Design of Structures

AgriMoon.Com
All About Agriculture…

Dr. K. Nagarajan
Prof. P. Natarajan

Design of Structures
-: Course Content Developed By :-

Dr. K. Nagarajan
Professor (SWC Engg.)
Water Technology centre, TNAU, Coimbatore

-:Content Reviewed by :-

Prof. P. Natarajan
Professor (Retd.)
Coimbatore

INDEX

MODULE 1.

LESSON 1. Structures and its kinds

1.1 INTRODUCTION

The Structural engineering is a branch of engineering which deals with structural analysis and structural design. The structural engineering plays an important role in civil engineering, mechanical engineering, electrical engineering, naval engineering, aeronautical engineering and in all the specialized phases of engineering. The structural analysis deals with the development of suitable arrangement of structural elements for the structures to support the external loads or the various critical combinations of the loads which are likely to act on the structure. The analysis also deals with the determination of internal forces developed in the various members, nature of stresses or critical combination of the stresses at the various points and the external reactions due to the worst possible combination of the loads. The structural design deals with the selection of proper material, proper sizes, proportions and shape of each member and its connecting details. The selection is such that it is economical and safe. The structural design further deals with the preparation of final layout of the structure and the design drawings are necessary for fabrication and construction.

1.2 DEFINITION

Construction or framework of structural elements (members) which gives form and stability, and resists stresses and strains. Structures have defined boundaries within which each element is physically or functionally connected to the other elements, and the elements themselves and their interrelationships are taken to be either fixed (permanent) or changing only occasionally or slowly.

1.3 CLASSIFICATION OF STRUCTURES

The structures may be classified as statically determinate structures and statically indeterminate structures. When the equation of statics ($\Sigma H=0$, $\Sigma V=0$ and $\Sigma M=0$) are enough to determine all the forces acting on the structure and in the structures are known as statically determinant structures. When the equation of equilibrium are not sufficient to determine all the forces acting on the structures and in the structure, then the structures are known as statically indeterminate structures. The equations of consistent deformations are added to the equations of equilibrium in order to analyze the statically indeterminate structures.

The structures are also classified as shell structures and framed structures. The shell roof covering of large buildings, air planes, rail road cars, ship wells, tanks etc are the examples of shell structures. The plates or sheets serve functional and structural purposes. The plates act as a load carrying elements. The plates are stiffened by frames which may or may not carry the principal loads. The framed structures are built by assemblies of elongated members. The truss frames, truss girders, rigid frames etc are the examples of framed structures. The main members are used for the transmission of loads.

Design of Structures

The structures may be further classified depending on the materials used as plastic structures, aluminium structures, timber structures, R.C.C structures and steel structures.

1.4 ADVANTAGES OF STEEL STRUCTURES

1. Steel has a high strength and so steel components have smaller sections for the same strength compared to corresponding components of other material.The existing steel structures and structural component may be strengthened by connecting additional sections or plates.

2. Steel members are gas and watertight, because of high density of steel.

3. Steel structures can be fabricated at site easily.

4. Steel structures have great durability and serve for many years.

5. Steel members can be readily disassembled or replaced.

6. The existing steel structures and structural component may be strengthened by connecting additional sections or plates.

1.5 DISADVANTAGES OF STEEL STRUCTURES

1. Steel structures are liable to corrosion and need painting frequently.

2. Steel structures have a low fire resistance and are liable to lose their strength and get deformed at high temperature.

1.6 STRUCTURAL STEEL

The structural steel is the steel used for the manufacture of rolled structural steel sections, fastenings and other elements for use in structural steel works. Steel is an alloy of iron, carbon and other elements in varying percentages. The strength, hardness and brittleness of steel increases and ductility of steel decreases with the increase of percentage of carbon. Depending on the chemical composition, the different type of steel are classified as mild steel, medium carbon steel, high carbon steel, low alloy steel and high alloy steel. The mild steel, medium carbon steel and low alloy steel are generally used for steel structures. The copper bearing quality of steel contains small percentage of copper contents. The corrosive resistance of such steel is increased.

Mild steel is used for the manufacture of rolled structural steel sections, rivets and bolts. The following operations can be done easily on mild steel 1.Cutting, 2. Punching, 3.Drilling, 4. Machining, 5. Welding and 6. Forging when heated. All structural steels used in general construction, coming within the purview of IS:800-84 shall, before fabrication, comply with one of the following Indian Standard specifications

1. IS : 226-1975 structural steel (standard quality)

2. IS : 1977-1975 structural steel (ordinary quality)

3. IS : 2062-1984 weldable structural steel

4. IS : 961-1975 structural steel (high tensile)

5. IS : 8500-1977 weldable structural steel (medium and high strength qualities)

1.6.1 IS : 226-1975 structural steel (standard quality).

The mild steel is designated as St 44-S for use in structural work. This steel is also available in copper bearing quality in which case it designated as St 44-SC. The copper content is between 0.20 and 0.35 per cent. The physical properties of structural steel are given below:

1. Unit weight of steel 78.430 to 79.000 kN/m^3

2. Young's modulus of elasticity, $E = 2.04$ to $2.18 \times 10^5 \ N/mm^2$

3. Modulus of rigidity, $G = 0.84$ to $0.98 \times 10^5 \ N/mm^2$

4. Coefficient of thermal expansion (or contraction) $\alpha = 12 \times 10^{-6}/\degree C$ or $6.7 \times 10^{-6}/\degree F$.

The tensile strength, yield stress and percentage elongation for IS : 226-1975 structural steel standard quality, determined in accordance with IS : 1608-1960. The steel confirming to IS : 226 is suitable for all types of steel structures subjected to static, dynamic and repeated cycles of loadings. It is also suitable for welding up to 20 mm thickness. When the thickness of element is more than 20 mm, it needs special precautions while welding.

1.6.2 IS : 1977-1975 structural steel (ordinary quality).

The steel which did not comply with IS : 226, was formerly called as steel of untested quality. The standards for such steel have been laid down in IS : 1977-75 (ordinary quality). There are two grades in this standard which are designated as St 44.0 and St 32.0. The steel St 44.0 is intended to be used for structures not subjected to dynamic loading other than wind loads e.g., platform roofs, office buildings, foot over bridge. The copper bearing quality is designated as St 44.0C.

The steel confirming to IS : 1977 is not suitable for welding and for the structures subjected to high seismic forces (earth quake forces). The steel structures using steel confirming to IS : 1977 must not be analyzed and designed by plastic theory.

1.6.3 IS : 2062-1984 weldable structural steel.

This structural steel intended to be used for members in structures subjected to dynamic loading where welding is employed for fabrication and where fatigue and great restraint are involved e.g., crane gantry girder, road and rail bridges etc,. it is designated as St 42-W and copper bearing quality is designated as St 42-WC. It is suitable for welding the elements of thickness between 28 mm and 50 mm. when the thickness of elements is less than 28 mm; it may be welded provided the limiting maximum carbon content is 0.22 per cent.

1.6.4 IS : 961-1975 structural steel (high tensile).

The high tensile steel forms a specific class of steel in which enhanced mechanical properties and in most of the cases increased resistance to atmospheric corrosion are obtained by the incorporation of low proportions of one or more alloying elements, besides carbon. These steels are generally intended for application where saving in weight can be effected by reason of their greater strength and atmospheric corrosion resistance. Standards of high tensile steel have been given in IS : 961-1975. It has been classified into two grades designated as St 58-HT and St 55-HTW. St 58-HT is intended for use in structures where fabrication is done by methods other than welding. St 55-HTW is intended for use in structures where welding is employed for fabrication. The high tensile steel is also available in copper bearing quality and two grades are designated as St 58-HTC and St 55-HTWC. The steel conforming to IS : 961 is suitable for bridges and general building construction.

1.6.5 IS : 8500-1977 weldable structural steel (medium and high strength qualities)

Various medium and high strength qualities of weldable structural steel are, Fe 440 (HT1 and HT2) Fe 540 (HT, HTA and HTB), Fe 570 HT, Fe 590 HT and Fe 640 HT.

1.7 PRODUCTION OF STEEL

The steel is produced in the form of ingots and converted to different shapes. In our country, Tata Iron and Steel Company, Indian Iron and Steel Company, Mysore Iron and Steel Company and Hindustan Steel produce steel at their plants

1.8 RECENT DEVELOPMENTS IN MATERIAL

A number of developments in material such as steel have been made recently. The weldable qualities of steel (IS : 2062) designated as St 42-W and IS : 961 designated as St-55-HTW are developed with the large scale use of welding. IS : 961 has been developed with high tensile strength and there is saving in weight due to enhanced mechanical properties. Its weldable quality is advantageous for composite construction.

LESSON 2. Rolled Structural Steel Sections

2.1 INTRODUCTION

The steel sections manufactured in rolling mills and used as structural members are known as rolled structural steel sections. The steel sections are named according to their cross sectional shapes. The shapes of sections selected depend on the types of members which are fabricated and to some extent on the process of erection. Many steel sections are readily available in the market and have frequent demand. Such steel sections are known as regular steel sections. Some steel sections are rarely used. Such sections are produced on special requisition and are known as special sections. 'ISI Handbook for Structural Engineers' gives nominal dimensions, weight and geometrical properties of various rolled structural steel sections.

2.2 TYPES OF ROLLED STRUCTURAL STEEL SECTIONS

The various types of rolled structural steel sections manufactured and used as structural members are as follows:

1. Rolled Steel I-sections (Beam sections).

2. Rolled Steel Channel Sections.

3. Rolled Steel Tee Sections.

4. Rolled Steel Angles Sections.

5. Rolled Steel Bars.

6. Rolled Steel Tubes.

7. Rolled Steel Flats.

8. Rolled Steel Sheets and Strips.

9. Rolled Steel Plates.

2.3 ROLLED STEEL BEAM SECTIONS

The rolled steel beams are classified into following four series as per BIS : (IS : 808-1989)

1. Indian Standard Joist/junior Beams   ISJB

2. Indian Standard Light Beams   ISLB

3. Indian Standard Medium Weight Beams   ISMB

4. Indian Standard Wide Flange Beams	ISWB

The rolled steel columns/heavy weight beams are classified into the following two series as per BIS (IS : 808-1989)

1. Indian Standard Column Sections	ISSC
2. Indian Standard Heavy Weight Beams	ISHB

The cross section of a rolled steel beam is shown in Fig. 2.1. The beam section consists of web and two flanges. The junction between the flange and the web is known as fillet. These hot rolled steel beam sections have sloping flanges. The outer and inner faces are inclined to each other and they intersect at an angle varying from 1½ to 8° depending on the section and rolling mill practice. The angle of intersection of ISMB section is 8°. Abbreviated reference symbols (JB, LB, MB, WB, SC and HB) have been used in designating the Indian Standard Sections as per BIS (IS 808-1989)

The rolled steel beams are designated by the series to which beam sections belong (abbreviated reference symbols), followed by depth in mm of the section and weight in kN per metre length of the beam, e.g., MB 225 @ 0.312 kN/m. H beam sections of equal depths have different weights per metre length and also different properties e.g., WB 600 @ 1.340 kN/m, WB 600 @ 1.450 kN/m, HB 350 @0.674 kN/m, HB 350 @0.724 kN/m.

I-sections are used as beams and columns. It is best suited to resist bending moment and shearing force. In an I-section about 80 % of the bending moment is resisted by the flanges and the rest of the bending moment is resisted by the web. Similarly about 95% of the shear force is resisted by the web and the rest of the shear force is resisted by the flanges. Sometimes I-sections with cover plates are used to resist a large bending moment. Two I-sections in combination may be used as a column.

2.4 ROLLED STEEL CHANNEL SECTIONS

The rolled steel Channel sections are classified into four categories as per ISI, namely,

1. Indian Standard Joist/Junior Channels	ISJC
2. Indian Standard Light Channels	ISLC
3. Indian Standard Medium Weight Channels	ISMC
4. Indian Standard Medium Weight Parallel Flange Channels	ISMCP

The cross section of rolled steel channel section is shown in Fig 2.2. The channel section consists of a web and two flanges. The junction between the flange and the web is known as fillet. The rolled steel channels are designated by the series to which channel section belong (abbreviated reference symbols), followed by depth in mm of the section and weight in kN per metre length of the channel, e.g., MC 225 @ 0.261 kN/m

Channels are used as beams and columns. Because of its shape a channel member affords connection of an angle to its web. Built up channels are very convenient for columns. Double

channel members are often used in bridge truss. The channels are employed as elements to resist bending e.g., as purlins in industrial buildings. It is to note that they are subjected to twisting or torsion because of absence of symmetry of the section with regards to the axis parallel to the web, i.e., yy-axis. Therefore, it is subjected to additional stresses. The channel sections are commonly used as members subjected to axial compression in the shape of built-up sections of two channels connected by lattices or batten plates or perforated cover plates. The built-up channel sections are also used to resist axial tension in the form of chords of truss girders.

As per IS : 808-1989, following channel sections have also been additionally adopted as Indian Standard Channel Secions

1. Indian Standard Light Channels with parallel flanges ISLC(P)

2. Medium weight channels   MC

3. Medium weight channels with parallel flanges   MCP

4. Indian Standard Gate Channels   ISPG

In MC and MCP channel sections, some heavier sections have been developed for their intended use in wagon building industry. The method of designating MC and MCP channels is also same as that for IS channels.

2.5 ROLLED STEEL TEE SECTIONS

The rolled steel tee sections are classified into the following five series as per ISI:

1. Indian Standard Normal Tee Bars   ISNT

2. Indian Standard Wide flange Tee Bars   ISHT

3. Indian Standard Long Legged Tee Bars   ISST

4. Indian Standard Light Tee Bars   ISLT

5. Indian Standard Junior Tee Bars   ISJT

The cross section of a rolled steel tee section has been shown in Fig. 2.3. The tee section consists of a web and a flange. The junction between the flange and the web is known as fillet. The rolled steel tee sections are designated by the series to which the sections belong (abbreviated reference symbols) followed by depth in mm of the section and weight in kN per metre length of the Tee, e.g., HT 125 @ 0.274 kN/m. The tee sections are used to transmit bracket loads to the columns. These are also used with flat strips to connect plates in the steel rectangular tanks.

A per IS: 808-1984, following T-sections have also been additionally adopted as Indian Standard T-sections.

1.	Indian Standard deep legged Tee bars	ISDT
2.	Indian Standard Slit medium weight Tee bars	ISMT
3.	Indian Standard Slit Tee bars from I-sections	ISHT

It is to note that as per IS 808 (part II) 1978, H beam sections have been deleted.

2.6 ROLLED STEEL ANGLE SECTIONS

The rolled steel angle sections are classified in to the following three series.

1.	Indian Standard Equal Angles	ISA
2.	Indian Standard Unequal Angles	ISA
3.	Indian Standard Bulb Angles	ISBA

Angles are available as equal angles and unequal angles. The legs of equal angle sections are equal and in case of unequal angle section, length of one leg is longer than the other. Thickness of legs of equal and unequal angle sections are equal. The cross section of rolled equal angle section, unequal angle section and that of bulb angle section is shown in Fig. 2.4. The bulb angle consists of a web a flange and a bulb projecting from end of web.

The rolled steel equal and unequal angle sections are designated by abbreviated reference symbols ∟ followed by length of legs in mm and thickness of leg, e.g.,

∟ 130 x 130 x 8 mm (∟ 130 130 @ 0.159 kN/m)

∟ 200 x 100 x 10 mm (∟ 200 100 @ 0.228 kN/m)

The rolled steel bulb angles are designated by BA, followed by depth in mm of the section and weight in kN per metre length of bulb angle.

Angles have great applications in the fabrications. The angle sections are used as independent sections consisting of one or two or four angles designed for resisting axial forces (tension and compression) and transverse forces as purlins. Angles may be used as connecting elements to connect structural elements like sheets or plates or to form a built up section. The angle sections are also used as construction elements for connecting beams to the columns and purlins to the chords of trusses in the capacity of beam seats, stiffening ribs and cleat angles. The bulb angles are used in the ship buildings. The bulb helps to stiffen the outstanding leg when the angle is under compression.

As per IS : 808-1984, some supplementary angle sections have also additionally adopted as Indian Standard angle sections. However prefix ISA has been dropped. These sections are designated by the size of legs followed by thickness e.g., ∟ 200 150 x 15.

2.7 ROLLED STEEL BARS

The rolled steel bars are classified in to the following two series:

1. Indian Standard Round Bars   ISRO

2. Indian Standard Square Bars   ISSQ

The rolled steel bars are used as ties and lateral bracing. The cross sections of rolled steel bars are shown in Fig. 2.5. The rolled steel bars are designated by abbreviated reference symbol RO followed by diameter in case of round bars and ISSQ followed by side width of bar sections. The bars threaded at the ends or looped at the ends are used as tension members.

2.8 ROLLED STEEL TUBES

The rolled steel tubes are used as columns and compression members and tension members in tubular trusses. The rolled steel tubes are efficient structural sections to be used as compression members. The steel tube sections have equal radius of gyration in all directions. The cross section of rolled steel tube is shown in Fig. 2.6.

2.9 ROLLED STEEL FLATS

The rolled steel flats are used for lacing of elements in built up members, such as columns and are also used as ties. The cross section of rolled steel flat is shown in Fig. 2.7. the rolled steel flats are designated by width in mm of the section followed by letters (abbreviated reference symbol) F and thickness in mm, e.g., 50 F 8. This means a flat of width 50 mm and thickness 8 mm. The rolled steel flats are used as lattice bars for lacing the elements of built up columns. The rolled steel flats are also used as tension members and stays.

2.10 ROLLED STEEL SHEETS AND STRIPS

The rolled steel sheet is designated by abbreviated reference symbol SH followed by length in mm x width in mm x thickness in mm of the sheet. The rolled steel strip is designated as ISST followed by width in mm x thickness in mm, e.g., SH 2000 x 600 x 8 and ISST 250 x 2.

2.11 ROLLED STEEL PLATES

The rolled steel plates are designated by abbreviated reference symbol PL followed be length in mm x width in mm x thickness in mm of the plates, e.g., PL 2000 x 1000 x 6.

The rolled steel sheets and plates are widely used in construction. Any sections of the required dimensions, thickness and configuration may be produced by riveting or welding the separate plates. The rolled plates are used in the web and flanges of plate girders, plated beams and chord members and web members of the truss bridge girders. The rolled steel plates are used in special plate structures, e.g., shells, rectangular and circular steel tanks and steel chimneys.

2.12 RECENT DEVELOPMENTS IN SECTIONS

The rolled steel beam sections with parallel faces of flanges are recently developed. These beam sections are called as parallel flange sections. These sections have increased moment of inertia, section modulus and radius of gyration about the weak axis. Such sections used as beams and columns have more stability. Theses sections possess ease of connections to other sections as no packing is needed as in beams of slopping flanges. The parallel flange beam sections are not yet rolled in our country.

New welded sections using plates and other steel sections are developed because of welding. The development of beams with tapered flanges and tapered depths is also due to welding. The open web sections and the castellated beams were also developed with the rapid use of welding.
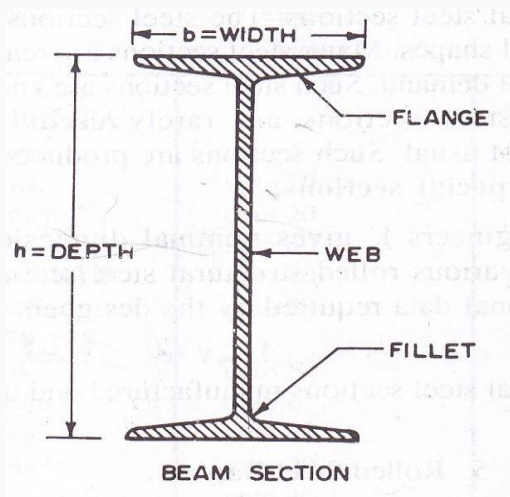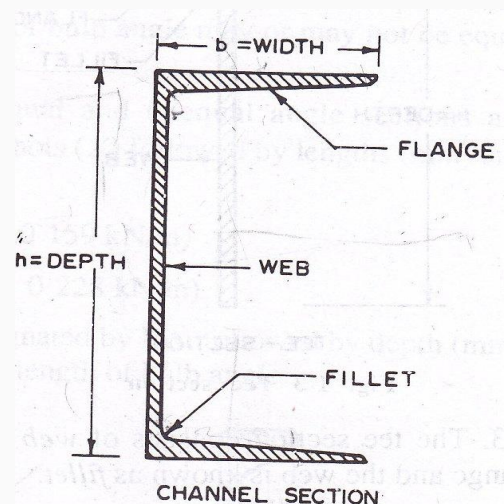
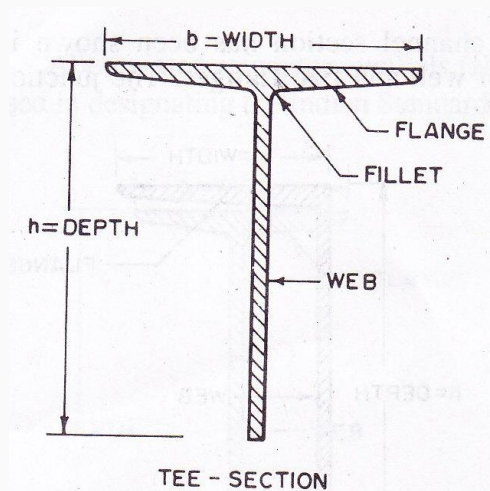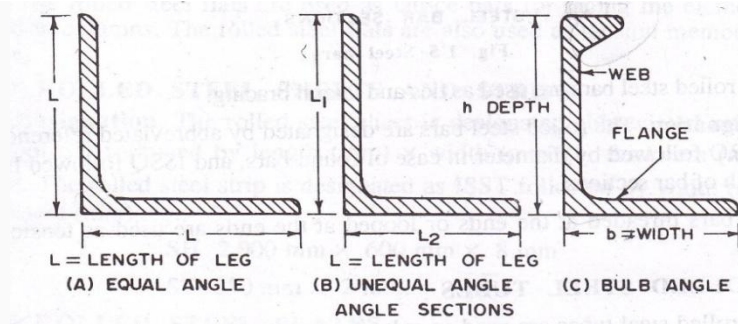Fig. 2.1 Beam section   Fig. 2.2 Channel section
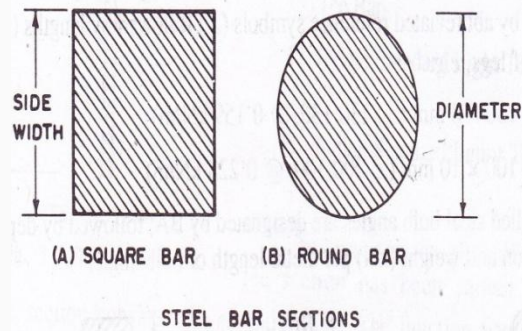
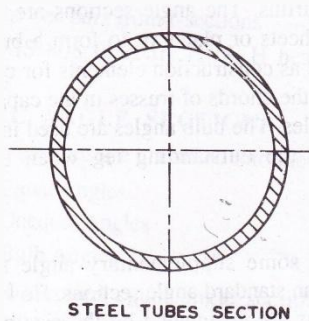Fig. 2.3 Tee section

Fig. 2.4 Angle section
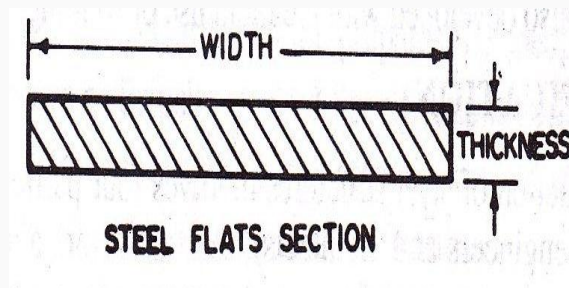

Fig. 2.5 Bar section

Fig. 2.6 Tube section

Fig. 2.7 Flat section

MODULE 2.

LESSON 3. Loads on structures

3.1 INTRODUCTION

The structures and structural members are designed to meet the functional and structural aspects. Both aspects are interrelated. The functional aspect takes in to consideration the purpose for which the building or the structure is designed. It includes the determination of location and arrangement of operating utilities, occupancy, fire safety and compliance with hygienic, sanitation, ventilation, special equipment, machinery or other features, incident to the proper functioning of the structures. In the structural aspect, it is ensured that the building or the structure is structurally safe, strong, durable and economical. The minimum requirements pertaining to the structural safety of buildings are being covered in codes dealing with loads by way of laying down minimum design loads which have to be assumed for dead loads, imposed loads, wind loads and other external loads, the structure would be required to bear. Unnecessarily, heavy loads without proper assessment should not be assumed. The structures are designed between two limits, namely, the structural safety and economy. The structures should be strong, stable and stiff.

Estimation of the loads for which a structure should be designed is one of the most difficult problems in structural design. The designer must be able to study the loads which are likely to be acting on the structure throughout its life time and the loads to which the structure may be subjected during a short period. It is also necessary to consider the combinations of loads for which the structure has to be designed.

3.2 TYPES OF LOADS

The loads to which a structure, will be subjected to consist of the following

1. Dead loads,

2. Live loads or imposed loads,

3. Wind load,

4. Snow load

5. Seismic load

6. Temperature effects

In addition o the above loads, following forces and effects are also considered while designing the structures.

1. Foundation movements

2. Elastic axial shortening

3. Soil and fluid pressures

4. Vibrations

5. Fatigue

6. Impact

7. Erection loads

8. Stress concentration effects

3.3 DEAD LOADS

Dead load of a structure means the weight of the structure itself. The dead load in a building will consist of the weight of all wall partitions, floors and roofs. Loads due to partition shall be estimated on the basis of actual constructional details of the proposed partitions and their positioning in accordance with plans and the loads thus estimated shall be included in the dead load for the design of the floors and the supporting structures. If the loads due to partitions cannot be actually computed for want of data, the floors and the supporting structures shall be designed to carry in addition to other loads a uniformly distributed dead load per square metre of not less than 33⅓ per cent of the weight per metre run of finished partitions over the entire floor area subjected to minimum uniformly distributed load of 1000 N/m^2 in the case of floors used for office purposes. Dead loads can be estimated using the unit-weight of materials used in building construction as per IS : 875 (part I) -1987

3.4 LIVE LOADS OR IMPOSED LOADS

Live loads are the loads which vary in magnitude and in positions. Live loads are also known as imposed or transient loads. Imposed loads consist of all loads other than dead loads. Live loads are assumed to be produced by the intended use of occupancy in building including the weight of movable partitions, distributed loads, concentrated loads, loads due to impact and vibration and snow loads. Live loads are expressed as uniformly distributed static loads. Live loads include the weight of materials stored, furniture and movable equipments. Efforts have been made at the international level to decide live loads on floors and these have been specified in the International standards (2103 Imposed floor loads in residential and public building and 2633 Determination of imposed floor loads in production buildings and warehouses). These codes have been published in the International Organization.

Code IS : 875 (part 2) -1987 defines the principal occupancy for which a building or part of a building is used or intended to be used. The buildings are classified according to occupancy as per IS : 875 (part 2)-1987.

3.5 WIND LOAD

The wind loads are the transient loads. The wind usually blows horizontal to the ground at high wind speeds. The vertical components of atmospheric motion are relatively small, therefore, the term wind denotes almost exclusive the horizontal wind. The winds of very

high speeds and very short duration are called Kal Baisaki or Norwesters occur fairly frequently during summer months over North East India.

The liability of a building or a structure to high wind pressure depends not only upon the geographical location and proximity of other obstructions to airflow but also upon the characteristics of the structure itself. In general, wind speed in the atmospheric boundary layer increases with height from zero at ground level to maximum at a height called the gradient height. The variation of wind with height depends primarily on the terrain conditions. However, the wind speed at any height never remains constant and it has been found convenient to resolve its instantaneous magnitude in to an average or mean value and a fluctuating component around this average value. The magnitude of fluctuating component of the wind speed is called gust, it depends upon averaging time. In general, smaller the averaging interval, greater is the magnitude of the gust speed. The wind load depends upon terrain, height of the structure and the shape and size of structure. It is essential to know the following terms to study the new concept of wind as described in IS : 875 (Part 3) – 1987

3.6 SNOW LOAD

The snow load depends upon latitude of place and atmospheric humidity. The snow load acts vertically and it is expressed in kN/m^2 of plan area. The actual load due to snow depends upon the shape of the roof and its capacity to retain the snow. When actual data for snow load is not available, snow load may be assumed to be 25 N/m^2 per mm depth of snow. It is usual practice to assume that snow load and maximum wind load will not be acting simultaneously on the structure.

3.7 SEISMIC LOAD (EARTHQUAKE LOAD)

It becomes essential to consider 'seismic load' in the design of structure, if the structure is situated in the seismic areas. The seismic areas are the regions which are geologically young and unstable parts and which have experienced earthquakes in the past and are likely to experience earthquakes in future. The Himalayan region, Indo Gangetic Plain, Western India, Cutch and Kathiawar are the places in our country which experience earthquakes frequently. Sometimes these earthquakes are violent also. Seismic load is caused by the shocks due to an earthquake. The earthquakes range from small tremors to severe shocks. The earthquake shocks cause movement of ground, as a result of which the structure vibrates. The vibrations caused because of earthquakes may be resolved in three perpendicular directions. The horizontal direction of vibration dominates over other directions. In some cases structures are designed for horizontal seismic forces only and in some case both horizontal seismic forces and vertical seismic forces are taken in to account. The seismic accelerations for the design may be arrived at from seismic coefficient, which is defined as the ratio of acceleration due to earthquakes and acceleration due to gravity. Our country has been divided in to seven zones for determining seismic coefficients. The seismic coefficients have also been recommended for different types of soils for the guidance of designers. IS : 1893-1962 Indian Standard Recommendations for Earthquake Resistant Design of Structure, may be referred to for actual design.

3.8 SOIL AND HYDROSTATIC PRESSURE

The pressure exerted by soil or water or both should be taken in to consideration for the design of structures or parts of structure which are below ground level. The soil pressure and hydrostatic pressure may be calculated from established theories.

3.9 ERECTION EFFECTS

The erection effects include all effects to which a structure or part of structure is subjected during transportation of structural members and erection of structural member by equipments. Erection effects also take in to account the placing or storage of construction materials. The proper provisions shall be made, e.g., temporary bracings, to take care of all stresses caused during erection. The stress developed because of erection effects should not exceed allowable stresses.

3.10 DYNAMIC EFFECTS (IMPACTS AND VIBRATIONS)

The moving loads on a structure cause vibrations and have also impact effect. The dynamic effects resulting from moving loads are accounted for, by impact factor. The live load is increased by adding to it the impact load. The impact load is determined by the product of impact factor and live load.

3.11 TEMPERATURE EFFECTS

The variation in temperature results in expansion and contraction of structural material. The range of variation in temperature varies from localities to localities, season to season and day to day. The temperature effects should be accounted for properly and adequately. The allowable stress should not be exceeded by stress developed because of design loads and temperature effects.

3.12 LOAD COMBINATIONS

All the parts of the steel structure shall be capable of sustaining the most adverse combination of the dead loads, prescribed live loads, wind loads, earthquake loads where applicable and any other forces or loads to which the steel structure may reasonably be subjected without exceeding the stress specified. The load combinations for design purpose shall be the one that produces maximum forces and effects and consequently maximum stresses from the following combinations

1. Dead load + Imposed (live) load

2. Dead load + Imposed (live) load + wind or earthquake loads and

3. Dead load + wind or earthquake loads

LESSON 4. Stresses on structures

4.1 INTRODUCTION

When a structural member is loaded, deformation of the member takes place and resistance is set up against deformation. This resistance to deformation is known as stress. The stress is defined as force per unit cross sectional area. The nature of stress developed in the structural member depends upon nature of loading on the member.

4.2 TYPES OF LOADS

The following are the various types of stresses:

1. Axial stress (direct stress) :   i. Tensile stress   ii. Compressive stress

2. Bearing stress

3. Bending stress

4. Shear stress

A member may be subjected to combined direct and bending stress. Such stress is known as combined stress. The tensile stresses are taken as positive and compressive stress as negative. This sign convention for stresses is convenient as a structural member elongates on application of tensile load and shortens on application of compressive load.

4.3 STRESS-STRAIN RELATIONSHIP FOR MILD STEEL

When a mild steel bar is subjected to a tensile load, it elongates. The elongation per unit length is known as strain. The stress is proportional to stain within limit of proportionality. The stress-strain relationship for mild steel can be studied by plotting stress-strain curve. The stress and load may be plotted on y-axis and strain may be plotted on x-axis as shown in Fig. 4.1.
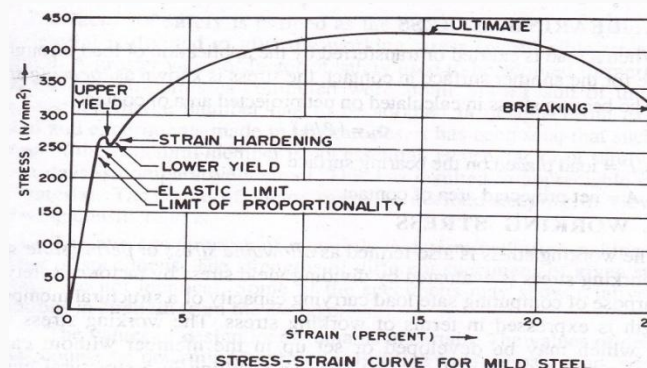

Fig. 4.1 Stress-strain curve

When the tensile load increases with increase in strain, stress-strain curve follows a straight line relationship up to 'Limit of proportionality'. The limit of proportionality is defined as stress beyond which straight line relationship ceases between stress and strain. Beyond the limit of proportionality stress approaches the elastic limit. The elastic limit is defined as the maximum stress up to which a specimen regains its original length on the removal of the applied load. There is hardly any distinct difference in the position of limit of proportionality and elastic limit. Practically, position of limit of proportionality coincides with the elastic limit. When the specimen is loaded beyond the elastic limit, the specimen does not resume its original length on the removal of applied load and a little strain is left in the specimen. This little strain is known as residual strain or permanent set.

When the tensile load further increases the stress reaches 'yield stress' and material starts yielding. The stress-strain curve suddenly falls showing a decrease in stress. The distinct position from where sudden fall of curve occurs marks the upper yield point and the position up to which fall of curve occurs is known as lower yield point. The material stretches suddenly at constant stress. The adjustment of stress takes place in the elements of material in between upper yield point and lower yield point. On further increase of load, stress increases with the increase of strain. However, strain increases more rapidly. Finally the load reaches the value of 'ultimate load'. The ultimate load is defined as maximum load, which can be placed prior to the breaking of specimen. The stress corresponding to the ultimate load is known as 'ultimate stress'. The stress-strain curve suddenly falls with rapid increase in strain and specimen breaks. The load corresponding to breaking position is known as 'breaking load'. The cross-section of specimen decreases. If actual breaking stress is computed on the basis of decreased cross-sectional area, the breaking stress will be found to be more than the ultimate stress.

The boundaries of grains of mild steel are composed of brittle material. This forms a rigid skeleton. The rigid skeleton prevents plastic deformation of the grains at low stress and shows upper yield point in stress-strain curve. At upper yield point, this rigid skeleton breaks down. As a result of this, the stress in material drops down without elongation from upper yield point to lower yield point. This is followed by sudden stretching of the material at constant stress from lower yield point up to strain hardening.

4.4 TENSILE STRESS

When a structural member is subjected to direct axial tensile load, the stress is known as tensile stress (σ_{at}). The tensile stress is calculated on net cross-sectional area of the member:

$$\sigma_{at} = (P_t / A_n)$$

Where P_t is the direct axial tensile load and A_n is the net cross-sectional area of the member.

 4.5 COMPRESSIVE STRESS

When a structural member is subjected to direct axial compressive load, the stress is known as compressive stress (σ_{ac}). The compressive stress is calculated on gross cross-sectional area of the member

$$\sigma_{ac} = (P_c / A_g)$$

Design of Structures

Where P_c is the direct axial compressive load and A_g is the gross cross-sectional area of the member

4.6 BEARING STRESS

When a load is exerted or transferred by the application of load through one surface for the another surface in contact, the stress is known as 'bearing stress'(σ_b). the bearing stress is calculated on net projected area of contact

$\sigma_b = (P/A)$

Where P is load placed on the bearing suface and A is the net projected area of contact.

4.7 WORKING STRESS

The working stress is also termed as allowable stress or permissible stress. The working stress is evaluated by dividing yield stress by factor of safety. For the purpose of computing safe load carrying capacity of a structural member, its strength is expressed in terms of working stress. The working stress is the stress which may be developed or set up in the member without causing structural damage to it. The actual stress resulting in a structural member from design loads should not exceed working stresses. This ensures the safety of structural member. The maximum working stresses are adopted from IS : 800-1984.

4.8 INCREASE IN PERMISSIBLE STRESS

A structure may be subjected to the different combinations of loads. These loads in combinations do not act for long period. Most of the national codes allow some increase in permissible stresses. Increase in permissible stresses as per IS : 800 is taken as follows:

1. When the effect of wind or seismic load is taken in to account, the permissible stress in steel are increased by 33⅓ percent.

2. For rivets, bolts and tension rods, the permissible stresses are increased by 25 per cent, when the effect of wind or seismic load is taken in to account.

The increased values of permissible stress must not exceed yield stress of the material.

4.9 FACTOR OF SAFETY

The factor of safety is defined as the factor by which the yield stress of the material is divided to give the working stress (permissible stress) in the material. A greater value of factor of safety results a larger cross-section of the member had to be adopted in design. If the factor of safety is comparatively small, results in appreciable saving in the material. The value of factor of safety is decided keeping in view of the following considerations.

1. The average strength of materials is determined after making test on number of specimens

2. The value of design loads remains uncertain

3. The values of internal forces in many structures depend upon methods of analysis

4. During fabrication, structural steel is subjected to different operations which causes the structural element are subjected to uncertain erection stress

5. The variations in temperatures and settlement of supports are uncertain

6. The failure of some small or some elements of a structure is less serious and less disastrous than the failure of large structure or main element of a structure

4.10 METHODS OF DESIGN

The following methods may be employed for the design of the steel frame work:

1. Simple design

2. Semi-rigid design

3. Fully rigid design and

4. Plastic design

4.10.1 Simple Design

This method is based on elastic theory and applies to structure in which the end connections between members are such that they will not develop restraint moments adversely affecting the members and the structures as a whole and in consequence the structure may be assumed to be pin jointed.

4.10.2 Semi-rigid design

This method permits a reduction in the maximum bending moment in beams suitably connected to their supports, so as to provide a degree of direction fixity. In the case of triangulated frames, it permits rotation account being taken of the rigidity of the connections and the moment of interaction of members. In cases where this method of design is employed, it is ensured that the assumed partial fixity is available and calculations based on general or particular experimental evidence shall be made to show that the stresses in any part of the structure are not in excess of those laid down in IS : 800-1984.

4.10.3 Fully rigid design

This method assumes that the end connections are fully rigid and are capable of transmitting moments and shears. It is also assumed that the angle between the members at the joint does not change, when it is subjected to loading. This method gives economy in the weight of steel used when applied in appropriate cases. The end connections of members of the frame shall have sufficient rigidity to hold virtually unchanged original angles between such members and the members they connect. The design should be based on accurate methods of elastic analysis and calculated stresses shall not exceed permissible stress.

4.10.4 **Plastic design**

The method of plastic analysis and design is recently (1935) developed and all the problems related to this are not yet decided. In this method, the structural usefulness of the material is limited up to ultimate load. This method has its main application in the analysis and design of statically indeterminate framed structures. This method provides striking economy as regards the weight of the steel. This method provides the margin of safety in terms of load factor which one is not less than provided in elastic design. A load factor of 1.85 is adopted for dead load plus live load and 1.40 is adopted for dead load, live load and wind or earthquake forces. The deflection under working load should not exceed the limits prescribed in IS : 800-1984.

