absorption Transition.

Due to sufficient absorption of energy by each absorption transition at event an atom is excited and one photon is subtracted from the incident beam of light (bunch of incident photon)

So we can expressed the equation for absorption process

$A + hv = A^*$

Where, hv - incident photon energy

A - An atom in lower energy state (E_1)

A* - An atom in excited state (E_2)

Rate of absorption transition (R_{12}) occur is proportional to the incident photon density ρ(v) and number of atom are in lower energy state (E_1) at Δt

R_{12} \[\propto\] ρ(v)

R_{12} \[\propto\] N_1

Hence,

$R_{12} = B_{12}N_1\, \rho(v)\, \Delta t$.….(1)

R_{12} - Rate of absorption transition

N_1 - Number of atoms in lower energy state.

ρ(v) - Incident photon density.

B_{12} - Constant of proportionality; known as Einstein coefficient for induced absorption

Δt - Time duration for absorption (excitation) process.

Here, N_1 is the population (more numbers of atoms) of atom at E_1, when more number of atoms are at lower energy state(E_1), then more atoms may jump into excited state (E_2). Further, when more photons are incident on the system, then more atoms may get excited to the higher energy level.

Normally, number of atoms in higher energy state (N_2) can't be greater than number of atoms in lower energy state (N_1). But, induce absorption involves the excitation of atom to the definite higher energy level ($hv = \Delta E$). As a result,N_1 decreases and N_2 increases.

Spontaneous Emission

- After the completion of the absorption process (transition), the atom in the excited state (E_2) can't stay in the excited state for a relative long time.
- The time of about 10^{-8} sec, the atom returns to the lower energy state (E_1) by releasing the photon of energy which was gained during absorption process. i.e $hv = E_2 - E_1$
- The emission of photon occurs on itself and without any external energy (force) to the excited atom. Hence, emission of photon by an atom without any external energy is called spontaneous emission.

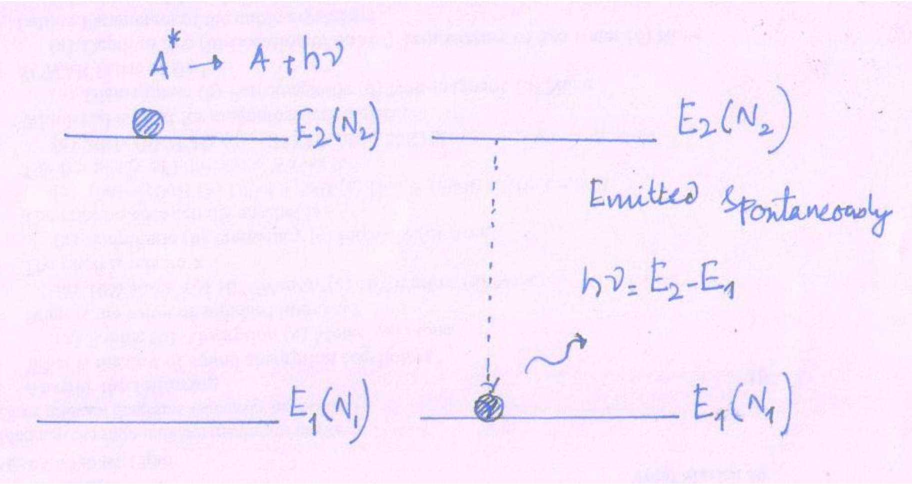


Fig16(4)

Expression for spontaneous process as

$A^* \rightarrow A + h\nu$

Due to absence of external energy; the rate of spontaneous (R_{21}) occurs depends upon the properties of energy states E_2 and E_1 only.

It is independent of photon density.

R_{21} \[\propto\] N2

$R_{21} = A_{21} N_2 \Delta t$ ……….. (2)

N_2- Number of atoms in excited state (E_2)

A_{21}- Constant of probability; Einstein coefficient for spontaneous emission.

Δt - Time duration for spontaneous transition.

It is a process of spontaneous emission that dominates (have a very strong influence over) in conventional light source.

Stimulated Emission

- Einstein reports to establish of equilibrium between matter and radiation required a new radiation process.
- It is called stimulated radiation process.

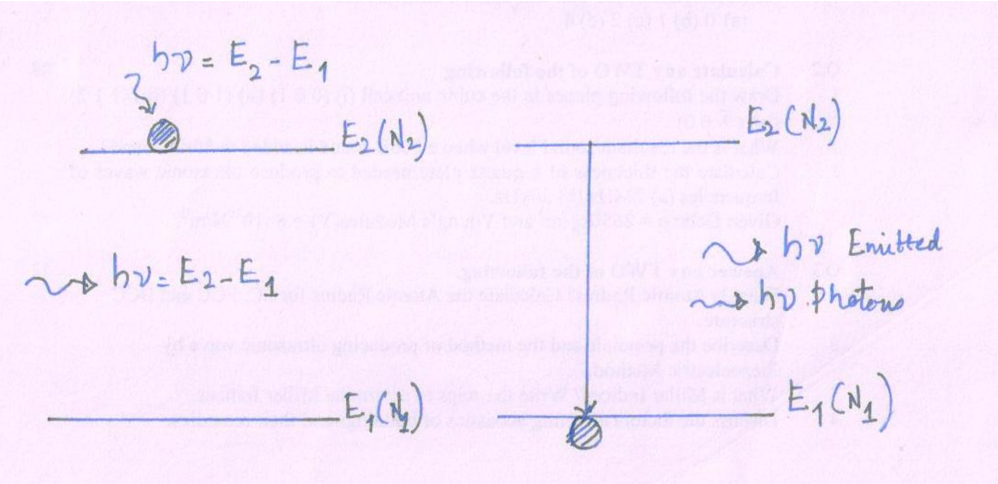


Fig16(5)

- It requires external radiation.

Let an atom in the excited state interact with a photon (external radiation) with energy $h\nu = E_2 - E_1$, an external photon induces (bring out) the excited atom to make a downward transition. But such downward transition well before the atom can make a spontaneous transition.

The atom emits the excess energy in the form of a photon; $h\nu = E_2 - E_1$ as it drops to lower energy state. Phenomenon of force photon emission by an external atom due to the role of external agency is called stimulated emission. The stimulated emission process may be expressed

$A^* + h\nu$ (external) $\rightarrow A + 2h\nu$

Rate of stimulated transition is proportional to external incident photon density $\rho(v)$ and the number of atom at energy level E_2 at Δt

R_{21} \[\propto\] $\rho(v)\ N_2\ \Delta t$

$\mathbf{R_{21} = B_{21}\ \rho(v)\ N_2\ \Delta t}$ …….. (3)

$\mathbf{B_{21}}$ - Constant of probability; known as Einstein coefficient for stimulated emission.

LESSON 26. Types of LASER

Steady State Equilibrium

Absorption transition, spontaneous transition and stimulated transition processes are happened simultaneously. However, under steady state condition the absorption process and emission process balance each other. Therefore, theoretically

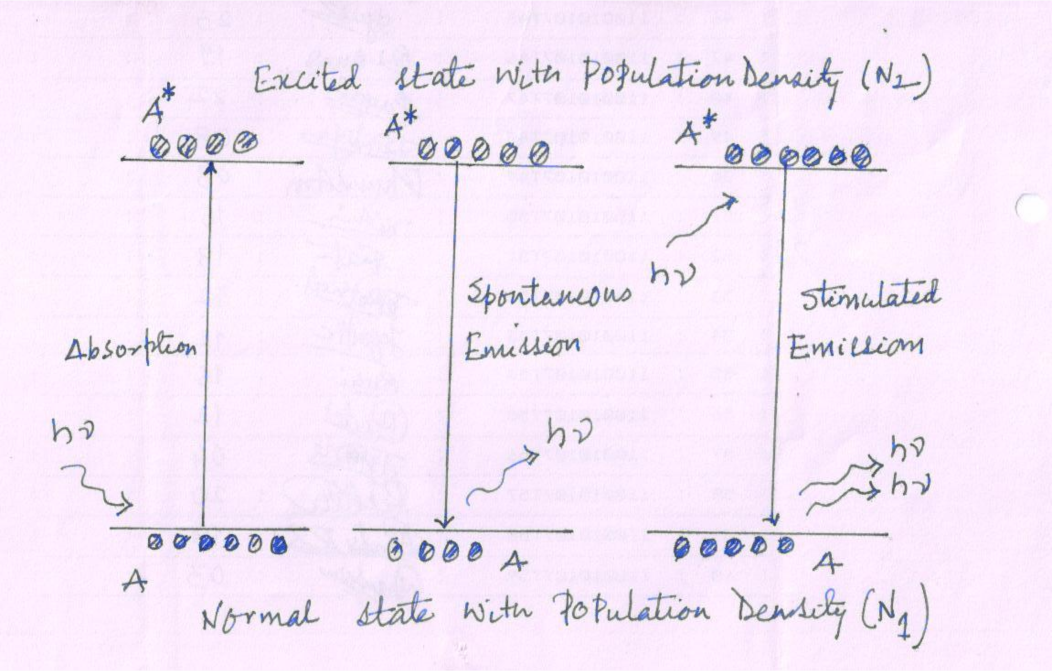


Fig17(1)

Rate of Absorption transition = Rate of spontaneous transition + Rate of stimulated transition

$\mathbf{R}_{12(\text{absortion})} = \mathbf{R}_{21(\text{spontaneous})} + \mathbf{R}_{21(\text{stimulated})}$

$B_{12}N_1\rho(\nu)\Delta t = A_{21} N_2 \Delta t + B_{21} \rho(\nu) N_2 \Delta t$

At thermal equilibrium condition,

Number of atoms in the lower energy state (N_1) are more than higher energy state (N_2)

Hence, $N_1 >> N_2$

Photon traveling through medium is more like to get absorbed compare to stimulate to an excited atom to emit a photon (stimulated emission). An atom that is at the excited state is more likely to jump the lower level on its own (spontaneous) way than (compare to) being stimulated by an external photon. Because of the external incident photon density is not sufficient to interact with the excited atom. So, the spontaneous emission dominates the stimulated emission. So, at steady state condition large population is available at lower energy state. As probability for absorption transition is equal to the probability for stimulated transition.

Population Inversion

At thermal equilibrium, population ratio is governed by the Boltzmann factor according to equation

$${{N_2} \over {N_1}} = {e^{ - {{h\nu } \over {kT}}}} \quad \text{OR} \quad {{N_1} \over {N_2}} = {e^{{{h\nu } \over {kT}}}}$$

$h\nu = E_2 - E_1$

N_2 - Number of atoms in the higher energy state

N_1 - Number of atom in lower energy state.

k - Boltzmann Factor

t – Temperature

It reports the population N_2 at the excited level E_2 will be for smaller than the population N_1 at the lower level E_1 hence; $N_1 >> N_2$

The condition in which there are more atoms in lower energy state and relatively lesser number of atoms in higher state is called normal state or state of equilibrium.

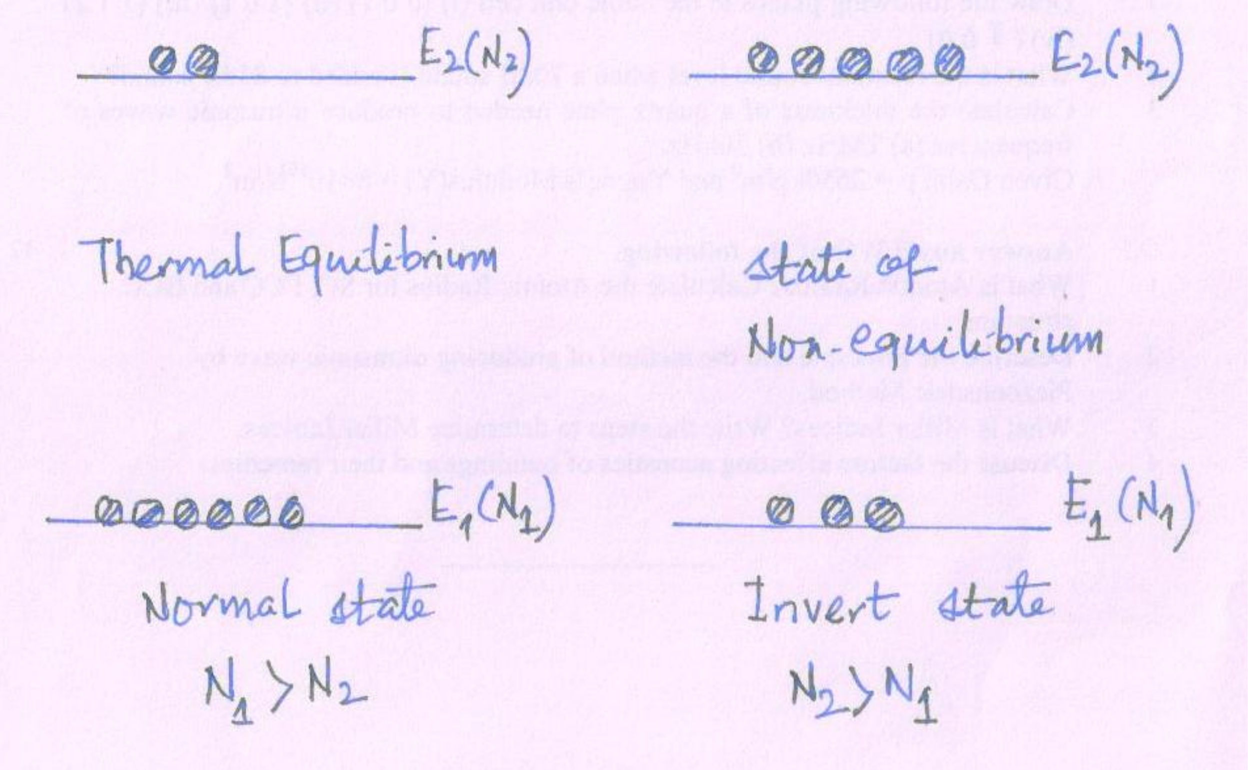


Fig17(2)

The condition of material in which the population of the upper energy level (N_2) far exceeds the population of the lower energy level (N_1).

In this situation, the population distribution between the level E_1 and E_2 is inverted and hence it is called as state the inverted state of non-equilibrium.

Invert state or non-equilibrium state only for a short time.

Population inversion is obtained by employing pumping technique, which transfer large number of atoms from lower energy state to higher energy

He-Ne Laser

- A Helium-Neon laser or He-Ne laser is a type of gas laser.
- The gain medium of He-Ne Laser consists of a mixture of helium and neon inside of a small bore capillary tube, usually excited by a DC electrical discharge.

Construction and operation

- The gain medium of the laser, a mixture of Helium and Neon gases, in approximately a 10:1 ratio, contained at low pressure in a glass envelope.
- The gas mixture is mostly helium, so that helium atoms can be excited and it collides with neon atoms, exciting some of them to the state that radiates 632.8nm .
- Without helium, the neon atoms would be excited mostly to lower excited states responsible for non-laser lines. A neon laser with no helium can be constructed but it is much more difficult without this means of energy coupling.
- The He-Ne laser that has lost enough of its helium (e.g., due to diffusion through the seals or glass) will most likely not lase at all since the pumping efficiency will be too low

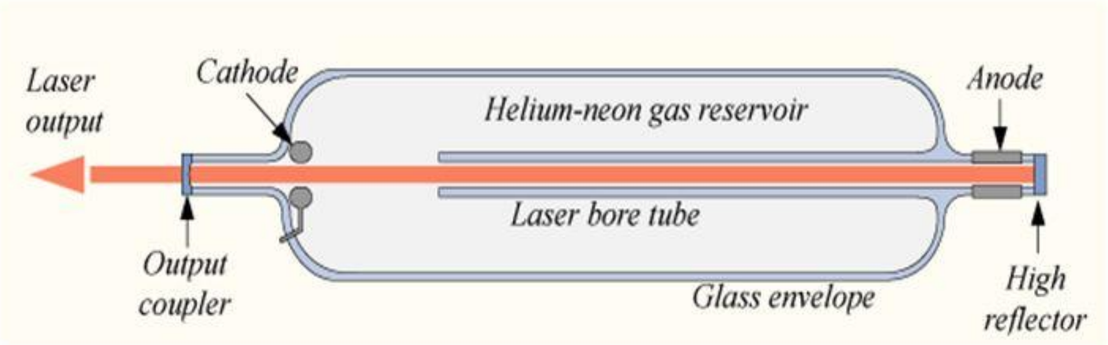


Fig.17 (3)

- The He-Ne lasers are relatively small devices having cavity lengths usually ranging from 15cm to 50cm . But sometimes up to about 1 meter to achieve the highest powers and optical output power levels ranging from 0.5 to 50mW .
- The energy or pump source of the laser is provided by a high voltage electrical discharge passed through the gas between electrodes (anode and cathode) within the tube.

- A DC current of 3 to 20 mA is typically required for continue operation.
- The optical cavityof the laser usually consists of two (fully and partially reflected) concave mirrors or one plane and one concave mirror.
- The population inversion and light amplification in a He-Ne laser plasma originates with inelastic collision of energetic electrons with ground state helium atoms in the gas mixture.

As shown in energy level diagram, these

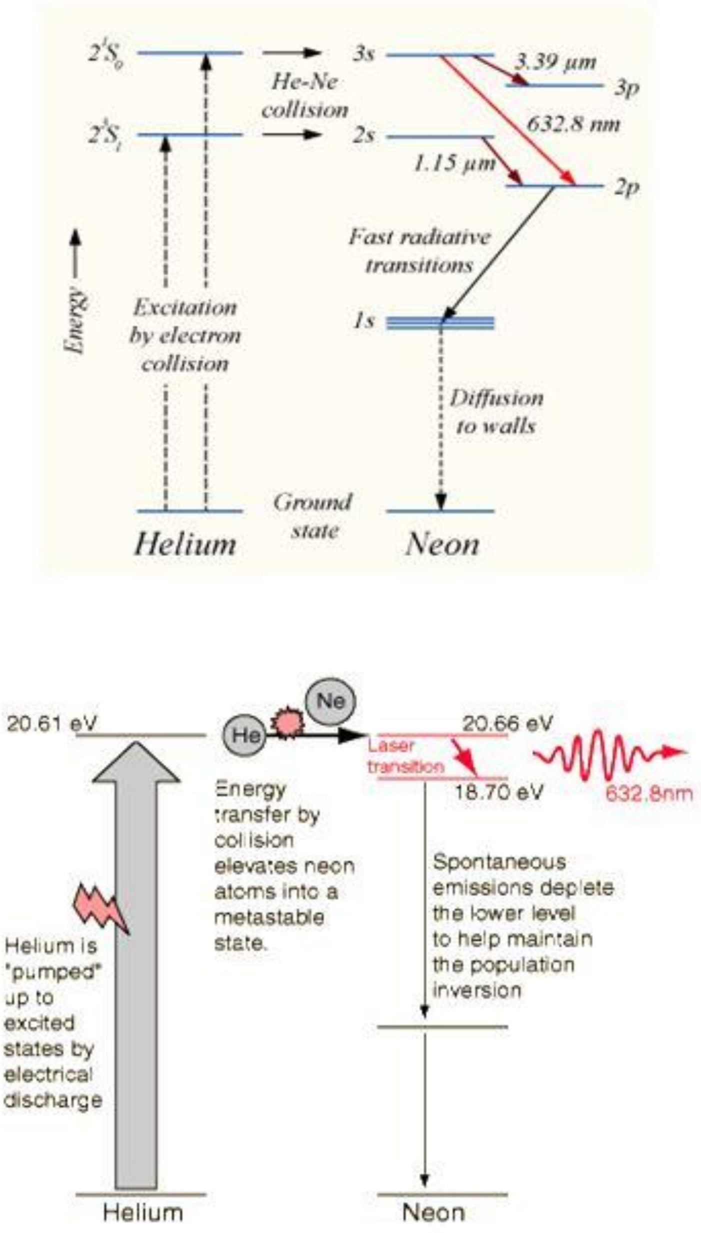


Fig.17 (4)