4.11 STABILITY OF STRUCTURE

According to the stability requirement, the stability of a structure as a whole against overturning is ensured so that the restoring moment is greater than the maximum overturning moment. The restoring moment shall be not less than the sum of 1.2 times the maximum overturning moment due to the characteristic dead load and 1.4 times the maximum overturning moment due to characteristic imposed loads.

The structure should have adequate factor of safety against sliding due to the most adverse combination of the applied loads. The structure shall have a factor of safety against sliding not less than 1.4 under the most adverse combination of the applied characteristic forces. In case only dead loads are acting, only 0.9 times the characteristic dead load shall be taken in to account.

To ensure stability at all times, account shall be taken of probable variations in dead load during construction, repair or other temporary measures. The wind and seismic loading shall be treated as imposed loading. In designing the framework of a building, provisions shall be made by adequate moment connections or by a system of bracings to effectively transmit all the horizontal forces to the foundations.

MODULE 3.

### LESSON 5. Riveted Connections

5.1 INTRODUCTION

In engineering practice it is often required that two sheets or plates are joined together and carry the load in such ways that the joint is loaded. Many times such joints are required to be leak proof so that gas contained inside is not allowed to escape. A riveted joint is easily conceived between two plates overlapping at edges, making holes through thickness of both, passing the stem of rivet through holes and creating the head at the end of the stem on the other side. A number of rivets may pass through the row of holes, which are uniformly distributed along the edges of the plate. With such a joint having been created between two plates, they cannot be pulled apart. If force at each of the free edges is applied for pulling the plate apart the tensile stress in the plate along the row of rivet hole and shearing stress in rivets will create resisting force. Such joints have been used in structures, boilers and ships. The following are the usual applications for connection.

1. Screws ,

2. Pins and bolts,

3. Cotters and Gibs,

4. Rivets,

5. Welds.

Of these screws, pins, bolts, cotters and gibs are used as temporary fastening i.e., the components connected can be separated easily. Rivets and welds are used as permanent fastenings i.e., the components connected are not likely to require separation.

5.2 RIVETS

Rivet is a round rod which holds two metal pieces together permanently. Rivets are made from mild steel bars with yield strength ranges from 220 N/mm^2 to 250 N/mm^2. A rivet consists of a head and a body as shown in Fig 5.1. The body of rivet is termed as shank. The head of rivet is formed by heating the rivet rod and upsetting one end of the rod by running it into the rivet machine. The rivets are manufactured in different lengths to suit different purposes. The size of rivet is expressed by the diameter of the shank.
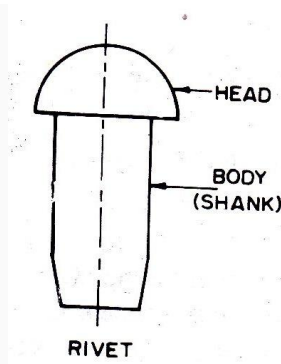
RIVET

Holes are drilled in the plates to be connected at the appropriate places. For driving the rivets, they are heated till they become red hot and are then placed in the hole. Keeping the rivets pressed from one side, a number of blows are applied and a head at the other end is formed. When the hot rivet so fitted cools it shrinks and presses the plates together. These rivets are known as hot driven rivets. The hot driven rivets of 16 mm, 18 mm, 20 mm and 22 mm diameter are used for the structural steel works.

Some rivets are driven at atmospheric temperature. These rivets are known as cold driven rivets. The cold driven rivets need larger pressure to form the head and complete the driving. The small size rivets ranging from 12 mm to 22 mm in diameter may be cold driven rivets. The strength of rivet increases in the cold driving. The use of cold driven rivets is limited because of equipment necessary and inconvenience caused in the field.

The diameter of rivet to suit the thickness of plate may be determined from the following formulae:

1. Unwins's formula $d = 6.05\, t^{0.5}$

2. The French formula $d = 1.5\, t + 4$

3. The German formula $d = (50\, t - 2)^{0.5}$

Where d= nominal diameter of rivet in mm and t= thickness of plate in mm.

5.3 RIVET HEADS

The various types of rivet heads employed for different works are shown in Fig. 5.2. The proportions of various shapes of rivet heads have been expressed in terms of diameter 'D' of the shank of rivet. The snap head is also termed as round head and button head. The snap heads are used for rivets connecting structural members. Sometimes it becomes necessary to flatten the rivet heads so as to provide sufficient clearance. A rivet head which has the form of a truncated cone is called a countersunk head. When a smooth flat surface is required, it is necessary to have rivets countersunk and chipped.

(A) SNAP HEAD (B) PAN HEAD (C) PAN HEAD WITH TAPERED NECK (D) ROUND COUNTER SUNK HEAD (E) COUNTER HEAD

RIVET HEADS
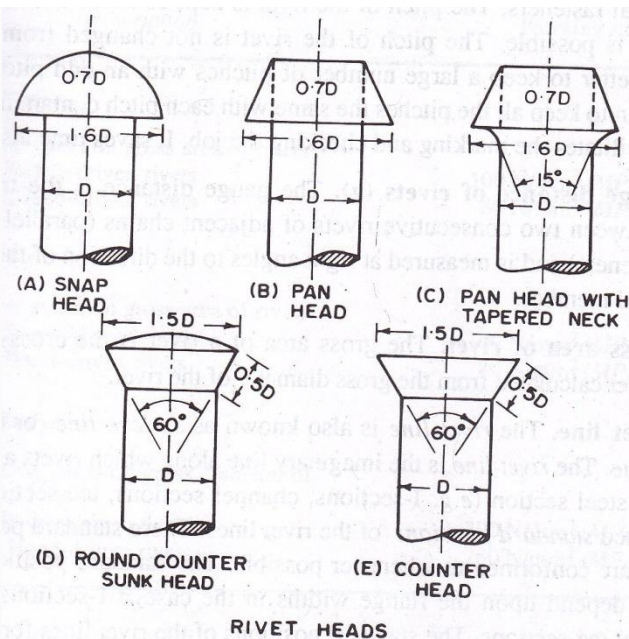
5.4 RIVET HOLES

The rivet holes are made in the plates or structural members by punching or drilling. When the holes are made by punching, the holes are not perfect, but taper. A punch damages the material around the hole. The operation known as reaming is done in the hole made by punching. When the hole are made by drilling, the holes are perfect and provide good alignment for driving the rivets. The diameter of a rivet hole is made larger than the nominal diameter of the rivet by 1.5 mm of rivets less than or equal to 25 mm diameter and by 2 mm for diameter exceeding 25 mm.

5.5 DEFINITIONS OF TERMS USED IN RIVETING

5.5.1 **Nominal diameter of rivet (d):**

The nominal diameter of a rivet means the diameter of the cold shank before driving.

5.5.2 **Gross diameter of rivet (D):**

The diameter of the hole is slightly greater than the diameter of the rivet shank. As the rivet is heated and driven, the rivet fills the hole fully. The gross or effective diameter of a rivet means the diameter of the hole or closed rivet. Strengths of rivet are based on gross diameter.

5.5.3 **Pitch of rivet (p):**

The pitch of rivet is the distance between two consecutive rivets measured parallel to the direction of the force in the structural member, lying on the same rivet line. Minimum pitch should not be less than 2.5 times the nominal diameter of the rivet. As a thumb rule pitch equal to 3 times the nominal diameter of the rivet is adopted. Maximum pitch shall not exceed 32 times the thickness of the thinner outside plate or 300 mm whichever is less.

5.5.4 **Gauge distance of rivets (g):**

The gauge distance is the transverse distance between two consecutive rivets of adjacent chains (parallel adjacent lines of fasteners) and is measured at right angles to the direction of the force in the structural member.

5.5.5 **Gross area of rivet:**

The gross area of rivet is the cross sectional area of a rivet calculated from the gross diameter of the rivet.

5.5.6 **Rivet line:**

The rivet line is also known as scrieve line or back line or gauge line. The rivet line is the imaginary line along which rivets are placed. The rolled steel sections have been assigned standard positions of the rivet lines. The standard position of rivet lines for the various sections may be noted from ISI Handbook No.1 for the respective sections. These standard positions of rivet lines are conformed to whenever possible. The departure from standard position of the rivet lines may be done if necessary. The dimensions of rivet lines should be shown irrespective of whether the standard positions have been followed or not.

5.5.7 **Staggered pitch:**

The staggered pitch is also known as alternate pitch or reeled pitch. The staggered pitch is defined as the distance measured along one rivet line from the centre of a rivet on it to the centre of the adjoining rivet on the adjacent parallel rivet line. One or both the legs of an angle section may have double rivet lines. The staggered pitch occurs between the double rivet lines.

5.6 TYPES OF JOINTS

Riveted joints are mainly of two types, namely, Lap joints and Butt joints.

5.6.1 **Lap Joint:** Two plates are said to be connected by a lap joint when the connected ends of the plates lie in parallel planes. Lap joints may be further classified according to number of rivets used and the arrangement of rivets adopted. Following are the different types of lap joints.

1. Single riveted lap joint (Fig.5.3),

2. Double riveted lap joint:

a. Chain riveted lap joint (Fig.5.4)   b. Zig-zag riveted lap joint (Fi.5.5)

(A) LAP JOINT (SINGLE RIVETED)

(A) CHAIN RIVETED LAP JOINT


(B) ZIG-ZAG RIVETED LAP JOINT
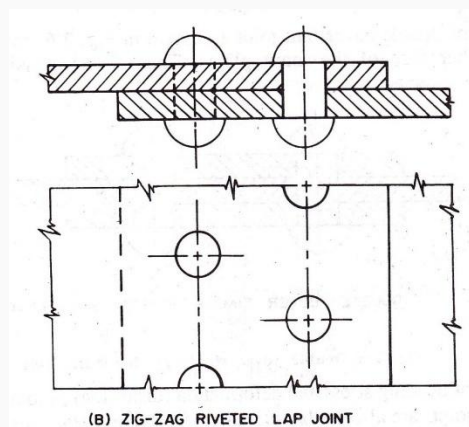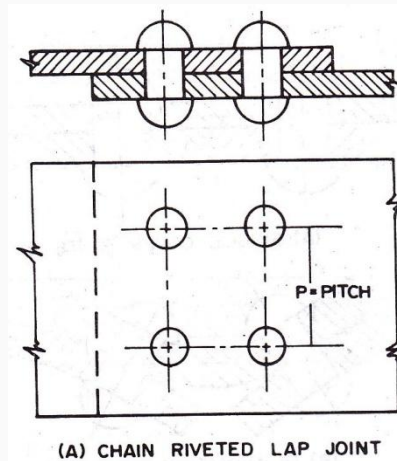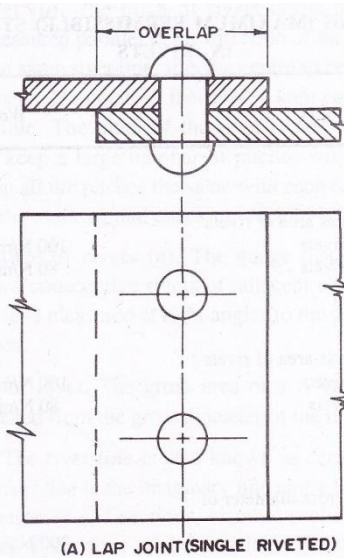
5.6.1 Butt Joint:

In a butt joint the connected ends of the plates lie in the same plane. The abutting ends of the plates are covered by one or two cover plates or strap plates. Butt joints may also be classified into single cover but joint, double cover butt joints. In single cover butt joint, cover plate is provided on one side of main plate (Fig.5.6). In case of double cover butt joint, cover plates are provided on either side of the main plate (Fig.5.7). Butt joints are also further classified according to the number of rivets used and the arrangement of rivets adopted.

1. Double cover single riveted but joint

2. Double cover chain riveted butt joint

3. Double cover zig-zag riveted butt joint

(A) SINGLE COVER PLATE BUTT-JOINT

DOUBLE COVER SINGLE RIVETED BUTT JOINT
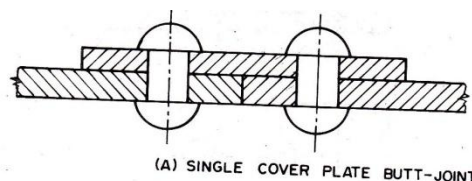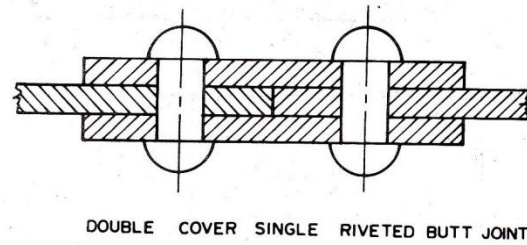
5.7 FAILURE OF A RIVETED JOINT

Failure of a riveted joint may take place in any of the following ways

1. Shear failure of rivets

2. Bearing failure of rivets

3. Tearing failure of plates

4. Shear failure of plates

5. Bearing failure of plates

6. Splitting/cracking failure of plates at the edges

5.7.1 **Shear failure of rivets :**

Plates riveted together and subjected to tensile loads may result in the shear of the rivets. Rivets are sheared across their sectional areas. Single shear occurring in a lap joint and double shear occurring in but joint (Fig.5.8)


(A)

(B)

SHEAR FAILURE OF PLATES.
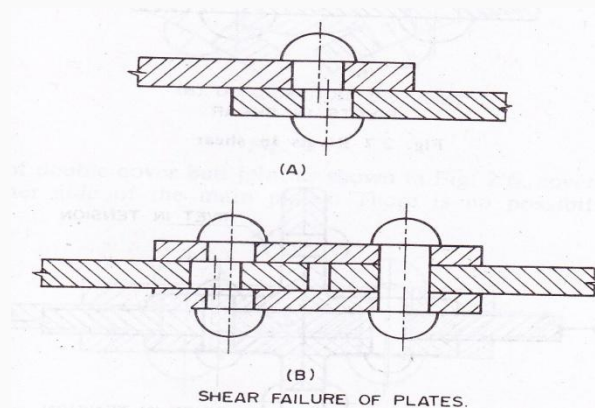
5.7.2 **Bearing failure of rivets:**

Bearing failure of a rivet occurs when the rivet is crushed by the plate (Fig.5.9)
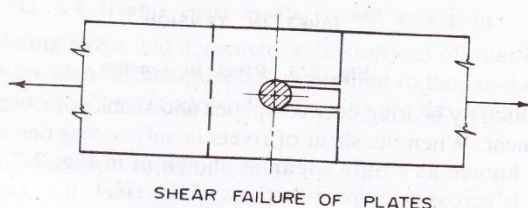

BEARING FAILURE OF RIVETS
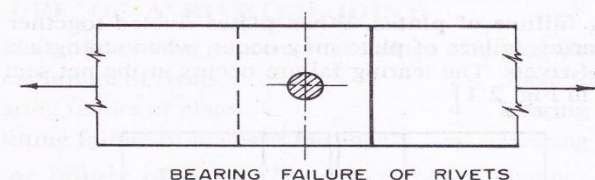
5.7.3 Tearing failure of plates :

When plates riveted together are carrying tensile load, tearing failure of plate may occur. When strength of the plate is less than that of rivets, tearing failure occurs at the net sectional area of plate (Fig.5.10)
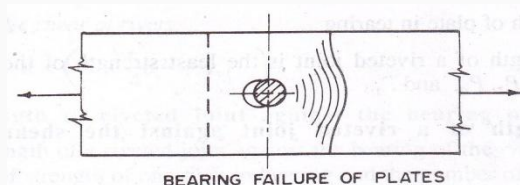
TEARING FAILURE OF PLATES
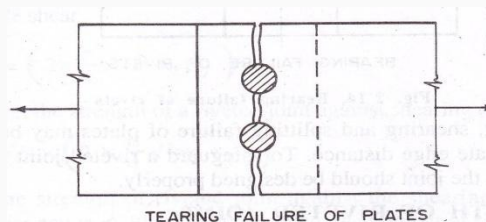
5.7.4 Shear failure of plates:

A plate may fail in shear along two lines as shown in Fig. 5.11. This may occur when minimum proper edge distance is not provided.

SHEAR FAILURE OF PLATES.

5.7.5 Bearing failure of plates:

Bearing failure of a plate may occur because of insufficient edge distance in the riveted joint. Crushing of plate against the bearing of rivet take place in such failure (Fig. 5.12)

BEARING FAILURE OF PLATES

5.7.6 Splitting/cracking failure of plates at the edges:

This failure occurs because of insufficient edge distance in the riveted joint. Splitting (cracking) of plate as shown in Fig. 5.13 takes place in such failure.

Shearing, bearing and splitting failure of plates may be avoided by providing adequate proper edge distance. To safeguard a riveted joint against other modes of failure, the joint should be designed properly.

SPLITTING FAILURE OF PLATES.
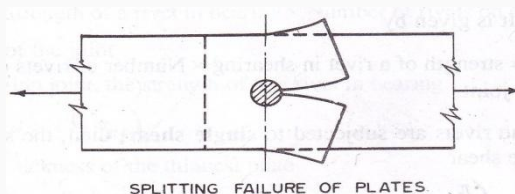
5.8 STRENGTH OF RIVETED JOINT

The strength of a riveted joint is determined by computing the following strengths:

1. Strength of a riveted joint against shearing   - P_s
2. Strength of a riveted joint against bearing   - P_b
3. Strength of plate in tearing   - P_t

The strength of a riveted joint is the least strength of the above three strength.

5.8.1 Strength of a riveted joint against shearing of the rivets:

The strength of a riveted joint against the shearing of rivets is equal to the product of strength of one rivet in shear and the number of rivets on each side of the joint. It is given by

P_s = strength of a rivet in shearing x number of rivets on each side of joint

When the rivets are subjected to single shear, then the strength of one rivet in single shear

$$= \frac{\pi}{4} D^2 p_s$$

Therefore, the strength of a riveted joint against shearing of rivets = $P_s = N \frac{\pi}{4} D^2 p_s$

Where N=Number of rivets on each side of the joint; D=Gross diameter of the rivet; p_s=Maximum permissible shear stress in the rivet(1025 ksc).

When the rivets are subjected to double shear, then the strength of one rivet in double shear

$= 2 \frac{\pi}{4} D^2 p_s$. Therefore, the strength of a riveted joint against double shearing of rivets,

$$P_s = N[2 \frac{\pi}{4} D^2 p_s]$$

When the strength of riveted joint against the shearing of the rivets is determined per gauge width of the plate, then the number of rivets 'n' per gauge is taken in to consideration. Therefore,

For single shear of rivets, $P_{s1} = n[\frac{\pi}{4} D^2 p_s]$

For double shear of rivets $P_{s2} = n[2 \frac{\pi}{4} D^2 p_s]$

5.8.2 Strength of riveted joint against the bearing of the rivets:

The strength of a riveted joint against the bearing of the rivets is equal to the product of strength of one rivet in bearing and the number of rivets on each side of the joint. It is given by,

P_b=Strength of a rivet in bearing x Number of rivets on each side of the joint

In case of lap joint, the strength of one rivet in bearing = D x t x p_b

Where D= Gross diameter of the rivet; t=thickness of the thinnest plate; p_b= maximum permissible stress in the bearing for the rivet (2360 ksc). In case of butt joint, the total thickness of both cover plates or thickness of main plate whichever is less is considered for determining the strength of a rivet in the bearing.

The strength of a riveted joint against the bearing of rivets P_b = N x D x t x p_b

When the strength of riveted joint against the bearing of rivets per gauge widh of the plate is taken into consideration, then, the number of rivets 'n' is also adopted per gauge. Therefore,

P_{b1} = n x D x t x p_b

5.8.3 Strength of plate in tearing

The strength of plate in tearing depends upon the resisting section of the plate. The strength of plate in tearing is given by Pt = Resisting section x p_t

Where p_t is the maximum permissible stress in the tearing of plate (1500 ksc). When the strength of plate in tearing per pitch width of the plate is P_{t1} = (p-D) x t x p_t

The strength of a riveted joint is the least of P_s, P_b, P_t. The strength of riveted joint per gauge width of plate is the least of P_{s1}, P_{b1}, P_{t1}.

5.9. STRENGTH OF LAP AND BUTT JOINT

The strength of riveted lap and butt joint given in the Fig. 5.14 is summarized as follows:
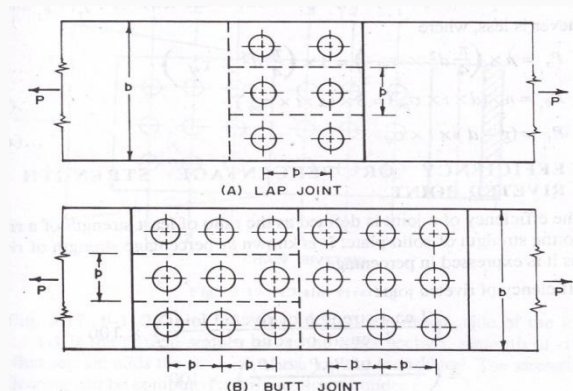
5.9.1 Strength of lap joint:

1. Strength of riveted joint against shearing $P_s = 6\frac{\pi}{4}D^2 p_s$

2. Strength of riveted joint against bearing P_b= 6 x D x t x p_b

3. Strength of riveted joint against tearing P_t = (b-3D) x t x p_t

4. Strength of riveted joint against shearing per gauge width $P_{s1} = 2\frac{\pi}{4}D^2 p_s$

5. Strength of riveted joint against bearing per gauge width P_{b1}= 2 x D x t x p_b

6. Strength of riveted joint against tearing per gauge width P_{t1} = (p-D) x t x p_t

5.9.2 **Strength of butt joint:**

1. Strength of riveted joint against shearing $P_s = 9 \times 2 \times \frac{\pi}{4} \times D^2 \times p_s$

2. Strength of riveted joint against bearing $P_b = 9 \times D \times t \times p_b$

3. Strength of riveted joint against tearing $P_t = (b-3D) \times t \times p_t$

4. Strength of riveted joint against shearing per gauge width $P_{s1} = 3 \times 2 \times \frac{\pi}{4} \times D^2 \times p_s$

5. Strength of riveted joint against bearing per gauge width $P_{b1} = 3 \times D \times t \times p_b$

6. Strength of riveted joint against tearing per gauge width $P_{t1} = (p-D) \times t \times p_t$

5.10 EFFICIENCY OR PERCENTAGE OF STRENGTH OF RIVETED JOINT

The efficiency of a joint is defined as the ratio of least strength of a riveted joint to the strength of solid plate. It is known as percentage strength of riveted joint as it is expressed in percentage.

Efficiency of riveted joint

$$\eta = \frac{Least\ strength\ of\ riveted\ joint}{Strength\ of\ solid\ plate} \times 100$$

$$\eta = \frac{Least\ of\ P_s,\ P_b\ or\ P_t}{P} \times 100$$

Where P is the strength of solid plate = b x t x p_t

Efficiency per pitch width

$$= \frac{(p-D) \times t \times p_t}{p \times t \times p_t} \times 100$$

$$= \frac{(p-D)}{p} \times 100$$

5.11 RIVET VALUE

The strength of a rivet in shearing and in bearing is computed and the lesser is called the rivet value (R).

LESSON 6. Design of Riveted Connections

6.1 INTRODUCTION

The perfect theoretical analysis for stress distribution in riveted connections cannot be established. Hence a large factor of safety is employed in the design of riveted connections. The riveted connections should be as strong as the structural members. No part in the riveted connections should be so overstressed. The riveted connections should be so designed that there is neither any permanent distortion nor any wear. These should be elastic. In general, the work of fabrication is completed in the workshops where the steel is fabricated.

6.2 ASSUMPTIONS FOR THE DESIGN OF RIVETED JOINT

Procedure for design of a riveted joint is simplified by making the following assumptions and by keeping in view the safety of the joint.

1. Load is assumed to be uniformly distributed among all the rivets

2. Stress in plate is assumed to be uniform

3. Shear stress is assumed to be uniformly distributed over the gross area of rivets

4. Bearing stress is assumed to be uniform between the contact surfaces of plate and rivet

5. Bending stress in rivet is neglected

6. Rivet hole is assumed to be completely filled by the rivet

7. Friction between plates is neglected

6.3 ARRANGEMENT OF RIVETS

Rivets in a riveted joint are arranged in two forms, namely, 1. Chain riveting, 2. Diamond riveting.

6.3.1 **Chain Riveting:** In chain riveting the rivets are arranged as shown in Fig. 6.1 and in the figure 1-1, 2-2 and 3-3 shows sections on either side of the joint. Section 1-1 is the critical section as compared to the other section. At section 2-2 is equal to the strength of plate in tearing at 2-2 plus strength of three rivets in bearing or shearing whichever is less at 1-1. At section 3-3 is equal to the strength of plate in tearing at 3-3 plus strength of rivets in bearing or shearing whichever is less (6 nos.).
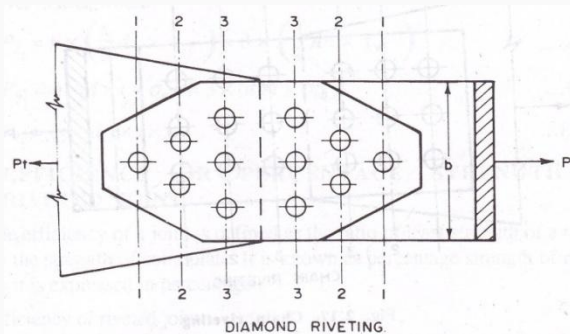
CHAIN RIVETING.
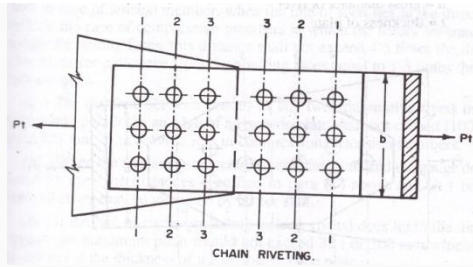
Therefore,

Strength of plate in tearing at 1-1 = (b-3D).t.p$_t$

Where b= width of the plate; D=Gross diameter of the rivet and t=Thickness of the plate.

When safe load carried by the joint (P) is known, width of the plate can be found as follows;

$$b = \left(\frac{P}{t \times p_t} + 3D\right)$$

6.3.2 **Diamond Riveting:** In diamond riveting, rivets are arranged as shown in Fig.6.2. All the rivets are arranged symmetrically about the centre line of the plate. Section 1-1 is the critical section. Strength of the plate in tearing in diamond riveting section 1-1 can be computed as follows

DIAMOND RIVETING.

$$P_t = (b - D).t.p_t$$

When the safe load carried by the joint (P) is known, width of the plate can be found as follows

$$b = \left(\frac{P}{t \times p_t} + D\right)$$

Where b=width of the plate, D=gross diameter of the rivet and t=thickness of the plate.

At section 2-2: All the rivets are stressed uniformly, hence strength of the plate at section 2-2 is

$$P_t = (b - 2D).t.p_t + strength\ of\ one\ rivet\ in\ shearing\ \&\ bearing\ whichever\ is\ less$$

At section 3-3,

$$P_t = (b - 3D).t.p_t + strength\ of\ three\ rivet\ in\ shearing\ \&\ bearing\ whichever\ is\ less$$

In diamond riveting there is saving of material and efficiency is more. Diamond riveting is used in bridge trusses generally.

6.4 SPECIFICATION FOR DESIGN OF RIVETED JOINT

6.4.1 **Members meeting at Joint:** The centroidal axes of the members meeting at a joint should intersect at one point, and if there is any eccentricity, adequate resistance should be provided in the connection.

6.4.2 **Centre of Gravity:** The centre of gravity of group of rivets should be on the line of action of load whenever practicable.

6.4.3 **Pitch:**

a. Minimum pitch: The distance between centres of adjacent rivets should not be less than 2.5 times the gross diameter of the rivet.

b. Maximum pitch: Maximum pitch should not exceed 12t or 200 mm whichever is less in compression member and 16t or 200 mm whichever is less in case of tension members, when the line of rivets lies along the line of action of force. If the line of rivets does not lie along the line of action of force, its maximum pitch should not exceed 32t or 300 mm whichever is less, where t is the thickness of the outside plate.

6.4.4 **Edge Distance:** A minimum edge distance of approximately 1.5 times the gross diameter of the rivet measured from the centre of the rivet hole is provided in the rivet joint. Table 6.1 gives the minimum edge distance as per recommendations of BIS in IS : 800-1984.

TABLE 6.1 EDGE DISTANCE OF HOLES

Gross diameter of rivet (mm)	Edge distance of Hole	
	Distance to sheared or Hand flame cut edge (mm)	Distance to rolled machine flame cut or planed edge (mm)
13.5 and below	19	17
15.5	25	22
17.5	29	25
19.5	32	29
21.5	32	29
23.5	38	32
25.5	44	38
29.0	51	44
32.0	57	51
35.0	57	51

6.5 DESIGN PROCEDURE FOR RIVETED JOINT

For the design of a lap joint or butt joint, the thickness of plates to be joined is known and the joints are designed for the full strength of the plate. For the design of a structural steel work, force (pull or push) to be transmitted by the joint is known and riveted joints can be designed. Following are the usual steps for the design of the riveted joint:

Step 1:

The size of the rivet is determined by the Unwin's formula

$$d = 6.04\sqrt{t}$$

Where d= nominal diameter of rivet in mm and t= thickness of plate in mm.

The diameter of the rivet computed is rounded off to available size of rivets. Rivets are manufactured in nominal diameters of 12, 14, 16, 18, 20, 22, 24, 27, 30, 33, 36, 39, 42 and 48 mm

Step 2:

The strength of rivets in shearing and bearing are computed. Working stresses in rivets and plates are adopted as per ISI. Rivet value R is found. For designing lap joint or butt joint tearing strength of plate is determined as follows

$$P_t=(p-D).t.p_t$$

Where p=pitch of rivets adopted, t=thickness of plate and p_t = working stress in direct tension for plate. Tearing strength of plate should not exceed the rivet value R (P_s or P_b whichever is less) or

$$(p - D).t.p_t \leq R$$

From this relation pitch of the rivets is determined.

Step 3:

In structural steel work, force to be transmitted by the riveted joint and the rivet value are known. Hence number of rivets required can be computed as follows

$$Number\ of\ rivets\ required\ in\ the\ joint = \frac{Force}{Rivet\ value}$$

The number of rivets thus obtained is provided on one side of the joint and an equal number of rivets is provided on the other side of joint also.

Step 4:

For the design of joint in a tie member consisting of a flat, width/thickness of the flat is known. The section is assumed to be reduced by rivet holes depending upon the

arrangements of the rivets to be provided, strength of flat at the weakest section is equated to the pull transmitted by the joint. For example, assuming the section to be weakened by one rivet and also assuming that the thickness of the flat is known we have

$$(b - D).t.p_t = P$$

Where b= width of flat, t=thickness of flat, pt=working stress in tension in plate and P=pull to be transmitted by the joint. From this equation, width of the flat can be determined.

Example 6.1: A single riveted lap joint is used to connect plate 10 mm thick. If 20 mm diameter rivets are used at 55 mm pitch, determine the strength of joint and its efficiency. Working stress in shear in rivets=80 N/mm^2 (MPa). Working stress in bearing in rivets=250 N/mm^2 (MPa). Working stress in axial tension in plates=156 N/mm^2.

Solution

Assume that power driven field rivets are used. Nominal diameter of rivet (D) is 20 mm and gross diameter of rivet is 21.5 mm.

Strength of rivet in single shear $\qquad = (\pi/4) \times 21.5^2 \times 80/1000$

$\qquad\qquad P_s \qquad = 29.044$ kN

Strength of rivet in bearing $\qquad\qquad = 21.5 \times 10 \times 250/1000$

$\qquad\qquad P_b \qquad = 53.750$ kN

Strength of plate in tension per gauge length = P_t=(p-D).t.p_t

$\qquad\qquad P_t \qquad = (55\text{-}21.5) \times 10 \times 156/1000$

$\qquad\qquad\qquad = 52.260$ kN

Strength of joint is minimum of P_s, P_b or P_t

Therefore, the strength of joint is $\qquad = 29.044$ kN

Efficiency of joint

$$\eta = \frac{Strength\ of\ joint\ per\ pitch\ length}{Strength\ of\ solid\ plate} \times 100$$

$$\eta = \frac{29.044 \times 10^3}{55 \times 10 \times 156} \times 100 = 33.85\ \%$$

Example 6.2: A double riveted double cover butt joint is used to connect plates 12 mm thick. Using Unwin's formula, determine the diameter of rivet, rivet value, pitch and efficiency of joint. Adopt the following stresses;

Working stress in shear in power driven rivets=100 N/mm² (MPa).

Working stress in bearing in power driven rivets=300 N/mm² (MPa).

For plates working stress in axial tension =156 N/mm².

Solution

Nominal diameter of rivet from Unwin's formula

$$d = 6.04\sqrt{t} = 6.04\sqrt{12} = 20.923 \, mm$$

Adopt nominal diameter of rivet = 22 mm; Gross diameter of rivet = 23.5 mm

Strength of rivet in double shear = $2 \times \frac{\pi}{4} \times 23.5^2 \times \frac{100}{1000} = 86.75 \, kN$

Strength of rivet in bearing = D x t x p_b = 23.5 x 12 x 300/1000 = 84.6 kN

The strength of a rivet in shearing and in bearing is computed and the lesser is called the rivet value (R). Hence the Rivet value is 84.6 kN.

Let p be the pitch of the rivets. P_t = (p-D) x t x p_t = ((p-23.5) x 12 x 156/100) =1.872 (p-23.5) kN

In double riveted joint,

Strength of 2 rivets in shear P_s = 2 x 86.75 = 173.5 kN

Strength of 2 rivets in bearing P_b = 2 x 84.6   = 169.2 kN

The pitch of the rivets can be computed by keeping $P_t = P_s$ or P_b whichever is less

Therefore   1.872 (p-23.5) = 169.2

p-23.5 = (169.2/1.872) = 90.385

p= 90.385 + 23.5 = 113.885 mm

Adopt pitch,   p= 100 mm

Efficiency of joint $\eta = \frac{(p-D)}{p} \times 100$

$$= \frac{(100 - 23.5)}{100} \times 100 = 76.5\,\%$$

Example 6.3: A double cover butt joint is used to connect plates 16 mm thick. Design the riveted joint and determine its efficiency.

Solution

Nominal diameter of rivet from Unwin's formula

$$d = 6.04\sqrt{t} = 6.04\sqrt{16} = 24.16\ mm$$

The hot driven rivets of 16 mm, 18 mm, 20 mm and 22 mm diameter are used for the structural steel works. Unwin's formula gives higher values. Hence, adopt nominal diameter of rivet = 22 mm; Gross diameter of rivet = 22 +1.5 = 23.5 mm

In double cover butt joint, rivets are in double shear. As per IS : 800-84,

Shear stress for power driven rivets=100 N/mm² (MPa).

Bearing stress for power driven rivets=300 N/mm² (MPa).

Strength of plate in tension =156 N/mm².

Strength of rivet in double shear = $2 \times \frac{\pi}{4} \times 23.5^2 \times \frac{100}{1000} = 86.75\ kN$

Strength of rivet in bearing = D x t x p_b = 23.5 x 16 x 300/1000 = 112.8 kN

The strength of a rivet in shearing and in bearing is computed and the lesser is called the rivet value (R). Hence the Rivet value is 86.75 kN.

Let p be the pitch of the rivets. P_t = (p-D) x t x p_t = ((p-23.5) x 16 x 156/100) =2.496 (p-23.5) kN

The pitch of the rivets can be computed by keeping P_t = P_s or P_b whichever is less

Therefore $\qquad$ 2.496 (p-23.5) = 86.75

$$(p-23.5) = (86.75/2.496) = 34.756$$

$$p= 34.756 + 23.5 = 58.256\ mm$$

Adopt pitch, $\qquad$ p= 55 mm

Adopt thickness of each cover plate t ≈ 5/8 x 16 ≈ 10 mm

Efficiency of joint $\qquad \eta = \frac{(p-D)}{p} \times 100$

$$= \frac{(55 - 23.5)}{55} \times 100 = 55.27\ \%$$

Example 6.4: Determine the strength of a double cover butt joint used to connect two flats 200 F 12. The thickness of each cover plate is 8 mm. Flats have been joined by 9 rivets in chain riveting at a gauge of 60 mm as shown in Fig. 6.3. What is the efficiency of the joint? Adopt working stresses in rivets and flats as per IS : 800-84.

DOUBLE COVER BUTT JOINT
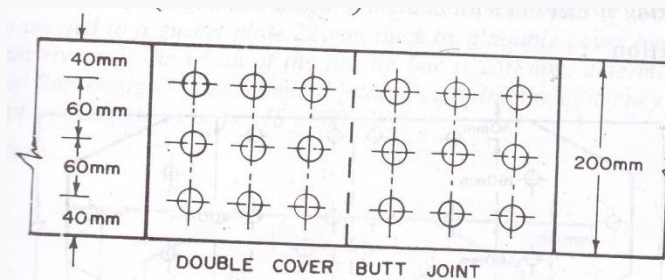
Solution

Size of flat used   = 200 F 12

Width of flat   = 200 mm

Thickness of flat   = 12 mm

Use power driven rivets

Nominal diameter of rivet from Unwin's formula

$$d = 6.04\sqrt{t} = 6.04\sqrt{12} = 20.923 \ mm$$

Adopt nominal diameter of rivet = 22 mm; Gross diameter of rivet D = 23.5 mm

Strength of rivet in double shear = $2 \times \frac{\pi}{4} \times 23.5^2 \times \frac{100}{1000} = 86.75 \ kN$

Strength of rivet in bearing = D x t x p_b = 23.5 x 12 x 300/1000 = 84.6 kN

Strength of joint in shear, P_s = 9 x 86.75   = 780.75 kN

Strength of joint in bearing P_b = 9 x 84.6   = 761.40 kN

Strength of plate in tearing P_t = (b-3D) x t x p_t

= ((200-3x23.5) x 12 x 156/1000)

= 242.42 kN

Strength of joint is minimum of P_s, P_b or P_t

Therefore, the strength of joint is   = 242.42 kN

Efficiency of joint

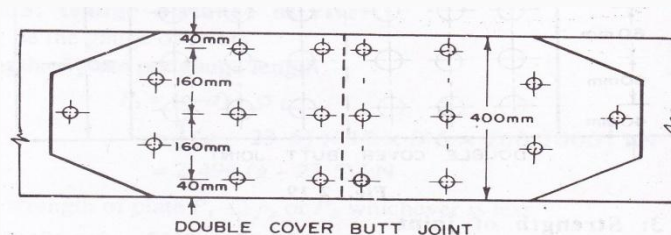$$\eta = \frac{Least\ of\ P_s,\ P_b\ or\ P_c}{Strength\ of\ solid\ plate} \times 100$$

$$\eta = \frac{242.42 \times 10^3}{200 \times 12 \times 156} \times 100 = 64.75\ \%$$

Example 6.5: In a truss girder of a bridge, a diagonal consists of a 16 mm thick flat and carries a pull of 750 kN and is connected to a gusset plate by a double cover butt joint. The thickness of each cover plate is 8 mm. Determine the number of rivets necessary and the width of the flat required. What is the efficiency of the joint? Sketch the joint. Take

Working stress in shear in power driven rivets=100 N/mm² (MPa).

Working stress in bearing in power driven rivets=300 N/mm² (MPa).

For plates working stress in axial tension =156 N/mm².