- The collisions excite helium atoms from the ground state to higher energy excited states, among them the \[2{\rm{S}}_1^3\] and \[2{\rm{S}}_0^1\] long-lived metastable states.
- Because of coincidence between the energy levels of the two He metastable states, and the 3s2 and 2s2 levels of neon, collisions between these helium metastable atoms and ground state neon atoms results in a selective and efficient transfer of excitation energy from the helium to neon.
- This excitation energy transfer process is given by the reaction equations:

He*(\[2{\rm{S}}_1^3\]) + Ne \[1{S_0}\] → He(\[1{S_0}\]) + Ne* \[2{S_2}\] + ΔE

He*(\[2{S^1}\]) + Ne \[1{S_0}\] + ΔE → He(\[1{S_0}\]) + Ne* \[3{S_2}\]

- Where *represents an excited state
- ΔE is the small energy difference between the energy states of the two atoms. Excitation energy transfer increases the population of the neon \[2{S_2}\] and \[3{S_2}\] levels manyfold.
- When the population of these two upper levels exceeds that of the corresponding lower level neon state $2P_4$, to which they are optically connected, population inversion is present.
- The medium becomes capable of amplifying light in a narrow band at 1.15 µm (corresponding to the \[2{S_2}\] to $2P_4$ transition) and in a narrow band at 632.8 nm (corresponding to the \[3{S_2}\] to $2P_4$ transition at 632.8 nm).
- The $2P_4$ level is efficiently emptied by fast radiative decay to the 1 \[S\] state, eventually reaching the ground state.
- The remaining step in utilizing optical amplification to create an optical oscillator is to place highly reflecting mirrors at each end of the amplifying medium.

A stabilized HeNe laser is also one of the benchmark systems for the definition of the meter.[7]

Application

- Red He-Ne lasers have many industrial and scientific uses.
- They are widely used in laboratory demonstrations in the field of optics.
- It produces same quality of spatial coherence and long coherence length at low cost and ease of operation compared to other visible lasers.
- Due to visible output, long coherence length and excellent spatial quality it is useful source for holography and as a wavelength reference for spectroscopy.
- Prior to the invention of cheap and abundant diode lasers, red He-Ne lasers were widely used in barcode scanners at supermarket checkout count.
- He-Ne lasers are generally used in educational and research optical laboratories.
- A consumer application of the red He-Ne laser is the Laser Disc player.

- The laser is used in the device to read the optical disk.

Ruby Laser

- Ruby is a crystal of Aluminium Oxide (Al_2O_3) in which some of the Aluminium ions (Al_3^+) is replaced by Chromium ions (Cr_3^+); this is done by doping small amounts of Chromium oxide (Cr_2O_3) in the melt of purified Al_2O_3
- These chromium ions give the crystal a pink or red color depending upon the concentration of chromium ions. It is a solid state laser.
- Laser rods are prepared from a single crystal of pink ruby which contains 0.05% (by weight) Chromium.
- Al_2O_3 does not participate in the laser action. It only acts as the host.
- The ruby crystal is in the form of cylinder.
- Length of ruby crystal is usually 2 cm to 30 cm and diameter 0.5 cm to 2 cm.
- As very high temperature is produced during the operation of the laser, the rod is surrounded by liquid nitrogen to cool the apparatus

Fig17(5 and 6).

Active medium or active center:

- Chromium ions act as active centers in ruby crystal. So it is the chromium ions that produce the laser.

Pumping source:

- A helical flash lamp filled with xenon is used as a pumping source.
- The ruby crystal is placed inside a xenon flash lamp.
- Thus, optical pumping is used to achieve population inversion in ruby laser.

Optical resonator system:

- The ends of ruby crystal are polished, grounded and made flat.
- The one of the ends is completely silvered while the other one is partially silvered to get the output.
- Thus the two polished ends act as optical resonator system.

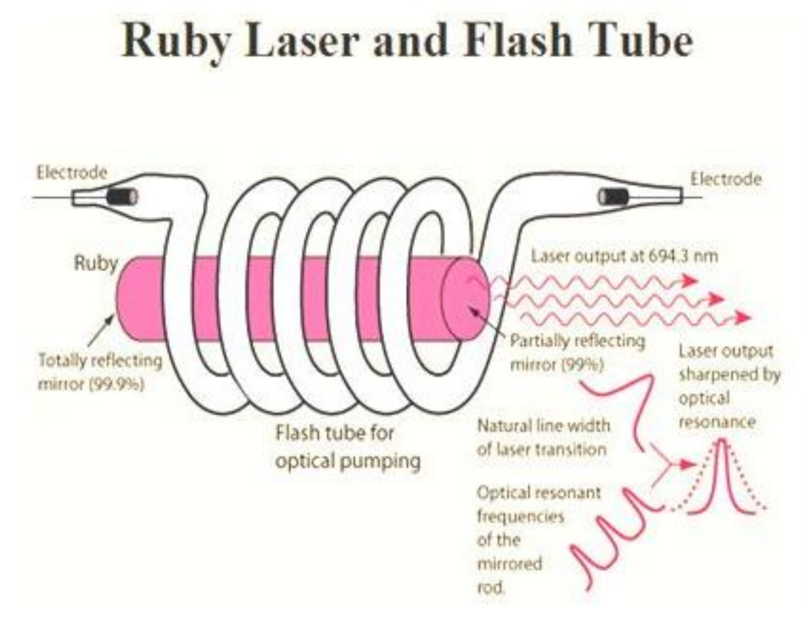


Fig 17(5)

Working

- Ruby is a three level laser system. Suppose there are three levels E_1 , E_2 and (E_3 & E_4).
- E_1 is the ground level, E_2 is the metastable level, E_3 and E_4 are the bands.
- E_3 & E_4 are considered as only one level because they are much closed to each other.

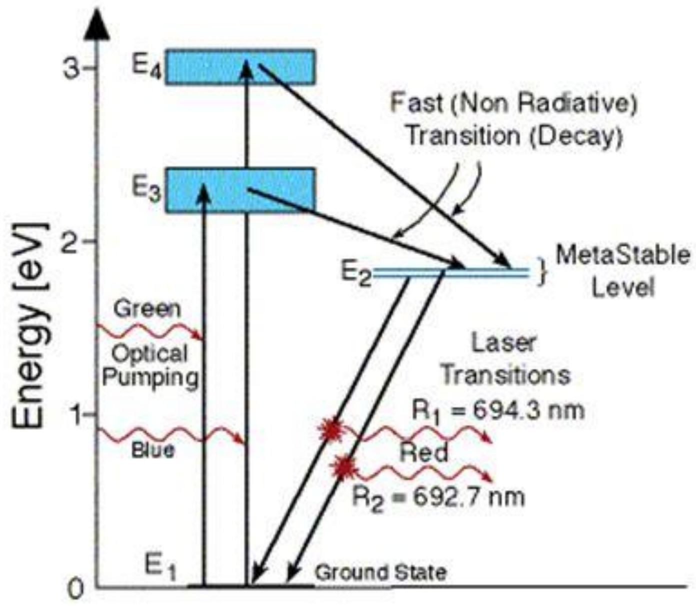


Fig 17(6)

Pumping:

- The ruby crystal is placed inside a xenon flash lamp.
- The flash lamp is connected to a capacitor which discharges a few thousand joules of energy in a few milliseconds.
- A part of this energy is absorbed by chromium ions in the ground state. Thus optical pumping raises the chromium ions to energy levels inside the bands E3 and E4 . This process is called stimulated absorption.
- The transition to bands E_3 and E_4 are caused by absorption of radiations corresponding to wavelengths approximately 6600 Å and 4000 Å respectively. The levels inside the bands E_3 and E_4 are also known as pumping levels.

Achievement of population inversion:

- \[Cr_3^ +\] ions in the excited state loose a part of their energy during interaction with crystal lattice and decay to the metastable state E_2 .
- Thus, the transition from excited states to metastable state is non-radiative transition or in other words there is no emission of photons.
- As E_2 is a metastable state, so chromium ions will stay there for longer time. Hence, the number of chromium ions goes on increasing in E_2 state, while due to pumping, the number in the ground state E_1 goes on decreasing.
- As a result, the number of chromium ions becomes more in excited state (metastable state) as compared to ground state E_1 .
- Hence, the population inversion is achieved between states E_2 and E_1 .

Achievement of laser:

- Few of the chromium ions will come back from E_2 to E_1 by the process of spontaneous emission by emitting photons (6943Å).
- This photon travels through the ruby rod and if it is moving in a direction parallel to the axis of the crystal, then it is reflected to and fro by the silvered ends of the ruby rod until it stimulates the other excited ions and cause it to emit a fresh photon in phase with the stimulating photon.
- Thus, the reflections will result in stimulated emission and it will result in the amplification of the stimulated emitting photons.
- This stimulated emission is the laser transition.
- The two stimulated emitted photons will knock out more photons by stimulating the chromium ions and their total number will be four and so on.

- This process is repeated again and again, thus photons multiply. When the photon beam become sufficiently intense, then a very powerful and narrow beam of red light of wavelength 6943 Å emerges through the partially silvered end of the ruby crystal.
- In the energy level diagram, E_2 is the upper laser level and E_1 is the lower laser level because laser beam is achieved in between these levels. Thus, the ruby laser fits into the definition of three level laser systems.

Drawbacks of ruby laser

- As the terminus of laser action is the ground state, it is difficult to maintain the population inversion which gives low efficiency.
- The ruby laser requires high power pumping source.
- Laser output is not continuous but occurs in the form of pulses (\[\mu s\]).
- The defects due to crystalline imperfection are also present in ruby laser.

Uses of ruby laser

- Ruby laser has very high output power of the order of 10^4 - 10^6 watts.
- It has wavelength of 6943 Å .
- Ruby lasers are used for holography, industrial cutting and welding.

LESSON 27. Application of LASER

MASER

A maser is a device that produces coherent electromagnetic waves through amplification by stimulated emission.

The word "maser" is derived from the acronym MASER: "Microwave Amplification by Stimulated Emission of Radiation".

The lower-case usage arose from technological development having rendered the original definition imprecise, because contemporary masers emit electromagnetic waves not just at microwave frequencies, but rather across a broader band of the electromagnetic spectrum.

Hence, the physicist Charles H. Townes suggested using "molecular" to replace "microwave" for contemporary linguistic accuracy.

Types of masers

Atomic beam masers

- Ammonia maser
- Free electron maser
- Hydrogen maser

Gas masers

- Rubidium maser

Solid state masers

- Ruby maser
- Whispering-gallery modes iron-sapphire maser

Holography

- It is a technique which enables three-dimensional images to be made.
- It involves the use of a laser, interference, diffraction, light intensity of recording and suitable illumination of the recording.
- It consists of an apparently random structure of either varying intensity and density or profile.

Difference between Holography and photography

- It is recording of information regarding the light that came from the original scene as scattered in a range of directions rather than from only one direction, as in a photograph.

- A photograph can be recorded using normal light sources like sunlight or electric lighting whereas a laser is required to record a hologram.
- A lens is required in photography to record the image, whereas in holography, the light from the object is scattered directly onto the recording medium.
- A holographic recording requires a second light beam (the reference beam) to be directed onto the recording medium.
- A photograph can be viewed in a wide range of lighting conditions, whereas holograms can only be viewed with very specific forms of illumination.
- When a photograph is cut in half, each piece shows half of the scene. When a hologram is cut in half, the whole scene can still be seen in each piece.
- A photograph is a two-dimensional representation that can only reproduce a rudimentary three-dimensional effect, whereas the reproduced viewing range of a hologram adds many more depth perception cues that were present in the original scene.
- A photograph clearly maps out the light field of the original scene. The developed hologram's surface consists of a very fine, seemingly random pattern, which appears to bear no relationship to the scene it recorded.

Physics of Holography

- For a better understanding of the process of Holography, It is necessary to understand interference and diffraction.
- Interference occurs when one or more wavefronts are superimposed.
- Diffraction occurs whenever a wavefront encounters an object.
- The process of producing a holographic reconstruction is explained below purely in terms of interference and diffraction.
- It is somewhat simplified but is accurate enough to provide an understanding of how the holographic process works.

(i) Recording a hologram

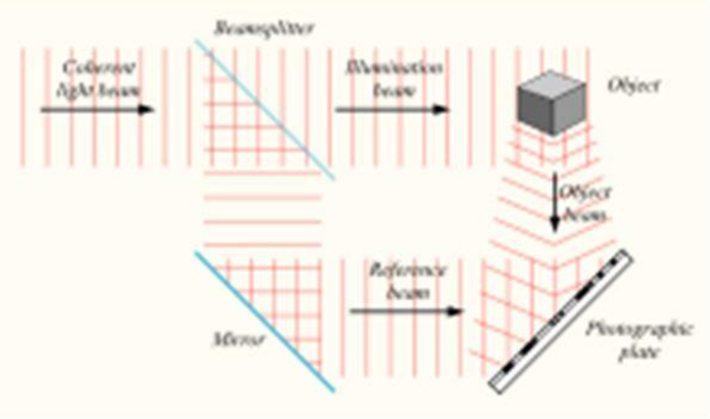


Fig.18 (1)

- To make a hologram, the following are required:
- A suitable object or set of objects
- A suitable laser beam
- Part of the laser beam to be directed so that it illuminates the object- the object beam)
- Another part so that it illuminates the recording medium directly- the reference beam, enabling the reference beam and the light which is scattered from the object onto the recording medium to form an interference pattern
- A recording medium which converts this interference pattern into an optical element which modifies either the amplitude or the phase of an incident light beam according to the intensity of the interference pattern.

(ii) Reconstructing and viewing the holographic image

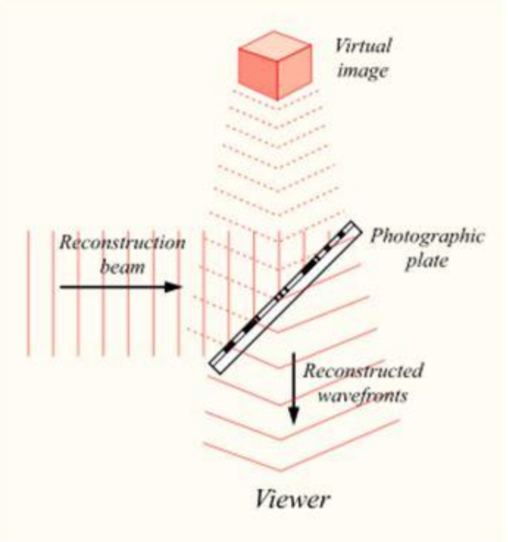


Fig.18 (2)

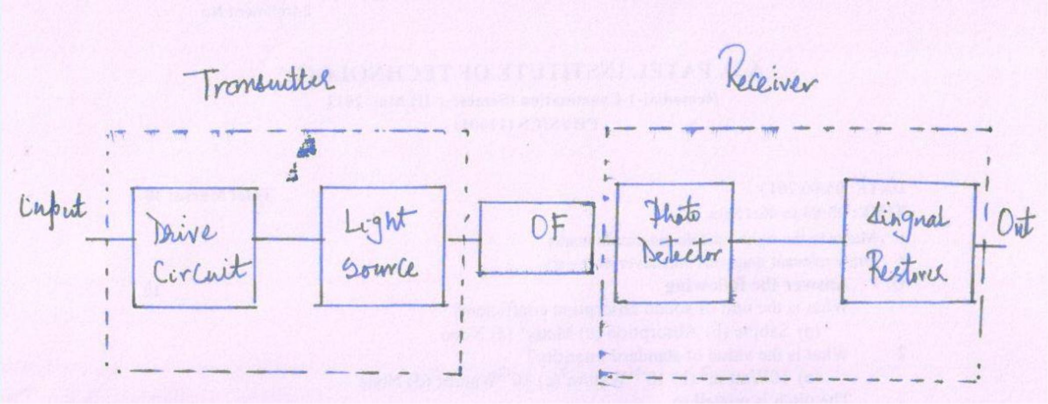
- When the hologram plate is illuminated by a laser beam identical to the reference beam which was used to record the hologram, an exact reconstruction of the original object wavefront is obtained. Fig.18(1) and (2)
- An imaging system (an eye or a camera) located in the reconstructed beam 'sees' exactly the same scene as it would have done when viewing the original.
- When the lens is moved, the image changes in the same way as it would have done when the object was in place.
- If several objects were present when the hologram was recorded, the reconstructed objects move relative to one another, i.e. exhibit parallax, in the same way as the original objects would have done.
- A holographic image can also be obtained using a different laser beam configuration to the original recording object beam, but the reconstructed image will not match the original exactly.

Applications

- Art
- Data storage
- Dynamic holography
- Hobbyist use
- Holographic interferometry
- Interferometric microscopy
- Sensors or biosensors
- Security
- Other applications

LESSON 28. Introduction to Optical Fiber System

Fiber Optic Communication System



A Fiber Optic Communication System has three major components;

Transmitter:

- It consists of a light source supported by suitable drive circuit.
- It converts electrical signal to light signals with the help of transducer.
    - Transducer converts a non-electrical message into an electrical signal and is fed to a light source.
    - Light source is very small either LED or LASER
    - Here, light waves are modulated with the electrical signals; the change in the intensity of LED or LASER beam gives analog modulation.
    - Such analog modulation is converted into digital (present of light pulse indicates as 1 and absence of light pulse indicates as 0) modulation by flashing of light source. A massage can be transmitted by number 1s or 0s.

Optical Fiber: transmit the signal

The optical signal travelling through the fiber will get weaker (attenuated) and twist out of shape (distorted) due to spreading over wide area (dispersion).