DOUBLE COVER BUTT JOINT

Solution

Nominal diameter of rivet from Unwin's formula

$$d = 6.04\sqrt{t} = 6.04\sqrt{16} = 24.16\ mm$$

The hot driven rivets of 16 mm, 18 mm, 20 mm and 22 mm diameter are used for the structural steel works. Unwin's formula gives higher values. Hence, adopt nominal diameter of rivet = 22 mm; Gross diameter of rivet = 22 +1.5 = 23.5 mm

Strength of rivet in double shear = $2 \times \frac{\pi}{4} \times 23.5^2 \times \frac{100}{1000} = 86.75\ kN$

Strength of rivet in bearing = D x t x p_b = 23.5 x 16 x 300/1000 = 112.8 kN

 The strength of a rivet in shearing and in bearing is computed and the lesser is called the rivet value (R). Hence the Rivet value is 86.75 kN.

 Number of rivets required to transmit pull of 750 kN   n= (750/86.75) = 8.67 ≈ 9 rivets.

Using diamond group of riveting, flat is weakened by one rivet hole. Strength of plate at section 1-1 in teaing

P_t = (b-d) x t x p_t = ((b-23.5) x 16 x 156/100) = 2.496 (b-23.5) kN

Since P = 750 kN,   2.496 (b-23.5) = 750

$$b=(750/2.496)+23.5 = 323.98 \text{ mm}$$

Hence provide 400 mm width of diagonal member. The design of joint is shown in Fig. 6.4.

Efficiency of the joint

$$\eta = \frac{(b-D) \times t \times p_t}{b \times t \times p_t} \times 100 = \frac{400-23.5}{400} \times 100 = 94.125\ \%$$

Example 6.6: A bridge truss diagonal carries an axial pull of 500 kN. It is to be connected to a gusset plate 22 mm thick by a double cover butt joint with 22 mm rivets. If the width of the tie bar is 250 mm, determine the thickness of flat. Design the economical joint. Determine the efficiency of the joint. Adopt working stresses in rivets and flats as per IS : 800-84.

Solution

Nominal diameter of rivet = 22 mm; Gross diameter of rivet = 23.5 mm

Strength of power driven rivet in double shear = $2 \times \frac{\pi}{4} \times 23.5^2 \times \frac{100}{1000} = 86.75\ kN$

Strength of power driven rivet in bearing = D x t x p_b = 23.5 x 22 x 300/1000 = 155.1 kN

The strength of a rivet in shearing and in bearing is computed and the lesser is called the rivet value (R). Hence the Rivet value is 86.75 kN.

Number of rivets required to transmit pull of 500 kN   n= (500/86.75) = 5.76 ≈ 6 rivets.

Provide six rivets in diamond group of riveting for efficient joint.

Let the thickness of flat be t mm

Strength of plate at weakest section P_t = (b-d) x t x p_t = ((250-23.5) x t x 156/100) = 500 kN

Therefore t = 14.151 mm; Adopt 16 mm thickness of flat. Keep 40 mm edge distance from centre of rivet and 85 mm distance between centre to centre of rivet lines as shown in the Fig. 6.5.

DIAMOND RIVETED DOUBLE COVER BUTT JOINT
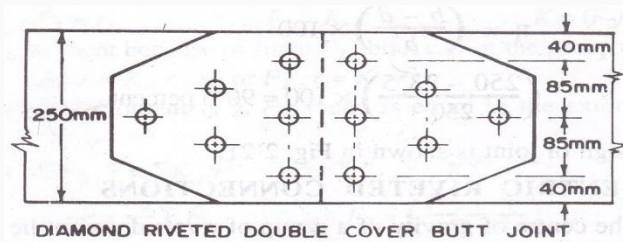
Efficiency of joint

$$\eta = \frac{(b-D) \times t \times p_t}{b \times t \times p_t} \times 100 = \frac{250-23.5}{250} \times 100 = 90.6\ \%$$

8. A welded structure has a better finish and appearance than the corresponding riveted structure.

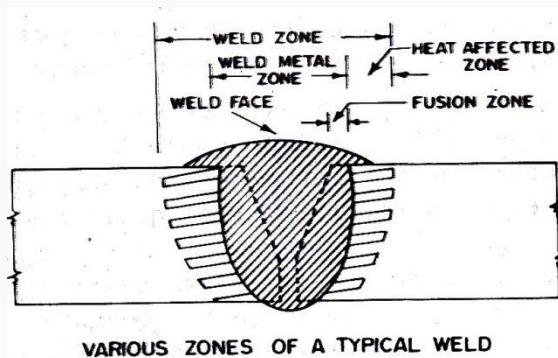9. Connecting angles, gusset plates, splicing plates can be minimized.

10. Steel bars in reinforced concrete structure may be welded easily so that lapping of bars may be avoided.

11. It is possible to weld at any point at any part of a structure, but riveting will always require enough clearance.

12. The process of welding does not involve great noise compared to the noise produced in the riveting process.

7.3 DISADVANTAGES OF WELDED CONNECTIONS

1. Welding requires skilled labor and supervision.

2. Testing a welded joint is difficult. An X-ray examination alone can enable us to study the quality of the connection.

3. Due to uneven heating and cooling, the welded members are likely to get warped at the welded surface.

4. Internal stresses in the welded zones are likely to be set up.

7.4 TYPES OF WELDED JOINTS

Welds may be classified into two main types namely butt-weld and fillet-weld.

7.4.1. Butt weld

This type of weld is used when the members are in same plane. Butt weld is also termed as groove weld. The butt weld is used to join structural members carrying direct compression or tension. It is used to make tee-joint and butt-joint. The following types of butt welds are in practice. These are named depending upon shape of the grove made for welding.

i. Square butt weld.

A square butt weld is a weld in the preparation of which the fusion faces lie approximately at right angles to the surfaces of the components to be joined and are substantially parallel to one another (Fig. 7.2 a & b).

(A) A SQUARE BUTT WELD (ONE SIDE)   (B) A SQUARE BUTT WELD (BOTH SIDES)
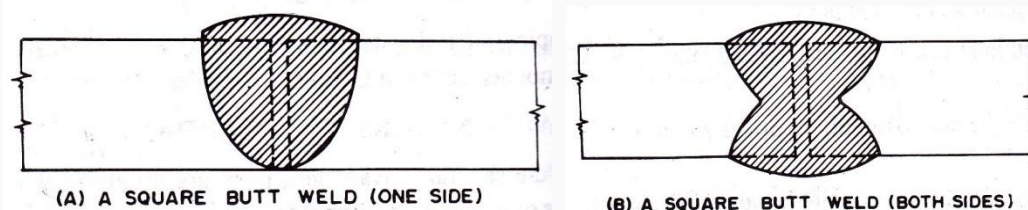
ii.Single V-butt weld

A single V-butt weld is a weld in the preparation of which the edges of both components are prepared so that in the cross-section, the fusion faces form a V as shown in Fig. 7.3.
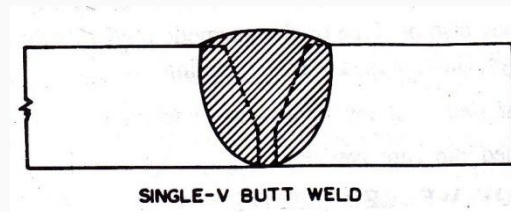
SINGLE-V BUTT WELD

iii.Double V-butt weld

A double V-butt weld is a weld in the preparation of which the edges of both components are double beveled so that in cross-section, the fusion faces form two opposing V's as shown in Fig. 7.4.
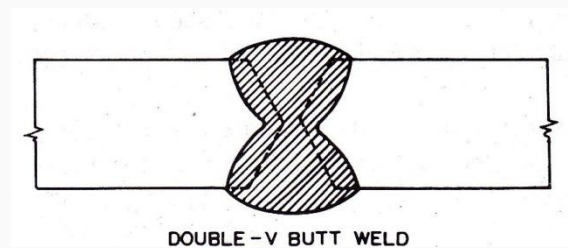
DOUBLE-V BUTT WELD

iv. Single U-butt weld

A single U-butt weld is a weld in the preparation of which the edges of both components are prepared so that in the cross section, the fusion faces form a U as shown in Fig. 7.5.
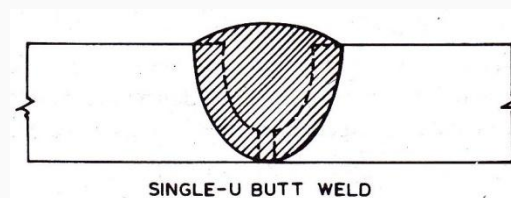
SINGLE-U BUTT WELD

v. Double U-butt weld

A double U-butt weld is a weld in the preparation of which the edges of both components are prepared so that in the cross section, the fusion faces form two opposing U's as shown in Fig. 7.6.

DOUBLE-U BUTT WELD
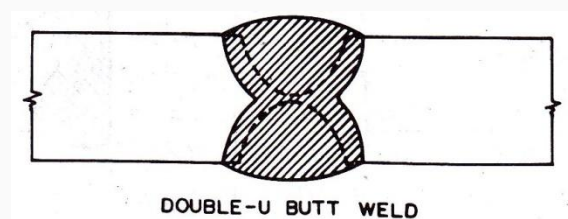
vi.Single J-butt weld

A single J-butt weld is a weld in the preparation of which the edges of one component are prepared so that in the cross section, the fusion faces is in the form a J and the fusion face of the other component is at right angles to the surface of the first component as shown in Fig. 7.7.
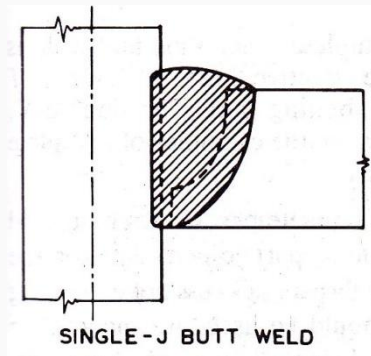
SINGLE-J BUTT WELD

vii.Double J-butt weld

A double J-butt weld is a weld in the preparation of which the edges of one component are prepared so that in the cross section, the fusion faces is in the form of two opposing J's and the fusion face of the other component is at right angles to the surface of the first component as shown in Fig. 7.8.

DOUBLE-J BUTT WELD
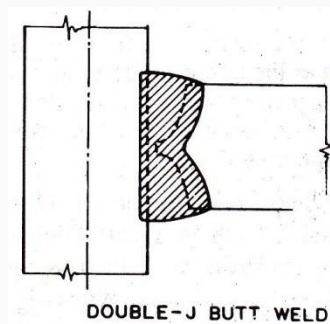
viii.Single bevel butt weld

A single bevel butt weld is a weld in the preparation of which the edge of one component is beveled and the fusion face of the other component is at right angles to the surface of the first component as shown in Fig. 7.9.

SINGLE-BEVEL BUTT WELD
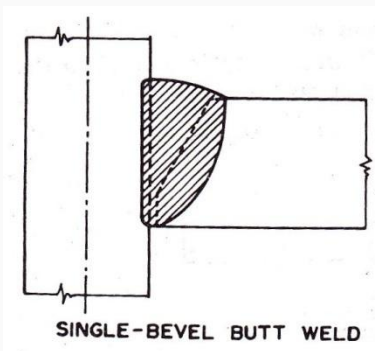
ix.Double bevel butt weld

A double bevel butt weld is a weld in the preparation of which the edges of one component are double beveled and the fusion face of the other component is at right angles to the surface of the first component as shown in Fig. 7.10.

DOUBLE-BEVEL BUTT WELD
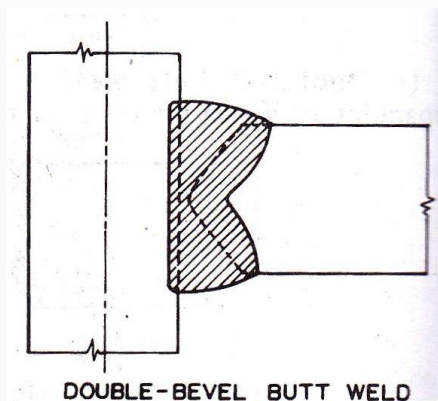
7.4.2. Specifications of the butt weld

i. Size of butt weld

The size of a butt weld is specified by the effective throat thickness. The effective throat thickness in case of complete penetration butt weld is taken as the thickness of thinner part joined. The double V, double U, double J and double bevel butt welds are the examples of complete penetration butt weld.

The effective throat thickness in case of incomplete penetration butt weld is taken as $7/8^{th}$ of the thickness of the thinner part joined. But for the purpose of stress calculation, a required effective throat thickness not exceeding $5/8^{th}$ of the thickness of thinner part joined should be used. An incomplete penetration butt weld is also termed as unsealed single butt weld. Single V, Single U, Single J, Single bevel butt joints are the examples of incomplete penetration butt weld. In incomplete penetration butt weld, the weld metal is not deposited intentionally through the full thickness of the joint. The unwelded portion in incomplete penetration butt weld, welded from both sides shall not be greater than $1/4^{th}$ of the thickness of thinner part joined and should be central in the depth of the weld.

The unsealed butt welds V, U, J and bevel types and incomplete penetration butt welds should not be used for highly stressed joints and joints subjected to dynamic, repeated or alternating forces. The shall also not be subjected to a bending moment about the longitudinal axis of the weld other than that normally resulting from the eccentricity of the weld metal relative to the parts joined.

ii. Effective length of butt weld

The effective length of butt weld is the length for which the specified size (throat thickness) of the weld exists.

iii. Effective area of butt weld

The effective area of a butt weld is taken as the product of the effective throat thickness and the effective length of butt weld.

iv. Reinforcement

The extra metal deposited above the surface of the parent metal as shown in Fig. 7.11 is called reinforcement. This reinforcement is provided to give sufficient surfaces convexity and to ensure full effectiveness at the joint. This requires a minimum practical surface convexity of 1.0 mm. This reinforcement should not exceed 3.0 mm. This is not considered as part of throat thickness. This reinforcement may also be removed if a flush surface is desired.
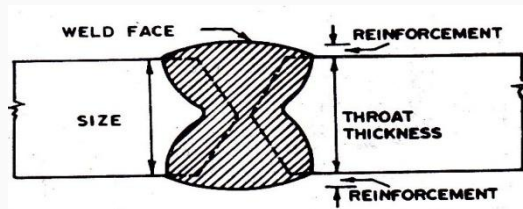
When the structural members of unequal thickness are butt welded and difference in thickness of members exceeds 25 per cent of the thinner part or 3.0 mm in metal arc welding and 6.0 mm or more in oxy-acetylene welding, the thicker part is beveled so that the slop of the surface from one part to the other is not steeper than one in five as shown in Fig. 7.12.A. Where this arrangement is not practicable, the weld metal should be built-up at the junction with the thicker part to dimension at least 25 per cent greater than that of the thinner part in metal arc welding as shown in 7.12.B. alternatively, the weld metal should be built-up to the dimensions of thicker members as shown in 7.12.C. In case of complete penetration butt weld, generally, deign calculations are not necessary, as these will usually provide the strength at the joint equal to the strength of the member connected.
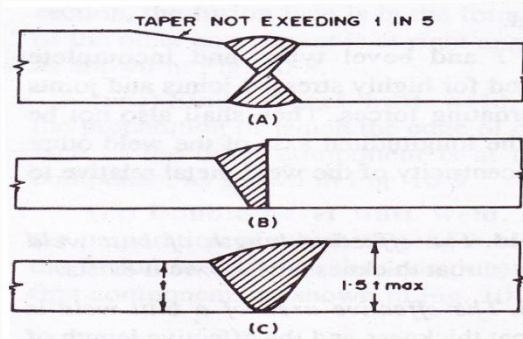
7.4.3 Fillet-weld

This type of weld is used when the members to be connected overlap each other. A fillet weld is a weld of approximately triangular cross section joining two surfaces approximately as right angles to each other in lap joint or tee joint. A fillet weld is shown in Fig. 7.13.A. When the cross section of fillet weld is 45°, isosceles triangle as shown in Fig. 7.13.B.I, it is known as a standard fillet weld. The standard 45° fillet weld is generally used. When the cross section of the fillet weld is 30° and 60° triangle as shown in Fig. 7.13.B.II, it is known as a special fillet weld.

(A) FILLET WELD

(i) STANDARD FILLET WELD

(ii) SPECIAL FILLET WELD
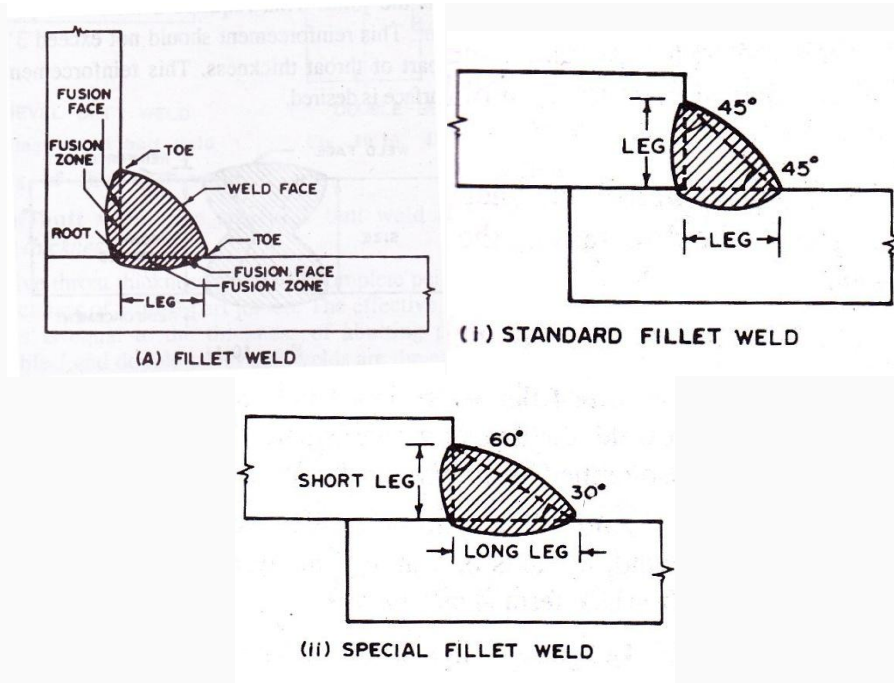
A fillet weld is termed as concave fillet weld or convex fillet weld or mitre fillet weld depending on the weld face in concave or convex or approximately flat as shown in Fig. 7.14, respectively. A fillet weld is termed as normal fillet weld or deep penetration fillet weld depending upon the depth of penetration beyond the root is less than 2.4 mm or more respectively.

(A) CONCAVE FILLET WELD

(B) CONVEX FILLET WELD

(C) MITRE FILLET WELD
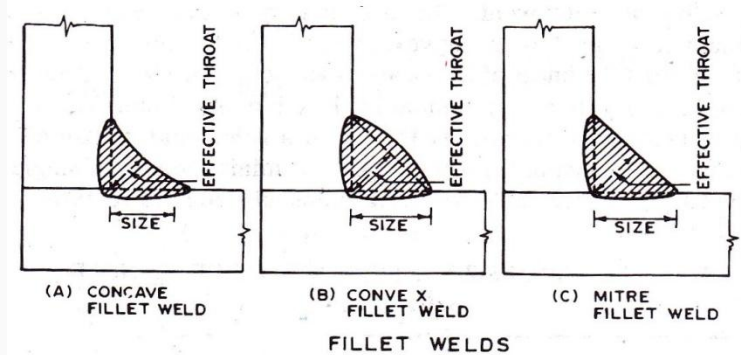
FILLET WELDS

The fillet welds are of three types as shown in Fig. 7.15.

TYPES OF FILLET WELD
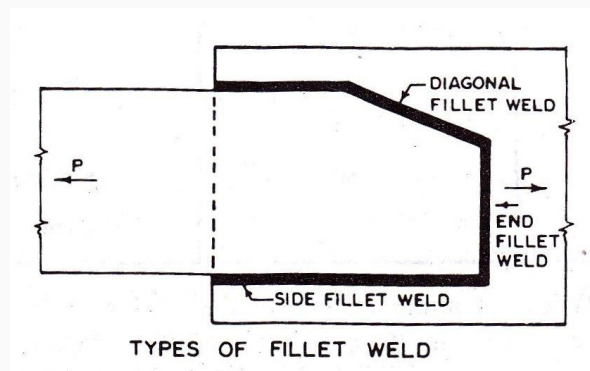
i.Side fillet weld

It is fillet weld stressed in longitudinal shear, i.e., a fillet weld, the axis of which is parallel to the direction of these applied loads. It is also termed as longitudinal fillet weld.

ii.End fillet weld

It is a fillet weld stressed in transverse shear, i.e., a fillet weld, the axis of which is at right angles to the direction of the applied load. It is also termed as transverse fillet weld.

iii.Diagonal fillet weld

It is a fillet weld the axis of which is inclined to the direction of the applied load

A fillet weld is placed on the sides or end of the base metal and it is subjected to shear along with tension or compression and usually bending.

7.4.4 Specification of fillet weld

i. Size of fillet weld

The size of normal fillet weld is specified as minimum leg length of a convex or mitre fillet weld or 1.414 times the effective throat thickness of a concave fillet weld. The size of deep penetration fillet weld is specified as minimum leg length plus 2.4 mm. the length of leg is the distance from the root to the toe of a fillet weld, measured along the fusion face.

The International Standard code has recommended the minimum size of the weld. If the thickness of thicker part is up to 10 mm, the minimum size of the welding is 3 mm. If the thickness of thicker part is in between 10 mm to 20 mm, the minimum size of the welding is 5 mm. If the thickness of thicker part is in between 20 mm to 32 mm, the minimum size of the welding is 6 mm. If the thickness of thicker part is above 32 mm, the minimum size of the welding is 10 mm. When the minimum size of the fillet weld is greater than the thickness of the thinner part, the minimum size of the weld should be equal to the thickness of thinner part. Where the thicker part is more than 50 mm, special precaution like preheating will have to be taken.

ii. Effective throat thickness

The effective throat thickness of a fillet weld is the perpendicular distance from the root to the hypotenuse of the largest isosceles right angled triangle that can be inscribed within the weld cross section. The effective throat thickness of a fillet weld shall not be less than 3 mm and shall generally not exceed 0.7 times the thickness of thinner part and equal to the thickness of thinner part under special circumstances.

$$Effective\ throat\ thickness = \frac{1}{\sqrt{2}} \times Size\ of\ weld = 0.7 \times Size\ of\ weld$$

In general, for the purpose of stress calculation,

$$Effective\ throat\ thickness = K \times Size\ of\ weld$$

Where K is a constant. The value of K for different angles between fusion faces is adopted as per Table 7.1 as recommended in IS:816-1969

TABLE 7.1. VALUE OF K FOR DIFFERENT ANGLES BETWEEN FUSION FACES

Angle between fusion faces	Value of constant K
60°-90°	0.70
91°-100°	0.65
101°-106°	0.60
107°-113°	0.55
114°-120°	0.50

A fillet weld is not used for joining parts, if the angle between fusion faces is greater than 120° or less than 60°.

iii.Effective length

The effective length of the weld is the length of the weld for which the specified size and throat thickness i.e., correctly proportioned cross section of the weld, exist. It is taken as the actual length minus twice the size of weld, since the specified size and throat thickness do not exist at the ends. The effective length of the weld is shown on the drawings. In practice the actual length of weld is made equal to the effective length shown on the drawing plus twice the weld size. The effective length of fillet weld should not be less than four times the size of the weld.

When the ends are returned as shown in Fig. 7.16, then the ends should be carried continuous around the corners for distance not less than twice the size of weld. This should be applied particularly to side and top fillet weld in tension.
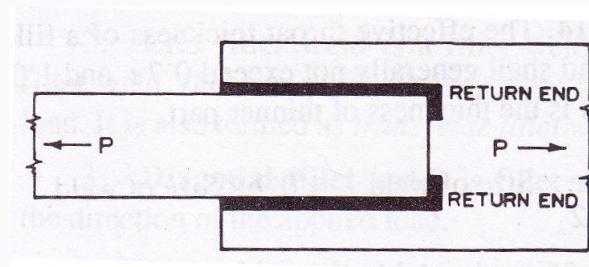
iv.Effective area

The effective area of a fillet weld is taken as the product of effective length and effective throat thickness.

7.5 WORKING STRESSES IN WELDS

Working stresses in welds, when welded joints are constructed with mild steel conforming to IS:226-1962 as parent metal and with electrodes conforming to IS:814-1974 are adopted as per Table 7.2 are recommended in IS:816-1969.

TABLE 7.2 WORKING STRESSES IN WELDS

Sl .No.	Kind of stresses	Max. Permissible value (N/mm²)
1	Tension on section through throat of butt weld	142
2	Compression on section through throat of butt weld	142
3	Fiber stresses in bending (tension and compression)	157.5
4	Shear on section through throat of butt and fillet weld	110
5	Plug weld	110

The maximum permissible value of stresses of shear and tension are reduced to 80 per cent of those given in Table 7.2, in case, the welding is done at site. When the effects of wind or earthquake forces are considered, then, maximum permissible values of stresses are increased by 25 per cent. It is to note that maximum permissible stresses given in the Table 7.2 are same as for the parent metal (mild steel IS:226-1962).

7.6 DESIGN OF WELDED JOINTS SUBJECTED TO AXIAL LOAD

The complete penetration butt weld does not require design calculations. In case of incomplete penetration butt weld, effective throat thickness of the weld is computed and welding is done up to the required length. In case of fillet weld, size of the weld is fixed keeping in view the minimum size of the weld as per IS:816-1969 recommends that when filet weld is applied to the square edge of member, the maximum size of weld should be less than the edge thickness by at least 1.5 mm as shown in Fig. 7.17. This avoids the washing down of edges of weld. When fillet weld is applied to the round toe of rolled steel sections, the maximum size of the weld should not exceed ¾ of the thickness of the section at the toe. When fillet weld is used for lap joint, then overlap of the members connected as shown in Fig. 7.17, should not be less than five times thickness of thinner part.
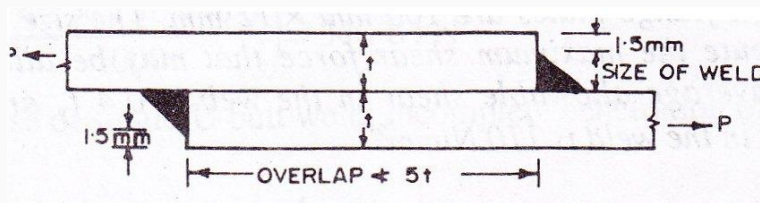
The strength of the fillet weld is determined per mm length for the size of the weld adopted. The effective length of the weld is then computed for the pull or thrust to be transmitted by the weld. In case, only side fillet welds are applied, the length of the each weld should not be less than perpendicular distance between them and spacing between them shall not be more than 16 times the thinner part.

 Example 7.1. Two plates 16 mm thick are joined by i. a double U butt weld, ii. A single U butt weld. Determine the strength of the welded joint in tension in each case. Effective length of weld is 150 mm. Allowable stress in butt weld in tension is 142 N/mm².

Design of Structures

Solution

i.In case of double U but weld, complete penetration of weld takes place

Effective throat thickness of weld   = 16 mm

Effective length of weld   = 150 mm

 Strength of single U butt weld = throat thickness x length of weld x permissible shear stress

                              = (16 x 150 x 142/1000)   = 340.8 kN

ii.In case of single U butt weld, incomplete penetration of butt weld takes place

Effective throat thickness   = 5/8 x 16   = 10 mm

Effective length of weld   = 150 mm

Strength of single U butt weld   = (10 x 150 x 142/1000)   = 213.0 kN

 Example 7.2. In a truss girder of a bridge, a tie as shown in Fig. 7.18 is connected to the gusset plate by fillet weld. Determine the strength of the weld. The size of the weld in the fillet weld is 6 mm.
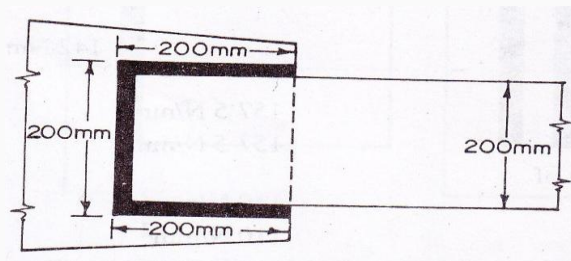
Solution

     Size of weld   = 6 mm

     Effective throat thickness   = 0.7 x 6 = 4.2 mm

     Effective length of fillet weld   = 200 + 200+ 200   = 600 mm

     Strength of fillet weld   = (4.2 x 600 x 110/1000)   = 277.2 kN

Example 7.3. In Example 7.2, the pull to be transmitted by the tie is 300 kN. Determine the necessary overlap of the tie.

Solution

     Size of weld   = 6 mm

     Effective throat thickness   = 4.2 mm

Pull transmitted by the end fillet weld   = (4.2 x 200 x 110/1000)   = 92.4 kN

Let *l* be the necessary overlap required, the pull transmitted by the side fillet is

$$= (4.2 \times 2 \times l \times 110/1000) \quad = 0.924\ l\ \text{kN}$$

Total pull transmitted = 92.4 + 0.924 *l*   = 300 kN

Therefore, the necessary overlap of the tie *l*   = 224.7 mm.

Example 7.4. The web plate of a built-up welded I-section is 200 mm x 12 mm and the flange plates are 100 mm x 12 mm. The size of fillet weld is 6 mm. Compute the maximum shear force that may be allowed at any section, if the average allowable shear in the web is 0.4 f_y and maximum allowable shear in the weld is 110 N/mm².

Solution

Moment of inertia of the built-up section (about xx axis)

I_{xx} = 1/12[10 x 22.4³ – 8.8 x 20³] x 10⁴   = 3499.52 x 10⁴ mm⁴

Intensity of shear stress (at weld section)

$$\tau_s = \left(\frac{F.A\bar{y}}{I_{xx} \cdot (2t)} \right)$$

Where,

$A\bar{y}$ = Moment of the area above the section about xx axis

F   = Shear force at the section

t   = Effective throat thickness of one weld

$$\tau_s = \left(\frac{F \times 100 \times 12 \times (100 + 6)}{3499.52 \times 10^4 \times 2 \times 0.7 \times 6} \right) = 0.11\ kN/mm^2$$

$$F = 254.21\ kN$$

The average shear stress in the web is 0.4 x 250 N/mm².

Allowable shear force in the web F_1=(200 x 12 x 0.4 x 250/1000)   = 240 kN.

The design drawing is Fig. 7.19.
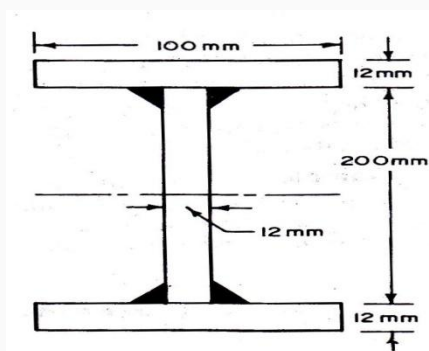
Example 7.5. Design a suitable longitudinal fillet weld to connect the plates as shown in Fig. 7.20 and to transmit a pull equal to the full strength of thin plate. Allowable stress in the weld is 110 N/mm² and tensile stress in the plate 0.6 f_y N/mm². The plates are 10 mm thick.
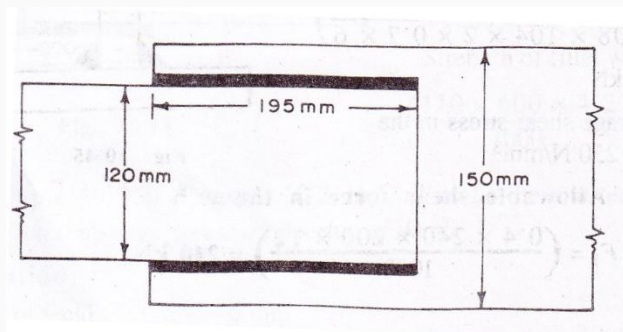
Solution

The minimum size of weld required for thickness up to 20 mm is 5 mm. The maximum size of fillet weld is limited by the thickness of the plate is (10-1.5)=8.5 mm. Provide 6 mm fillet weld.

Pull transmitted by 1 mm weld   = (1 x 0.7 x 6 x 110/1000)   = 0.462 kN

Tensile strength of thin plate   = (120 x 10 x 0.6 x 250/1000)   = 180 kN

Necessary length of the weld   = (180 / 0.462)   = 389.61 mm

Provide 195 mm longitudinal weld on each side.

Check: Length of the weld 195 mm is greater than perpendicular distance 120 mm between welds

Example 7.6. Two plates 120 mm x 10 mm are overlapped and connected together by transverse fillet weld to transmit pull equal to full strength of the plate. Design the suitable welding. Allowable stress in the weld is 110 N/mm². Allowable stress in tension in the plate is 0.6 f_y N/mm².

Solution

Minimum size of weld   = 5mm

Maximum size of weld = (10-1.5)   = 8.5 mm

Total length of two welds   = 240 mm

Total load transmitted by 6 mm weld   = (240 x 0.7 x 6 x 110/1000) = 110.88 kN

Maximum pull that can be transmitted by the plate = (120 x 10 x 0.6 x 250/1000)   =180 kN

To transmit the pull equal to the full strength of plate, provide additional weld by plug weld. Provide two rectangular plug welds 30 mm x 15 mm as shown in Fig. 7.21 which satisfies the specification.
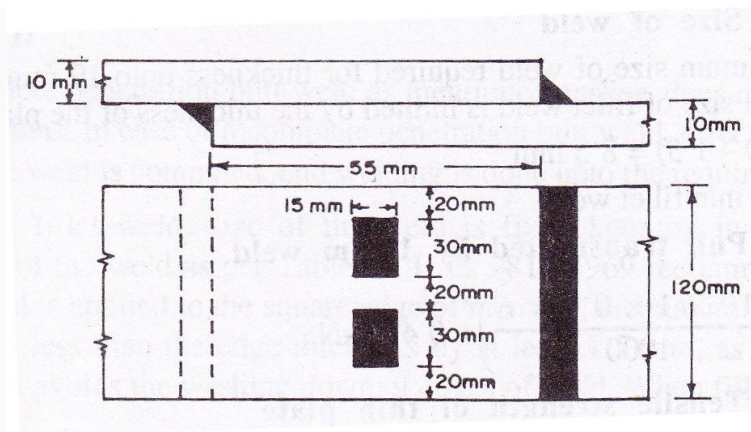
Strength of two plug welds   = (2 x 30 x 15 x 110/1000)   = 99 kN.

Total pull now transmitted   = (110.88 + 99) = 209.88 kN > 180 kN.   Hence satisfactory.

Example 7.7. A tie member consists of two MC 225, @ 0.250 kN/m. The channels are connected to either side of a gusset plate 12 mm thick. Design the welded joint to develop the full strength of the tie. The overlap limited to 400 mm.

Solution

From ISI Handbook No. 1, for MC 225, @ 0.250 kN/m

Thickness of web   = 6.4 mm

Thickness of flange   = 12.4 mm

Sectional area   = 3301 mm²

Tensile strength of each channel section   = (3301 x 0.6 x 250/1000)   = 495.15 kN

Provide 4 mm weld

Strength of weld per mm length   = (1 x 0.7 x 4 x 110/1000)   = 0.308 kN

Total length of fillet weld necessary to connect one channel section = (495.15/0.308)

$$= 1607.6 \text{ mm}$$

The overlap of channe is limited to 400 mm. the width of slot should not be less than 3 times thickness (3 x 6.4 = 19.2 mm). Provide two slots 20 mm side. The distance between edge of the slot and edge of channel or between adjacent slots also should not be less than twice the thickness (2x6.4=12.8 mm). Provide these distances as shown in Fig. 7.22. Let x be the length of the slot.

The total length of the weld   = 800 + 225 + 4x – 2x20   = 1607.6 mm

Therefore, the length of the slot x = 155.65 mm ≈ 160 mm long fillet welding is done as shown in Fig. 7.22.
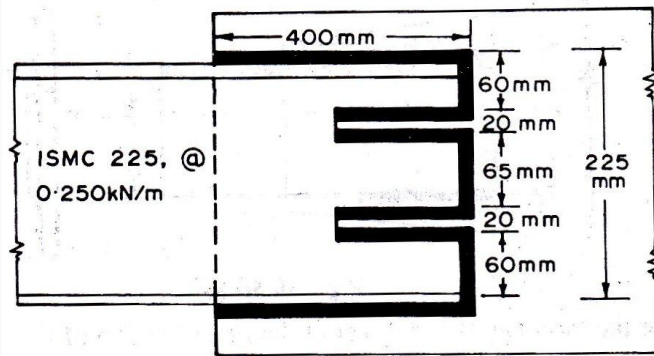
LESSON 8. Tension Member

8.1 INTRODUCTION

A tension member is a member which carries mainly a tensile force in the direction parallel to its longitudinal axis. A tension member is also called as a tie member or simply a tie. In some cases tension member also subjected to bending either due to eccentricity of the longitudinal load or due to transverse loads acting in addition to the main longitudinal load. A tension member is one of the most commonly occurring types of structural members. Tension members may occur either as minor tension members such as bars, flats, rods etc. or as major tension members of roof and bridge trusses

8.2 MINOR TYPES OF TENSION MEMBERS

The minor types of tension members are shown in Fig. 8.1.
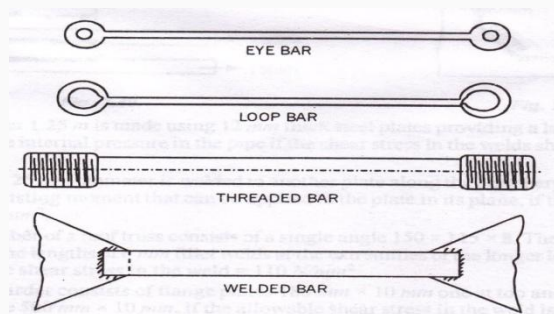
i.Eye-bars

These members are used where flexible end connections are desired. They are used as the members of pin-connected truss bridges. Eye bars are made by first upsetting each end of a bar of rectangular section to a nearly round shape and then boring holes of the desired sizes on the enlarged ends. A pin is passed through the eye or the hole in the bar and also through corresponding holes in the other members meeting at the joint. The pin provides means of transmission of load from the eye bar to the other members at the joint.

ii.Loop bars

These are made by bending each end of a bar of square or round section, back upon the bar itself and then welding it so as to form a loop. Stress transmission is exactly similar to that in the eye-bar.

iii.Threaded bars

These consist of round bars whose ends are threaded. Nuts are attached on the threaded ends after the bar has been placed in its proper position. The ends of the rod are first upset and then threaded so that the sectional area at the root of the threads is not less than the sectional

area of the bar. After upsetting, usually the sectional area at the ends will be about 20 per cent greater than the sectional area of the bar. If a non-upset threaded bar is to be selected, the designer must select a bar in which the diameter at the root of the threads will be at least 1.5 mm greater than the normally required diameter.

iv.Welded bars

These are flat bars carrying light tensile loads and welded at their ends.

8.3 MAJOR TYPES OF TENSION MEMBERS

Single angle tension members are commonly used in roof trusses carrying light loads. They are also used as bracings for members of composite section. A single angle member transfers its load eccentrically to the gusset plate and is hence also subjected to bending moment. This factor should also be taken into account in the design.

Double angle tension members are often used connected on either side of a gusset plate at the end. If provided in this manner eccentric load transfer to the gusset plate will be avoided and hence the member will be practically free from bending stresses. These are most commonly used in roof trusses and foot bridge trusses.

Double channel tension members may also be used in a manner similar to double angle members. In view of considerably greater depth of web available two or even three rows of rivets can be provided. These members therefore require less length of gusset plate.

Besides the above two angle members, four angle members with or without a plate; two channel members may be used as tension members in more heavily loaded bridge trusses. The major types of tension members are shown in Fig. 8.2.
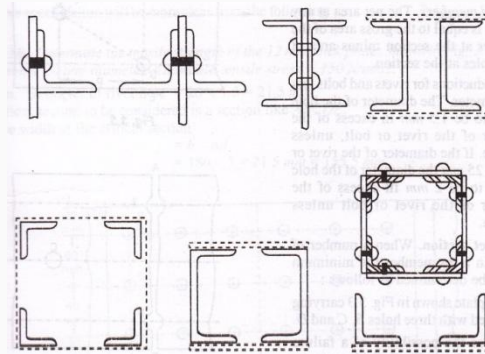
8.4 PERMISSIBLE TENSILE STRESS

For a tension member, it is necessary that the intensity of tensile stress on the net section of the member shall be less than the permissible limit. The I.S. specification has recommended the following permissible stresses.