Output light from the fiber directs to the semiconductor photodiode, which converts the light signals to electrical signals

Receiver:

- It captures the signals (light signals) at the other end of the fiber and converts them to electrical signals.

- The electrical signals are amplified
- Receiver is programmed to accept such digital signals. It can re-construct the original massage.

Physical Structure

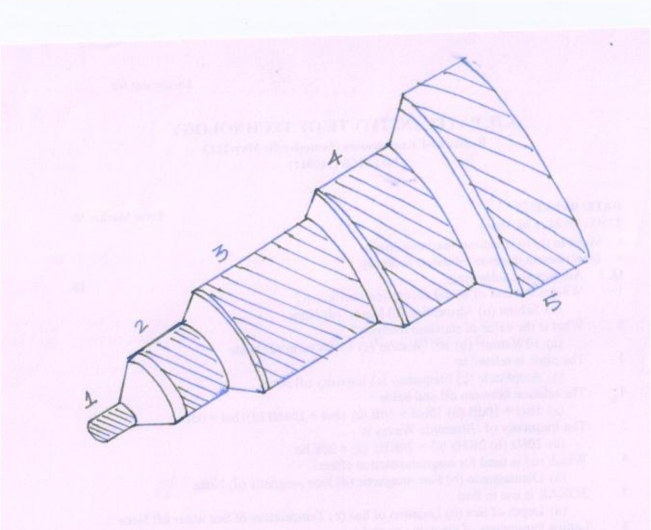


Fig19(2)

Core

- It is inner light carrying member.
- The innermost cylindrical region is the light guiding region.
- The diameter of the core is of the order of 8.5 μm to 62.5 μm
- Refractive index of the core is n_1.

Cladding

- It is middle layer, which serves to confine the light to the core.
- It is surrounded by a coaxial middle region.
- The diameter of the cladding is of the order of 125 μm -200 μm
- Refractive index of cladding is n_2
- Refractive index of cladding (n_2) is always greater than Refractive index of the core (n_1).

Buffer Jacket

- The outermost region is called the sheath or protective buffer coating.
    - It is plastic coating given to the cladding for extra protection.

- This coating is applied during the manufacturing process.
- It surrounds the cladding, which protects the fiber from physical damage and environment effect.
- The buffer coating can vary in size from 250 \[\mu m\] -900 \[\mu m\]
- It avoids fiber leakage excessive optical attenuation.

Kevlar

- It is yarn type of material.
- It is strength member that provides toughness and tensile strength of the cable.

Polyurethane

- Outermost jacket covers all inner layers and provides flexibility, ruggedness and avoid moisture to come in contact with the fiber.

Basic Principle

Transmission of light through an optical fiber is based on the phenomenon of Total Internal Reflection (TIR). TIR obeys the laws of reflection.

Let a medium having a lower refractive index (n_2) is said to be an optically rarer medium and a medium having a higher refractive index (n_1) is said to be an optically denser medium.

When a beam of light travels from a **denser (n_1) to rarer medium (n_2)**.

The incident ray makes angle with normal in the denser medium is called the angle of incident. Some rays are partially internally reflected in the same medium.

The ray is transmitted (refracted) in the rarer medium. It is bent away from the normal in the rarer medium and makes the angle   with respect to the normal known as refracted angle.

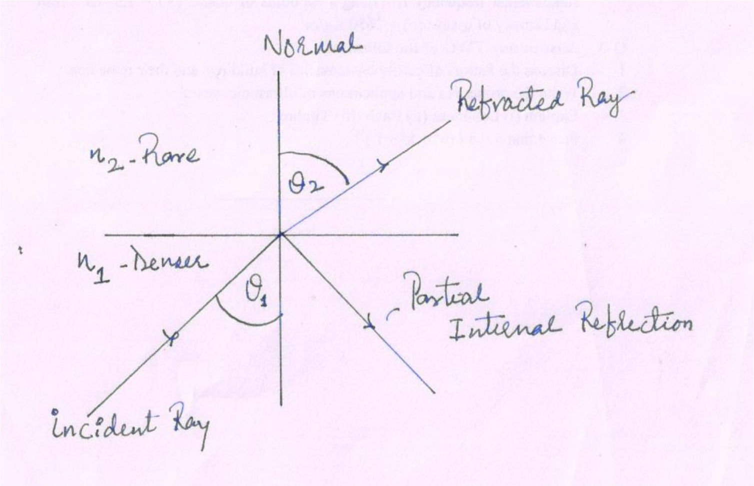


Fig 19(3a).

Here, beam of light changes the medium; however we can apply the Snell's Law

$$ {{\sin {\theta _1}} \over {\sin {\theta _2}}} = {{{n_2}} \over {{n_1}}} $$

Where,

θ_1 - Angle of incident in denser medium

θ_2 - Angle of refraction in rarer medium

n_1 - Refractive index of denser medium ($n_1 > n_2$)

n_2 - Refractive index of rarer medium

Now, the angle on incident (θ_1) in the denser medium is increased, the refracted angle (θ_2) increases and the transmitted rays bend more and more away from the normal. However, at some particular position of the angle of incident in denser medium, the refracted ray glides along the boundary surface so that the refracted angle becomes 90° with respect to the normal

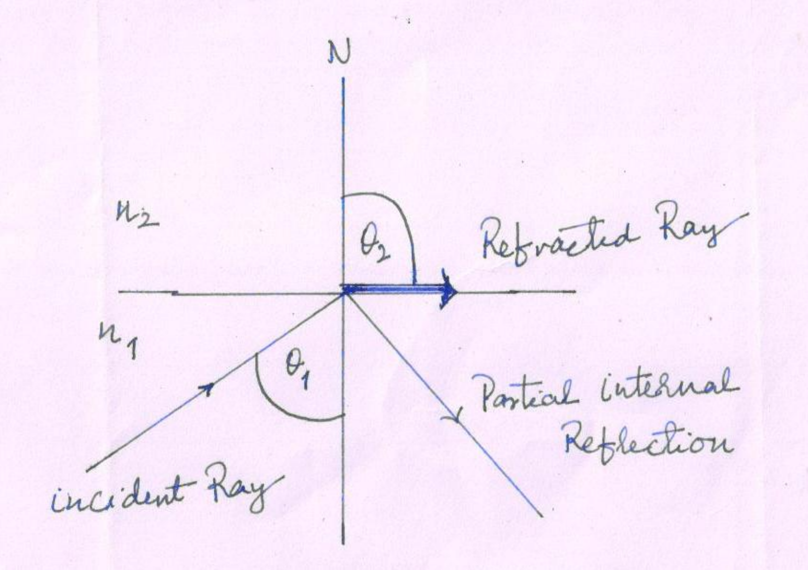


Fig 19(3b).

The particular position of the angle of incident (θ_1) in denser medium is called critical angle (θ_c). Again we can apply the Snell's Law

$$ {{\sin {\theta _1}} \over {\sin {\theta _2}}} = {{{n_2}} \over {{n_1}}} $$

If, $\theta_2 = 90^o$ at that time $\theta_1 = \theta_c$ = critical angle

$$ {{\sin {\theta _c}} \over {\sin 90^\circ }} = {{{n_2}} \over {{n_1}}} $$

$$ \sin {\theta _c} = {{{n_2}} \over {{n_1}}} $$

Suppose, the rarer medium (n_2) is air, so $n_2 = 1$

$\sin {\theta _c} = {1 \over {{n_1}}}$

Critical angle (θ_c) = ${\sin ^{ - 1}}\left({{1 \over {{n_1}}}} \right)$

The particular position of the angle of incident (θ_1) in denser medium is greater than θ_c i.e. $\theta_1 > \theta_c$, the rays (refracted and partial internal reflected) are reflected back into the denser medium. There are no refracted rays at all

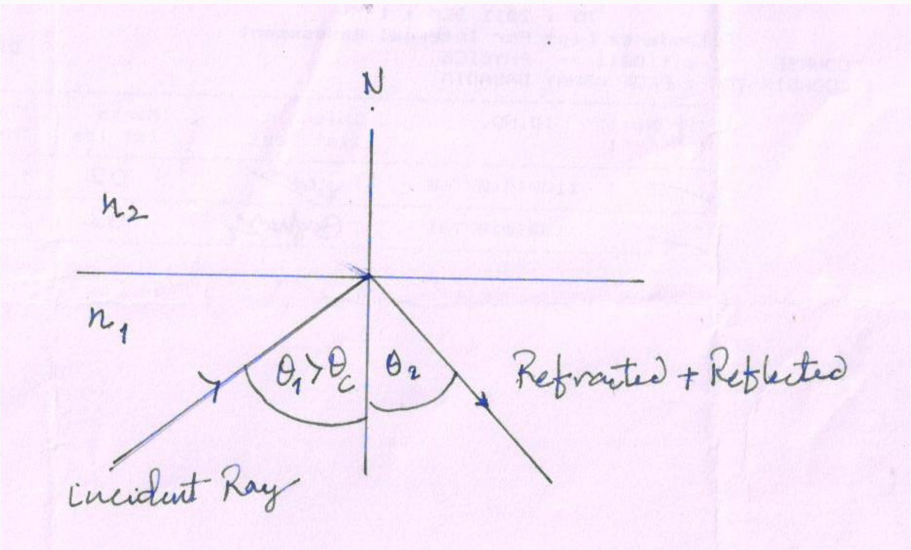


Fig 19(3c).