The permissible stress in axial tension on the net effective area of the section shall not exceed 0.6 f_y, where f_y is the minimum yield stress of steel.

For example, if f_y=250 N/mm², Safe stress in axial tension = 0.6 x 250 = 150 N/mm².

Design of Structures

8.5 NET SECTIONAL AREA

The maximum stress for a tension member occurs at the section where the area is a minimum. The net area for tension members should be determined as follows:

1. Threaded rods: The sectional area at the root of the threads is regarded of the net area.

2. Riveted members: The net area at any section is equal to the gross area of the member at the section minus area of rivet holes at the section.

In making deduction for rivets and bolts less than 25 mm in diameter, the diameter of the hole shall be assumed to be 1.5 mm in excess of the nominal diameter of the rivet or bolt, unless specified otherwise. If the diameter of the rivet or bolt is greater than 25 mm the diameter of the hole shall be assumed to be 2 mm in excess of the nominal diameter of the rivet or bolt unless specified otherwise.

Minimum net section: When a number of holes are present in a tie member the minimum net section should be determined as follows:
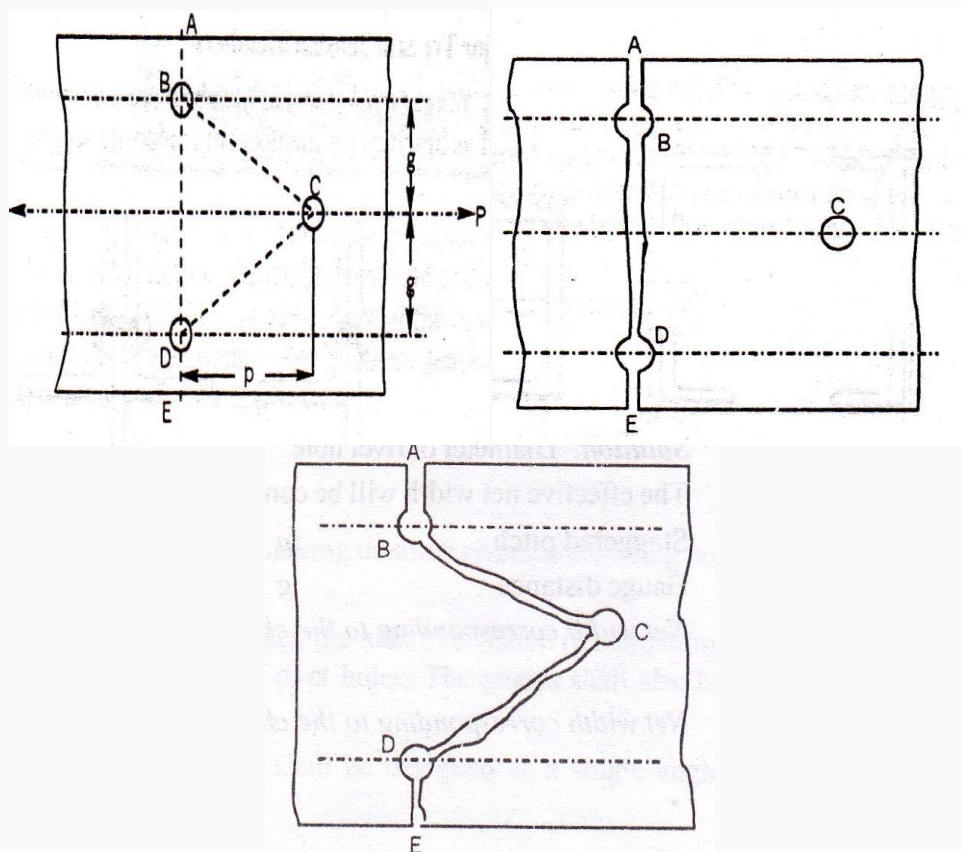
Consider the plate shown in Fig 8.3 carrying a pull P and provided with three holes, B, C and D. If we consider the possibility of a failure along the section ABDE (Fig. 8.4), the net area at the section equals the gross area of the plate minus area of two rivet holes. There may be possibility of a failure along the section ABCDE (Fig. 8.5). This can occur when the staggered pitch p is within a certain limit. For a case like this the net effective width b_e can be determined by the following rule:

$$b_e = b - nd + n_z \left(\frac{p^2}{4g}\right)$$

Where,   b = gross width of the plate

n = number of rivet holes along any possible failure section

n_z = number of zigzag lines along the line of failure

p = staggered pitch, i.e. the distance between any two consecutive rivet measured parallel to the direction of stress in the member.

g = the gauge, i.e. the distance between the same two consecutive rivets in a chain line measured at right angles to the direction of stress in the members.

For a failure along the section ABCDE, the effective net width

$$b_e = b - 3d + 2\left(\frac{p^2}{4g}\right)$$

Effective net area = $b_e.t$

$$Maximum\ stress\ intensity\ in\ the\ plate = \frac{Load\ on\ the\ plate}{Minimum\ effective\ net\ area}$$

8.6 NET EFFECTIVE SECTION FOR ANGLES AND TEES IN TENSION

The I.S 800 specification has stated the following in connection with the determination of the net effective section for angles and tees in tension.

a. Singles angles:

In the case of single angles in tension connected by one leg only (Fig. 8.6),
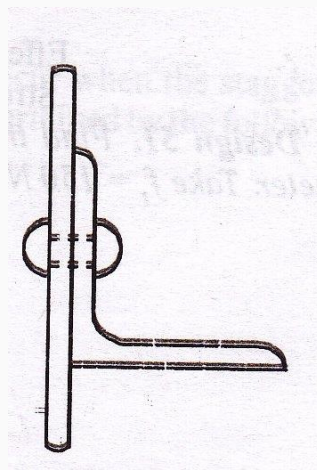
The net effective section of the angle = $A_1 + A_2K$

Where, A_1 = Effective cross sectional area of the connected leg,

A_2 = The gross-sectional area of the unconnected leg, and

$$K = \frac{3A_1}{3A_1 + A_2}$$

b. Double angles back to back (and single tees)

When   only one leg of each angle is connected to the same side of a gusset (or when the flange of the tee is connected to the gusset) as shown in Fig. 8.7,
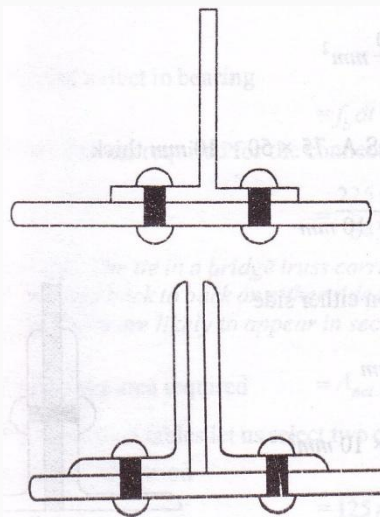
The net effective section = = $A_1 + A_2K$

Where, A_1 = Effective cross sectional area of the connected leg,

   A_2 = The gross-sectional area of the unconnected leg, and

$$K = \frac{5A_1}{5A_1 + A_2}$$

The angles shall be connected together along their lengths with tacking or stitch rivets at a spacing not exceeding 32 times the thickness or 300 mm whichever is less.

1. When the double angles are connected to each side of a gusset, the area to be taken in computing the mean tensile stress shall be the full gross area minus the area of rivet holes. The angles shall also be connected together along their lengths with tacking rivets.

2. When the double angles are not tack riveted each angle shall be designed as a single angle connected through one leg only.

The area of the leg of an angle shall be taken as the product of the thickness and the length from the outer corner minus half the thickness and the area of the leg of a tee as the product of the thickness and the depth minus the thickness of the flange.

 8.7 LUG ANGLES

To accommodate the necessary number of rivets for the connection of a member to a gusset plate a certain length of the member is utilized. When the load in the member is large many rivets may be required for the connection and the length of the member utilized for the

connection itself may be large and may involve a considerable length of gusset plate. By using lug angles the length of the connection can be decreased.
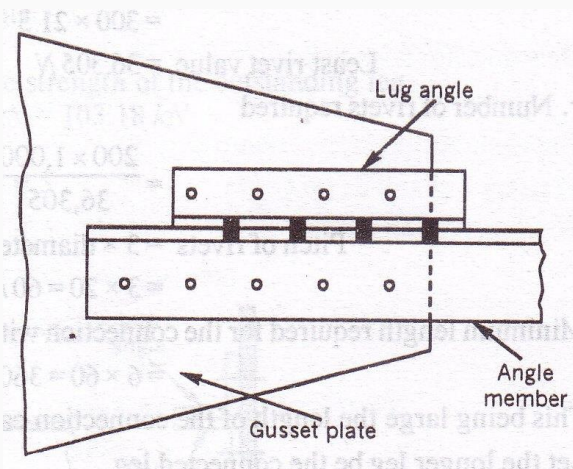
Fig 8.8 shows a lug angle used for the connection of an angle member to the gusset plate. Similarly for the connection of a channel member two lug angles may be used as shown in Fig. 8.9.
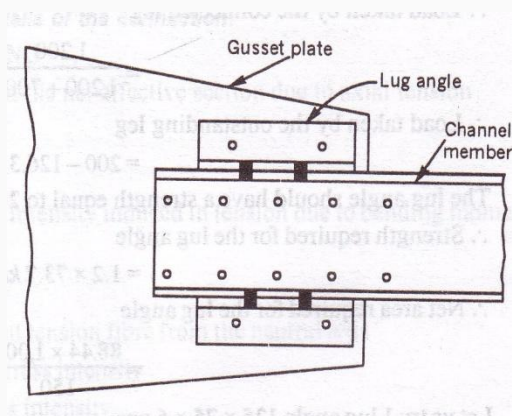
In the case of angle members, the lug angles and their connections to the gusset or other supporting member shall be capable of developing a strength not less than 20 per cent in excess of the force in the outstanding leg of the angle and the attachment of the lug angles to the angle member shall be capable of developing 40 per cent in excess of that force.

Lug angles connecting a channel shaped member shall, as for as possible, be deposed symmetrically with respect to the section of the member. The lug angles and their connection to the gusset or other supporting member shall be capable of developing a strength of not less than 10 per cent in excess of the force not accounted for by the direct connection of the member and the attachment of the lug angles to the member shall be capable of developing 20 per cent in excess of that force.

In no case shall less than two bolts or rivets be used for attaching the lug angle to the gusset or other supporting member.

MODULE 5.

LESSON 9. Design of Tension Member

9.1 INTRODUCTION

When a tension member is subjected to axial tensile force, then the distribution of stress over the cross-section is uniform. The complete net area of a member is effectively used at the maximum permissible uniform stress. Therefore, a tensile member subjected to axial tensile force is used to be efficient and economical member. The procedure of the design of a tension member is explained below with help of example problems.

9.2 STEPS TO BE FOLLOWED IN THE DESIGN OF A TENSION MEMBER

The following steps may be followed in the design of axially loaded tension members.

1. Corresponding to the loading on the structure of which the tension member is a part, the tensile force in the member is first computed.

2. The net area required for the member is determined by dividing the tensile force in the member by the permissible tensile stress.

3. Now, a suitable section having gross area about 20 per cent to 25 per cent greater than the estimated area is selected. For the member selected deductions are made for the area of rivet holes and the net effective area of the section is determined. If the net area of the section of the member so determined is greater than the net area requirement estimated in step i, the design is considered safe.

4. The slenderness ratio of a tension member shall not exceed 400. In the case of a tension member liable to reversal of stress due to the action of wind or earthquake, slenderness ratio shall not exceed 350. If the reversal of stress is due to loads others than wind or earthquake, the slenderness ratio shall not exceed 180.

Example 9.1: Determine the tensile strength of the 12 mm thick plate shown in Fig 9.1. Rivets used for the connection are 20 mm diameter. Allowable tensile stress is 150 N/mm².

Solution

Diameter of the rivet hole   = 20 + 1.5 = 21.5 mm

The critical section to be considered is a section like ABCDE.

Effective width at critical section   = b – nd = 180 – (3 x 21.5) = 115.5 mm

Effective net area   = 115.5 x 12 mm²   = 1386 mm²

Strength of plate   = 1386 x 150   = 207900 N   = 207.9 kN.

Example 9.2: Find the strength of the 12 mm thick plate shown in Fig. 9.2. All the holes are 21.5 mm as gross diameter. Take f_t=150 N/mm².
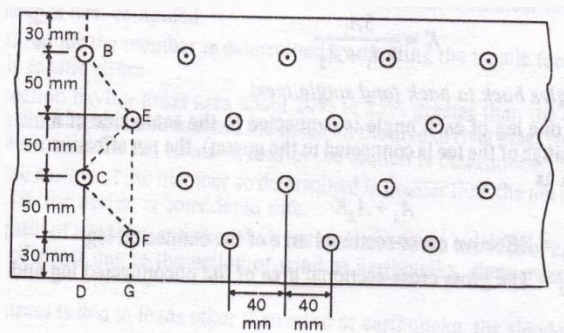
Solution

Gross diameter of rivet hole   = 21.5 mm

The effective net width will be computed along the various chain lines

Staggered pitch   p = 40 mm

Gauge distance   g = 50 mm

Net width corresponding to the chain ABCD   = 210 – (2 x 21.5)   = 167 mm

Net width corresponding to the chain ABECFG

$$= b - nd + n_z\frac{p^2}{4g} = 210 - (4 \times 21.5) + \left(3 \times \frac{40^2}{4 \times 50}\right) = 148 \text{ mm}$$

Net width corresponding to the chain ABEFG

$$= b - nd + n_z\frac{p^2}{4g} = 210 - (3 \times 21.5) + \left(1 \times \frac{40^2}{4 \times 50}\right) = 153.5 \text{ mm}$$

Net width corresponding to the chain ABECD

$$= b - nd + n_z\frac{p^2}{4g} = 210 - (3 \times 21.5) + \left(2 \times \frac{40^2}{4 \times 50}\right) = 161.5 \text{ mm}$$

Therefore minimum net width   = 148 mm

Safe load   = Safe stress x Area of minimum net effective section

= 150 x 148 x 12   = 2,66,400 N = 266.4 kN.

Example 9.3: The tension member of a roof truss consist of two unequal angles 70 x 45 x 8 with the longer legs connected by 16 mm diameter rivets. Find the safe tension for the member, the angles being one on either side of the gusset plate.

Solution

Gross area of 2 angles,   70 x 45 x 8   = 1,712 mm²

Area of 2 rivet holes   2 x (16 + 1.5) x 8   = 280 mm²

Net area of the member   1712 – 280   = 1,432 mm²

Therefore safe tension for the member $\qquad$ = 150 x 1432 $\quad$ = 2,14,800 N $\quad$ = 214.8 kN.

Example 9.4: The tension member of a roof truss carries a maximum axial tension of 250 kN. Design the section. Diameter of connecting rivets = 20 mm. Safe stress in tension = 150 N/mm².

Solution

Net area required $\qquad A_{net} = \dfrac{250 \times 1000}{150} = 1667\ mm^2$

Referring to steel tables, let us choose two angles of I.S.A. 75 x 50 x 10 mm thick.

Gross area $\quad$ = 75 mm x 50 mm x 10 mm $\quad$ = 2,300 mm².

Let us assume that the angles are placed back to back on either side of the gusset plate (Fig. 9.3) with two rivets.
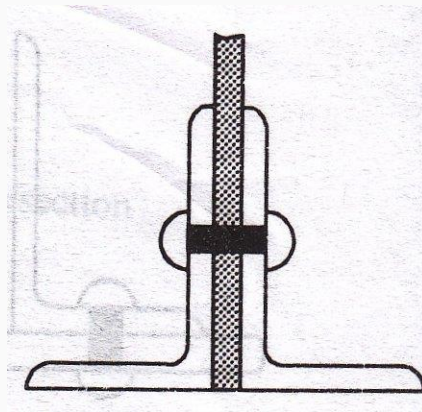
Diameter of rivet hole = 20 + 1.5 = 21.5 mm

Net area provided $\quad$ = 2,300 – (2 x 21.5 x 10) $\qquad$ = 1,870 mm².

But the area required is only 1,667 mm². Hence the section is safe.

Example 9.5: The tie of a truss carries an axial tension of 225 kN. Design the section of the member and also the connection of the member to 10 mm thick gusset plate. Use 20 mm diameter rivets.

Solution

Net area required $\qquad A_{net} = \dfrac{225 \times 1000}{150} = 1500\ mm^2$

Referring to steel tables, let us choose two angles of I.S.A. 75 x 50 x 8 mm thick.

Gross area $\quad$ = 75 mm x 50 mm x 8 mm $\quad$ = 1,872 mm².

Let us assume that the angles are placed back to back on either side of the gusset plate (Fig. 9.4).
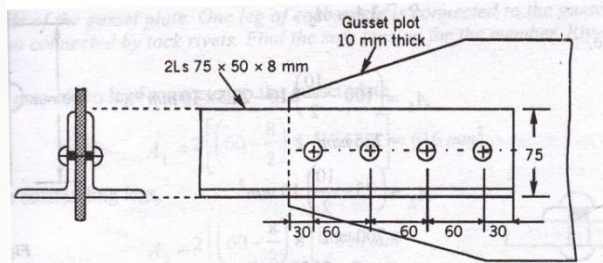
Diameter of rivet hole $= 20 + 1.5 = 21.5$ mm

Net area provided $\quad = 1,872 - (2 \times 21.5 \times 8) \qquad = 1,528$ mm^2.

But the area required is only 1,500 mm^2. Hence the section is safe.

Strength of a rivet in double shear $= \dfrac{2 f_s \pi d^2}{4} = \dfrac{2 \times 100 \times \pi \times 21.5^2}{4} = 72,610\ N$

Strength of a rivet in bearing $= f_b dt = 300 \times 21.5 \times 10 = 64,500\ N$

Number of rivets required for the connection $= \dfrac{225 \times 10^3}{64500} \approx 4\ rivets$

Example 9.6: The tie in a bridge truss carries an axial tension of 350 kN. The member is to consist of two channels connected back to back on either side of a gusset plate. The diameter of rivets used for the connection is 16 mm. Two rivets are likely to appear in section. Design the member. Safe stress in tension is 150 N/mm^2.

Solution

Net area required

Referring to steel tables, let us select two channels ISLC, 125 (Fig. 9.5)
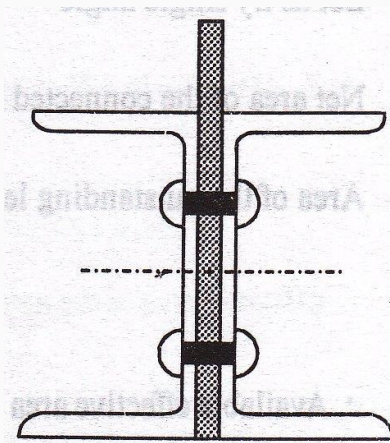
Dimensions of section $\qquad = 125$ mm x 65 mm

$\qquad t_f \qquad = 6.6$ mm

$\qquad t_w \qquad = 4.4$ mm

Gross area $\qquad A_g \qquad = 1,367$ mm^2 per channel

Diameter of rivet hole   = 16 + 1.5   = 17.5 mm

Making allowance for four rive holes,

Net area provided   = 2 x 1367 – (4 x 17.5 x 4.4) = 2,426 mm².

But the required area is only 2,333 mm². Therefore, the section selected is safe.

Example 9.7: The tension member of a roof truss consist of a single ISA 100 x 75 x 10 mm thick, connected at the end to a gusset plate with the longer leg vertical with 20 mm diameter rivet. Find the safe tension the member can withstand. Permissible tensile stress may be taken as 150 N/mm².

Solution

Diameter of rivet hole   = 20 + 1.5   = 21.5 mm

Net effective area provided

Where, A_1 = Net sectional area of the connected leg,

         A_2 = Area of the unconnected leg, and

$$K = \frac{3A_1}{3A_1 + A_2}$$

In our case,

$$A_1 = \left(100 - \frac{10}{2}\right) 10 - 21.5 \times 10 = 735\ mm^2$$

$$A_2 = \left(75 - \frac{10}{2}\right) 10 = 700\ mm^2$$

$$K = \frac{3 \times 735}{3 \times 735 + 700} = 0.759$$

$$A_{eff} = 735 + 700 \times 0.759 = 1,266\ mm^2$$

Safe axial tension   = A_{eff} x Safe stress

         = 1,266 x 150 = 1,89,900 N   = 189.9 kN.

The tension member is shown in Fig. 9.6.
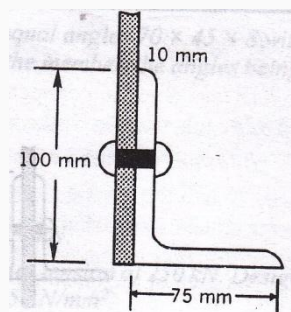
Example 9.8: Design a single angle tension member to sustain a tension of 1,30,000 N. Use 18 mm diameter rivets.

Solution

The single angle tension member is shown in Fig. 9.7.
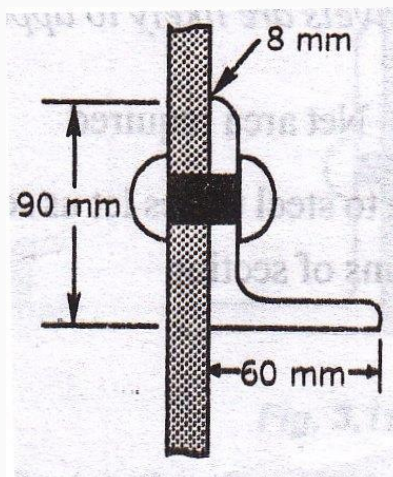
Effective area required $\qquad A_{eff} = \frac{1,30,000}{150} = 866.7 \ mm^2$

Approximate gross area required $\qquad = A_{eff} + 30 \ \% \ more$

$\qquad\qquad\qquad = 866.7 + (0.3 \ x \ 866.7) \qquad = 1,127 \ mm^2$

Let us try single angle 90 mm x 60 mm x 8 mm,

Net area of the connected leg $\qquad A_1 = (90\text{-}4) \ x \ 8 - (19.5 \ x \ 8) \qquad = 532 \ mm^2$

Area of the outstanding leg $\qquad A_2 = (60\text{-}4) \ x \ 8 \qquad\qquad = 448 \ mm^2$

$$K = \frac{3A_1}{3A_1 + A_2} = \frac{3 \times 532}{(3 \times 532) + 448} = 0.78$$

Therefore, available effective area $= A_{eff} = A_1 + A_2 K = 532 + 0.78 \times 448 = 881.44 \ mm^2$

Safe tension for the member $= 150 \ x \ 881.44 = 1,32,216 \ N.$

But actual tension in the member is only 1,30,000 N. Hence the design is safe.

Example 9.9: A tension member consist of two angles 60 x 60 x 8 the angles being placed back to back on the same side of the gusset plate. One leg of each angle is connected to the gusset plate. The outstanding legs are also connected by tack rivets. Find the safe tension for the member. Rivets are 16 mm in diameter.

Solution

The tension member is shown in Fig. 9.8.
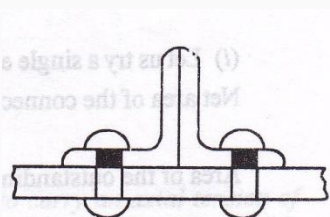
Net area of the legs connected to the gusset plate $A_1 = 2\left[\left(60 - \frac{8}{2}\right)8 - 17.5 \times 8\right] = 616\ mm^2$

Net area of the outstanding legs, $\qquad A_2 = 2\left[\left(60 - \frac{8}{2}\right)8\right] = 896\ mm^2$

$$K = \frac{5A_1}{5A_1 + A_2} = \frac{5 \times 616}{(5 \times 616) + 896} = 0.775$$

$$A_{eff} = A_1 + A_2 K = 616 + 0.775 \times 896 = 1310.4\ mm^2$$

Therefore, safe tension for the member $\qquad = 150 \times 1310.4 \qquad = 1,96,560\ N = 196.56\ kN.$

Example 9.10: A tension member of a truss consists of a single ISA 125 x 75 x 10 mm carrying a load of 200 kN. If 20 mm diameter rivets be used, design the connection to the gusset plate using a lug angle. Take f_s=100 N/mm² and f_b=300 N/mm².

Solution

Finished diameter of rivet $\qquad = 20 + 1.5 \qquad = 21.5\ mm$

Strength of one rivet in single shear $= f_s \frac{\pi d^2}{4} = 100 \times \frac{\pi 21.5^2}{4} = 36,305\ N$

Strength one rivet in bearing $= f_b dt = 300 \times 21.5 \times 10 \qquad = 64,500\ N$

Rivet value $\qquad = 36,305\ N$

Therefore number of rivets required $= \frac{200 \times 1,000}{36,305} \approx 6\ rivets$

Pitch of rivets = 3 x diameter of rivets $\qquad = 3 \times 20 = 60\ mm$

This being large the length of the connection can be decreased by using a lug angle,

Let the longer leg be the connected leg,

Area of the connected leg $\qquad = (125-5)10 \qquad = 1,200\ mm^2$

Area of the outstanding leg $\quad = (75-5)10 \qquad = 700\ mm^2$

Therefore load taken by the connected leg

Therefore load taken by the outstanding leg $= 200 - 126.3 = 73.7\ kN$

Connection to the gusset plate using a lug angle is shown in Fig. 9.9.
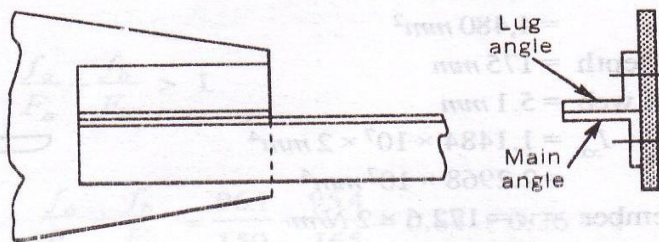
The lug angle should have a strength equal to 20 % greater than the strength of the outstanding leg.

Therefore strength required for the lug angle = 1.2 x 73.7 kN $\qquad = 88.44\ kN$

Ne area required for the lug angle $\qquad = (88.44 \times 1000/150) = 589.6\ mm^2$

Design of Structures

Let us try 1 lug angle 125 x 75 x 6 mm

Gross area   = 1,166 mm^2

Net area   = gross area – area of one rivet hole   = 1,166 – (21.5 x 6)   = 1,037 mm^2

Number of rivets required for connecting the 125 mm leg of the lug angle with the gusset plate

$$= \frac{Load\ in\ lug\ angle}{Least\ rivet\ value}$$

Rivet value in bearing on 6 mm plate = 300 x 21.5 x 6   = 38,700 N

Rivet value in single shear   = 36,305 N

Number of rivets for connecting the lug angle to the gusset plate $= \frac{88.44 \times 1000}{36,305} \approx 3$

We will therefore provide 3 rivets for connecting the 125 mm leg of the lug angle to the gusset plate. Number of rivets required to connect the 125 mm leg of the main angle to the gusset plate

$$= \frac{126.3 \times 1000}{36,305} \approx 4\ rivets$$

Strength required for connection between lug angle and the main angle is equal to 1.4 times the strength of the outstanding leg.

Therefore, strength required for this connection   = 1.4 x 73.7   = 103.18 kN

Number of rivets required = (103.18 x 1000/36,305) ≈ 3

LESSON 10. Design of Columns

10.1 INTRODUCTION

A column is defined as a structural member subjected to compressive force in a direction parallel to its longitudinal axis. The term stanchions and posts are also used for columns. In truss bridge girders, end compression members are termed as end posts. Columns are commonly classified as short and long columns. This classification is arbitrary and there is no absolute way to determine the exact limits for each classification.

10.2 AXIALLY LOADED COLUMNS

In an axially loaded column, the load is applied at the centroid of the section and in a direction parallel to the longitudinal axis of the column. The terms centrally loaded and concentrically loaded are also used for axially loaded columns. An axially loaded column as defined by the structural engineers transmits a compressive force without an explicit design requirement to carry lateral loads or end moments.

An ideal column is assumed initially to be perfectly straight and is centrally loaded. Consider a case of a slender ideal column. The column is vertically fixed at the base and free at the upper end and subjected o an axial load P as shown in Fig. 10.1. The column is assumed to be perfectly elastic. When the value of load P is less than critical load and stress is within the limit of proportionality, the column remains straight. The column is in stable equilibrium. If a small lateral load is applied at the free end, the column defect. On withdrawal of the lateral load, the column resumes its vertical position and deflection vanishes. When the axial load P gradually increased, a stage will be reached when the vertical position of the column is in the unstable equilibrium. If a small lateral load is applied, a deflection will be produced, which will not vanish on withdrawal of lateral load.

The axial load which is sufficient to keep the column in such a slight deflected shape is called critical load. Critical load is also called a buckling load or crippling load. The buckling load is defined as the load at which a member or a structure as a whole collapses in service (or buckles in load test). The buckling is defined as the sudden bending, warping, curling or crumbling of the elements or members under compressive stresses. The direction of buckling of a column depends upon flexural rigidity EI, of the column. It buckles in a direction perpendicular to the axis about which the moment of inertia of the section is minimum.
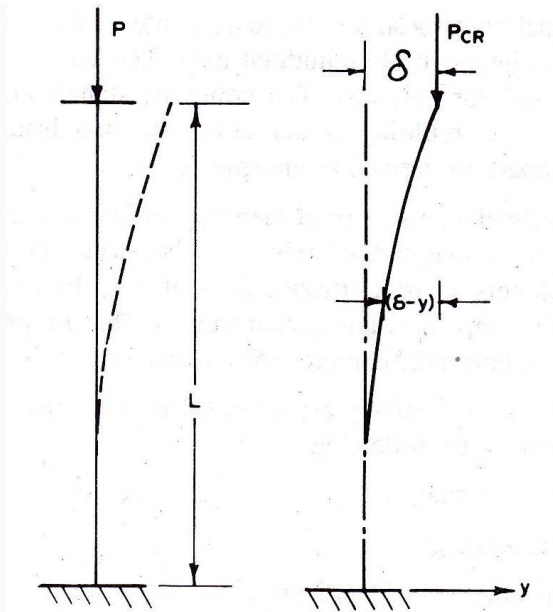
In about 1759, a Swiss mathematician Prof. Leonhard Euler derived the most popular column formula. The critical load for the column as shown in Fig 10.1 was determined as follows.

The differential equation of the deflected shape of the column is

$$EI \frac{d^2y}{dx^2} = +M \ (Hogging \ moment + ve)$$

The bending moment at any point on the deflected shape $M = +P(\delta - y)$

Therefore

$$EI \frac{d^2y}{dx^2} = P(\delta - y)$$

$$\frac{d^2y}{dx^2} - \frac{P}{EI}(\delta - y) = 0$$

Let $n = (\frac{P}{EI})^{1/2}$, Then the differential equation becomes,

$$\frac{d^2y}{dx^2} + n^2 y - n^2 \delta = 0$$

The general solution of this equation is $y = A.\sin(nx) + B.\cos(nx) + \delta$

At $\left(x = 0, \ y = 0, \ \frac{dy}{dx} = 0 \right)$, $B = -\delta$, $A = 0$,

$$y = \delta(1 - \cos nx)$$

At $x = l, \ y = \delta$ This condition is satisfied when $\delta \cos(nl) = 0$

From this, either $\delta = 0 \ or \ nl = 0$

If δ is zero, buckling of column does not occur, if cos nl=0, then

$$nl = \frac{\pi}{2}(2n - 1) \quad where \ n = 1,2,3, ...$$

For n=1, the values of nl is smallest. It is equal to $\pi/2$

Therefore $\qquad\qquad P_{cr} = (\frac{\pi^2 EI}{4l^2})$

The values of critical loads for other end conditions can be determined from this case. For a column, hinged at both ends,

$$P_{cr} = (\frac{\pi^2 EI}{l^2})$$

Where P_{cr}=Critical load (or Buckling load).

Since each half of the column is in the same position as the whole of the column. This is called fundamental case of buckling of a bar.

10.3 EFFECTIVE LENGTH OF COMPRESSION MEMBER

The effective length of a compression member depends upon end restraint conditions. The end restraint conditions are of two types as given below:

1. Position restraint

2. Direction restraint

10.3.1 **Position restraint**

In position restraint, end of the column is not free to change its position but rotation about the end of the column can take place e.g., hinged end of column as shown in Fig. 10.2.A

10.3.2 **Direction restraint**

In direction restraint, end of the column is free to change its position but rotation about the end of the column cannot take place.

When an end of a column is having restraint in position and direction both, then end is not free to change its position and the rotation about the end of the column also cannot take place as shown in Fig. 10.2.B

(A) POSITION RESTRAINT

(B) POSITION AND DIRECTION RESTRAINT
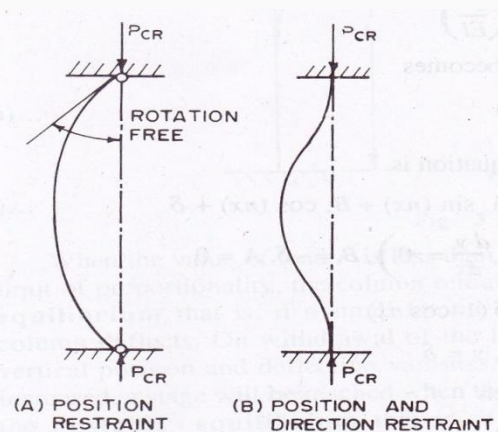
There are various possible combinations of restraints about either or both axes. The restraint conditions at the two ends of a column may be different or may be same. Following are the ideal cases of the end conditions:

1. Both ends of column hinged

2. Both ends of column fixed

3. One end of column fixed and the other end hinged

4. One end of column fixed and the other end free.

The deflected shapes of columns under critical loads have been shown in Fig. 10.3. A, B, C and D respectively. The actual lengths have been indicated by 'L'. Case one, both ends of the column hinged have been considered as standard case. The effective length (l) of a column is expressed in terms of equivalent length of compression members, hinged at both ends. It is the length of column between two adjacent points of zero moments and is represented by 'l'. It is also called as unsupported length.

IDEAL (STANDARD) CASES OF END CONDITIONS
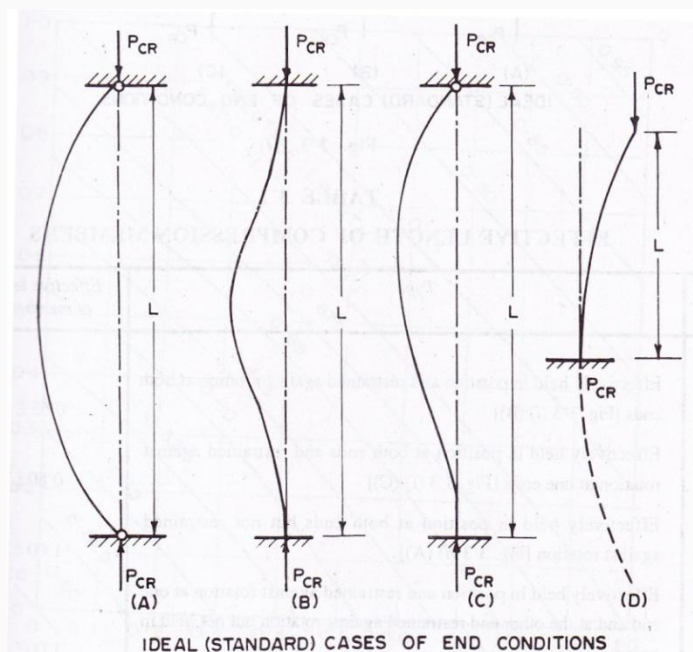
The ideal end conditions cannot be achieved in actual practice. The effective length of a compression member is adopted as per Table 10.1 as recommended by BIS in IS:800-1984 for different types of compression members. The effective length as given in this table will be adequate in most of the cases and the same may also be adopted where the column directly form part of the frame structures.

Table 10.1 EFFECTIVE LENGTH OF COMPRESSION MEMBERS

Sl. No.	Type	Effective length of member (l)
1	Effectively held in position and restrained against rotation at both ends [Fig. 10.3.B]	0.65 L
2	Effectively held in position at both ends and restrained against rotation at one ends [Fig. 10.3.C]	0.80 L
3	Effectively held in position at both end but not restrained against rotation [Fig. 10.3.A]	1.00 L
4	Effectively held in position and restrained against rotation at one end and at the other end restrained against rotation but not held in position [Fig. 10.4.A]	1.00 L
5	Effectively held in position and restrained against rotation at one end and at the other end partially restrained against rotation but not held in position. [Fig. 10.4.B]	1.50 L
6	Effectively held in position at one end but not restrained against rotation, at the other end restrained against rotation but not held in position. [Fig. 10.4.C]	2.00 L
7	Effectively held in position and restrained against rotation at one end but not held in position [Fig. 10.3.D] or restrained against rotation at the other end.	2.00 L

Note: For battened struts, the effective length l is increased by 10 per cent.

Where the exact frame analysis is not done, the effective length of columns in the frame structures may be found from the ratio of effective length to the unsupported length (l/L) from Fig. 10.5 when the relative displacements of the column is prevented (i.e. when there is no sway) and from Fig. 10.6 when the relative lateral displacement of the ends is not prevented (i.e. without restraint against sway viz., the sway occurs), when sway occurs, IS:800-1984 recommends that the effective length ratio, (l/L) may not be taken to be less than 1.2.

In Fig. 10.5 and Fig. 10.6,

EFFECTIVE LENGTH RATIOS FOR A COLUMN IN A FRAME WITH NO SWAY

EFFECTIVE LENGTH RATIOS FOR A COLUMN IN A FRAME WITHOUT RESTRAINT AGAINST SWAY
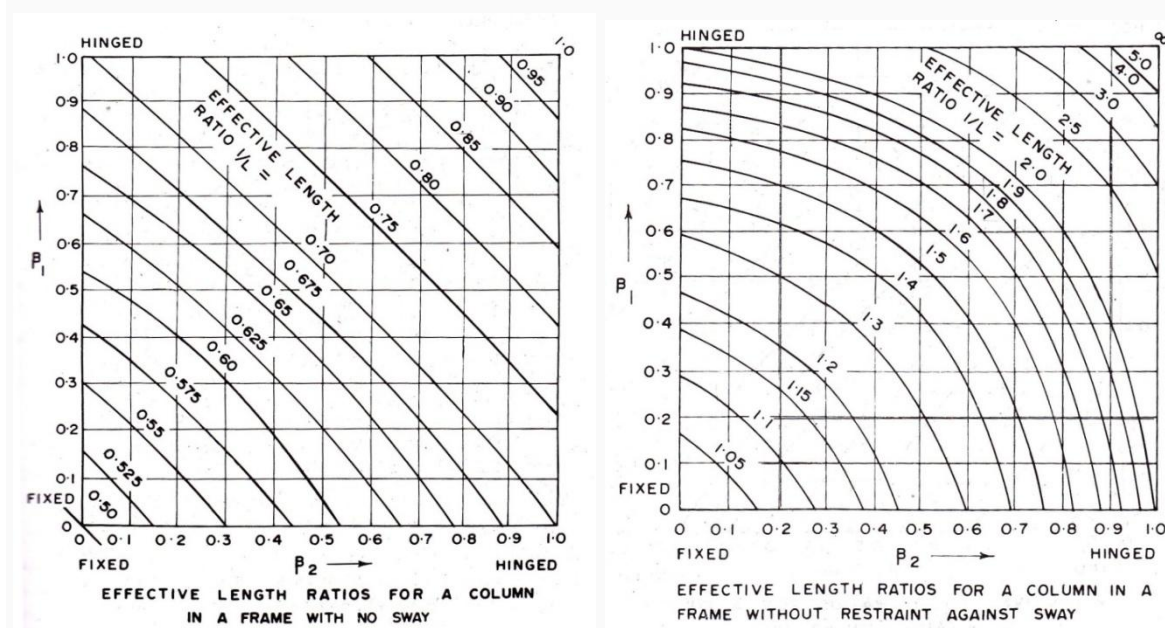
$$\beta_1 = \left(\frac{\Sigma k_c}{\Sigma k_c + \Sigma k_b}\right) \qquad So\ also \quad \beta_2 = \left(\frac{\Sigma k_c}{\Sigma k_c + \Sigma k_b}\right)$$

Where, the summation is to be done for the members framing into a joint at top and bottom respectively, and

k_c = flexural stiffness of the column and

k_b = flexural stiffness of the beam

Fig. 10.5 and Fig. 10.6 are from the paper titled as Effective lengths of columns in multistory buildings by Professor R.H.Wood, published in the structural Engineer Vol. 52, No. 7 July 1974. It is worthwhile to note that IS;800-1984 'code of practice for general construction in steel' and IS:456-1978 'code of practice for plain and reinforced concrete' have recommended the same effective lengths for the columns with similar support conditions.

10.4 EFFECTIVE SECTIONAL AREA

The gross cross-sectional area is the area as calculated from the specified size of the member or part thereof. The effective sectional area of a compression member is the gross cross-sectional area of the member. The deduction is not made for members connected by rivets, bolts and pins. If the holes are not filled by the fastening material, then deduction is made for unfilled holes. The effective area is however modified when the ratio of the outstand to thickness exceeds the limits specified by BIS. The deduction is also made from the gross cross-sectional area for excessive effective plate width (if any) to determine the effective sectional area.

10.5 RADIUS OF GYRATION

The radius of gyration of a section is a geometrical property of the section and it is denoted by r

$$r = (I/A)^{1/2}$$

where I=moment of inertia of the section about the axis.

r=radius of gyration of the section about the axis

A=effective sectional area of the section.

10.6 SLENDERNESS RATIO OF COMPRESSION MEMBER

The slenderness ratio of a compression member is defined as ratio of effective length of compression member (l) to appropriate radius of gyration (r)

$$Slenderness\ ratio,\ \lambda = l/r$$

The radii of gyration about various axes of rolled steel sections can be obtained from structural steel section tables (ISI Handbook No.1). The minimum radius of gyration is used

for computing the maximum slenderness ratio. For built-up compression members, value of radius of gyration is calculated. The slenderness ratio for a compression member should be as small as possible so that the material may be stressed to its greatest possible limit. The maximum slenderness ratio of compression members should not exceed the value given in Table 10.2. These limits have been laid down by BIS in IS:800-1984.

The end restraints of columns are often different in the two principal planes. The different moments of inertia of the column cross-section in these planes are sometimes desirable to achieve approximately equal slenderness ratios. The intermediate supports are provided to the columns for this purpose.

Table 10.2 MAXIMUM SLENDERNESS RATIO OF COMPRESSION MEMBER ($\lambda=l/r$)

Sl. No.	Type of member	$\lambda=l/r$
1	A member carrying comprehensive loads resulting from dead loads and imposed loads	180
2	A member subjected to compression forces resulting from wind/earthquake forces provided that the deformation of such members does not adversely affect the stress in any part of the structure	250
3	A member normally acting as a tie in a roof truss or a bracing system but subjected to possible reversal of stress resulting from the action of wind or earthquake forces	350

The intermediate supports reduce the unsupported length of the columns. When the unsupported lengths of columns are reduced then the smaller sections may be used at a higher average stress. Sometimes, the intermediate supports are furnished only in one direction, for example, a rolled steel I-section column is having its continuous length up to two storeys. At the level of one storey, intermediate support is provided by connecting beams with the web. The radius of gyration, r_{yy} of the section, about yy-axis (axis parallel to the web) is much smaller than the radius of gyration r_{xx} of the section, about xx-axis.

By providing the intermediate support, the effective length of the column become different in two different directions. The effective length of column, l_{yy} for bending about yy-axis is found by considering the length of column between one-storey only. The effective length of column l_{xx} for bending about xx-axis is found by considering the length of column between two storeys. It is seen that the effective length of column l_{yy} is much smaller than that of l_{xx}. The values of slenderness ratio (l_{yy}/r_{yy}) and (l_{xx}/r_{xx}) about two directions may be made approximately equal. As such the use of sections with different values of radii of gyration in two directions may be made economical. When a column is subjected to different bending moments in two directions then, the greater value of r may be kept in the direction of greater moment. The intermediate supports in the weak direction make the use of I-section and channel section economical.

10.7 COLUMN FORMULAE FOR AXIAL STRESS IN COMPRESSION

The strength of a column depends upon large number of variables. The efforts are made to obtain a design formula by fitting a curve to experimentally found buckling loads for the intermediate range of the slenderness ratio. It is tried to draw a curve which may merge with the Euler hyperbola in the very slender column range on one side and with the material yield strength for the zero length on the other side.

A perfectly straight column of perfectly homogeneous material (i.e. an ideal column) is subjected to an axial load. The primary object is to find the average axial stress in compression, which corresponds to the allowable load. The average axial stress is uniform across the section. It is given by

$$\sigma_{ac} = (P_a / A)$$

Where P_a = allowable load and A = cross-sectional area of the column

The required cross-sectional area for a given design load may be found conveniently in case σ_c is known

$$A_{reqd} = (P / \sigma_c)$$

Where P is the design load

10.8 DESIGN OF AXIALLY LOADED COMPRESSION MEMBER

When a column or compression member is designed, for given load, actual length of the member and its support conditions, the cross-sectional shape of the member is determined. The cross-sectional shape of axially loaded compression member depends largely on whether the compression member is long or short and whether it carries a small load or a large load. It is difficult to decide, whether a column is short or long. It is arbitrarily decided.

When the slenderness ratio of a column is less than 60, it may be considered as a short column. When the slenderness ratio is between 60 and 180, the column may be considered as long column. Following are the length and load categories arbitrarily made for design of compression members:

1. Short compression members with small loads
2. Short compression members with large loads
3. Long compression members with small loads
4. Long compression members with intermediate load.

The strength of axially loaded compression member depends upon slenderness ratio (l/r_{min}). For the design of axially loaded compression member load to be carried, the length of compression member and end conditions are known. The effective length of the compression member for the given end conditions is computed. The radius of gyration of compression member is not known as the cross-sectional shape of the compression member is not known. The allowable working stress in compression can be found when the slenderness ratio is known. There is no direct method of designing a compression member. The compression member is designed by trial and error method. The design of compression member is also done by using safe load tables, if available.

ISI handbook No.1 provides tables for safe concentric loads on rolled steel column sections (HB-sections) for bending about xx-axis and yy-axis. The effective length of column is determined knowing the end conditions. The values of safe concentric loads corresponding to respective effective lengths are given for various sizes of HB-sections. A column section having safe axial load equal to or slightly greater than the required load on the column is selected.

Design of Structures

Design procedure: Following are the usual steps in design of compression members.

Step 1. The slenderness ratio for the compression member and the value of yield stress for the steel are assumed. For the rolled steel beam section compression members, the slenderness ratio varies from 70 to 90. For struts, the slenderness ratio varies from 110 to 130. For compression members carrying large loads, the slenderness ratio is about 40.

Step 2. The effective sectional area (A) required for compression member is determined.

$A = (P/\sigma_c)$, Where P is the design load to be carried by the member

Step 3. From the steel section tables, section for the compression member of the required area is selected. The section for the compression member is selected such that it has the largest possible radius of gyration for the required sectional area. It should also be most economical section.

Step 4. Knowing the geometrical properties of the section slenderness ratio is computed and allowable axial stress in compression is found from IS:800-1984 for the quality of steel assumed.

Step 5. The safe load carrying capacity of the compression member is determined.

The section selected for the compression member is revised in case the safe load carrying of the compression member is less than or much larger than the load to be carried by it.

Example 10.1 A rolled steel beam section HB 350 @0.674 kN/m is used as a stanchion. If the unsupported length of the stanchion is 4 m, determine safe load carrying capacity of the section.

Solution:

Step 1: Properties of I-section

HB 350 @ 0.674 kN/m section is used as a stanchion. From the steel tables, the geometrical properties of the section are as follows:

Sectional area $A = 8591 \ mm^2$

Radius of gyration $r_{xx} = 149.3 \ mm$

Radius of gyration $r_{yy} = 53.4 \ mm$

 Step 2: Slenderness ratio

Minimum radius of gyration $r_{min} = 53.4 \ mm$

Unsupported length   l= 4 m

Slenderness ratio of the stanchion $\frac{l}{r_{min}} = \left(\frac{4 \times 1000}{53.4}\right) \approx 75$

Step 3: Safe load

From IS:800-1984 for l/r=75 and the steel having yield stress, f_y=260 N/mm², allowable working stress in compression σ_{ac} =109 N/mm² (MPa)

The safe load carrying capacity of the stanchion $P = (\sigma_{ac}A) = \left(\frac{109 \times 8591}{1000}\right) = 936.42\ kN$

Example 10.2 In Example 10.1, in case standard column section SC 25°,@85.6 kg/m is used as a column, determine the safe load carrying capacity of the section.
Solution:

Step 1: Properties of I-section

From IS:808-1964, the geometrical properties of the section are as follows:

Sectional area   A = 109 x 10² mm²

Radius of gyration r_{xx}= 107 mm

Radius of gyration r_{yy}= 54.6 mm

 Step 2: Slenderness ratio

Minimum radius of gyration r_{min}= 54.6 mm

Unsupported length   l= 4 m = 4000 mm

Slenderness ratio of the stanchion $\frac{l}{r_{min}} = \left(\frac{4 \times 1000}{54.6}\right) = 73.26$

Step 3: Safe load

From IS:800-1984 for l/r=73.26 and the steel having yield stress, f_y=260 N/mm², allowable working stress in compression

$$\sigma_{ac} = (115-(12/10 \times 3.26)) = 111.088\ N/mm² \text{ (MPa)}$$

The safe load carrying capacity of the stanchion $P = (\sigma_{ac}A) = \left(\frac{111.088 \times 109 \times 100}{1000}\right) = 1210.86\ kN$

Example 10.3 A built-up column consist of three rolled steel beam sections WB 450 @0.794 kN/m, connected effectively to act as one column as shown in Fig. 10.7. Determine the safe load carrying capacity of built-up section, if unsupport length of column is 4.25 m.

Design of Structures
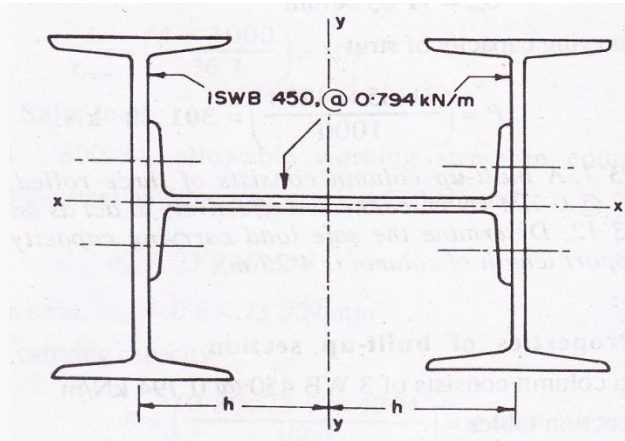
Solution:

Step 1: Properties of built-up section

The built-up column consist of 3 WB 450 @0.794 kN/m

From steel section tables, area of section   =(3 x 101.15 x 100)   = 30345 mm²

Moment of inertia of built-up section about xx-axis

I_{xx}=[2 x 35057.6 + 1706.7] x 10⁴   = 71821 x 10⁴ mm⁴

Moment of inertia of built-up section about yy-axis

I_{yy}=[35057.6 x 10⁴ + 2 x 1706.7 x 10⁴ + 2Ah²] = 38471 x 10⁴ + 2Ah² mm⁴

Where A=sectional area of one I-section

I_{yy}=[38471.0 + 2 x 101.15(22.5 + ½ x0.92)²] x 10⁴ mm⁴

= 145115.79 x 10⁴ mm⁴

Step 2: Slenderness ratio

I_{xx} is less than I_{yy}. Therefore r_{xx} is Minimum.

Minimum radius of gyration $r_{min} = \left(\frac{71821.9 \times 10^4}{30345}\right)^{1/2} = 154\ mm$

Unsupported length   l= 4.25 m = 4250 mm

Slenderness ratio of the column $\frac{l}{r_{min}} = \left(\frac{4.25 \times 1000}{154.0}\right) = 27.6$

Step 3: Safe load

From IS:800-1984 for l/r=27.6 and the steel having yield stress, f_y=260 N/mm², allowable working stress in compression

σ_{ac} =151.72 N/mm² (MPa)

Safe load carrying capacity of built-up section $P = (\sigma_{ac}A) = \left(\frac{151.72 \times 303.45}{1000}\right) = 46.04\ kN$

Example 10.4 Design a rolled steel beam section column to carry an axial load 1100 kN. The column is 4 m long and adequately restrained in position but not in direction at both ends:

Design: The slenderness ratio for the column and the value of yield stress for the steel to be used may be assumed as 80 and 260 N/mm² respectively.

Step 1. Selection of trial section

Allowable stress as per IS:800-1984, $\quad\quad\quad \sigma_{ac} = 103$ N/mm²

Effective sectional area required $= \left(\dfrac{1100\times1000}{103}\right) = 10679.61\ mm^2$

Effective length of the column is 4 m

Step 2: Properties of trial section

From steel section tables, try HB 450,@0.872 kN/m section

Sectional area $\quad$ A=11114 mm², $\quad$ r_{xx}=187.8 mm, $\quad\quad\quad$ r_{yy}=51.8 mm

Step 3. Slenderness ratio

Therefore $\quad$ r_{min}=51.8 mm

Slenderness ratio $\dfrac{l}{r_{min}} = \left(\dfrac{4.0\times1000}{51.8}\right) = 77.2$

Step 4. Check for safe load

From IS:800-1984 for l/r=77.2 and the steel having yield stress, f_y=260 N/mm², allowable working stress in compression $\quad\quad \sigma_{ac}$ =106.36 N/mm² (MPa)

Safe load carrying capacity of the column $\quad P = (\sigma_{ac}A) = \left(\dfrac{106.36\times11114}{1000}\right) = 1182.08\ kN$

The column section lighter in weight than this is not suitable. Hence the design is satisfactory.

Alternatively:

Step 2: Properties of trial section

From IS:808-1984, try SC 250,@85.6 kg/m (standard column section)

A= 109 x 100 mm², $\quad\quad\quad$ r_{xx}=107.0 mm, $\quad\quad\quad$ r_{yy}=54.6 mm

Step 3: Slenderness ratio

Effective length of column is 4000 mm. Minimum radius of gyration, r_{min}=54.6 mm.

Therefore, Slenderness ratio $\frac{l}{r_{min}} = \left(\frac{4000}{54.6}\right) = 73.26$

Step 4. Check for safe load

From IS:800-1984 for l/r=73.26 and the steel having yield stress, f_y=260 N/mm², allowable

working stress in compression $\sigma_{ac} = \left(115 - \frac{12}{10} \times 3.26\right) = 111.088$ N/mm² (MPa)

Safe load carrying capacity of the column $P = (\sigma_{ac} A) = \left(\frac{111.088 \times 109 \times 100}{1000}\right) = 1210.86\ kN$

Example 10.5 A column 5 m long is to support a load 4500 kN. The ends of the column are effectively held in position and direction. Design the column if rolled steel beams and 18 mm plates are only available.

Design:

Step 1: Selection of trial section

Length of column   L=5m

Effective length of column is (0.65 x 5) =3.25 m

In order to support large load, the slenderness ratio for the built-up column and the value of yield stress for the steel may be assumed as 40 and 260 N/mm², respectively.

Allowable working stress in compression σ_{ac} =145 N/mm² (MPa)

Effective sectional area required $= \left(\frac{4500 \times 1000}{145}\right) = 31034.48\ mm^2$

Step 2: Properties of trial section

From steel section tables, try HB 450,@0.925 kN/m section, r_{xx}=185.0 mm, r_{yy}=50.8 mm

For the columns carrying large loads, r_{xx}= r_{yy}

Step 3. Slenderness ratio

Therefore r_{min} for the plated built-up column may be estimated as 185 mm

Slenderness ratio $\frac{l}{r_{min}} = \left(\frac{3.25 \times 1000}{185}\right) = 17.567$

Step 4. Area of plates

From IS:800-1984 for l/r=17.567 and the steel having yield stress, f_y=260 N/mm², allowable working stress in compression σ_{ac} =154.486 N/mm² (MPa)

Sectional area required $= \left(\frac{4500 \times 1000}{154.486}\right) = 29128.85\ mm^2$

Area of HB 450,@0.925 kN/m is 11789 mm²

Area to be provided by two cover plates   = 17339.74 mm²

Area to be provided by one plate   = 8669.87 mm²

Plates available   = 18 mm

Width required $\qquad$ = (8669.87/18) =481.66 mm

Provide 700 mm width of cover plate

Step 5: Check for outstanding width; thickness ratio for cover plate

Outstanding width $\quad$ = ½ (700-140)=560/2=280 mm

Thickness $\qquad$ = 18 mm

$$\left(\frac{Outstanding\ width}{Thickness}\right) = \frac{280}{18} = 15.55 < 16 \ which\ is\ satisfactory$$

Step 6: Check for load carrying capacity

1. Properties of section

I_{yy} of plates $\qquad$ = 2 x (1/12) x 1.8 x 70^3 x 10^4 mm^4 $\quad$ = 102900 x 10^4 mm^4

I_{yy} of H-section $\qquad$ = 3045 x 10^4 mm^4

Iyy of compound section $\quad$ = 105945 x 10^4 mm^4

Area of section $\qquad$ = (117.89 + 2 x 1.8 x 100 mm^2 = 369.89 x 100 mm^2

$$r_{yy} = \left(\frac{105945 \times 10^4}{369.9 \times 100}\right)^{1/2} = 169.2 \ mm$$

r_{xx}=185 mm (for I-section alone)

2. Slenderness ratio of column section

$$\frac{l}{r_{min}} = \left(\frac{3.25 \times 1000}{169.2}\right) = 19.21$$

From IS:800-1984 for l/r=19.21 and the steel having yield stress, f_y=260 N/mm^2, allowable working stress in compression $\quad$ σ_{ac} =154.158 N/mm^2 (MPa)

$$\text{Safe load carrying capacity } P = (\sigma_{ac}A) = \left(\frac{154.158 \times 36989}{1000}\right) = 5702.15 \ kN$$

Hence, the design is satisfactory. Provide HB 450,@0.925 kN/m with two plates 700 mm x 18 mm. One plate is connected with each flange of I-section. The design drawing is given in Fig. 10.8

Step 4. Area of plates

From IS:800-1984 for l/r=17.567 and the steel having yield stress, f_y=260 N/mm², allowable working stress in compression σ_{ac} =154.486 N/mm² (MPa)

$$\text{Sectional area required} = \left(\frac{4500 \times 1000}{154.486}\right) = 29128.85 \; mm^2$$

Area of HB 450,@0.925 kN/m is 11789 mm²

Area to be provided by two cover plates   = 17339.74 mm²

Area to be provided by one plate   = 8669.87 mm²

Plates available   = 18 mm

Width required   = (8669.87/18) =481.66 mm

Provide 700 mm width of cover plate

Step 5: Check for outstanding width; thickness ratio for cover plate

Outstanding width   = ½ (700-140)=560/2=280 mm

Thickness   = 18 mm

$$\left(\frac{Outstanding \; width}{Thickness}\right) = \frac{280}{18} = 15.55 \; < 16 \; which \; is \; satisfactory$$

Step 6: Check for load carrying capacity

1. Properties of section

I_{yy} of plates   = 2 x (1/12) x 1.8 x 70³ x 10⁴ mm⁴   = 102900 x 10⁴ mm⁴

I_{yy} of H-section   = 3045 x 10⁴ mm⁴

I_{yy} of compound section   = 105945 x 10⁴ mm⁴

Area of section   =(117.89 + 2 x 1.8 x 100 mm² = 369.89 x 100 mm²

MODULE 6.

LESSON 11. Design of Compression Members

11.1 INTRODUCTION

A strut is defined as a structural member subjected to compression in a direction parallel to its longitudinal axis. The term strut is commonly used for compression members in roof trusses. A strut may be used in a vertical position or in an inclined position in roof trusses. The compression members may be subjected to both axial compression and bending.

When compression members are overloaded then their failure may take place because of one of the following:

1. Direct compression

2. Excessive bending

3. Bending combined with twisting

The failure of column depends upon its slenderness ratio. The load required to cause above mentioned failures decreases as the length of compression member increases, the cross-sectional area of the member being constant.

11.2 COMMON SECTIONS OF COMPRESSION MEMBERS

The common sections used for compression members are shown in Fig. 11.1 with their approximate radii of gyration. A column or a compression member may be made
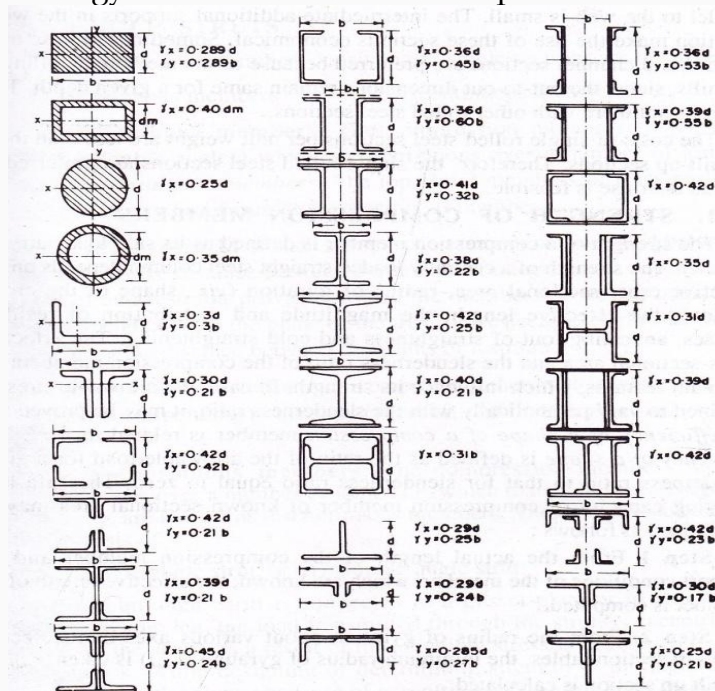
APPROXIMATE RADIUS OF GYRATION FOR COMMON SECTIONS FOR COMPRESSION MEMBERS

of many different sections to support a given load. Few sections satisfy practical requirement in a given case. A tubular section is most efficient and economical for the column free to buckle in any direction. The radius of gyration r for the tubular section in all the directions remains same. The tubular section has high local buckling strength. The tubular sections are suitable for medium loads. However, it is difficult to have their end connections. A solid round bar having a cross-sectional area equal to that of a tubular section has radius of gyration, r much smaller than that of tube. The solid round bar is less economical than the tubular section. The solid round bar is better than the thin rectangular section or a flat strip. The radius of gyration of flat strip about its narrow direction is very small. Theoretically, the rods and bars do resist some compression. When the length of structural member is about 3 m, then the compressive strengths of the rods and bars are very small.

Single angle sections are rarely used except in light roof trusses, because of eccentricity at the end connections. Tee-sections are often used in roof trusses. The single rolled steel I-section and single rolled steel channel section are seldom used as column. The value of radius of gyration r, about the axis parallel to the web is small. The intermediate additional supports in the weak direction make the use of these sections economical. Sometimes the use of I-sections and channel sections are preferred because of the method of rolling at the mills, since, the out-to-out dimensions remain same for a given depth. This failure is not there with other rolled steel sections. The costs of single rolled steel sections per unit weight are less than those of built-up sections. Therefore the single rolled steel sections are preferred so long as their use is feasible.

11.3 STRENGTH OF COMPRESSION MEMBERS

The strength of a compression member is defined as its safe load carrying capacity. The strength of a centrally loaded straight steel column depends on the effective cross-sectional area, radius of gyration (viz., shape of the cross-section), the effective length, the magnitude and distribution of residual stresses, annealing, out of straightness and cold straightening. The effective cross-sectional area and the slenderness ratio of the compression members are the main features, which influence its strength. In case, the allowable stress is assumed to vary parabolically with the slenderness ratio, it may be proved that the efficiency of a shape of a compression member is related to A/r^2. The efficiency of a shape is defined as the ratio of the allowable load for a given slenderness ratio to that for slenderness ratio equal to zero. The safe load carrying capacity of compression member of known sectional area may be determined as follows:

Step 1. From the actual length of the compression member and the support conditions of the member, which are known, the effective length of the member is computed.

Step 2. From the radius of gyration about various axes of the section given in section tables, the minimum radius of gyration (r_{min}) is taken. r_{min} for a built up section is calculated.

Step 3. The maximum slenderness ratio (l/ r_{min}) is determined for the compression member.

Step 4. The allowable working stress (σ_{ac}) in the direction of compression is found corresponding to the maximum slenderness ratio of the column from IS:800-1984.

Design of Structures

Step 5. The effective sectional area (A) of the member is noted from structural steel section tables. For the built up members it can be calculated.

Step 6. The safe load carrying capacity of the member is determined as $P=(\sigma_{ac}.A)$, where P=safe load

11.4 ANGLE STRUTS

The compression members consisting of single sections are of two types:

1. Discontinuous members

2. Continuous members

11.4.1 **Continuous members**

The compression members (consisting of single or double angles) which are continuous over a number of joints are known as continuous members. The top chord members of truss girders and principal rafters of roof trusses are continuous members. The effective length of such compression members is adopted between 0.7 and 1.0 times the distance between the centres of intersections, depending upon degree of restraint provided. When the members of trusses buckle in the plane perpendicular to the plane of the truss, the effective length shall be taken as 1.0 times the distance between the points of restraint. The working stresses for such compression members is adopted from IS:800-1984 corresponding to the slenderness ratio of the member and yield stress for steel.

11.4.2 **Discontinuous members**

The compression members which are not continuous over a number of joints, i.e., which extend between two adjacent joints only are known as discontinuous members. The discontinuous members may consist of single angle strut or double angle strut. When an angle strut is connected to a gusset plate or to any structural member by one leg, the load transmitted through the strut, is eccentric on the section of the strut. As a result of this, bending stress is developed along with direct stress. While designing or determining strength of an angle strut, the bending stress developed because of eccentricity of loading is accounted for as follows:

i.Single angle strut

(A) SINGLE ANGLE STRUT CONNECTED WITH ONE RIVET.

Design of Structures
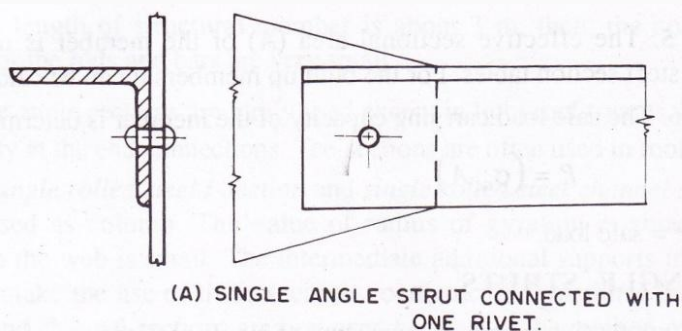
1. When single angle discontinuous strut is connected to a gusset plate with one rivet as shown in Fig. 11.2.A, its effective length is adopted as centre to centre of intersection at each end and the allowable working stress corresponding to the slenderness ratio of the member is reduced to 80 per cent. However, the slenderness ratio of such single angle strut should not exceed 180.

2. When a single angle discontinuous strut is connected with two or more number of rivets or welding as shown in Fig. 11.2.B, its effective length is adopted as 0.85 times the length of strut centre to centre of intersection of each end and allowable working stress corresponding to the slenderness ratio of the member is not reduced.

(B) SINGLE ANGLE STRUT CONNECTED TWO OR MORE RIVETS.
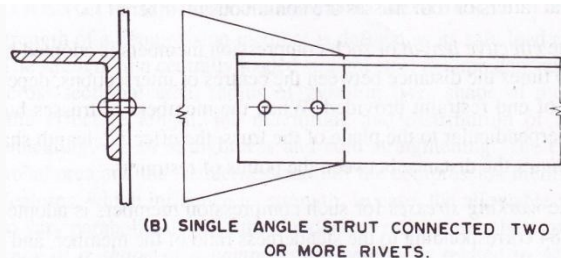
ii.Double angle strut

1. A double angle discontinuous strut with angles placed back to back and connected to both sides of a gusset or any rolled steel section by not less than two rivets or bolts or in line along the angles at each end or by equivalent in welding as shown in Fig. 11.3.A, can be regarded as an axially loaded strut. Its effective length is adopted as 0.85 times the distance between intersections, depending on the degree of restraint provided and in the plane perpendicular to that of the gusset, the effective length 'l' shall be taken as equal to the distance between centres of the intersections. The tacking rivets should be provided at appropriate pitch.

2. The double angles, back to back connected to one side of a gusset plate or a section by one or more rivets or bolts or welds as show in Fig. 11.3.B, these are designed as single angle discontinuous strut connected by single rivet or bolt.

If the struts carry in addition to axial loads, loads which cause transverse bending, the combined bending and axial stress shall be checked as described for the columns subjected to eccentric loading. The tacking rivets should be provided at appropriate pitch.

The tacking rivets are also termed as stitching rivets. In case of compression members, when the maximum distance between centres of two adjacent rivets exceeds 12 t to 200 mm whichever is less, then tacking rivets are used. The tacking rivets are not subjected to calculated stress. The tacking rivets are provided throughout the length of a compression member composed of two components back to back. The two components of a member act together as one piece by providing tacking rivets at a pitch in line not exceeding 600 mm and such that minimum slenderness ratio of each member between the connections is not greater than 40 or 0.6 times the maximum slenderness ratio of the strut as a whole, whichever is less.

Design of Structures

In case where plates are used, the tacking rivets are provided at a pitch in line not exceeding 32 times the thickness of outside plate or 300 mm whichever is less. Where the plates are exposed to weather the pitch in line shall not exceed 16 times the thickness of the outside plate or 200 mm whichever is less. In both cases, the lines of rivets shall not be apart at a distance greater than these pitches.

The single angle sections are used for the compression members for small trusses and bracing. The equal angle sections are more desirable usually. The unequal angle sections are also used. The minimum radius of gyration about one of the principal axis is adopted for calculating the slenderness ratios. The minimum radius of gyration of the single angle section is much less than the other sections of same cross-sectional area. Therefore, the single angle sections are not suitable for the compression member of long lengths. The single angle sections are commonly used in the single plane trusses (i.e., the trusses having gusset plates in one plane). The angle sections simplify the end connections.

The tee-sections are suitable for the compression members for small trusses. The tee-sections are more suitable for welding.

Example 11.1 A single angle discontinuous strut ISA 150 mm x 150 mm x 12 mm (ISA 150 150,@0.272 kN/m) with single riveted connection is 3.5 m long. Calculate safe load carrying capacity of the section.

Solution:

Step 1: Properties of angle section

ISA 150 mm x 150 mm x 12 mm (ISA 150 150,@0.272 kN/m) is used as discontinuous strut. From the steel tables, the geometrical properties of the section are as follows:

Sectional area $\qquad$ A = 3459 mm^2

Radius of gyration $\qquad$ $r_{xx}= r_{yy}=149.3$ mm

Radius of gyration $\qquad$ $r_{uu}= 58.3$ mm, $r_{vv}=29.3$ mm

Step 2: Slenderness ratio,

Minimum radius of gyration $r_{min}= 29.3$ mm

Effective length of strut l= 3.5 m

Slenderness ratio of the strut

Step 3: Safe load

From IS:800-1984 for l/r=119.5 and the steel having yield stress, f_y=260 N/mm^2, allowable working stress in compression σ_{ac} =64.45 N/mm^2 (MPa)

For single angle discontinuous strut with single riveted connection, allowable working stress

Design of Structures

$0.80\ \sigma_{ac} = (0.80 \times 64.45) = 51.56\ N/mm^2$.

The safe load carrying capacity $\qquad P = (\sigma_{ac}A) = \left(\dfrac{51.56 \times 3459}{1000}\right) = 178.346\ kN$

Example 11.2 In case in Example 11.1, a discontinuous strut 150 x 150 x 15 angle section is used, calculate the safe load carrying capacity of the section.

Solution:

Step 1: Properties of angle section

Angle section 150 mm x 150 mm x 15 mm is used as discontinuous strut. From the steel tables, the geometrical properties of the section are as follows:

Sectional area $\qquad$ A = 4300 mm²

Radius of gyration $\qquad$ $r_{xx} = r_{yy} = 45.7$ mm

Radius of gyration $\qquad$ $r_{uu} = 57.6$ mm, $r_{vv} = 29.3$ mm

Step 2: Slenderness ratio,

Minimum radius of gyration $r_{min} = 29.3$ mm

Effective length of strut l = 3.5 m

Slenderness ratio of the strut $\qquad \dfrac{l}{r_{min}} = \left(\dfrac{3.5 \times 1000}{29.3}\right) = 119.5$

Step 3: Safe load

From IS:800-1984 for l/r=119.5 and the steel having yield stress, f_y=260 N/mm², allowable working stress in compression σ_{ac} =64.45 N/mm² (MPa)

For single angle discontinuous strut with single riveted connection, allowable working stress

$0.80\ \sigma_{ac} = (0.80 \times 64.45) = 51.56\ N/mm^2$.

The safe load carrying capacity $\qquad P = (\sigma_{ac}A) = \left(\dfrac{51.56 \times 4300}{1000}\right) = 221.708\ kN$

Example 11.3 In Example 11.1, if single angle discontinuous strut is connected with more than two rivets in line along the angle at each end, calculate the safe load carrying capacity of the section.

Design of Structures

Solution:

Step 1: Properties of angle section

Discontinuous strut ISA 150 mm x 150 mm x 12 mm (ISA 150 150,@0.272 kN/m) is used with double riveted connections. From the steel tables, the geometrical properties of the section are as follows:

Sectional area   A = 3459 mm²

Radius of gyration r_{xx}= r_{yy}=149.3 mm

Radius of gyration r_{uu}= 58.3 mm, r_{vv}=29.3 mm

Length of strut between centre to centre of intersection L=3.50 m

Step 2: Slenderness ratio,

Minimum radius of gyration r_{min}= 29.3 mm

Effective length of discontinuous strut double riveted   0.85 x L= 0.85 x 3.5 = 2.975 m

Slenderness ratio of the strut $\frac{l}{r_{min}} = \left(\frac{2.975 \times 1000}{29.3}\right) = 101.5$

Step 3: Safe load

From IS:800-1984 for l/r=101.5 and the steel having yield stress, f_y=260 N/mm², allowable working stress in compression σ_{ac} =71.65 N/mm² (MPa)

Allowable working stress for discontinuous strut double riveted is not reduced.

The safe load carrying capacity $P = (\sigma_{ac} A) = \left(\frac{71.65 \times 3459}{1000}\right) = 247.84 \ kN$

Example 11.4 A double angle discontinuous strut ISA 125 mm x 95 mm x 10 mm (ISA 125 95,@0.165 kN/m) long legs back to back is connected to both the sides of a gusset plate 10 mm thick with 2 rivets. The length of strut between centre to centre of intersections is 4 m. Determine the safe load carrying capacity of the section.

Solution:

Step 1: Properties of angle section

The double angle discontinuous strut 2 ISA 125 mm x 95 mm x 10 mm (ISA 125 95,@0.165 kN/m) is shown in Fig. 11.4. Assume the tacking rivets are used along the length. From the steel tables, the geometrical properties of (two angle back to back) the sections are as follows:

Sectional area   A = 4204 mm²

Radius of gyration r_{xx}= 39.4 mm

95

Design of Structures

Angles are 10 mm apart

Radius of gyration $\quad r_{yy}$= 40.1 mm

Length of strut between centre to centre of intersection L=4 m

Step 2: Slenderness ratio,

Minimum radius of gyration r_{min}= 39.4 mm

Effective length of discontinuous strut $\quad$ 0.85 x L= 0.85 x 4.0 = 3.40 m

Slenderness ratio of the strut $\dfrac{l}{r_{min}} = \left(\dfrac{3.4 \times 1000}{39.4}\right) = 86.3$

Step 3: Safe load

From IS:800-1984 for l/r=86.3 and the steel having yield stress, f_y=260 N/mm², allowable working stress in compression σ_{ac} =95.96 N/mm² (MPa)

The safe load carrying capacity $P = (\sigma_{ac}A) = \left(\dfrac{95.96 \times 4204}{1000}\right) = 403.416 \ kN$

Example 11.5 In Example 11.4, if double discontinuous strut is connected to one side of a gusset, determine safe load carrying capacity of the strut.

Solution:

Step 1: Properties of angle section

The double angle discontinuous strut 2 ISA 125 mm x 95 mm x 10 mm (ISA 125 95,@0.165 kN/m) connected to one side of a gusset is shown in Fig. 11.5. Assume the tacking rivets are used along the length. From the steel tables, the geometrical properties of (two angle back to back) the sections are as follows:
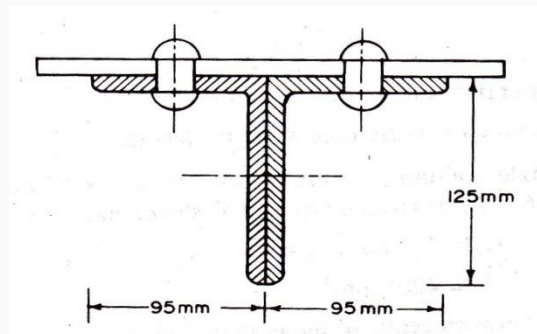
Sectional area $\quad$ A = 4204 mm²

Radius of gyration $\quad r_{xx}$= 39.4 mm

Distance back to back of angles is zero

Design of Structures

Radius of gyration r_{yy}= 36.7 mm

Effective length of strut whether single riveted or double riveted L=4 m

Step 2: Slenderness ratio,

Minimum radius of gyration r_{min}= 36.7 mm

$$\frac{l}{r_{min}} = \left(\frac{4 \times 1000}{36.7}\right) = 109$$

Slenderness ratio of the strut

Step 3: Safe load

From IS:800-1984 for l/r=109 and the steel having yield stress, f_y=260 N/mm², allowable working stress in compression σ_{ac} =73.9 N/mm² (MPa)

For above strut, allowable working stress   0.80 σ_{ac} = (0.80 x 73.9) = 59.12 N/mm².

$$P = (\sigma_{ac}A) = \left(\frac{59.12 \times 4204}{1000}\right) = 248.54 \ kN$$

The safe load carrying capacity

Example 11.6 In Example 11.4, double angle strut is continuous and connected with a gusset plate with single rivet; determine safe load carrying capacity of the strut.