Thus,

- If $\theta_1 < \theta_c$, the refracts into the rarer medium
- If $\theta_1 = \theta_c$, the ray just grazes the interface of rarer-to-denser media
- If $\theta_1 > \theta_c$, the ray is totally reflected (refracted + partial internal reflected) into the denser medium

The phenomenon in which the light is totally reflected from a denser-to-rarer medium boundary is known as total internal reflection.

LESSON 29. Theory of Optical Fiber- Index Fiber

Ray Model

Modes; it can be observed as possible number of allowed paths of light in an optical fiber. Exact, an axial directed path, all other paths are “Zigzag”

These zigzag rays get repeatedly reflected at the interface of core-cladding of fiber and phase shift occurs. As a result, constructive interference occurred. Hence, light ray paths along which the waves are in phase inside the fiber is called modes.

Number of modes propagating in the fiber is depending upon fractional refractive index (Δ). Hence, RI of core increases, mode of propagating are increase.

Mode of propagation (Number of mode) depends on the ratio ${d \over \lambda }$;

d - Diameter of core.

λ - Wavelength of the light is being transmitted.

A mode is a path taken by a ray of light through an optical fiber.

It is mathematically expression to understand the concept of propagation of light waves in an optical fiber.

The number of mode of propagation within the fiber depends on;

The diameter of the core- narrower the fiber core lesser is the number of mode of propagation.

The number of modes supported for propagation in the fiber is determined by a parameter called V-number. It is given by

V = ${{2\pi d} \over \lambda }\sqrt {n_1^2 - n_2^2}$

Or

V = ${{2\pi d} \over \lambda }\left({NA} \right)$

V – V number; depends upon characteristics of fiber and the wavelength of light propagation through it.

d: Diameter of the core

λ: Wavelength of light

For step index fiber; the maximum of modes (N_m) = ${{{V^2}} \over 2}$,

Single Mode Step Index Fiber V < 2.405

Multimode Mode Step Index Fiber V > 2.405

For Gradded Index fiber; the maximum of modes (N_m) = \[{{{V^2}} \over 4}\]

The classification of the fiber on the basis of

- Material
- Refractive Index(RI)
- Path(Mode) of propagation of light through light

Before discussion of the mode of the fiber, we must to resolve the concept of refractive index based fiber structure.

Refractive Index Profile is "a graphical representation between Refractive Index and Distance from center of the core".

The graphical representation shows Refractive Index on one axis; Refractive Index Axis

Distance from the center of the core on other axis; Core Axis

(1) Step Index Fiber(SI)

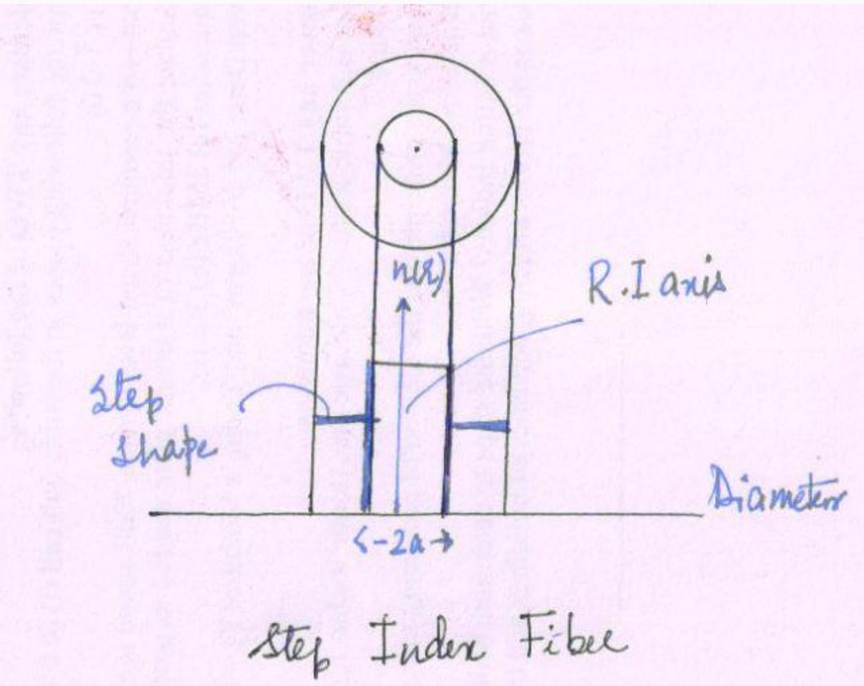


Fig20(1)

- Refractive Index of the core is constant along radial direction.
- RI falls to a lower value at the cladding and core boundary.

- There is an abrupt change in the refractive index at the core-cladding interface and hence the refractive index profile takes place the shape of step.

(2) Graded Index(GRIN) Fiber

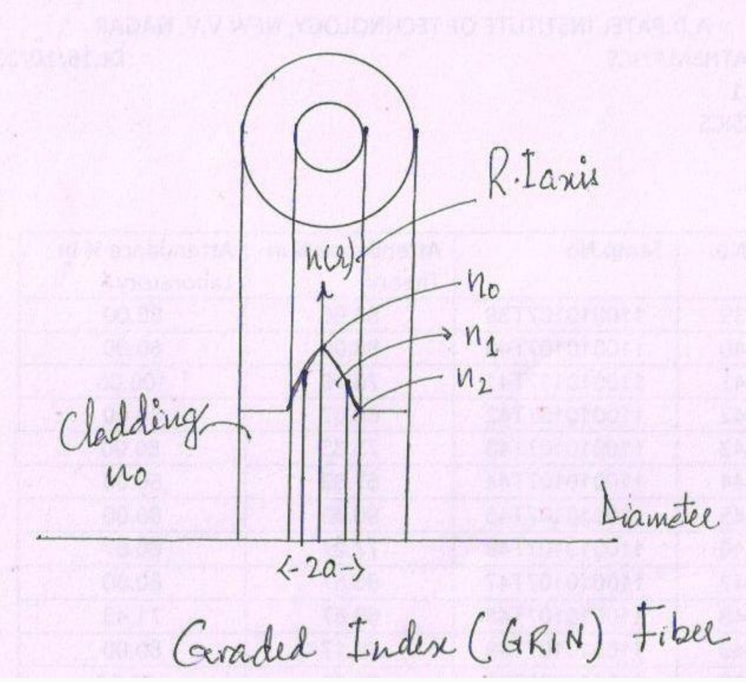


Fig20(2)

- The RI of the core is not constant but varies smoothly over the diameter of the core.
- It has a maximum value at the center and decreases gradually toward outer edge of the core.
- At the core-cladding interface the RI of the core matches with the RI of cladding i.e n_3 \[\cong\] n_2 (RI of cladding)

LESSON 30. Theory of Optical Fiber - Mode of Propagation

Type of Mode

Fiber thickness is fixed;

The mode, that propagate at angles close to the critical angle are higher order modes.

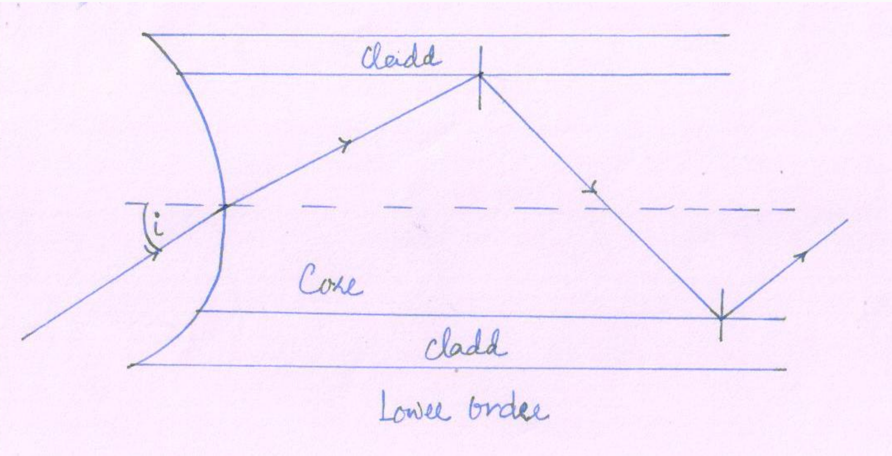


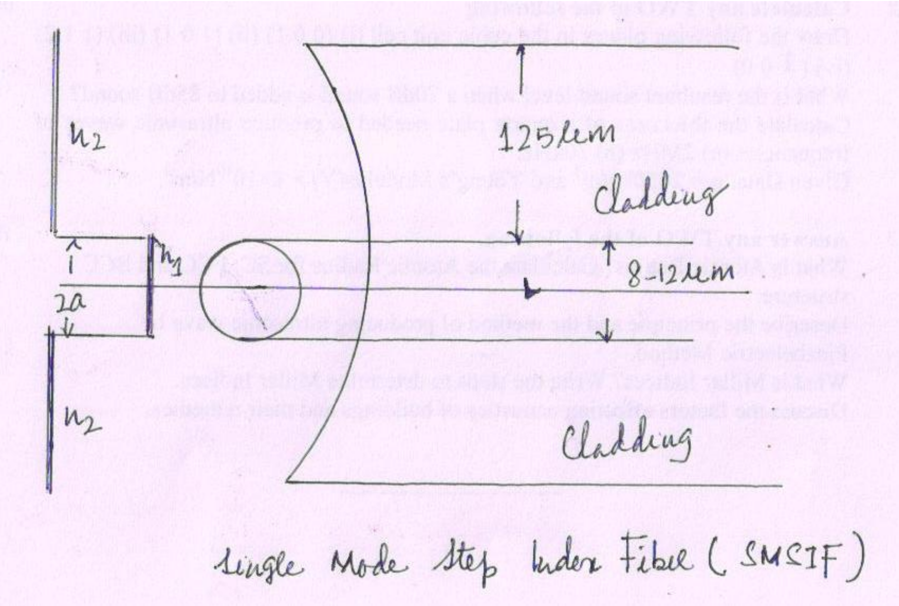
Fig.20(3, 3a)

Fields are distributed more toward the edge of the wave guide and tend to send light energy into cladding. As result energy may loss.

It has transverse longer path and hence it takes larger time than the lower order mode to cover given length of fiber. As a result higher order mode arrives at the end of fiber later than lower mode.

The mode, that propagates with angle larger than the critical angle are called lower order mode. Fields are concentrated near center of the fiber.

Single Mode Step Index Fiber (SMSIF)



Single Mode Step Index Fibre (SMSIF)

Fig 20(4)

Structure:

- SMSIF has a very fine thin of core diameter of 8-12 \[\mu m\]
- It is prepared by Ge doped silicon.
- The core is surrounded by a thick cladding of lower refractive index compared to core.
- Cladding is prepared by composed Silica lightly doped with phosphorus Oxide.
- The diameter of cladding is 125 \[\mu m\] .
- This fiber is surrounded by opaque protective sheath.
- RI of the fiber changes abruptly at core-cladding interface.
- The variation of RI of SI fiber as a function of radial distance (n(r)) can be mathematically.

$n(r) = n_1$ ($r < a$; inside the core region)

$n(r) = n_2$ ($r > a$; inside the core region)

Propagation of light through (SMSIF)

- Light travel in SMSIF is single path.
- It is zero order modes.
- Fractional Refractive Index (Δ) and numerical aperture (NA) are very small for SMSIF; Δ is small obtained by reducing the radius of the fiber and change the difference between $n_1 - n_2$.

- The low numerical aperture (NA) ; low acceptance angle. As a result, Coupling into fiber is very difficult. As a result, costly laser diodes are required to launch the light into the SMSIF.

Features:

- Support only single mode of propagation.
- Core diameter is small.
- RI of core abruptly falls to a lower value at the core-cladding interface.
- It has very small numerical aperture (NA) .
- The attenuation is least.
- Monochromatic and coherent light sources are required to launch the light into fiber.

Advantage:

- No degradation of signal.
- It is more suitable for communication.

Disadvantage:

- Manufacturing and operating of SMSIF is more difficult.
- The fiber is costlier than other.
- Launching of light into fiber is difficult.

Multimode Step Index Fiber (MMSIF)

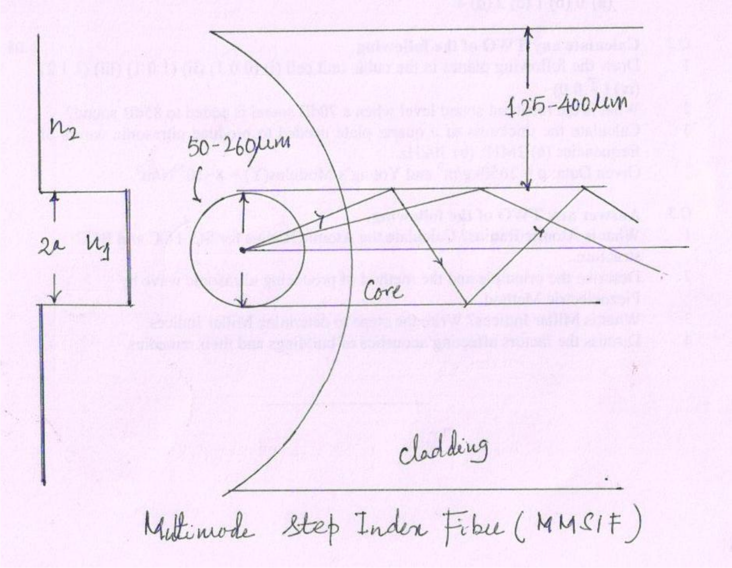


Fig 20(5)

- MMSIF is very much similar to SMSIF with respect to RI profile.
- The diameter of core is larger than SMSIF; 50-100 \[\mu m\] , which is very large compared to the wavelength of light.
- The cladding diameter is about 150-250 \[\mu m\]

Propagation of light through (MMSIF)

- MMSIF allow limited number of guided modes.
- The direction of polarization, alignment of electric and magnetic fields will be different for different modes.
- The many zigzag paths are permitted to propagate in the MMSIF; as a result the path length will be long for higher zigzag pattern along fiber axis and the path length will be lower for low zigzag pattern along fiber axis. As a result the lower order mode reaches earlier than higher order mode to the end of fiber.

Features:

- It supports more modes of propagation.
- Core diameter is 50-100 \[\mu m\] ; large diameter compared to SMSIF.
- RI profile is identical to SMSIF.
- It has large numerical aperture (NA) .
- Attenuation is high.
- Larger numerical aperture allows more number of modes which causes dispersion (spread over larger area).
- Incoherent sources like LED can be used as a light source for launching a light into fiber.
- Lower order mode reaches earlier than higher order mode to the end of the fiber.

Advantages:

- MMSIF is relatively easy to manufacture and less expensive.
- LED or LASER source can be used.
- Launching of light into fiber is easier than other.
- It is easy to couple with other multimode fiber.

Disadvantage:

- Transmission is less efficient.

- It is less suitable for long distance communication.

Graded Index Multimode Fiber (GRINMMF)

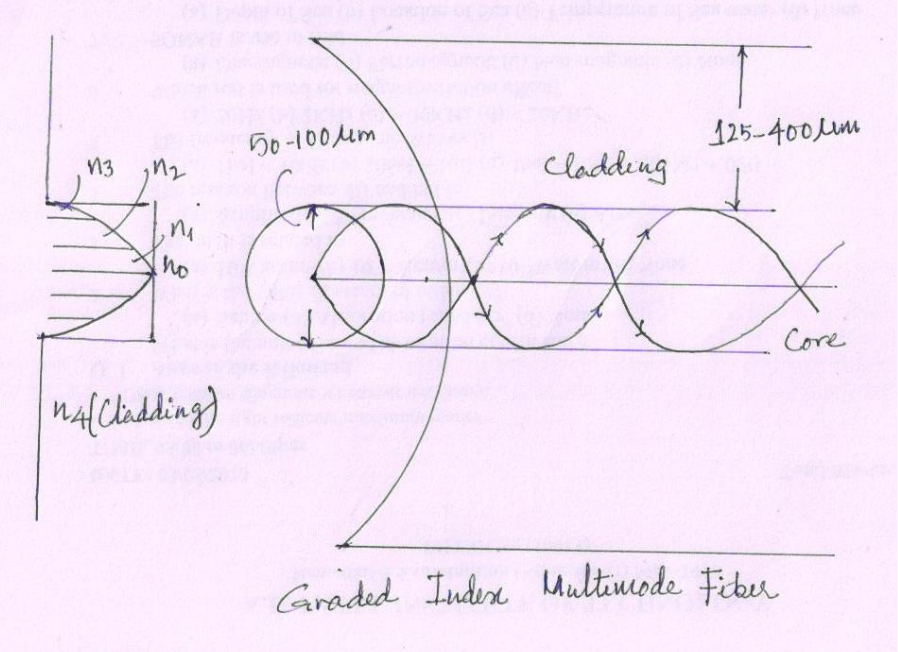


Fig20(6)

Structure:

- GRINMMF has core consisting of concentric layer of different refractive indices plus allows more than one mode into the core.
- RI of core varies with radial distance from fiber axis.
- It has high value at the center of the core and reduces with increasing radial distance from the axis.
- The size of the graded index fiber is about similar to multimode step index fiber.

The variation of refractive index of the core with the radius measured from center is give by

$$n(r) = n_1 \sqrt{1 - 2\Delta\left(\frac{r}{a}\right)^{\alpha}} \ ; r < a \text{ Inside core}$$

$$n(r) = n_2, \ r > a \ ; r > a \text{ Outside core}$$

n_1 - Maximum RI at the core axis.

\[a\] The radius of core

\[a\] The grading profile index number which varies from 1 to ∞ ; when \[a\] = 2, the index profile is parabolic and is preferred for different applications.

Propagation of light through Graded Index Multimode Fiber (GRINMMF)

- A light ray goes from a region of higher refractive index (denser) to the region of lower refractive index (rarer), it is bent away from the normal.
- This process continuous till the condition for TIR is achieved.
- Then ray travels back toward the core axis, being continuously refracted.
- The turning around may take place even before reaching the core-cladding interface. Thus continuous refraction is followed by total internal reflection and again continuous refraction towards the axis. Consequently, all rays traveling through the fiber, irrespective of their modes of travel, will have almost the same optical path length and reach the output end of the fiber at same time.

The numerical aperture of a graded index fiber is given by

$$NA = \sqrt{n^2(r) - n_2^2}$$

$$NA = n_1\sqrt{2\Delta\left[1 - \left(\frac{r}{a}\right)^{\alpha}\right]}$$

Feature:

- It supports more of propagation
- Core diameter is large.
- Refractive Index of the core varies radials with distance from the fiber axis.
- NA is large.
- All the modes reach the output end nearly at the same time.

Application of Optical Fiber

- Illumination and Image Transmission
- Fiber Endoscopy
- Optical Communication
- Medical Application
- Military Application
- Optical Fiber Communication System
    - i. Local Area Network
    - ii. Long –haul Communication

LESSON 31. Basic concept of Photometry

Photometry

A light is a form of energy. So, it is measurable quantity. But the absolute measurement of light energy is not possible because, it is invariably accompanied by heat energy.