Solution:

Step 1: Properties of angle section

The double angle discontinuous strut 2 ISA 125 mm x 95 mm x 10 mm (ISA 125 95,@0.165 kN/m) is singly riveted as shown in Fig. 11.4. Assume the tacking rivets are used along the length. From the steel tables, the geometrical properties of (two angle back to back) the sections are as follows:
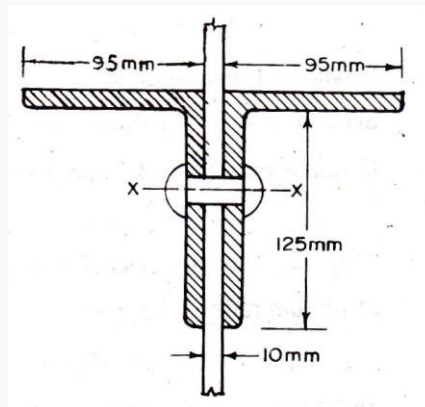
Sectional area   A = 4204 mm²

Radius of gyration r_{xx}= 39.4 mm

Angles are 10 mm apart

Design of Structures

Radius of gyration r_{yy}= 40.1 mm

Length of strut between centre to centre of intersection L=4 m

Step 2: Slenderness ratio,

Minimum radius of gyration r_{min}= 39.4 mm

Effective length   L= 4 m

Slenderness ratio of the strut
$$\frac{l}{r_{min}} = \left(\frac{4 \times 1000}{39.4}\right) = 101.5$$

Step 3: Safe load

From IS:800-1984 for l/r=101.5 and the steel having yield stress, f_y=260 N/mm², allowable working stress in compression σ_{ac} =71.65 N/mm² (MPa)

$$P = (\sigma_{ac} A) = \left(\frac{71.65 \times 4204}{1000}\right) = 301.22 \, kN$$

The safe load carrying capacity

Example 11.7 Design a single angle discontinuous strut to carry 110 kN load. The length of the strut between centre to centre of intersections is 3.25 m.

Design:

Step 1: Selection of trial section

Assuming that the angle strut is connected to the gusset plate with two or more than two rivets.

Effective length of strut   l=0.85L=(0.85 x 3.25 x 1000) = 2762.5 mm.

The slenderness ratio for the single angle discontinuous strut and value of yield stress for the steel may be assumed as 130 and 260 N/mm², respectively.

Therefore, allowable stress in compression for strut σ_{ac} =57 N/mm² (MPa)

Effective sectional area required
$$= \left(\frac{110 \times 1000}{57}\right) = 1929.82 \, mm^2$$

The equal angle section is suitable for single angle strut. It has maximum value for minimum radius of gyration.

Step 2: Properties of trial section

From steel section tables, try ISA 110 mm x 110 mm x 10 mm (ISA 110 110@0.165 kN/m)

Sectional area A=2106 mm², $r_{xx}=r_{yy}$=33.6 mm, r_{uu}=42.5 mm, r_{vv}=21.4 mm

Design of Structures

Therefore r_{min}=21.4 mm

Step 3: Slenderness ratio

Slenderness ratio $\dfrac{l}{r_{min}} = \left(\dfrac{2762.5}{21.4}\right) = 129.09$

Step 4: Safe load

From IS:800-1984, allowable working stress in compression for the steel having yield stress as 260 N/mm² σ_{ac} =57.56 N/mm² (MPa)

The safe load carrying capacity $P = (\sigma_{ac}A) = \left(\dfrac{57.56 \times 2106}{1000}\right) = 121.22\ kN$

The angle section lighter in weight than this is not suitable. Hence the design is satisfactory.

Step 5: Check for width of outstanding leg

Width of outstanding leg to thickness ratio $P = (\sigma_{ac}A) = \left(\dfrac{57.56 \times 2106}{1000}\right) = 121.22\ kN$

Hence, satisfactory. Provide ISA 110 mm x 110 mm x 10 mm (ISA 110 110@0.165 kN/m) for discontinuous strut.

Alternatively:

Step 2: Properties of trial section

From IS:808-1984, try angle section 120 x 120 x 10 (@ 18.2 kg/m)

Sectional area, A=2320 mm², $r_{xx}=r_{yy}$=36.7 mm, r_{uu}=46.3 mm, r_{vv}=23.6 mm

Step3: Slenderness ratio

Effective length of strut is 2762.5 mm

Minimum radius of gyration r_{min}=23.6 mm

Slenderness ratio $\dfrac{l}{r_{min}} = \left(\dfrac{2762.5}{23.6}\right) = 117.055$

Step 4: Safe load carrying capacity

From IS:800-1984 for l/r=117.055 and the steel having yield stress, f_y=260 N/mm², allowable working stress in compression

$\sigma_{ac} = \left(73 - \dfrac{9}{10} \times 7.55\right) = 66.205\ N/mm^2\ (MPa)$

$$P = (\sigma_{ac}A) = \left(\frac{66.205 \times 2320}{1000}\right) = 153.596 \, kN$$

Safe load carrying capacity

The angle section lighter in weight than this is not suitable. Hence the design is satisfactory.

Example 11.8 Design a double angle discontinuous strut to carry 150 kN load. The length of strut between centre to centre of intersections is 4 m

Design:

Step 1: Selection of trial section

Assuming that the strut is connected to both sides of gusset 10 mm thick by two or more than two rivets.

Length of strut   L= 4.00 m

Effective length of strut   l=0.85L=(0.85 x 4) = 3.40 m.

The slenderness ratio of a double angle discontinuous strut and the value of yield stress for the steel may be assumed as 120 and 260 N/mm², respectively.

Therefore, allowable stress in compression σ_{ac} =64 N/mm² (MPa)

$$= \left(\frac{150 \times 1000}{64}\right) = 2343.75 \, mm^2$$

Effective sectional area required

Step 2: Properties of trial section

From steel section tables (properties of two angles back to back), try 2 ISA 100 mm x 65 mm x 8 mm (2 ISA 100 65,@0.099 kN/m)

Sectional area A=2514 mm², r_{xx}=31.6 mm,

For angles having 10 mm distance back to back and long legs vertical r_{yy}=27.5 mm

Therefore r_{min}=27.5 mm

Step 3: Slenderness ratio

$$\frac{l}{r_{min}} = \left(\frac{3.40 \times 1000}{27.5}\right) = 123.6$$

Slenderness ratio

Step 4: Safe load

From IS:800-1984, allowable working stress in compression for the steel having yield stress as 260 N/mm² σ_{ac} =61.48 N/mm² (MPa)

The safe load carrying capacity $$P = (\sigma_{ac} A) = \left(\frac{61.48 \times 2541}{1000}\right) = 156.22\ kN$$

The angle section lighter in weight than this is not suitable. Hence the design is satisfactory. Provide 2 ISA 100 mm x 65 mm x 8 mm for the strut. Provide tacking rivets 18 mm in diameter at 500 mm spacing.

LESSON 12. Design of Colum Bases-Slab Base

12.1 INTRODUCTION

The columns are supported on the column bases. The column bases transmit the column load to the concrete or masonry foundation blocks. The column load is spread over large area on concrete or masonry blocks. The intensity of bearing pressure on concrete or masonry is kept within the maximum permissible bearing pressure. The safety of the structure depends upon stability of foundation. The column bases should be designed with utmost care and skill. In the column bases, intensity of pressure on concrete block is assumed to be uniform. The column bases shall be of adequate strength, stiffness and area to spread the load upon the concrete, masonry, other foundation or other supports without exceeding the allowable stress on such foundation under any combination of the load and bending moments. The column bases are of two types;

1. Slab base, and
2. Gusseted bases

The column footings are designed to sustain the applied loads, moments and forces and the induced reactions. The column load is spread over large area, so that the intensity of bearing pressure between the column footing and soil does not exceed the safe bearing capacity of the soil. it is ensured that any settlement which may occur shall be as nearly uniform as possible and limited to an accepted small amount. The column load is first transmitted to the column footing through the column base. It is then spread over the soil through the column footing. The column footings are of two types;

1. Independent footings, and
2. Combined footings.

12.2 SLAB BASE

The slab base as shown in Fig. 12.1 consists of cleat angles and base plate. The column end is faced for bearing over the whole area.

The gussets (gusset plates and gusset angles) are not provided with the column with slab bases. The sufficient fastenings are used to retain the parts securely in plate and to resist all moments and forces, other than the direct compression. The forces and moments arising during transit, unloading and erection are also considered. When the slab alone distributes the load uniformly the minimum thickness of a rectangular slab is derived as below;

PRESSURE DISTRIBUTION
SLAB BASE

SLAB BASE (PLAN)
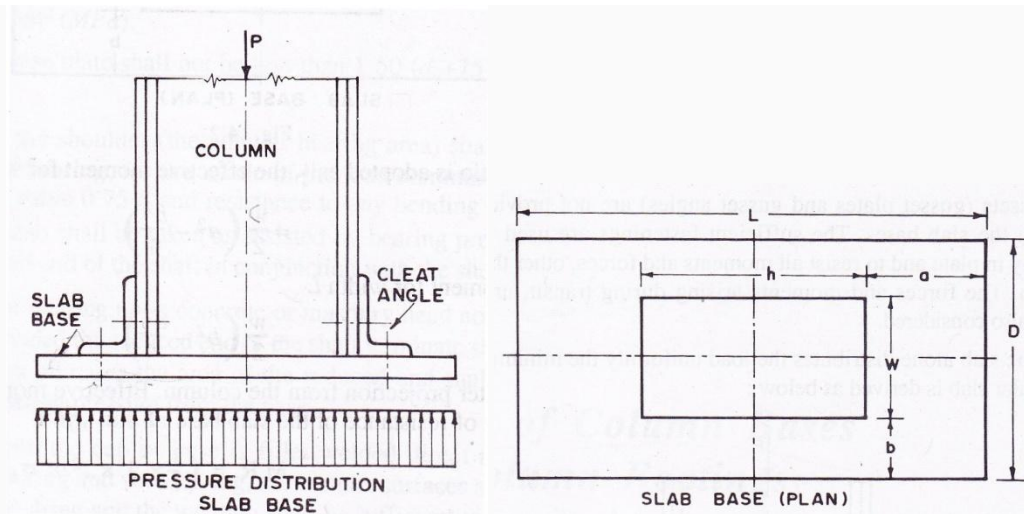
The column is carrying an axial load P. Consider the load distributed over area h x w and under the slab over the area L x D as shown in Fig. 12.2.

Let   t=Thickness of the slab

        W=Pressure or loading on the underside of the base

        a=Greater projection beyond column

        b=Lesser projection beyond column

σ_{bs}= Allowable bending stress in the slab bases for all steels, it shall be assumed as 185 N/mm²

Consider a strip of unit width.

Along the xx-axis
$$M_{xx} = \left(\frac{wa^2}{2}\right)$$

Along the yy-axis
$$M_{yy} = \left(\frac{wb^2}{2}\right)$$

If Poison ratio is adopted as ¼ the effective moment for width D
$$= \frac{w}{2}\left(a^2 - \frac{b^2}{4}\right)$$

Effective moment for width L
$$= \frac{w}{2}\left(b^2 - \frac{a^2}{4}\right)$$

Since a is greater projection from the column, the effective moment for width D is more. Moment of resistance of the slab base of unit width

$$M.R = \left(\frac{1}{6} \times 1 \times t^2 \times \sigma_{bs}\right)$$

Therefore $\left(\frac{1}{6} \times 1 \times t^2 \times \sigma_{bs}\right) = \frac{w}{2}\left(a^2 - \frac{b^2}{4}\right)$

Hence the thickness of the slab base $t = \left[\frac{3w}{\sigma_{bs}}\left(a^2 - \frac{b^2}{4}\right)\right]^{1/2}$ (as per IS: 800-1984)

For solid round steel column, where the load is distributed over the whole area, the minimum thickness of square base as per IS:800-1984 is given by

$$t = 10\left[\frac{90w}{16\sigma_{bs}}\left(\frac{B}{B - d_o}\right)\right]^{1/2}$$

Where t= Thickness of plate in mm

W=Total axial load in kN

B=Length of the side of base of cap in mm

d_0=Diameter of the reduced end (if any) of the column in mm

σ_{bs}= Allowable bending stress in steel (is adopted as 185 N/mm²)

The allowable intensity of pressure on concrete may be assumed as 4 N/mm². When the slab does not distribute the load uniformly or where the slab is not rectangular, separate calculation shall be made to show that stresses are within the specified limits.

When the load on the cap or under the base is not uniformly distributed or where end of the column shaft is not machined with the cap or base, or where the cap or base is not square in plan, the calculations are made on the allowable stress of 185 N/mm² (MPa). The cap or base plate shall not be less than 1.50 (d_o+75) mm in length or diameter.

The area of the shoulder (the annular bearing area) shall be sufficient to limit the stress in bearing, for the whole of the load communicated to the slab to the maximum value 0.75 f_y and resistance to any bending communicated to the shaft by the slab shall be taken as assisted by bearing pressures developed against the reduced and of the shaft in conjunction with the shoulder.

The bases foe bearing upon concrete or masonry need not be machined on the underside provided the reduced end of the shaft terminate short of the surface of the slab and in all cases the area of the reduced end shall be neglected in calculating the bearing pressure from the base.

In cases where the cap or base is fillet welded direct to the end of the column without boring and shouldering, the contact surfaces shall be machined to give a perfect bearing and the welding shall be sufficient to resist transmitting the forces specified above. Where the full length T butt welds are provided no machining of contact surfaces shall be required.

Example 12.1 A column section HB 250,@ 0.510 kN/m carries an axial load of 600 kN. Design a slab for the column. The allowable bearing pressure on concrete is 4 N/mm². The allowable bending stress in the slab base is 185 N/mm²(MPa).

Design of Structures

Design:

Step 1: Area of slab base required

Axial load of column = 600 kN

It is assumed uniformly distributed under the slab

Area of the slab base required $= \left(\frac{600 \times 1000}{4}\right) = 15 \times 10^4 mm^2$

The length and width of slab base are proportioned so that projections on either side beyond the column are approximately equal.

Size of column section HB 250,@ 0.510 kN/mm is 250 mm x 25 mm

Area of slab base $= (250+2a)(250+2b)$ mm²

Step 2: Projections of base plate

Let projections a and b are equal

Area of slab $(250+2a)^2 = 15 \times 10^4$. Therefore a=68.45 mm

Provide projections   a=b=70 mm

Provide slab base (250 + 2 x 70) (250 + 2 x 70) = 390 mm x 390 mm

Area of slab provided = 390 x 390 = 1,52,100 mm²

Intensity of pressure from concrete under slab $w = \left(\frac{600 \times 1000}{152100}\right) = 3.945 \ N/mm^2$

Step 3: Thickness of slab base:

Thickness of slab $= \left[\frac{3 \times 3.945}{185}\left(70^2 - \frac{70^2}{4}\right)\right]^{1/2} = 15.33 \ mm$

Provide 16 mm thick slab base. The fastenings are provided to keep the column in position.

Example 12.2 A column section SC 250,@ 85.6 carries an axial load of 600 kN. Design a slab base for the column. The allowable bearing pressure on concrete is 4 N/mm². The allowable bending stress in the slab base is 185 N/mm²(MPa).

Design:

Step 1: Area of slab base required

Axial load of column is 600 kN. It is assumed uniformly distributed under the slab.

Area of slab base required $= \left(\frac{600 \times 1000}{4}\right) = 15 \times 10^4 mm^2$

The length and width of slab base are proportioned so that the projections on either side beyond the column are approximately equal.

Size of column section SC 250,@ 85.6 kg/m = 250 mm x 250 mm

Area of slab base $\qquad$ = (250 + 2a)(250+2b) mm²

Step 2: Projections of base plate

Let the projections a and b be equal.

Area of slab $\quad$ (250+2a)² = 15 x 10⁴. Therefore a=68.45 mm

Provide projections $\quad$ a=b=70 mm

Provide slab base (250 + 2 x 70) (250 + 2 x 70) = 390 mm x 390 mm

Area of slab provided = 390 x 390 = 1,52,100 mm²

Intensity of pressure from concrete under slab

Step 3: Thickness of slab base:

Thickness of slab $= \left[\frac{3 \times 3.945}{185}\left(70^2 - \frac{70^2}{4}\right)\right]^{1/2} = 15.33\ mm$

Provide 16 mm thick slab base. The fastenings are provided to keep the column in position.

MODULE 7.

LESSON 13. Steel Beams

13.1 INTRODUCTION

A beam is defined as a structural member subjected to transverse loads. The plane of transverse load is parallel to the plane of symmetry of the cross-section of the beam and it passes through the shear centre, so that the simple bending occurs. The transverse loads produce bending moments and shear forces in the beams at all the section of the beam.

The term joist is use for beams of light sections. Joist support floor construction; they do not support other beams. The term subsidiary beam or secondary beam is also used for the beams supporting floor construction. Main beams are the supporting joists for subsidiary beams. These are called floor beams in buildings. The term girder is most commonly used in buildings. Any major beam in a structure is known as a girder.

In the roof trusses, horizontal beams spanning between the two adjacent trusses are known as purlins. The beams resting on the purlins are known as common rafter or simply rafters. In the buildings the beams spanning over the doors, windows and other openings in the walls are known as lintels. The beams at the outside wall of a building, supporting its share of the floor and also wall upto the floor above it are known as spandrel beams. The beams framed to two beams at right angles to it and usually supporting joists on one side of it; used at openings such as stair wells are known as headers. The beams supporting the headers are termed as trimmers. The beams supporting the stair steps are called as stringers.

In the brigde floors, the longitudinal beams supported by the floor beams are also called as stringers. In the mill buildings, the horizontal beams spanning between the wall columns and supporting wall covering are called as girts. The beams are also called simply supported, overhanging cantilever, fixed and continuous depending upon nature of supports and conditions.

13.2 ROLLED STEEL SECTIONS USED AS BEAMS

The rolled steel I-sections, channel sections, angle sections, tee-sections, flat sections and bars as shown in Fig. 13.1 are the regular sections, which are used as beams. The rolled steel I-sections as shown in Fig. 13.1.A are most commonly used as the beams and as such thses sections are also termed as beam sections. The rolled steel I-sections are symmetrical sections. In these sections more material is placed near top and bottom faces, i.e., in the flanges as compared to the web portion. The rolled steel I-sections provide large moment of inertia about xx-axis with less cross sectional area. The rolled steel I-sections provide large moment of resistance as compared to the other sections and as such these are most efficient and economical beam sections. The rolled steel wide flange beams as shown in Fig. 13.1.B provide additional desirable features. As the name indicates, the flanges of the sections are wide. These sections provide greater lateral stability and facilitate the connections of flanges to other members. I-sections and wide flange beam sections have excellent strength.

ROLLED STEEL SECTIONS USED AS BEAMS
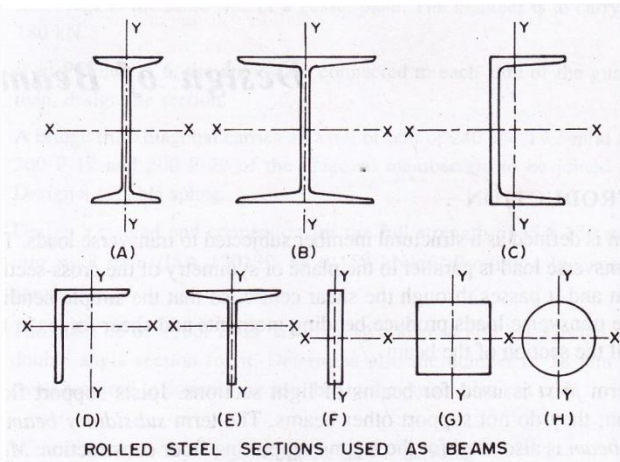
The rolled steel channel sections as shown in Fig. 13.1.C are used as purlins and other small structural member. The channel sections have reasonably good lateral strength and poor lateral stability. The channel sections are unsymmetrical sections about yy-axis. When the channel sections are loaded and supported by vertical forces passing through the centroid of the channel, then the channel sections bend and twist if these are laterally unsupported, except for the special case, wherein the loads act normal to the plane of web, causing bending in the weakest direction. The rolled steel angle sections as shown in Fig. 13.1.D are also used as purlins and so other small structural members. The angle sections act as unsymmetrical sections about both xx-axis and yy-axis.

The rolled steel tee-sections as shown in Fig. 13.1.E are used as beams in the rectangular water tanks. The angles and tee-sections are used for light loads. The rolled steel flats and bars as shown in 13.1.F, G and H are very rarely used. These sections are weak in resisting bending. Most commonly the beams are loaded in the direction perpendicular to xx-axis, so that the bending of beams occurs about strong and xx-axis becomes neutral axis. The beams are very rarely loaded in the direction perpendicular to yy-axis. In such cases, yy-axis becomes neutral axis.

In cases of bending of the beams about one axis, the load is considered to be applied through the shear centre of the beam sections. In case, the loading passes through the shear centre, the section may be analyzed for simple bending and shear. The shear centre for the beam section is at the centre of area and this load position produces simple bending about either axis. When the load does not pass through the shear centre as in channels, angles and some built-up sections, a torsional moment is produced along with the bending moment and both are considered to avoid over stressing of the member. For such sections, a special load device may be used so that the load passes through shear centre of the section and the torsional moment may be avoided.

In addition to the above, expanded or castellated beams as shown in Fig. 13.2.B are used. The castellated beams are light beams and light beams and these are economically used for the light construction. The castellated beams are made by splitting the web of rolled steel I-sections in a predetermined pattern as shown in Fig. 13.2.A. The splitted portions are rejoined in such a manner as to produce a regular pattern of opening in the web.

13.3 BENDING STRESS

The bending stress is also termed as flexural stress. When the beams are loaded, they bend and bending stresses are setup at all the sections. The established theory of bending is expressed in the following formula:

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

Where,　　　　M = Bending moment

I = Moment of inertia

= Bending stress at any point

y = Distance from the neutral axis to the point under consideration

R = Radius of curvature of the beam

The above equation holds good when the plane of bending coincides with the plane of symmetry of the beam section. The bending of beam occurs in the principal plane of the beam section. The simple bending of beam occurs, i.e., the bending is produced by the application of pure couples at the ends of the beam. In such bending the deflection of beam does not occur due to shear. In the above equation, it is assumed that the vertical sections of the beam plane before bending remain plane after bending. Then stress in any given fibre is proportional to its strain, i.e., Hooke's law holds good. For the material of beam, the value of E is same for the complete beam.

When the load is acting downward in a simply supported beam, then the distribution of bending stress for any section of beam is as shown in Fig. 13.3. The bending stress varies linearly. The bending stress is zero at the neutral axis. When the load is acting downward, the bending stress is compressive above the neutral axis of section and tensile below it and these are denoted by $\sigma_{bc.cal}$ and $\sigma_{bt.cal}$ respectively. The bending stress is maximum at the extreme fibre.

$$\sigma_{bmax} = \left(\frac{M}{I} \times y_{max}\right) = \left(\frac{M}{I/y_{max}}\right) = \left(\frac{M}{Z}\right)$$

Where, Z is the section modulus ($Z=I/y_{max}$), y_{max} is the distance from the neutral axis to the extreme fibre and $\sigma_{b.max}$ is the maximum bending stress.

The maximum bending stress in the beam section (if compressive) should be less than the allowable bending compressive stress and (if tensile); should be less than the allowable bending tensile stress. When the section of beam is symmetrical about the neutral axis then the value of y_{max} is equal to half the depth of section and the maximum bending stress in compression and in the tension at the extreme fibres are equal. When the beam section is not symmetrical about the neutral axis, then there are two distance y_1 and y_2 to the two extreme fibres from the nutral axis. The bending stresses at the extreme top and bottom are not equal. Then, the values of $Z_1=(I/y_1)$ and $Z_2=(I/y_2)$ both should be calculated and compared with the section modulus, Z of the beam section provided.

The total compressive force 'C' above the neutral axis is equal to the tensile force 'T', for the beam in equilibrium. These two forces act in opposite directions and form a couple. This couple resists the bending moment and this moment is known as moment of resistance 'M$_r$'. the moment of resistance of a beam section is the moment of the couple which is set up at the section by the longitudinal forces C and T created in the beam due to bending.

M$_r$=(C x Lever arm)=(T x Lever arm)

For the beam in equilibrium, the moment of resistance 'M$_r$' would be equal to the maximum bending moment 'M' at any section (M$_r$=M).

13.4. ALLOWABLE STRESS IN BENDING

The allowable bending stress, σ_{bc} in the design of rolled steel beam section considerably depends on the geometrical properties of the section and the lateral support. In case of flange width/flange thickness ($\frac{1}{2}b_f/t_f$) and the depth of section/thickness of web (h/t$_w$) ratios not adequate, the elements of beam section will tend to buckle at low compressive stresses (which will be due to bending combined with axial loads). If the compression flange is not laterally supported (i.e., supports at intervals or uniformly) along the compression zone, it will either buckle in plane or cut-of plane coupled with twisting.

The rolled sections are produced with adequate ($\frac{1}{2}b_f/t_f$) and (h/t$_w$) ratios such that the buckling of flange or web does not occur. The designers may provide supports at intervals or uniformly along he compression flange such that its buckling is avoided. The calculated bending compressive stress $\sigma_{bc.cal}$ and bending tensile stress $\sigma_{bt.cal}$ in the extreme fibres should not exceed the maximum permissible bending stress in compression (σ_{bc}) or in tension σ_{bt} as below.

σ_{bc} or σ_{bt} = 0.66 f$_y$

The structural steel used in general construction may have yield stress as 220, 230, 240, 250, 260, 280, 300, 320, 340, 360, 380, 400, 420, 450, 480, 510 or 540 N/mm^2 (M Pa). The structural steels having these values of yield stress are also used in flexural members. The maximum permissible bending compressive stress in beams and channels with equal flanges have been given separately in IS:800-1984. For an I-beam or channel with equal flanges bending about the axis of maximum strength (xx-axis), the maximum bending compressive stress on the extreme fibre, calculated on the effective section shall not exceed the values of maximum permissible bending compressive stress, σ_{bc}.

The safe compressive stress for a given grade of steel depends on a number of parameters as given below.

Let   D=Overall depth of the beam

d$_1$=Clear distance between the flanges

l=Effective length of the compression flange (Table 13.1)

r_y=Radius of gyration of the section about its axis of minimum strength (yy-axis)

T=Mean thickness of the compression flange

t=Web thickness.

For the rolled steel sections, the mean thickness is that which one is given in ISI Handbook No.1. In the case of compound girders, with curtailed flanges, D shall be taken as the overall depth of the girder at the point of maximum bending moment and T shall be taken as the effective thickness of the compression flange and shall be calculated as

T = K_1 x mean thickness of the horizontal portion of the compression flange at the point of maximum bending moment where K_1 = a co-efficient given in Table 13.2

Table 13.1 EFFECTIVE LENGTH OF COMPRESSION FLANGE (l).

Sl. No.	End conditions	Effective length (l)
i	Unrestrained against lateral bending (i.e., free to rotate in plan at the bearing)	l=span
ii	Partially restrained against lateral bending (i.e., not free to rotate in plan at the bearings)	l=0.85 x span
iii	Fully restrained against lateral bending (i.e., not free to rotate in plan at the bearings)	l=0.70 x span

Restraint against torson can be provided by

i) Web or flange cleats, or

ii) Bearing stiffeners acting in conjection with the bearing of the beam, or

iii) Lateral end frames or other external supports to the ends of the compression flanges, or

iv) Their being built into wall.

Where the ends of the beams are not restrained against torsion or where the load is applied to the compression flange and both the load and the flange are free to move laterally, the above values of the effective length shall be increased by 20 percent.

The end constraint element shall be capable of safely resisting, in addition to wind and other applied external forces a horizontal force acting at the bearing in a direction normal to the compression flange of the beam at the level of the centroid of the flange and having a value to not less than 2.5 per cent of the maximum force occurring in the flange.

For cantilever beams of projecting length L, the effective length l to be used shall be taken as follows:

a)	Built in at the support, free at the end	1=0.85 L
b)	Built in at the support restrained against torsion at the end by continuous construction	1=0.75 L
c)	Built in at the support, restrained against lateral deflection and torsion at the free end	1=0.5 L
d)	Continuous at the support unrestrained against torsion at the support and free at the end	1=3 L
e)	Continuous at the support with partial restraint against torsion of the support and free at the end	1=2 L
f)	Continuous at the support, restrained against torsion at the support and free at the end	1= L

If there is a degree of fixity at the end, the effective length shall be multiplied by 0.5/0.85 in (b) and (c) above, and by 0.75/0.85 in (d), (e) and (f) above.

13.5 MAXIMUM PERMISSIBLE BENDING COMPRESSIVE STRESS IN BEAMS AND PLATE GIRDERS

For beams and plate girders bent about the x-x axis the maximum bending compressive stress on the extreme fibre, calculated on the effective section shall not exceed the maximum permissible bending compressive stress σ_{bc} calculated from the following formula.

$$\sigma_{bc} = 0.66 \frac{f_{cb} f_y}{\left[(f_{cb})^n + (f_y)^n \right]^{1/n}}$$

Where, $\quad f_{cb} = elastic\ critical\ stress\ in\ bending$

$$f_y = yield\ stress\ of\ the\ steel$$

$$n = a\ factor\ assumed\ as\ 1.4$$

The elastic critical stress f_{cb} for beams and plate girders with I_y smaller than I_x shall be calculated using the following formula.

$$f_{bc} = K_1 (X + K_2 Y) \frac{C_2}{C_1}$$

Where, $X = Y \sqrt{1 + \frac{1}{20} \left(\frac{lT}{r_y.D} \right)^2}$ N/mm² and $\quad Y = \frac{26.5 \times 10^5}{\left(\frac{l}{r_y} \right)^2}$ N/mm²

K_1= a coefficient to allow for reduction in thickness or breadth of flanges between points of effective lateral resistant and depends on ψ_1 the ratio of the total area of both flanges at the point of least bending moment to the corresponding area at the point of greatest bending moment between such points of resistant. Values of K_1 for different values of ψ are given in the Table 13.2.

K_2= a coefficient to allow for the inequality of flanges, and depends on ω, the ratio of the moment of inertia of the compression flange alone to that of the sum of the moments of inertia of the flanges, each calculated about its own axis parallel to the y-y axis of the girder, at the point of maximum bending moment. Values of K_2 for different values of ω are given in the Table 13.3.

C_1, C_2=respectively the lesser and the greater distances from the section neutral axis to the extreme fibres.

I_x=moment of inertia of the whole section about the axis normal to the plane of bending (x-x axis) and

I_y= moment of inertia of the whole section about the axis lying in the plane of bending (y-y axis)

Table 13.2 VALUES OF K_1 FOR BEAMS WITH CURTAILED FLANGES

ψ	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
K_1	1.0	1.0	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2

Table 13.3 VALUES OF K_2 FOR BEAMS WITH UNEQUAL FLANGES

Ω	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
K_2	0.5	0.4	0.3	0.2	0.1	0	-0.2	-0.4	-0.6	-0.8	-1.0

Values of X and Y for appropriate values of D/T and l/r_y can be taken from standard text books on design of steel structures. Values of f_{cb} shall be increased by 20 per cent when $\frac{T}{t} \not> 2$ and $\frac{d_1}{t} \not> \frac{1.344}{\sqrt{f_y}}$. The maximum permissible bending stress in tension σ_{bt} or in compression σ_{bc} in beams bent about the axis of minimum strength shall not exceed $0.66 f_y$ where f_y is the yield stress of steel.

(A) SPLITTING PATTERN

(B) CASTELLATED BEAMS

(A) SECTION OF BEAM   (B) DISTRIBUTION OF BENDING STRESS
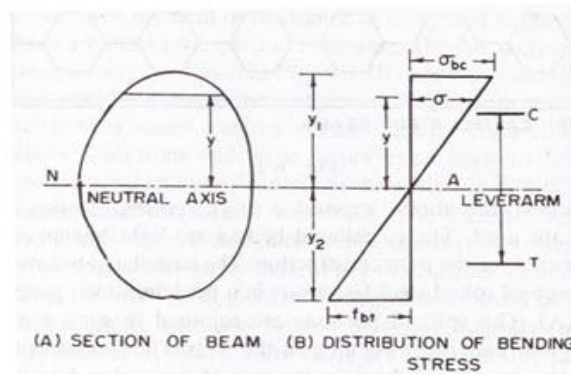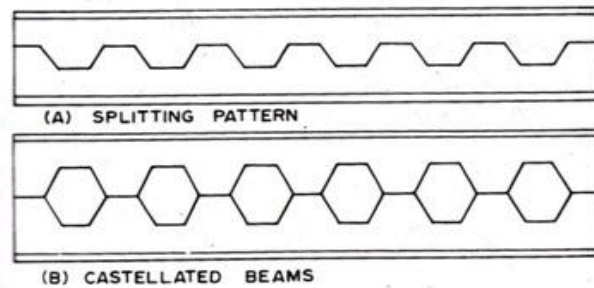
LESSON 14. Design of Steel Beams

14.1 INTRODUCTION

The following points should be considered in the design of a beam.

1. Bending moment consideration: The section of the beam must be able to resist the maximum bending moment to which it is subjected.

2. Shear force consideration: The section of the beam must be able to resist the maximum shear force to which it is subjected.

3. Deflection consideration: The maximum deflection of a loaded beam should be within a certain limit so that the strength and efficiency of the beam should not be affected. Limiting the deflection within a safe limit will also prevent any possible damage to finishing. As per the I.S. code, generally the maximum deflection should not exceed 1/325 of the span.

4. Bearing stress consideration: The beam should have enough bearing area at the supports to avoid excessive bearing stress which may lead to crushing of the beam or the support itself.

5. Buckling consideration: The compression flange should be prevented from buckling. Similarly the web, the beam should also be prevented from crippling. Usually these failures do not take place under normal loading due to proportioning of thickness of flange and web. But under considerably heavy loads, such failures are possible and hence in such cases the member must be designed to remain safe against such failures

14.2 SHEAR AND BEARING STRESSES

When the beams are subjected to loads, then, these are also required to transmit large shear forces either at supports or at concentrated loads. For simply supported beams, the shear force is maximum at the supports. The values of shear force at the concentrated loads also remain large. Due to shear force, the shear stresses are setup along with the bending stresses at all sections of the beams. The shear stress at any point of the cross-section is given by

$$\tau_v = \left(\frac{F.Q}{I.t}\right)$$

Where is the shear stress,

F = the shear force at cross-section,

Q = Static moment about the neutral axis of the portion of cross-sectional area beyond the location at which the stress is being determined.

I = Moment of inertia of the section about the neutral axis

t = Thickness of web (width of section at which the stress is being determined)

RECTANGULAR   SHEAR STRESS   I- BEAM   SHEAR STRESS
BEAM
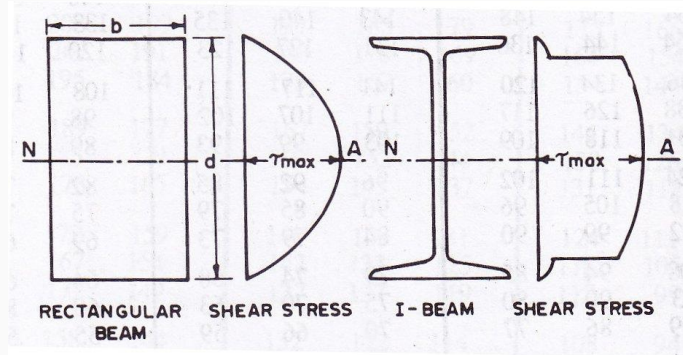
The distribution of shear stresses for rectangular section of beam and I-beam section are shown in Fig. 14.1. The maximum shear stress occurs at the neutral axis of the section. The maximum shear stress in a member having regard to the distribution of stresses in conformity with the elastic behavior of the member in flexure (bending) should not exceed the value of maximum permissible shear stress, τ_{vm} found as follows.

$$\tau_{vm}=0.45f_y$$

Where f_y is the yield stress of structural steel to be used. It is to note that in the case of rolled beams and channels, the design shear is to be found as the average shear. The average shear stress for rolled beams or channels calculated by dividing the shear force at the cross-section by the gross-section of the web. The gross-cross-section of the web is defined as the depth of the beam or channel multiplied by its web thickness.

Average shear stress for rectangular beam is given by $\tau_v = \left(\dfrac{F}{b \times d}\right)$

Average shear stress for rectangular beam is given by $\tau_v = \left(\dfrac{F}{h \times t_w}\right)$

For rolled steel beams and channels, it is assumed that shear force is resisted by web only. The portion of shear resisted by the flanges is neglected. The average shear stress $\tau_{va.cal}$, in a member calculated on the gross cross-section of web (when web buckling is not a factor) should not exceed in case of unstiffened web of the beam,

$$\tau_{va}= 0.4\,f_y$$

The allowable shear stress as per AISC, AASHTO and AREA specifications are as follows:

Specifications	Allowable shear stress
AISC	$0.40\,f_y$
AASHTO	$0.33\,f_y$
AREA	$0.35\,f_y$

When the beams are subjected to co-existent bending stresses (tension or compression) and shear stress, then the equivalent stress, $\sigma_{e.cal}$ is obtained from the following formula

$$\sigma_{e.cal} = [\sigma_{bt.cal}^2 + 3\tau_{vm.cal}^2]^{1/2} \qquad \text{or} \qquad \sigma_{e.cal} = [\sigma_{bc.cal}^2 + 3\tau_{vm.cal}^2]^{1/2}$$

The equivalent stress due to co-existent bending (tension or compression) and shear stresses should not exceed the maximum permissible equivalent stress σ_e found as under

$$\sigma_e = 0.90\, f_y$$

When the bearing stress σ_p is combined with tensile or compressive bending and shear stresses under the most unfavourable conditions of loading, the equivalent stress $\sigma_{e.cal}$ obtained as below should not exceed

$$\sigma_e = 0.90\, f_y$$

$$\sigma_{e.cal} = [\sigma_{bt.cal}^2 + \sigma_{p.cal}^2 + \sigma_{bt.cal} \cdot \sigma_{p.cal} + 3\tau_{vm.cal}^2]^{1/2}$$

$$\sigma_{e.cal} = [\sigma_{bc.cal}^2 + \sigma_{p.cal}^2 + \sigma_{bc.cal} \cdot \sigma_{p.cal} + 3\tau_{vm.cal}^2]^{1/2}$$

$\sigma_{bc.cal}$, $\sigma_{bt.cal}$, $\tau_{vm.cal}$ and $\sigma_{p.cal}$ are the numerical values of the co-existent bending (compression or tension), shear and bending stresses. When bending occurs about both the axes of the member, $\sigma_{bt.cal}$ and $\sigma_{bc.cal}$ should be taken as the sum of the two calculated fibre stresses, σ_e is the maximum permissible equivalent stress.

The bearing stress in any part of a beam when calculated on the net area of contact should not exceed the value of σ_p calculated as below

Where σ_p is the maximum permissible bearing stress and f_y is the yield stress.

14.3 EFFECTIVE SPAN AND DEFLECTION LIMITATION

The effective span of a beam shall be taken as the length between the centres of the supports, except in cases where the point of application of the reaction is taken as eccentric to the support, then, it shall be permissible to take the effective span as the length between the assumed points of application of reaction.

The stiffness of a beam is a major consideration in the selection of a beam section. The allowable deflections of beams depend upon the purpose for which the beams are designed. The maximum deflections for some standard cases are given below. In these formulae W is the total load on the beam in case of uniformly distributed load and each concentrated load in the case of concentrated loads.

Deflection in beam carrying uniformly distributed load over the whole span $= \dfrac{5}{384} \dfrac{wl^4}{EI}$

Deflection in beam carrying a point load at the centre $= \dfrac{1}{48} \dfrac{wl^3}{EI}$

Deflection in beam carrying two point loads (uniformly spaced) $= \dfrac{23}{648} \dfrac{wl^3}{EI}$

Deflection in beam carrying three point loads (uniformly spaced) $= \dfrac{19}{384} \dfrac{wl^3}{EI}$

Deflection in beam carrying a number of point loads W each (uniformly spaced)

$$= n\left\{3 - \tfrac{1}{2}\left(1 + \tfrac{4}{n^2}\right)\right\}\dfrac{wl^3}{EI}$$

Deflection in beam carrying a symmetrical triangular load $= \dfrac{1}{60} \dfrac{wl^3}{EI}$

The large deflections of beams are undesirable for the following reasons:

1. When the loads are primarily due to human occupants especially in the case of public meeting places, large deflections result in noticeable vibratory movement. This produces an uncomfortable sensation to the occupants.