Photometry

- Branch of optics
- It deals with “measurement of light”, other word we can resolve Photo-Light and Metry-Measurement.
- Deals with the luminous energy emitted by a source (energy emission capacity)
- Visible point of view, It is comparison of relative brightness of two sources or two illuminated surfaces(light scattering capacity of surfaces)

We know that light makes object visible and light is a energy. Basically, by which objects are seen means we can say that the light either “reflected by the objects” or light is emitted by the objects”

Hence there are two types of objects

(i) Self –luminous objects; which give light of their own e.g sun, burning flame and lighted lamp

(ii) Non-luminous objects; which do not give the light of their own e.g tress, table, books, planets and moon

Fundamental concept

Luminous Flux

- The amount of light (visible part of the spectrum) which flows from a source of flows from a illuminating surface per second.
- It is part of the total visible radiation which can affect our eyes.
- It is the rate of flow of the energy e.g. it is one king of power.
- Denoted by F or Ø
- Unit is Lumen

Justification of Lumen

It is luminous flux radiated by uniform sources of one candle power through unit solid angle. i.e point source of one candle power radiates luminous flux through unit solid angle.

The luminous flux which spread out from uniform point source of one candle power placed at the center of a sphere of one meter radius and falls on $1m^2$

Luminous Intensity (Illumination Power) (I)

Luminous flux is emitted by the source of candle power in all direction. The emitted of luminous flux in all direction per unit solid angle is called luminous intensity or illumination power.

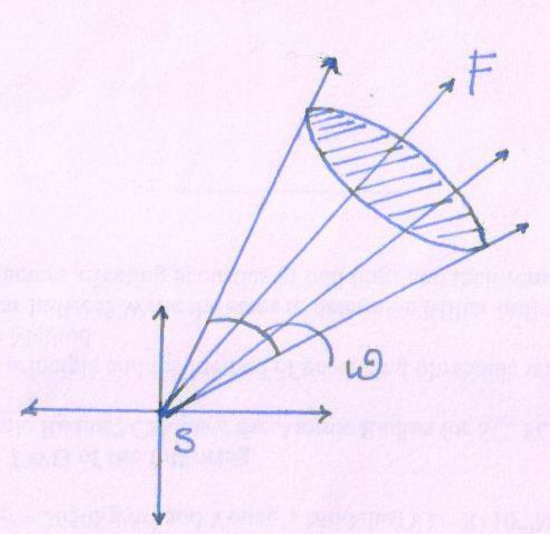


Fig 21(1)

Let F or Ø is luminous flux is radiated by a source within a solid angle (ω) in any direction.

Hence, Luminous Intensity $\left(I \right) = {{F or \emptyset } \over \omega }$

F or Ø in Lumen ω- in steradians

I $= {{Lumen} \over {Steradians}}$ or candela

Candela; Source (light source) radiates ${1 \over {60}}$ of luminous intensity per cm^2

Illumination OR Intensity of Illumination of a surface

The illumination or intensity of illumination at a point on a surface is defined as the luminous flux received on the unit area of the surface surrounding the point.

Let F Luminous flux in lumen

A Area in met^2

So, intensity of illumination $= {F \over A}$ $= {{lumen} \over {me{t^2}}}$

Brightness OR Luminance of surface

Let light is incident on the surface some part of the light is reflected from surface and it is distributed over a wide range of directions. The brightness of the surface in a given direction is measured by the luminous flux per meter square which is coming from the surface in a particular direction.

It is denoted by B

It is different from illumination (E) which is luminous flux per meter square incident on the surface

Different surfaces have different brightness because it is reflecting capacity of the surface. Hence, different material has different reflecting coefficient.

Reflection Coefficient (r) = \[{B \over E}\] = $\frac{B}{\frac{F}{A}}$ = \[{AB \over F}\]

B Brightness (luminescence of surface)

E Illumination (intensity of illumination of surface)

Brightness of the surface may be due to transmission of the luminous flux (F) through it

So, Transmission Coefficient $(t) = \frac{F}{E} = \frac{F}{\frac{F}{A}}$

F - Luminous flux per meter square transmitted through the substance

E - Luminous flux or illumination per meter square incident on the surface.

LESSON 32. Law of Photometry

Law of Illumination

Inverse Square Law

“The intensity of illumination of surface (E) or illumination of surface (E) is inversely proportional to the square of the distance between the surface and source”.

Hence, intensity of illumination of surface

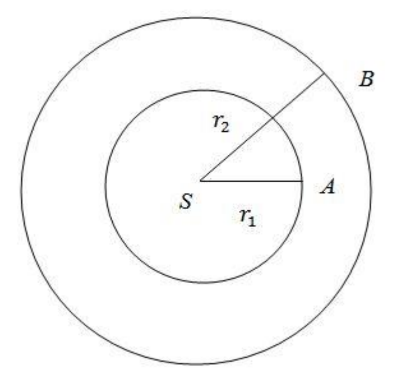
\[\left(E \right)\] \[\propto\] \[{1 \over {distbetweenthesourceandsurface}}\]

\[\left(E \right)\] \[\propto\] \[{1 \over {{r^2}}}\]

Physical significance of inverse square law

According to the following figure suppose the source(s) is at center (o). It emits the light in all directions.

Now, the Q amount of the light is emitted in unit time and it incident on the two surfaces A and B. Surface A is at distance r_1 and surface B is at distance r_2 from source respectively.



The intensity of illumination of surface E = \[{{Theamountoflightwhichflowfromilluminatingsurfacepersec} \over {Areaofilluminatiom}}\]

For surface A, E_1 = \[{Q \over A}\] = \[{Q \over {4\pi r_1^2}}\]

A Surface area of the circle

For surface B, E_2 = \[{Q \over {4\pi r_2^2}}\] ……… (1)

But illumination power (I) = \[{{LuminousFlux} \over {SolodAngle}}\]

I = \[{Q \over {4\pi }}\] …… (2)

From equation (1) and (2)

$$E_1 = \frac{I}{r_1^2} \text{ and } E_2 = \frac{I}{r_2^2}$$

Lambert's Cosine Law

"The illumination is directly proportional to the "Cosine of the angle made by the normal to the illuminating surface with the direction of the incident flux".

- Note: Cosine angle either angle between the normal and the direction of the incident flux or the angle between two plane surfaces (one is normal incident surface and other is bend surface)

Lambert derived a relation between normal illumination E_n and slanting illumination E for the same distance.

Let S is point source at distance r from surface area ds (original)

I is luminous intensity or illumination power of the point source(S)

dω is solid angle; subtended by ds and slanting area are same at point source(S)

θ is the angle between normal to the surface area ds and the direction of incident flux.

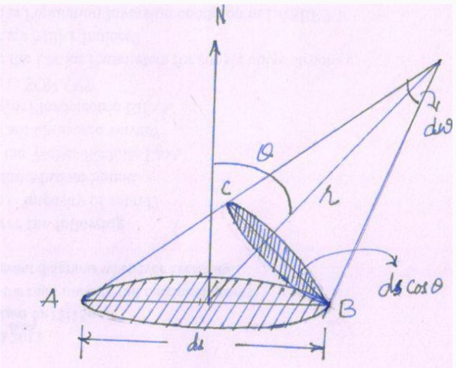


Fig 21(2)

According to resolved figure,

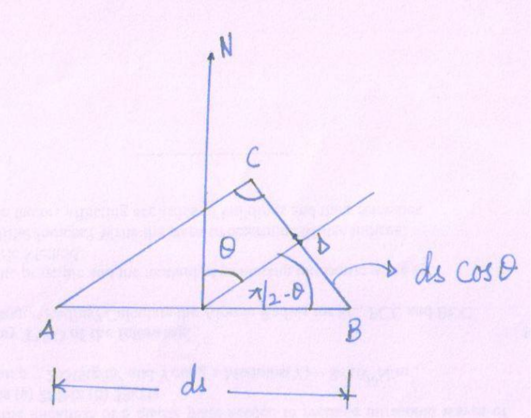


Fig 21(3)

Let $\sin\left(\frac{\pi}{2}-\theta\right)=\frac{\frac{x}{2}}{\frac{ds}{2}}$

$ds\cos\theta$ = x = BD

Luminous Intensity (I) = ${F \over w}$

But dF are fluxes in solid angle dω

I = ${dF \over dw}$

Solid angle dω = ${{ds\cos\theta} \over {r^2}}$

dF = I dω

dF = ${{Ids\cos\theta} \over {r^2}}$

I = ${{r^2 dF} \over {ds\cos\theta}}$

Expression reports

Light illumination with intensity I and making solid angle ${{ds\cos\theta} \over {r^2}}$ w.r.t area ds cos θ . Due to that solid angle, luminous flux dF is slanting with angle θ and focusing to surface ds (original).

Light illuminates with intensity I from source. Source is slanting mode w.r.t. normal. It is going to illuminates original surface ds. So, intensity fluxes dF illuminating the surface with slanting angle cos θ and with dF flux is illuminating surface ds.

Illumination or Intensity of illumination of surface E = ${F \over A}$

Hence E = ${dF \over dS}$

E = ${{Ids\,\mathrm{Cos}\,\theta} \over {ds\,r^2}}$

E = ${{I\,\mathrm{Cos}\,\theta} \over {r^2}}$ …… (2)

For Normal (direct) illumination

En = ${I \over {r^2}}$

E = En Cos θ…… (3)

Equations (2) and (3) are the two form of Lambert's Law