2. The large deflections may result in cracking of ceiling plaster, floors or partition walls.

3. The large deflection indicate the lack of rigidity. It may cause vibrations and over-stresses under dynamic loads.

4. The large deflections may cause the distortions in the connections. The distortions cause secondary stresses.

5. The large deflections may cause poor drainage, which will lead to ponding of water and therefore increase the loads.

14.3.1 Limiting vertical deflection

The deflection of a member is calculated without considering the impact factor or dynamic effect of the loads causing the deflection. The deflection of a member shall not be such as to impair the strength or efficiency of the structure and lead to damage to finishing. Generally, the maximum deflection for a beam shall not exceed 1/325 of the span. This limit may be exceeded in cases where greater deflection would not impair the strength or efficiency of the structure or lead to damage to finishing. The deflection of the beams may be decreased by increasing the depth of beams, decreasing the span, providing greater and restraint or by any other means.

14.3.2 Limiting horizontal deflection

At the caps of columns in single storey buildings, the horizontal deflection due to lateral force should not ordinarily exceed 1/325 of the actual length 'l' of the column. This limit may be exceeded in cases where the grater deflection would not impair the strength and efficiency of the structure or lead to damage to finishing. According to AISE specifications, the deflections of beams and girders for live load and plastered ceiling should not exceed 1/360 of the span.

Design of Structures

14.4 LATERALLY SUPPORTED BEAMS

The laterally supported beams are also called laterally restrained beams. When lateral deflection of the compression flange of a beam is prevented by providing effective lateral support (restraint), the beam is said to be laterally supported. The effective lateral restraint is the restraint which produces sufficient resistance in a plane perpendicular to the plane of bending to restrain the compression flange of a beam from lateral buckling to either side at the point of application of the restraint. The concrete slab encasing the top flange, so that the bottom surface of the concrete slab is flush with the bottom of the top flange, is shown in Fig. 14.2.A. It provides a continuous lateral support to the top flange of the beam. When other beams frame at frequent intervals into the beam in questions as shown in Fig. 14.2.B, lateral support is provided at each point of connection but main beam should still be checked between the two supports.

(A) CONTINUOUS LATERAL SUPPORT   (B) LOCAL LATERAL SUPPORT

ADEQUATE LATERAL SUPPORT
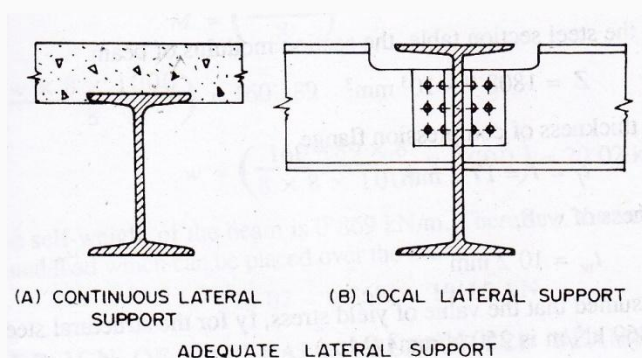
In the laterally supported beams, the value of allowable bending compressive stress remains unaltered and the reduction in its value is not made. Bending comprehensive stress is taken equal to the allowable bending tensile stress, ($\sigma_{bc}=\sigma_{bt}=0.66f_y$). The adequate lateral support is provided to safeguard against the lateral-torsional bucking. In case of doubt for adequate lateral support, the beams should be designed as laterally unsupported. In case the concrete slab holds the top flange (compression flange) of the beam from one side only, then, the lateral support is not credited. The concrete slab simply resting over the top flange of the beam without shear connectors also does not provide an lateral support. Sometimes, the plank or bar grating is attached to the top flange of beam by means of bolts. When the bolts are firmly fastened, then, they provide adequate lateral support temporarily. Even then, bolts have temporary nature of connections. It is possible that the bolts might be omitted or removed. As such, the top flange should not be considered laterally supported fully. The beams having lateral support from other members may buckle between points of lateral support. Therefore, the laterally unsupported length of beam is kept short.

14.5 DESIGN OF LATERALLY SUPPORTED BEAMS

The design of beams is generally governed by the maximum allowable bending stress and the allowable deflection. Its design is controlled by shear only when the spans are short and loads are heavy. The members are selected such that the sections are symmetrical about the plane of loading and the unsymmetrical bending and torsion are eliminated. The design of beams deals with proportioning of members, the determination of effective section modulus, maximum deflection and the shear stress. In general, the rolled steel sections have webs of

sufficient thickness such that the criterion for design is seldom governed by shear. The following are the usual steps in design of laterally supported beams:

Step 1. For the design of beams, load to be carried by the beam, and effective span of the beam are known. The value of yield stress, f_y for the structural steel to be used is also known. For the rolled steel beams of equal flanges as given in ISI Handbook no.1, the ratio of mean thickness of the compression flange ($T=t_f$) to the thickness of web used to be less than 2.00. Also the ratio of the depth of web d_1 to the thickness of web is also smaller than 85. The ends of compression flange of a laterally supported beam remain restrained against lateral bending (i.e., not free to rotate in plan at the bearings).

In the beginning of design, the permissible bending stress in tension, σ_{bt} or in compression, σ_{bc} may be assumed as 0.66 f_y. The bending compressive stress, σ_{bc} and the bending tensile stress, σ_{bt} are equal for the laterally supported beam.

Step 2. The maximum bending moment M and the maximum shear force F in the beam are calculated. The required section modulus for the beam is determined as $Z=(M/\sigma_{bc})$

Step 3. From the steel section tables, a rolled steel beam section, a rolled steel beam section, which provides more than the required section modulus is selected. The steel beam section shall have (D/T) and (l/r_y) ratios more than 8 and 40 respectively. As such the trial section of beam selected may have modulus of section, Z more than that required. Some of the beam sections of different categories have almost the same value of the section modulus Z. It is necessary to note the weight of beam per meter length and the section modulus, Z. The beam section selected should be such that it has minimum weight and adequate section modulus, Z.

Step 4. The rolled steel beam section is checked for the shear stress. The average and maximum shear stresses should not exceed the allowable average and maximum values of shear stresses.

Step 5. The rolled steel beam is also checked for deflection. The maximum deflection should not exceed the limiting deflection.

ISI Handbook no.1 provides tables for allowable uniform loads on beams and channels used as flexural members with adequate lateral support for compression flange. The values of allowable uniform loads corresponding to respective effective spans are given for various beams and channel sections. For given span and total uniformly distributed load found, rolled beam or channel section may be selected from these tables. The rolled steel I-sections and wide flange beam sections are most efficient sections. These sections have excellent flexural strength and relatively good lateral strength for their weights.

 Example 14.1 The effective length of compression flange of simply supported beam MB 500,@0.869 kN/m is 8 m. Determine the safe uniformly distributed load per meter length which can be placed over the beam having an effective span of 8 meters. Adopt maximum permissible stresses as per IS 800-1984. The ends of beam are restrained against rotation at the bearings.

Solution:

Step 1: Permissible bending stress

MB 500,@0.869 kN/m has been used as simply supported beam. The effective span of beam is 8 m. The effective length of compression flange is also 8 m.

From the steel section table, the section modulus of beam $\quad\quad\quad Z=1808.7 \times 10^3$ mm³

Mean thickness of compression flange $\quad\quad\quad\quad\quad t_f =T= 17.2$ mm

Thickness of web $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad t_w=10.2$ mm

It is assumed that the value of yield stress, f_y for the structural steel of MB 500,@0.869 kN/m is 250 N/mm²(MPa).

Ratio $\quad\quad \left(\frac{T}{t_w} = \frac{17.2}{10.2}\right) = 1.686 < 2.00$

Ratio $\quad\quad \left(\frac{d_t}{t_w}\right) = \left(\frac{h-2h_2}{t_w}\right) = \left(\frac{500-2\times37.95}{10.2}\right) = \left(\frac{h_1}{t_w}\right) = \left(\frac{424.1}{10.2}\right) = 41.578 < 85$

Ratio $\quad\quad \left(\frac{D}{T}\right) = \left(\frac{500}{17.0}\right) = 29.07$

The effective length of compression flange is 8 m.

Ratio $\quad\quad \frac{l}{r_y} = \left(\frac{0.7\times8\times1000}{35.2}\right) = 159.1$

From IS: 800-1984, the maximum permissible bending stress, for above ratios (by linear interpolation) σ_{bc}=65.121 N/mm²(MPa)

Step 2: Load supported over beam

$$M_r = (\sigma_{bc} \times Z) = \left(\frac{88.566 \times 1808.7 \times 10^3}{1000 \times 1000}\right) = 160.189 \ m-kN$$

MB 500,@0.869 kN/m can resist maximum bending moment equal to moment of resistance. Therefore the maximum bending moment M=160.189 m-kN

Step 3: Load supported over beam

The effective span of the beam is 8 meters. Let w be the uniformly distributed load per meter length. The maximum bending moment, M for the beam occurs at the centre..

$$M = \left(\frac{w.l^2}{8}\right) = 160.189 = \left(\frac{w \times 8 \times 1000}{8}\right)$$

Therefore $w = \left(\frac{160.189\times8}{8\times8\times1000} \times 1000\right) = 20.02 \ kN/m$

The self-weight of the beam is 0.869 kN/m. Therefore, the safe uniformly distributed load which can be placed over the beam $\quad$ (20.02-0.869)=19.15 kN.

Example 14.2 Design a simply supported beam to carry a uniformly distributed load of 44 kN/m. The effective span of beam is 8 meters. The effective length of compression flange of the beam is also 8 m. The ends of beam are not free to rotate at the bearings.

Design:

Step 1: Load supported, bending moment and shear force

Uniformly distributed load   = 44 kN/m

Assume self weight of beam   = 1.0 kN/m

Total uniformly distributed load   w= 45 kN/m

The maximum bending moment, M occurs at the centre

$$M = \left(\frac{w.l^2}{8}\right) = \left(\frac{45 \times 8 \times 8 \times 1000}{8 \times 1000}\right) = 360\ m - kN$$

The maximum shear force, F occurs at the support $F = \left(\frac{w.l}{2}\right) = \left(\frac{45 \times 8}{2}\right) = 180\ kN$

Step 2: Permissible bending stress

It is assumed that the value of yield stress, f_y for the structural steel is 250 N/mm² (MPa). The ratios (T/t_w) and (d_1/t_w) are less than 2.0 and 85 respectively. The maximum permissible stress in compression or tension may be assumed as $\sigma_{bc} = \sigma_{bt} = (0.66 \times 250) = 165$ N/mm²

Section modulus required, $Z = \frac{M}{\sigma_{bc}} = \left(\frac{360 \times 1000 \times 1000}{165}\right) = 2181.82 \times 10^3\ mm^3$

The steel beam section shall have (D/T) and (l/r_y) ratios more than 8 and 40 respectively. The trial section of beam selected may have modulus of section, Z more than that needed (about 25 to 50 per cent more).

Step 3: Trial section modulus

1.50 x 2181.82 x 10³ mm³=3272.73 x 10³ mm³

From steel section tables, try WB 600,@1.337 kN/m

Section modulus, Z_{xx}=3540.0 x 10³ mm³

Moment of inertia, I_{xx}=106198.5 x 10⁴ mm⁴

Thickness of web, t_w=11.2 mm

Thickness of flange,   T=t_f=21.3 mm

Depth of section,   h=600 mm

Step 4: Check for section modulus

$$\frac{D}{T} = \frac{600}{21.3} = 28.169$$

$$\frac{T}{t_w} = \frac{21.3}{11.2} = 1.901 < 2.0, \ also \ \left(\frac{d_1}{t_w} < 85\right)$$

The effective length of compression flange of beam is 8 m.

$$\frac{l}{r_y} = \left(\frac{0.7 \times 8 \times 10.0}{52.5}\right) = 106.66$$

From IS: 800-1984, the maximum permissible bending stress σ_{bc}=118.68 N/mm²(MPa)

Section modulus required

$$\left(\frac{360 \times 1000 \times 1000}{118.68}\right) = 3033.34 \times 10^3 mm^3 < 3540 \times 10^3 mm^3 provided$$

Further trial may give more economical section.

Step 5: Check for shear force

Average shear stress, $\tau_{v.cal} = \left(\frac{F}{h \times t_w}\right) = \left(\frac{180 \times 1000}{600 \times 11.2}\right) = 26.78 \ N/mm^2$

Permissible average shear stress

$$0.4 \times f_y = (0.4 \times 250) = 100 \ N/mm^2 > Actual \ average \ shear \ stress$$

Step 6: Check for deflection

Maximum deflection of the beam

$$y_{max} = \left(\frac{5}{384} \times \frac{w.l^4}{EI}\right) = \frac{5}{384} \times \left(\frac{45 \times 8 \times 8^3 \times 1000^3 \times 1000}{2.047 \times 10^5 \times 106198.5 \times 10^4}\right) = 24.53 \ mm$$

Allowable deflection $\frac{1}{325} \times span = \left(\frac{1}{325} \times 8000\right) = 24.60 \ mm$

The maximum deflection is less than allowable deflection, hence the beam is safe. Provide WB 600,@1.337 kN/m

MODULE 8.

LESSON 15. Cement Concrete

15.1 INTRODUCTION

Cement concrete consist of hard inorganic materials called aggregates such as gravel, sand, crushed stone, slag etc. cemented together with Portland cement and water. It is a mixture of aggregates and cement-water paste. The cement-water paste has its role to bind the aggregates to form a strong rock like mass after hardening as a consequence of the chemical reaction between cement and water. Aggregates are classified into fine aggregates and coarse aggregates. Fine aggregates consist of sand whose particle size does not exceed 4.75 mm. Course aggregates consist of gravel, crushed stone etc., of particle size more than 4.75 mm. the cement-water paste acts as a slurry which fills the voids in sand forming mortar. The mortar so formed fills the voids in the coarse aggregates. When the above materials are mixed together so as to form a workable mixture, it can be moulded or cast into beams, slabs etc. A few hours after mixing, the materials undergo a chemical combination and as a consequence, the mixture solidifies and hardens, attaining greater strength with age.

Even though the structural design may be absolutely safe and satisfactory, if the concrete made in the construction of the structure is defective, the result will be a weak structure liable to failure and it cannot be rectified or made up and will remain forever. Concrete possesses a high compressive strength and is usually more economical than steel and is not subjected to corrosive weathering and such effects. Hence concrete is used in all present day constructions. But, concrete has a poor tensile strength and is liable to be craked when subjected to tension. It also develops shrinkage stresses. Hence, by providing steel reinforcement within the concrete mass at the time of pouring, the resulting member will have both the properties of concrete and steel. By reinforcing the concrete with steel, the defects of concrete are made good.

15.2 REQUIREMENTS OF GOOD CONCRETE

In making sound and durable concrete the prime requirements are the following:

1. The aggregates should be hard and durable.

2. The aggregates shall be properly graded in size from fine to course.

3. Cement should be of sufficient quantity to produce the required water-tightness and strength

4. The water used while mixing shall be free from organic material or any deleterious minerals.

5. The quantity of water should be such as to produce the needed consistency.

6. Mixing should be done thoroughly so as to produce homogeneity.

7. Concrete should fill every part in the form. This is done by ramming or puddling.

8. Until the concrete is thoroughly hard it is necessary to ensure that the temperature of concrete is maintained above the freezing point. This is done to avoid retarded hardening.

15.3 GRADES OF CONCRETE

In construction certain standard mixes alone are used. A set of mix for concrete should be well defined either in terms of the proportion of cement, fine and course aggregate or in terms of the 28-days compressive strength requirements

15.3.1 **Concrete mix in terms of proportions of the components.**

The usual mixes and its uses are given below

The mix 1:3:6 is used for mass concreting and the rear sides of dams.

The mix 1:2:4 is used for general reinforced concrete work. The revised I.S. Code has recommended the mix 1:1½:3 for general reinforced concrete work.

The mix 1:1½:3 is used for front faces of dams, water tanks, columns etc.

The mix 1:1:2 is used for piles.

15.3.2 **Concrete mix based on the 28-days cube strength**

The usual grades of concrete are given in the following table.

Table 15.1 Grades of concrete based on the 28-days cube strength

Group	Grade designation		Specified characteristic compressive strength of 150 mm cube at 28 days (N/mm²)
Ordinary concrete	M 10	(1:3:6) approximately	10
	M 15	(1:2:4) approximately	15
	M 20	(1:1½:3) approximately	20
Standard concrete	M 25	(1:1:2) approximately	25
	M 30		30
	M 35		35
	M 40		40
	M 45		45
	M 50		50
	M 55		55
High strength concrete	M 60		60
	M 65		65
	M 70		70
	M 75		75
	M 80		80

The I.S. 456:2000 code has recommended that the minimum grade of concrete for plain and reinforced concrete work is M 20. In the above table characteristic strength means the strength of material below which not more than 5 per cent of the test results are expected to fall.

15.4. PROPERTIES OF CONCRETE

Strength, durability and workability may be considered as the main properties of concrete. In addition, good concrete should be able to resist wear and corrosion and it should be water-tight, compact and economical.

15.4.1 Increase in strength with age

Where it can be shown that a member will receive its full design load/stress within a period of 28-days after the casing of the member (for example, in foundation and lower columns in multistory buildings), the characteristic compressive strength given in the Table 15.1 may be increased by multiplying by the factors given in the Table 15.2.

Table 15.2 Age factor

Minimum age of member when full design load/stress is expected in months	Age factor
1	1.00
3	1.10
6	1.15
12	1.20

Note:

1. No increase in respect of age at loading should be allowed where high alumina cement concrete is used.

2. Where members are subjected to lower direct load during construction, they should be checked for stresses resulting from combination of direct load and bending during construction

3. The permissible stresses or design strengths shall be based on the increased value of the compressive strength.

15.4.2 Tensile strength of concrete

The flexural and split tensile strengths shall be obtained as described in I.S.516 and I.S.5816 respectively. When the designer wishes to use an estimate of the tensile strength from the compressive strength, the following formula may be used:

Flexural strength $\quad f_{cr} = 0.7 \sqrt{f_{ck}} \, N/mm^2$

Where f_{ck} = characteristic compressive strength of concrete in N/mm^2.

15.4.3 Shearing strength of concrete

Concrete really has a large shearing strength. Generally the shearing strength as such does not have any worthy significance, since the shear stresses are in all cases limited to much lower values so as to protect the concrete against failure by diagonal tensile stresses.

The modulus of elasticity is normally related to the compressive strength of concrete. The modulus of elasticity is primarily influenced by the elastic properties of the aggregate and to a lesser extent by the conditions of curing and age of the concrete, the mix proportions and the type of cement. In the absence of test data, the modulus of elasticity for structural concretemay be assumed as $E_c = 5000\sqrt{f_{ck}}$

Where E_c=The short term static modulus of elasticity in N/mm² and

f_{ck}=The characteristic cube strength of concrete in N/mm²

15.4.4 Durability

Concrete can be made durable by using good quality materials (cement, aggregates and water) by reducing the extent of voids by suitable grading and proportion of the materials, by using adequate quantity of cement and low water cement ratio thereby ensuring concrete of increased permeability. In addition, thorough mixing, desired placing, adequate compaction and curing of the concrete is equally important to have durable concrete.

15.4.5 Workability

Workability is the most elusive property of concrete and is quit difficult to define and measure. In the simplest form a concrete is said to be workable if it can be easily mixed, handled, transported, placed in position and compacted. A workable concrete mix must be fluid enough so that it can be compacted with minimum labour. A workable concrete does not result in bleeding or segregation. Bleeding of concrete takes place when the excess of water in the mix comes up at the surface and segregation is caused when coarse aggregates have a tendency to separate from the fine aggregates.

15.4.6 Shrinkage of concrete

Concrete undergoes shrinkage as it loses moisture by evaporation. The withdrawal of moisture is never uniform throughout the concrete mass, resulting in differential shrinkage. Such differential shrinkage produces internal stresses. In ordinary concrete the amount of shrinkage depends on the exposure condition and the quality of concrete. Exposure to wind increases shrinkage. A humid atmosphere decreases shrinkage low humidity increases shrinkage. For a given environment, the total shrinkage of concrete is most influenced by the total amount of water present in the concrete at the time of mixing and to a leasser extent by the cement content. In the absence of test data, the approximate value of the total shrinkage strain may be taken as 0.0003 (For additional information see IS 1343)

As concrete is subjected to loading, the initial strain in concrete at low unit stresses is nearly elastic. But over a period of time this strain increases even under constant load. This increases the deformation of a loaded member which takes place over a period of time is called creep. Factors contributing to creep are, loading at early age, using concrete of high water cement ratio, exposing concrete to drying conditions. Creep co-efficient is the ratio of the ultimate creep strain to the elastic strain at the age of loading. In the absence of experimental data the creep co-efficient may be taken as per the Table 15.3. The ultimate creep strain does not include the elastic strain.

Table 15.3 Creep co-efficient

Age at loading	Creep co-efficient
7 days	2.2
28 days	1.6
1 year	1.1

15.4.7 Thermal expansion

The co-efficient of thermal expansion for concrete may be taken at the values given in Table 15.4.

Table 15.4 Thermal expansion

Types of Aggregate	Co-efficient of expansion for concrete per degree C
Quartzite	1.2 to 1.30 x 10^{-5}
Sandstone	0.9 to 1.20 x 10^{-5}
Granite	0.7 to 0.95 x 10^{-5}
Basalt	0.8 to 0.95 x 10^{-5}
limestone	0.6 to 0.90 x 10^{-5}

15.5 STRENGTH OF CONCRETE

Strength of concrete means the ultimate resisting capacity against the action of loads. Under standardized sample and loading conditions, concrete can be characterized by the load intensity it can resist. Generally the crushing stress of 150 mm cube of concrete is taken to represent the strength of concrete. These days the crushing stress of 150 mm cube of concrete is taken to represent the grade or the quality of concrete.

In the earlier days it was believed that the strength of concrete would increase by using more cement and greater compaction. The role of water was taken only to render concrete to a sufficient plastic state to be compacted easily. Now it is well known that the compressive strength of fully compacted concrete is inversely proportional to the water-cement ratio. To day it is established that the strength of concrete can be enhanced by decreasing the water-cement ratio, increasing the fineness of cement, by age of concrete, by increasing the size of aggregate and by proper grading and shape of aggregates.

15.6 PERMISSIBLE STRESSES IN CONCRETE (I.S. 456-2000)

The I.S. code has specified the permissible stresses in concrete for various grades. They are given in the Table 15.5

Table 15.5 Permissible stress in concrete in compression and average bond

Grade of concrete	Bending stress in compression σ_{cbc} N/mm^2	Direct stress in compression N/mm^2	Average bond stress for plain tension bars N/mm^2
M 10	3.0	2.5	-
M 15	5.0	4.0	0.6
M 20	7.0	5.0	0.8
M 25	8.5	6.0	0.9
M 30	10.0	8.0	1.0
M 35	11.5	9.0	1.1
M 40	13.0	10.0	1.2
M 45	14.5	11.0	1.3
M 50	16.0	12.0	1.4

Note: 1. The bond stress given above shall be increased by 25 per cent for bars in compression.

2. The bond stress given above shall be increased by 40 per cent for deformed bars.

Table 15.6 Permissible direct stress in tension in concrete based on equivalent concrete area

Grade of concrete	Permissible tensile stress N/mm²
M 10	1.2
M 15	2.0
M 20	2.8
M 25	3.2
M 30	3.6
M 35	4.0
M 40	4.4
M 45	4.8
M 50	5.2

$$Tensile\ stress = \frac{Tension}{(Actual\ concrete\ area) + (m \times steel\ area)}$$

15.7 MODULAR RATIO m

This is the ratio of modulus of elasticity for steel to the modulus of elasticity for concrete. As per the code, the modular ratio shall be taken as $m = \dfrac{280}{3 \times \sigma_{cbc}}$

Where σ_{cbc} = Permissible compressive stress in concrete in bending. The values of modular ratio for different grades of concrete are given in the Table 15.7.

Table 15.7 Modular ratio for different grades of concrete

Grade of concrete	σ_{cbc} N/mm²	$m = \dfrac{280}{3 \times \sigma_{cbc}}$
M 10	3.0	31.11
M 15	5.0	18.67
M 20	7.0	13.33
M 25	8.5	10.98
M 30	10.0	9.33
M 35	11.5	8.11
M 40	13.0	7.18
M 45	14.5	6.44
M 50	16.0	5.83

15.8 PERMISSIBLE SHEAR STRESS IN CONCRETE

The I.S. code has introduced the concept of nominal shear stress defined by the following relation.

$$\tau_v = \frac{V}{bd}$$

Nominal shear stress in beams or slabs of uniform depth

Where V= Shear force due to design loads

Design of Structures

b = Breadth of the member (or breadth of web)

d = Effective depth

In the case of beams of varying depth
$$\tau_v = \frac{V \pm \frac{M}{d}\tan\beta}{bd}$$

Where, M=Bending moment at the section and

β = angel between the top and bottom edges of the beam

The negative sign in the formula should be used when the bending moment increases numerically in the same direction as the effective depth increases; and the positive sign in the formula should be used when the bending moment decreases numerically in this direction. The permissible shear stress in concrete in beams without shear reinforcement is given in the Table 15.8.

Table 15.8 Permissible shear stress in concrete in beams

| $\frac{100 A_{st}}{bd}$	Permissible shear stress in concrete τ_c in N/mm² for various grades of concrete					
	M 15	M 20	M 25	M 30	M 35	M 40 and above
0.15	0.18	0.18	0.19	0.20	0.20	0.30
0.25	0.22	0.22	0.23	0.23	0.23	0.23
0.50	0.29	0.30	0.31	0.31	0.31	0.32
0.75	0.34	0.35	0.36	0.37	0.37	0.38
1.00	0.37	0.39	0.40	0.41	0.42	0.42
1.25	0.40	0.42	0.44	0.45	0.45	0.46
1.50	0.42	0.45	0.46	0.48	0.49	0.49
1.75	0.44	0.47	0.49	0.50	0.52	0.52
2.00	0.44	0.49	0.51	0.53	0.54	0.55
2.25	0.44	0.51	0.53	0.55	0.56	0.57
2.50	0.44	0.51	0.55	0.57	0.58	0.60
2.75	0.44	0.51	0.56	0.58	0.60	0.62
3.00 and above	0.44	0.51	0.57	0.60	0.62	0.63

Note: A_{st} is that area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at supports where the full area of tension reinforcement may be used.

15.9 REINFORCEMENT

Reinforcement used shall confirm to the requirements of I.S. 432 specification for mild steel and high tensile steel bars and hard drawn steel wire for concrete reinforcements (Revised). The reinforcement shall be free from loose mill scale, loose rust, oil and grease or any such harmful matter, immediately before placing the concrete. The reinforcement shall be placed and positioned strictly following the requirements shown in the structural drawings. Bending bars are expected to fulfill certain definite functions and hence bars must be bent so that there is good advantage of the worked design. Bending bars shall be done with greater caution. Often this job does not receive that much attention which it deserves. There are instances of failure of structures for want of correct bending even though the designs worked out are not faulty.

15.10 THE NECESSITY OF BENDING REINFORCEMENT

Bars are bent under different circumstances. They may be bent to form hooks so as to develop proper anchorage. Sometimes bars have to be bent so as to form loops as in the case of stirrups as shear reinforcement. They may also be bent up to form necessary reinforcement for hogging bending moments. The following are the types of bend we normally come across and are in Fig. 15.1
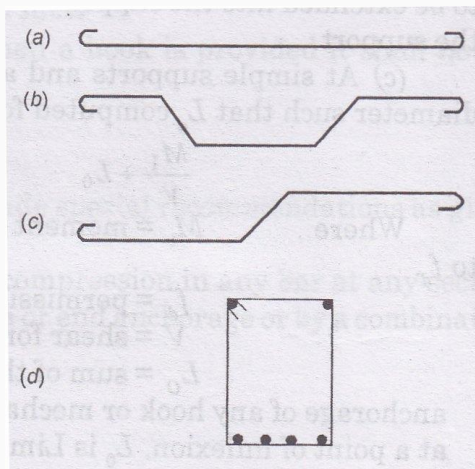
1. Hooks at the end of mild steel bars in beams.

2. Bars bent up at ends and hooked in beams for resisting diagonal tension.

3. Bars which serve for positive bending moment which are bent up to resist negative bending moment.

4. Bars bent to form loops to serve as shear reinforcement.

15.10.1 **Splices in tensile reinforcement**

Splices at point of maximum tensile stress shall be avoided wherever possible; splices where used shall be welded, lapped or otherwise fully developed. In any case the splice shall transfer the entire computed stress from bar to bar. Lapped splices in tension shall not be used for bars of sizes larger than 36 mm diameter and such splices shall preferably be welded. For contact splices, spaced laterally closer than 12-bar diameters or located closer than 150 mm or 6-bar diameters from the outside edge, the lap shall be increased by 20 percent or stirrups or closely spaced spirals shall enclose the splice for its full length. Where more than one half of the bars are spliced within a length of 40-bar diameters or where splices are made at points of maximum stress special precaution shall be taken such as increasing the length of the lap and/or using spirals of closely spaced stirrups around and for the length of the splice.

15.10.2 **Splices in compression reinforcement**

Where lapped splices are used, the lap length shall confirm to the requirements. Welded splices may be used instead of lapped splices. Where bar size exceeds 36 mm diameter welded splices shall preferably be used. In bars required for compression only the

compressive stress may be transmitted by bearing the square cut ends held in concrete contact by a suitably welded sieve or mechanical device.

In columns where longitudinal bars are offset at a splice, the slope of the inclined portion of the bar with the axis of the column shall not exceed 1 in 6 and the portions of the bar above and below the offset shall be parallel to the axis of the column. Adequate horizontal support at the offset bands shall be treated as a matter of design and shall be provided by metal ties, spirals or parts of the floor construction. Metal ties or spirals so designed shall be placed near (not more than eight-bar diameters from) the point of bend. The horizontal thrust to be resisted shall be assumed as 1½ times the horizontal component of the nominal stress in the inclined portion of the bar. Offset bars shall be bent before they are placed in forms.

15.11 TYPES AND GRADES OF REINFORCEMENT BARS

The Table 15.9 shows the various types of steel permitted for use as reinforcement bars and their characteristic yield strengths f_y. The Table 15.10 shows the permissible stresses in steel reinforcement.

Table 15.9 Various types of steel permitted for use as reinforcement bars

Types of steel	I.S. code	Bar diameter	Characteristic yield strength f_y N/mm²
Mild steel (plain bars)	I.S.: 432 (part I)	Up to 20 mm Over 20 mm	250 240
Mild steel (hot rolled deformed bars)	I.S.: 1139	Up to 20 mm Over 20 mm	250 235
Medium tensile steel (plain bars)	I.S.: 432 (part I)	Up to 20 mm 20 mm to 40 mm	350 340
Medium tensile steel (hot rolled deformed bars)	I.S.: 1139	Up to 20 mm 20 mm to 40 mm Over 40 mm	350 340 320
High yield strength steel (hot rolled deformed bars)	I.S.: 1139	All sizes	415
High yield strength steel (cold twisted deformed bars)	I.S.: 1786	All sizes	50
Hard drawn steel wire fabric	I.S.: 1566 and I.S.: 432 (Part II)	All sizes	480

Table 15.10 Permissible stresses in steel reinforcement (I.S. 456)

Types of stress in steel reinforcement	Permissible stress in N/mm^2		
	Mild steel bars confirming to grade I of I.S.432 (Part I) or deformed mild steel bars confirming to I.S.:1139	Medium tensile steel confirming to I.S.432 (Part I) or deformed medium tensile steel bars confirming to I.S.:1139	High yield strength deformed bars conforming to I.S. 1139 or I.S. 1786 grade F 415 (Ribbed Torsteel)
Tension (σ_{st} or σ_{sv}) a. Up to 20 mm b. Over 20 mm	140 130	Half the guaranteed yield stress subject to a maximum of 190	230 230
Compression in column bars (σ_{sc})	130	130	190
Compression in bars in a beam or slab when the compressive resistance of the concrete is taken into account	The calculated compressive stress in the surrounding concrete multiplied by 1.5 times the modular ratio or σ_{sc} whichever is lower		
Compression in bars in a beam or slab when the compressive resistance of the concrete is not taken into account: a. Up to 20 mm b. Over 20 mm	 140 130	Half the guaranteed yield stress subject to a maximum of 190	 190 190

15.12 JOINING OR LAPPING

15.12.1 **Length of lap**

The length of lap in reinforcement shall not be less than:

a. For bars in tension:

$$bar\ diameter \times \frac{actual\ tensile\ stress}{4\ times\ the\ permissible\ average\ bond\ stress\ or\ 30 - bar\ diameter\ whichever\ is\ greater}$$

b. For bars in compression:

$$bar\ diameter \times \frac{actual\ compressive\ stress}{5\ times\ the\ permissible\ average\ bond\ stress\ or\ 24 - bar\ diameter\ whichever\ is\ greater}$$

15.12.2 **Minimum spacing of reinforcement**

The minimum horizontal distance between parallel reinforcement shall not be less than the following:

1. Diameter of bar when bars are of the same diameter and diameter of the thickest bar when bars of more than one size are used
2. Maximum size of coarse aggregate plus 5 mm. a greater distance should be provide when convenient.

The vertical distance between two horizontal main bars shall be not less than 15 mm, two-thirds the nominal maximum size of aggregate or the maximum size of bar whichever is greatest.

15.12.3 **Cover**

All reinforcements shall have a cover of concrete and the thickness of such a cover exclusive of plaster or other decorative finish shall be as follows:

1. At each end of a reinforcing bar–not less than 25 mm nor less than twice the diameter of such bar.

2. For longitudinal reinforcement in a column–not less than 40 mm nor less than the diameter of bar. In the case of columns of minimum dimension of 200 mm or less, whose bars do not exceed 12 mm dia-25 mm cover may be used.

3. For longitudinal reinforcement in beams–not less than 25 mm nor less than diameter of such bar.

4. For tensile, compressive, shear and other reinforcement in a slab–not less than 15 mm nor diameter of reinforcement.

5. For any other reinforcement–not less than 15 mm nor less than the diameter of reinforcement.

For reinforced concrete members totally immersed in sea water, the cover shall be 50 mm more than that specified above. For concrete of grade over M 25 the additional thickness of cover may be reduced to half the value stipulated above. In all cases the cover should not exceed 75 mm.

15.13 USEFUL CONSTANT OF STEEL

Modulus of elasticity $\quad\quad\quad\quad\quad$ 2×10^5 N/mm^2

Poisson's ratio $\quad\quad\quad\quad\quad\quad$ 0.25 to 0.30

Modulus of rigidity $\quad\quad\quad\quad\quad$ 7×10^4 to 8×10^4 N/mm^2

Coefficient of thermal expansion $\quad$ 12×10^{-6} per °C

Specific weight $\quad\quad\quad\quad\quad\quad$ 7.85×10^4 N/m^3

Table 15.11 Area, Perimeter and weights of round bars

Diameter of bar mm	Area mm^2	Perimeter mm	Weight N/m
5	20	15.7	1.47
6	28	18.8	2.16
8	50	25.1	3.83
10	79	31.4	6.08
12	113	37.7	8.73
16	201	50.3	15.50
18	254	56.5	19.62
20	314	62.8	24.23
22	380	69.1	29.23
25	491	78.5	37.77
28	616	88.0	47.38
32	804	100.5	61.90

LESSON 16. Analysis of Singly Reinforced Section

16.1 INTRODUCTION

In working stress method it will be assumed that concrete and steel are elastic and they are subjected to such stresses that the components remain elastic and the maximum stresses included in the components do not exceed the allowable stresses. This method has certain shortcomings. For concrete, the relation between stress and strain is not linear but follows a curve. Though the stress-strain relation is linear for mild steel it is not so in the case of high yield strength deformed bars which are most commonly used in practice. This method does not provide a true factor of safety against failure or objectionable deformation. The method ignores the effect of creep and shrinkage of concrete.

A reinforced concrete member shall be designed for all conditions of stresses that may occur and in accordance with the principles of mechanics. The characteristic property of a reinforced concrete member is that its components namely concrete and steel act together as a single unit as long as they remain in the elastic condition i.e., the two components are bound together so that there can be no relative displacement between them.

16.2 REINFORCED CONCRETE MEMBERS SUBJECTED TO DIRECT LOAD

Let us consider a reinforced concrete member of length l

Let $\quad$ A_{sc}=area of steel and

A_c =area of concrete
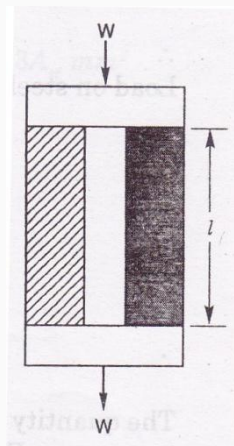
For the sake of analysis the two components have been shown separately in Fig. 16.1. Let the member be subjected to an axial load W. Let W_s and W_c be the load components transmitted to steel and concrete respectively. Assuming both the components undergo same change in length.

Strain in steel = Strain in concrete

$$\frac{w_s}{A_{sc}E_s} = \frac{w_c}{A_cE_c} = \frac{w_s+w_c}{A_{sc}E_s+A_cE_c} = \frac{W}{A_{sc}E_s+A_cE_c} \qquad \cdots \text{(i)}$$

Where, E_s and E_c are the elastic moduli for steel and concrete respectively.

From equation (i), $\quad \frac{W_s}{A_{sc}} = \frac{E_s}{E_c} \times \frac{W_c}{A_c}$

But, $\frac{W_s}{A_{sc}}$ = Stress in steel and $\frac{W_c}{A_c}$ = Stress in concrete

Let $\frac{E_s}{E_c} = m$ = modular ratio between steel and concrete

Therefore, $\qquad$ Stress in steel = m x stress in concrete

Again from equation (i) $\qquad W_s = \left[\frac{A_{sc}E_s}{A_{sc}E_s+A_cE_c}\right]W$

and $\qquad W_c = \left[\frac{A_cE_c}{A_{sc}E_s+A_cE_c}\right]W$

putting $\qquad E_s = mE_c$ the above relations can be reduced to

$$W_s = \left[\frac{mA_{sc}}{mA_{sc}+A_c}\right]W$$

$$W_c = \left[\frac{A_c}{mA_{sc}+A_c}\right]W$$

Suppose, $A_{sc}=A_c=A$ $\qquad W_s = \left[\frac{m}{m+1}\right]W \qquad$ and $\qquad W_c = \left[\frac{1}{m+1}\right]W$

If m=18 and the two components have the same area,

$$W_s = \left[\frac{18}{19}\right]W \qquad \text{and} \qquad W_c = \left[\frac{1}{19}\right]W$$

Hence we find that the steel member is subjected to a greater load than concrete. This explains, that when steel is provided in combination with concrete, it will be very useful in sharing a considerable part of the load on the composite member.

16.3 EQUIVALENT OR TRANSFORMED CONCRETE AREA

Let f_s and f_c be the stresses in steel and concrete. Since strains are equal in steel and concrete,

$\frac{f_s}{E_s} = \frac{f_c}{E_c} \qquad$ therefore, $\qquad f_s = \frac{E_s}{E_c}f_c \qquad$ and $\qquad f_c = mf_c$

Load on steel + load on concrete = Total load i.e., $f_sA_{sc} + f_cA_c = W$ and $mf_cA_{sc} + f_cA_c = W$

Therefore $f_c = \left[\frac{W}{A_c+mA_{sc}}\right]$ and $f_s = m\left[\frac{W}{A_c+mA_{sc}}\right]$

The quantity (A_c+mA_{sc}) is called the equivalent concrete area

Equivalent concrete area = A_c = Actual concrete area + (m x steel area)

16.4 COMPOSITE MEMBER SUBJECTED TO BENDING

Consider and reinforced concrete beam simply supported on a span l. Let us consider the mid section of the beam. Let b be the width of the beam and d the depth of the beam from the top compression edge to the centre of steel. Fig. 16.2 shows the strain and stress diagrams for concrete at the section. We assume that the strain varies linearly consistent with the assumption that transverse sections which were plane before bending remain plane after bending. With the assumption of no slip between steel and the surrounding concrete, it

means, that the strains in steel and the surrounding concrete are equal. Suppose the strain in concrete at the level of the steel is e, then the strain in steel is also equal to e. the corresponding stresses in concrete and steel at the same level will be eE_c and eE_c respectively. Thus we find the stress in steel is again modular ratio times the stress in concrete at the level of steel.

Fig. 3.2.
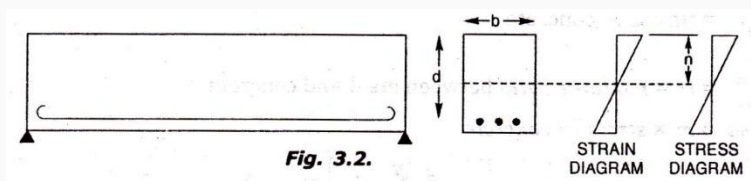
We know that in the case of beams of homogeneous material the neutral axis is a centroidal axis. In the case of composite section the neutral axis can be determined by considering the equivalent or transformed section of one material only. The neutral axis therefore is the centroidal axis of this equivalent section. This treatment is satisfactory when the components of the beam remain as one unit in the elastic condition. For instance in the case of a composite beam considering of a timber beam strengthened by steel plates, the analysis can be made by replacing steel by its equivalent timber section and taking the neutral axis at the centroid of the transformed section. But concrete is a material strong in compression and weak in tension. After the concrete is strained beyond a limit it develops cracks in tension zone. After this stage the concrete in the tension zone becomes ineffective to offer tensile resistance. This condition shifts the position of the neutral axis. Hence, the neutral axis has to be determined ignoring the concrete in the tension zone.

Example 16.1 A 300 mm x 300 mm R.C. member reinforced with 1257 mm² of steel supports an axial compressive load of 440 kilonewtons. Calculate the stresses in concrete and steel. Take m=13.33

Solution

A = 300 x 300 = 90000 mm²

A_{sc}=1257 mm² therefore A_c = 90000-1257 = 88743 mm²

Equivalent concrete area   = A_e = A_c+mA_{sc}=88743+(13.33x1257)=105498.81 mm²

Compressive load   = W = 440 kilonewtons = 440000 newtons

Stress in concrete $f_c = \dfrac{W}{A_c} = \dfrac{440000}{105498.81} = 4.17 \ N/mm^2$

Stress in steel $f_{sc} = mf_c = 13.33 \times 4.17 = 55.59 \ N/mm^2$

Example 16.2 A 250 mm x 250 mm reinforced concrete member has to support an axial compressive load of 400 kilonewtons. If the stress in concrete is not to exceed 4 N/mm², calculate the area of steel required. Take m=13.33.

Solution

A   = 250 x 250   = 62500 mm²

A_{sc} = area of steel =?

A_c = (62500-A_{sc})

Equivalent concrete area A_e = A_c+mA_{sc} =(62500-A_{sc})+(13.33 A_{sc}) mm²=62500+13.33A_{sc} mm²

Limiting the stress in concrete to 5 N/mm²

$$f_c = \frac{W}{A_c} = \frac{400000}{62500 + 13.33 A_{sc}} = 5 \ N/mm^2$$

$$62500+13.33A_{sc} = 80000$$

$$A_{sc} = 1312.83 \ mm^2$$

16.5 SINGLY REINFORCED BEAMS

A singly reinforced beam is a beam provided with longitudinal reinforcement in the tension zone only. The analysis and design of a reinforced concrete member subjected to bending are based on the following assumptions:

1. Plane sections transverse to the centre line of a member before bending remain plane sections after bending.

2. Elastic modulus of concrete has the same value within the limits of deformation of the member.

3. Elastic modulus for steel has the same value within the limits of deformation of the member.

4. The reinforcement does not slip from concrete surrounding it.

5. Tension is borne entirely by steel.

6. The steel is free from initial stresses when embedded in concrete.

7. There is no resultant thrust on any transverse section of the member

Of the above assumption, the assumption that plane sections transverse to the centre line of a member before bending remain plane sections after bending is further explained below.
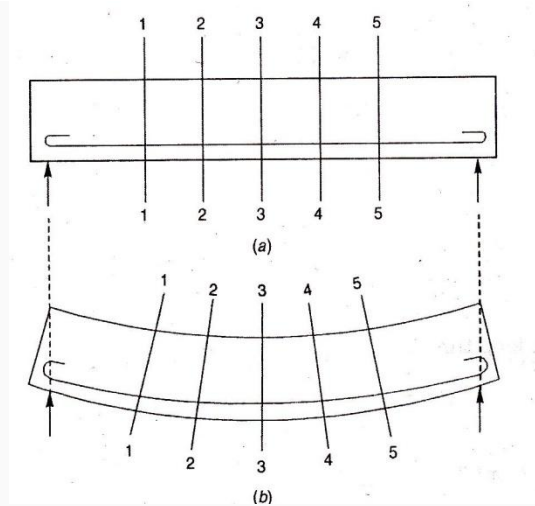
(a)

(b)

Fig. 16.3.a. shows a beam subjected to an external loading. Consider sections 1-1, 2-2, 3-3, 4-4, 5-5, etc, which are at right angles to the centre line of the member. After the beam bends, the various fibres are subjected to deformations of such amounts that these planes respectively occupy the new positions shown in Fig.16.3.b. Fig.16.4.a. shows a simply supported singly reinforced beam. Consider two sections 1-1 and 2-2 unit distance apart. Let the beam subjected to an external loading. Fig.16.4.b. shows the deflected form of the beam. The upper most fibre of concrete AC deforms to A_1C_1. A fibre BD of concrete at the level of the reinforcement deforms to B_1D_1.

(a)

(b)
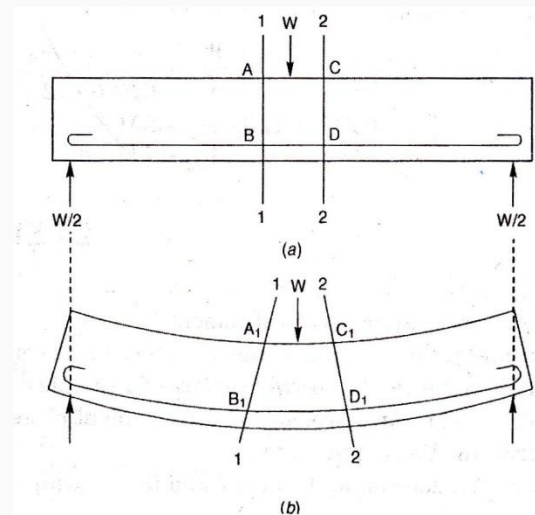
We have strain in concrete in the top fibre = $AC - A_1C_1 = e_c$

Similarly, strain in concrete just surrounding the steel = $B_1D_1 - BD = e_t$

Since there is no slip between steel and the concrete surrounding it, the strain in steel is also equal to e_t.

Therefore stress in steel   (t)   = $E_s e_t = m E_c e_t$

Stress in concrete surrounding steel   = $c_t = E_c e_t = E_c (1/E_s)$

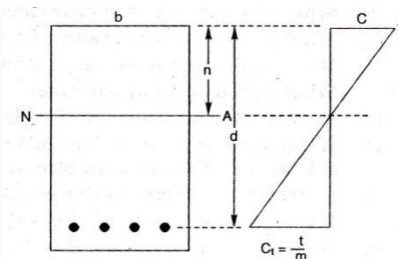Therefore $c_t = t/m$.

Design of Structures


Fig. 16.5 shows the stress distribution in concrete across the section of the beam.

16.6 EFFECT OF REINFORCEMENT IN CONCRETE

Concrete has a very low tensile strength. Tensile strength of concrete is about one-tenth of its compressive strength. Hence in structural design, it is assumed that the tensile strength of concrete is nil. Thus it is necessary to reinforce (strengthen) concrete components which are subjected to tension. Such reinforcement is accomplished by providing steel bars which are meant to resist the entire tension.
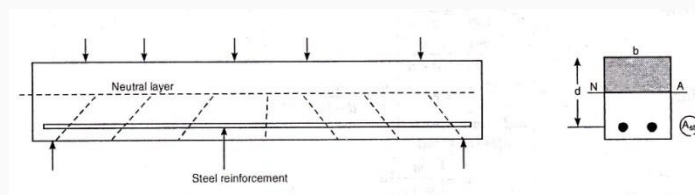
Consider the reinforced concrete beam shown in F.g.16.6. If the loading on the beam is gradually increased, the lower layer of the beam elongate and when such elongation exceeds the ultimate tensile strain of concrete, crakes will occur. We know the strain in section is proportional to the distance from the neutral axis. Hence as the loads are increased the tensile stresses also increase and consequently the crakes in number spread upward toward the neutral axis. These crakes will be at right angles to the direction of the maximum principal tensile stress in the concrete. Thus, the inclination of these cracks in concrete is related to bending, shear and axial stresses the section subjected to. As the concrete gets cracked, it will no longer be able to resist or transmit tensile forces. As this happens, the tensile force in the bottom of the beam has to be resisted entirely by the steel reinforcement, while the compressive forces at the top are resisted by concrete. Thus, for all practical purposes, the effective cross-section which resists flexure is as shown in Fig.16.6. The part of the section shown shaded above the neutral axis becomes the compressive zone, while the steel reinforcement alone offers the required tensile resistance resisting the tensile stresses below the neutral axis.

16.7 NEUTRAL AXIS

The neutral axis for a beam section is the line of intersection of the neutral layer with the beam section. This is a straight line dividing the cross-section into a tension and a compression zone. One of the basic assumptions made in the analysis of reinforced concrete beams is that the tension is borne completely by steel. Hence, it is important to note that in determining the neutral axis, the concrete in the tension zone should not be taken in to account. The tension should be considered as resisted entirely be the steel.

If the area of the reinforcement is A_{st} and the tensile stress in the reinforcement is t,

The total tension resisted, $T = A_{st} t = A_{st} mc_t = (mA_{st}) c_t$

Hence, a reinforcement of area A_{st} can be regarded as equivalent to an area (mA_{st}) of the concrete. Let b and d be the breadth and effective depth of the beam section. Effective depth means the depth from the compression edge to the centre of the tensile reinforcement. Let n be the depth of neutral axis.

One of the assumption in the analysis is that there is no resultant thrust on the section i.e., the total compression is equal to the total tension. Therefore,

Compression area x average compressive stress=Area of tensile reinforcement x stress in steel.

$$bn\frac{c}{2} = A_{st}t \qquad \text{... (i)}$$

Further, by geometry of the stress diagram (Fig.16.5) we have $\quad \frac{c}{t/m} = \frac{n}{d-n} \qquad$ ... (ii)

Equating the moments of area of the compression and tension zones about the neutral axis,

$$\frac{bn^2}{2} = mA_{st}(d-n) \qquad \text{... (iii)}$$

Solving the above quadratic equation in n, we get, $\quad n = \frac{-mA_{st} + \sqrt{(mA_{st})^2 + 2bdmA_{st}}}{b} \qquad$ ... (iv)

Besides the above results, the following result will be found interesting.

We know that, $\frac{c}{t/m} = \frac{n}{d-n}$, $\qquad$ therefore $\frac{mc}{t} = \frac{n}{d-n}$

Putting, $\frac{t}{c} = r$, $\frac{mc}{r} = \frac{n}{d-n}$ $\qquad$ therefore $md - mn = nr$ $\qquad$ and $\qquad n(m+r) = md$

Therefore $\quad n = \left[\frac{m}{m+r}\right]d$

Putting, $n = n_1 d$, $\qquad$ we have, $\qquad n_1 = \frac{m}{m+r} \qquad$ ... (v)

As per IS code, $\quad m = \frac{280}{3\sigma_{cbc}}$, where σ_{cbc}= permissible compressive stress in concrete in bending. in N/mm^2, For M 20 concrete, $\sigma_{cbc} = 7 \ N/mm^2$ and,

$$m = \frac{280}{3 \ X \ 7} = \frac{40}{3} = 13.333$$

Accordingly, $\qquad n_1 = \frac{13.333}{13.333 + r}$

Fig 16.7 shows the values of n_1 for various values of r. Alternative expression for n:

$$n_1 = \frac{m}{m+r} = \frac{m}{m+\frac{t}{c}} \qquad \text{therefore,} \qquad n_1 = \frac{mc}{mc+t}$$

Position of neutral axis for a given percentage of steel:

Rearranging equation (iii), we get, $\frac{n^2}{2} + A_{st}\frac{mn}{b} - A_{st}\frac{md}{b} = 0$

Let the percentage of steel be p% therefore $A_{st} = \frac{p}{100}(bd)$

Substituting in the above equation, $\frac{n^2}{2} + \frac{mnpd}{b} - \frac{mpd^2}{100} = 0$,

Therefore $\quad \frac{n^2}{2} + \frac{mpd}{100}(n-d) = 0 \qquad$ therefore, $\qquad p = \frac{50n^2}{md(d-n)}$

Putting, $\qquad n = n_1 d$, $\qquad p = \frac{50n_1^2}{m(1-n_1)} \qquad$ ... (vi)

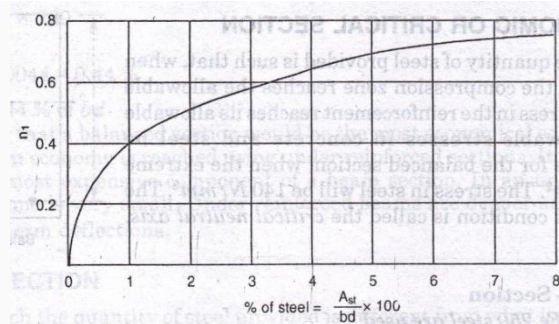Fig. 16.8 shows the relation between the percentage of steel and the position of neutral axis for m=13.333.

16.8. LEVER ARM

This is the distance between the line of action of the resultant compression and the line of action of the resultant tension. The line of action of the resultant compression is at the level of the centroid of the compressive stress diagrams. i.e. at the depth of $\frac{n}{3}$ from the compression edge.But the resultant tension is at the level of the reinforcement since the tensile resistance of concrete is ignored.

$$\text{Therefore Lever arm} = a = d - \frac{n}{3} \qquad\qquad ...(viii)$$

Sometimes the lever arm is expressed as the product of a coefficient a_1 and the effective depth

$$a = a_1 d, \qquad a = d - \frac{n}{3}, \qquad a_1 d = d - \frac{n}{3}, \qquad a_1 = 1 - \frac{1}{3}\frac{n}{d},$$

$a_1 = 1 - \frac{n_1}{3}$, Fig. 16.9 shows the value of lever arm factor for different values of n₁.

16.9 MOMENT OF RESISTANCE

This is the resisting moment offered by a beam section to resist the bending moment at the section.

Moment of resistance = Total compression or total tension x lever arm

$$M.R. = bn\frac{c}{2}\left(d - \frac{n}{3}\right) \quad \text{and} \quad M.R. = A_{st}t\left(d - \frac{n}{3}\right)$$

The moment of the resistance can also be expressed as follows:

$$M.R. = M = bn\frac{c}{2}\left(d - \frac{n}{3}\right)$$

Put $n = n_1 d$ therefore $M.R. = M = bn_1 d\frac{c}{2}\left(d - \frac{n_1 d}{3}\right)$

Therefore, $M = \frac{1}{2}n_1\left(1 - \frac{n_1}{3}\right)cbd^2$ therefore $M = Qbd^3$ where $Q = \frac{1}{2}n_1\left(1 - \frac{n_1}{3}\right)c$.

## 16.10. BALANCED OR ECONOMIC OR CRITICAL SECTION

This is a section, in which the quantity of steel provided is such that, when the most distant concrete fiber in the compression zone reaches the allowable stress in compression, the

tensile stress in the reinforcement reaches its allowable stress. For example let the allowable stresses in concrete and steel be 7N/mm² and 140 N/mm². Hence for the balanced section, when the extreme stress in concrete reaches 7N/mm².The stress in steel will be 140 N/mm². The neutral axis corresponding to this condition is called the critical neutral axis. Let n_c be the depth of critical neutral axis as shown in the Fig. 16.10
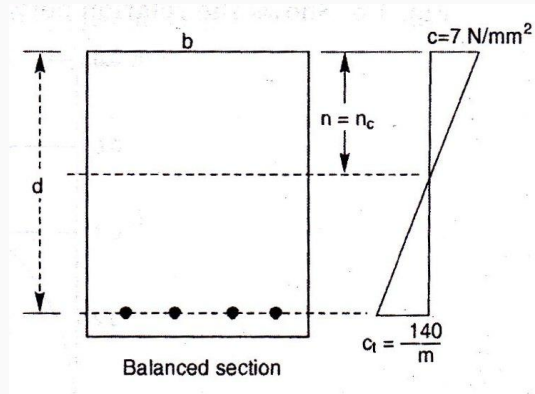
Balanced section

Analysis of the Balanced – Section

(i) When M 20 concrete and Fe 250 steel are used (Fig. 16.11),   = 7 N/mm², = 140 N/mm²

Balanced section
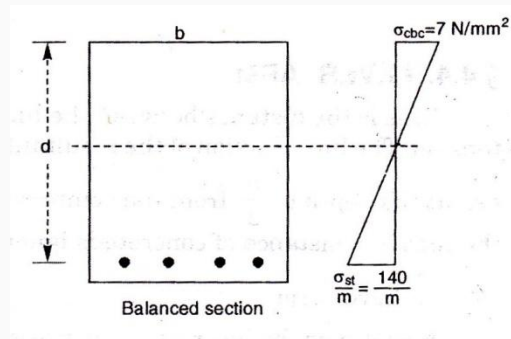
$m = \dfrac{280}{3\sigma_{cbc}}$ therefore $m\sigma_{cbc} = \dfrac{280}{3}$

By the geometry of the stress diagram, $\dfrac{m\sigma_{cbc}}{\sigma_{st}} = \dfrac{n_c}{d-n_c}$

$\dfrac{280}{3 \times 140} = \dfrac{n_c}{d-n_c}$ therefore $n_c = 0.4\ d$

Neutral axis factor $n_1 = 0.40$, Lever arm $a = d - \dfrac{n}{3} = d - \dfrac{0.4\ d}{3} = \dfrac{13}{15}d = 0.87\ d$

Moment of resistance = M.R. $= bn_c.\dfrac{c}{2}\ a = b(0.4d)\dfrac{7}{2} . \dfrac{13}{15}d = 1.213\ bd^2$

Total compression = Total tension.   Therefore, $bn_c.\dfrac{\sigma_{cbc}}{2} = A_{st}\sigma_{st}$

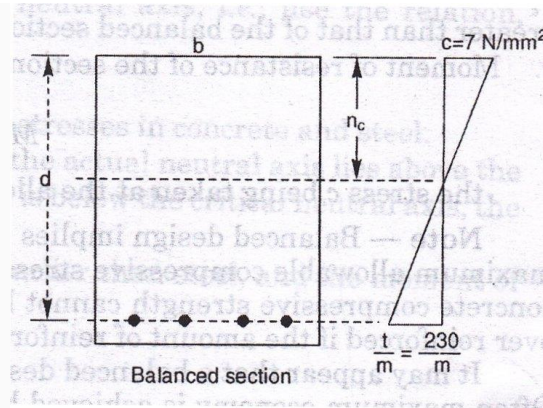$b(0.4d)\dfrac{7}{2} = A_{st} \times 140$, $\dfrac{A_{st}}{bd} = 0.01 = 1\ \%$, $A_{st} = 1\%$ of bd

(ii) When M 20 concrete and Fe 415 steel used (Fig. 16.12),

$\sigma_{cbc} = 7\ N/mm^2, \sigma_{st} = 230\ N/mm^2$

$m = \dfrac{280}{3\sigma_{cbc}}$,   therefore $m\sigma_{cbc} = \dfrac{280}{3}$

By the geometry of the stress diagram, $\dfrac{m\sigma_{cbc}}{\sigma_{st}} = \dfrac{n_c}{d-n_c}$ and $\dfrac{280}{3 \times 230} = \dfrac{n_c}{d-n_c}$

Therefore, $n_c = \dfrac{28}{97}d = 0.29\ d$

Balanced section

Neutral axis factor = n_1 = 0.29

Lever arm $\quad a = d - \dfrac{n}{3} = d - \dfrac{28}{97}d = \dfrac{263}{291}d = 0.90d$

Moment of resistance $\quad$ M.R. $= bn_c\dfrac{c}{2}a = b\dfrac{28}{97}d.\dfrac{7}{2}.\dfrac{263}{291}d = 0.913\,bd^2$

Total compression = Total tension, $\quad bn_c.\dfrac{\sigma_{cbc}}{2} = A_{st}\sigma_{st}$

$b\dfrac{28}{97}d.\dfrac{7}{2} = A_{st} \times 230$; $\quad \dfrac{A_{st}}{bd} = 0.0044 = 0.44\%$, therefore $A_{st}= 0.44\%$ of bd

It may appear that a balanced section would be the most economical solution to design. This is not really true. In reality the maximum economy is reached using under reinforced sections. It is important to recognize that the steel reinforcement is the most expensive component of a beam section. In an under reinforced beam the ratio of volume of steel to total volume is very small. Under reinforced beams are deeper and stiffer and are not subjected to objectionable short or long term deflections.
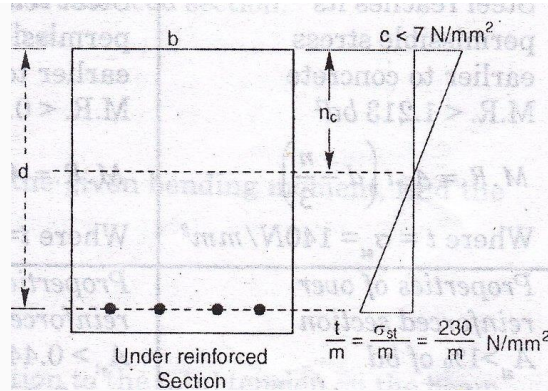
16.11 UNBALANCED SECTION

This is a section in which the quantity of steel provided is different from what is required for the balanced section. Unbalanced sections may be classified into under-reinforced and over-reinforced sections.

16.11.1 **Under-Reinforced Section**

This is a section in which the quantity of steel provided is less than what is required for a balanced section. In this case when the stress in steel reaches its permissible value, the corresponding extreme compressive stress reached in concrete will be less than its permissible value. For example, taking the permissible stresses in concrete and steel as 7 N/mm² and 230 N/mm², we find that when the stress in steel reaches 230 N/mm² , the extreme compressive stress in concrete will be less than 7 N/mm² (Fig. 16.13). The depth n of the actual neutral axis will be less than the depth n_c of the critical neutral axis. The moment of resistance of the section will be less than that of the balanced section.
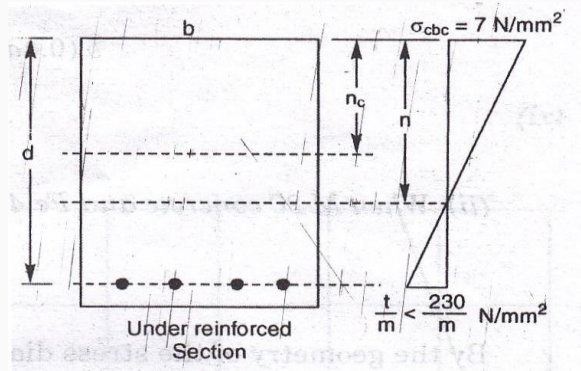
Under reinforced Section

Therefore, Moment of resistance of the section is given by,

$$\text{M.R.} = A_{st}t\left(d - \frac{n}{3}\right) = A_{st}\sigma_{st}\left(d - \frac{n}{3}\right)$$

the stress t being taken at the allowable stress for steel .

16.11.2 **Over-Reinforced Section**

This is a section in which the quantity of steel provided is more than what is required for a balanced section. In this case when the extreme compressive stress in concrete reaches its permissible value, the corresponding tensile stress in steel will be less than its permissible value. For example, taking the permissible stresses in concrete and steel as 7 N/mm² and 230 N/mm² (See Fig. 6.14). The depth n of the actial neutral axis will be greater than the depth of the critical neutral axis. The moment of resistance of the section will be greater than that of the balanced section.

Under reinforced Section

Therefore Moment of resistance of the section is given

$$\text{M.R.} = bn\frac{c}{2}\left(d - \frac{n}{3}\right) = bn\frac{\sigma_{cbc}}{2}\left(d - \frac{n}{3}\right)$$

The stress c being taken at the allowable compressive stress for concrete .

Balanced design implies that there is just exactly enough amount of steel reinforcement to develop the maximum allowable compressive stress in concrete. If the amount of steel reinforcement provided is lesser, then the concrete compressive strength cannot be

developed, and the section becomes under reinforced. The section becomes over reinforced if the amount of reinforcement provided is more than what is needed for a balanced section.

It may appear that a balanced design may prove to be the most economical design. This need not be true always. Often maximum economy is achieved by using under reinforced sections. This is because the reinforcing steel is the most expensive component of the section. Under reinforced beams, have greater depth providing greater stiffness and such beams are not subjected to objectionable deflections. The properties of reinforced section is summarized in Table 16.1

Table 16.1 Summary of Results

M 20 concrete Fe 250 steel	M 20 concrete Fe 415 steel	M 25 concrete Fe 250 steel	M 25 concrete Fe 415 steel
Permissible Stresses	Permissible Stresses	Permissible Stresses	Permissible Stresses
$\sigma_{cbc}=7\ N/mm^2$, $\sigma_{st}=140\ N/mm^2$ $m=\dfrac{280}{3\sigma_{cbc}}=\dfrac{280}{3\times7}=13.33$	$\sigma_{cbc}=7\ N/mm^2$, $\sigma_{st}=230\ N/mm^2$ $m=\dfrac{280}{3\sigma_{cbc}}=\dfrac{280}{3\times7}=13.33$	$\sigma_{cbc}=8.5\ N/mm^2$, $\sigma_{st}=140\ N/mm^2$ $m=\dfrac{280}{3\sigma_{cbc}}=\dfrac{280}{3\times8.5}=10.98$	$\sigma_{cbc}=8.5\ N/mm^2$, $\sigma_{st}=230\ N/mm^2$ $m=\dfrac{280}{3\sigma_{cbc}}=\dfrac{280}{3\times8.5}=10.98$
Properties of the balanced section	Properties of the balanced section	Properties of the balanced section	Properties of the balanced section
$n=n_c=0.4\ d$ $a=0.87\ d$ $M.R=1.213bd^2$ $A_{st}=1\%\ of\ bd$	$n=n_c=0.29d$ $a=0.90\ d$ $M.R=0.913bd^2$ $A_{st}=0.44\%\ of\ bd$	$n=n_c=0.4d$ $a=0.87\ d$ $M.R=1.479\ bd^2$ $A_{st}=1.21\%\ of\ bd$	$n=n_c=0.29d$ $a=0.90\ d$ $M.R=1.109\ bd^2$ $A_{st}=0.53\%\ of\ bd$
Properties of under reinforced section	Properties of under reinforced section	Properties of under reinforced section	Properties of under reinforced section
$A_{st}<1\%\ of\ bd$ $n<n_c$ Steel reaches its permissible stress earlier to concrete M.R. $<1.213\ bd^2$ $MR=A_{st}t\left(d-\frac{n}{3}\right)$ Where $t=\sigma_{st}=140\ N/mm^2$	$A_{st}<0.44\%\ of\ bd$ $n<n_c$ Steel reaches its permissible stress earlier to concrete M.R. $<0.913\ bd^2$ $MR=A_{st}t\left(d-\frac{n}{3}\right)$ Where $t=\sigma_{st}=230\ N/mm^2$	$A_{st}<1.21\%\ of\ bd$ $n<n_c$ Steel reaches its permissible stress earlier to concrete M.R. $<1.479\ bd^2$ $MR=A_{st}t\left(d-\frac{n}{3}\right)$ Where $t=\sigma_{st}=140\ N/mm^2$	$A_{st}<1.53\%\ of\ bd$ $n<n_c$ Steel reaches its permissible stress earlier to concrete M.R. $<1.109\ bd^2$ $MR=A_{st}t\left(d-\frac{n}{3}\right)$ Where $t=\sigma_{st}=230\ N/mm^2$
Properties of over reinforced section	Properties of over reinforced section	Properties of over reinforced section	Properties of over reinforced section
$A_{st}>1\%\ of\ bd$ $n<n_c$ M.R. $<1.213\ bd^2$	$A_{st}>0.44\%\ of\ bd$ $n<n_c$ M.R. $<0.913\ bd^2$	$A_{st}>1.21\ of\ bd$ $n<n_c$ M.R. $<1.479\ bd^2$	$A_{st}>0.53\%\ of\ bd$ $n<n_c$ M.R. $<1.109\ bd^2$

MODULE 9.

LESSON 17. Design of Singly Reinforced Section

17.1 INTRODUCTION

In singly-reinforced beams we come across the following types of problems:

Type A Data: Dimensions of the section, permissible stresses in concrete and steel, area of tensile steel and modular ratio.

Required: Moment of resistance of the section.

This type of problem may be solved as follows:

1. First determine the position of the actual neutral axis by equating the moment of the concrete area in compression about the neutral axis to the moment of equivalent tension area about the neutral axis i.e. use the relation,

    $\frac{bn^2}{2} = mA_{st}(d-n)$ and find n.

2. Find the position of critical neutral axis corresponding to the given safe stresses in concrete and steel.

3. Ascertain whether the section is under-reinforced or over-reinforced. If the actual neutral axis lies above the critical neutral axis, the section is under-reinforced. But, if the actual neutral axis is below the critical neutral axis, the section is over-reinforced

4. If the section is over-reinforced concrete attains its permissible stress earlier than steel, and the moment of resistance is given by

$$\text{M.R.} = bn\frac{c}{2}\left(d - \frac{n}{3}\right)$$

Taking,   c = σ_{cbc} = permissible stress in concrete

and   n = depth of actual neutral axis.

If the section is under-reinforced, steel attains its permissible stress earlier than concrete and the moment of resistance is given by

$$\text{M.R.} = A_{st}t\left(d - \frac{n}{3}\right)$$

taking,   t = = permissible stress in steel

and   n = depth of actual neutral axis.

Type B Data : Dimensions of the section, Area of reinforcement, Bending moment M and modular ratio.

Required : Stresses in concrete and steel.

This type of problem may be solved as follows:

1. Determine the position of the actual neutral axis.

2. Find the stress in concrete by equating the moment of resistance to the given bending moment i.e., use the relation,

$$bn\frac{c}{2}\left(d - \frac{n}{3}\right) = M \text{ and find } c.$$

3. Find the stress in steel from the relation.

$$\frac{mc}{t} = \frac{n}{d-n}$$

Type C Data : Permissible stresses in concrete and steel, Bending moment M and modular ratio.

Required: To design the section.

This type of problem may be solved as follows: The beam will be designed as a balanced section

1. Determine the depth of critical neutral axis in terms of the effective depth d.

i.e., use the relation, $\frac{m\sigma_{cbc}}{\sigma_{st}} = \frac{n_c}{d-n_c}$ and find n_c in terms of d.

2. Choose a convenient width b. By equating the moment of resistance to the given bending moment, find the effective depth

i.e, use the relation $bn_c \frac{\sigma_{cbc}}{2}\left(d - \frac{n_c}{3}\right) = M$ and find d.

3. Find the area of steel by equating the total compression on the beam section to the total tension on the beam section.

i.e. use the relation, $bn_c \frac{\sigma_{cbc}}{2} = A_{st}\sigma_{st}$ and find A_{st}

The following problems illustrate the above types of problems.

Example 17.1 A singly reinforced beam 250 mm wide and 380 mm deep to the centre of reinforced with 3 bars of 18 mm diameter. Determine the depth of neutral axis and the maximum stress in concrete when the stress in steel is 150 N/mm². Take m = 13.33.

Solution. $b = 250\ mm,\ d = 380\ mm,\ A_{st} = 3 \times \frac{\pi}{4}(18)^2 = 764.4\ mm^2$

Position of neutral axis (see Fig. 17.1)
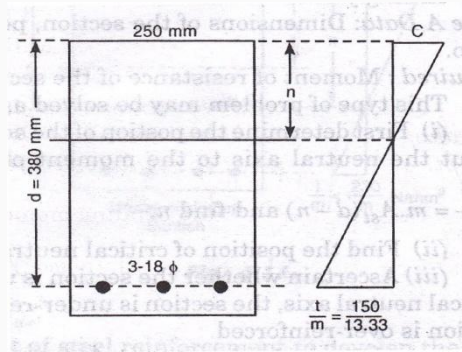
Taking moments about the neutral axis,

$$250\frac{n^2}{2} = 13.33 \times 763.4\ (380 - n)$$

$$n^2 + 81.409\,n - 30935.41 = 0$$

$$n = 139.83\ mm$$

$$\frac{mc}{t} = \frac{n}{d-n}$$

$$c = \frac{t}{m} \cdot \frac{n}{d-n}$$

When the stress in steel is 150 N/mm² the corresponding maximum compressive stress in concrete

$$c = \frac{150}{13.33} \cdot \frac{139.83}{(380 - 139.83)} = 6.55\ N/mm^2$$

Example 17.2 The cross-section of a singly-reinforced concrete beam is 300 mm wide and 400 mm deep to the centre of the reinforcement which consists of three bars of 12 mm diameters. If the stresses in concrete and steel are not to exceed 7 N/mm² and 230 N/mm², determine the moment of resistance of the section. Take m =13.33

Solution. Area of steel $A_{st} = 3 \times 113 = 339\ mm^2$

Position of actual neutral axis (see Fig. 17.2)
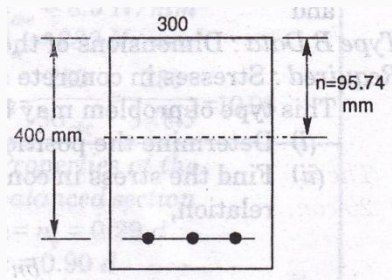
Taking moments about the neutral axis,

$$300\frac{n^2}{2} = 13.33 \times 339 \, (400 - n)$$

Therefore $\quad n^2 + 30.1258\,n - 12050.32 = 0$

Therefore $\qquad n = 95.74 \, mm$

Depth of critical neutral axis

$$n_c = 0.29\,d = 0.29 \times 400 = 116 \text{ mm}$$

Therefore $\quad n < n_c$

Hence the section is under-reinforced and steel will reach its permissible stress earlier to concrete.

$$\text{M.R.} = A_{st}\sigma_{cbc}\left(d - \frac{n}{3}\right) = 339 \times 230 \left(400 - \frac{95.74}{3}\right)$$

$$= 28.70 \times 10^6 \, Nmm = 28.70 \, kNm$$

Example 17.3 The cross-section of a singly-reinforced concrete beam is 300 mm wide and 400 mm deep to the centre of the reinforcement which consists of four bars of 16 mm diameter. If the stresses in concrete and steel are not exceed 7 N/mm² respectively, determine the moment of resistance of the section. Take m = 13.33.

Solution. $\qquad A_{st} = 4 \times 201 = 804 \, mm^2$

Position of actual neutral axis (see Fig. 17.3)
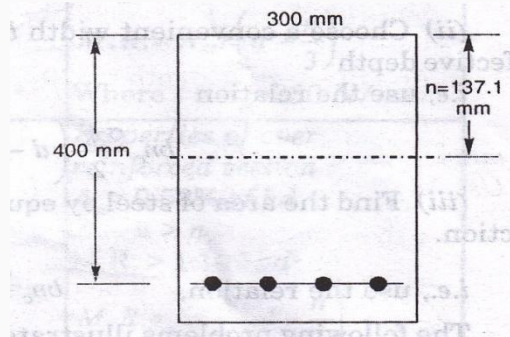
Taking moments about the neutral axis

$$\frac{300n^2}{2} = 13.33 \times 804 \, (400 - n)$$

$$n^2 + 71.449n - 28579.52 = 0$$

Therefore $\quad n = 137.1mm$

Depth of critical neutral axis

$$n_c = 0.4\,d = 0.4 \times 100 = 160 \text{ mm}$$

$$n < n_c$$

Therefore, the section is under-reinforced and steel will reach its permissible earlier to concrete.

$$\text{M.R.} = A_{st}\sigma_{cbc}\left(d - \frac{n}{3}\right) = 804 \times 140 \left(400 - \frac{137.1}{3}\right) Nmm$$

$$= 39.88 \times 10^6 \, Nmm = 39.88 \, kNm$$

Example 17.4 A singly-reinforced rectangular beam 350 mm wide has a span of 6.25 m and carries an all inclusive load of 16.30 kN/m. If the stresses in concrete and steel shall not exceed 7 N/mm² and 230 N/mm² find the effective depth and the area of the tensile reinforcement. Take m=13.33.

Solution. Maximum bending moment $= \dfrac{16.30 \times 6.25^2}{8} = 79.59 kNm$

Depth of critical Neutral axis $\dfrac{m\sigma_{cbc}}{\sigma_{st}} = \dfrac{n_c}{d-n_c}$, $\dfrac{13.33 \times 7}{230} = \dfrac{n_c}{d-n_c}$

Therefore $n_c = 0.2886d$

Therefore Lever arm, $a = d - \dfrac{n_c}{3} = d - \dfrac{0.2886\,d}{3} = 0.9038\,d$

Moment of resistance $= bn_c \dfrac{\sigma_{cbc}}{2}.a = b\,(0.2886d)\,3.50 \times 0.9038\,d = 0.9129bd^2$

Equating the M.R. to the B.M.,

$$0.9129bd^2 = 0.9129 \times 350d^2 = 79.59 \times 10^6$$

$$d = 499\ mm$$

Total compression = Total tension

$$bn_c \dfrac{\sigma_{cbc}}{2} = A_{st}\sigma_{st}$$

$$A_{st} = \dfrac{bn_c\sigma_{cbc}}{2\times\sigma_{st}} = \dfrac{350 \times (0.2886 \times 499)7}{2\times 230} = 767\ mm^2$$

Example 17.5 A singly reinforced beam has a span of 5 meters and carries a uniformly distributed load of 25 kN/m. The width of the beam is chosen to be 300 mm. Find the depth and the steel area requited for a balanced section. Use M 20 concrete and Fe415 steel

Solution Maximum bending moment M $= \dfrac{25 \times 5^2}{8} = 78.125$ kNm

The section is a balanced section (see Fig. 17.4).

300 mm

570 mm

2 – 18 mm d
1 – 16 mm d
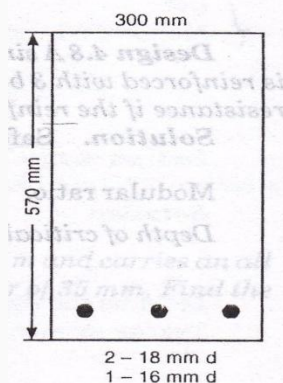
Equating balanced M.R. to bending moment

0.913 bd² = 0.913 x 300 d² = 78.123 x 10⁶

Therefore $\qquad\qquad$ d = 534 mm

Area of steel required = $A_{st} \dfrac{78.125 \times 10^6}{230 \times 0.90 \times 534}$ = 706.7 mm²

Provide 2 bars of 18 mmΦ and 1 bar 16 mmΦ

Area of steel provided = 2 (254) + 201 = 709 mm²

Overall depth of the beam = 534 + 9 + 25 = 568 mm

Let us provide an overall depth of 570 mm

Actual effective depth = 570-34 = 536 mm

Example 17.6 Design a singly reinforced beam section subjected to a maximum bending moment of 55.35 kNm. The width of the beam may be made two third the effective depth. Use M 20 concrete and Fe415 steel.

Solution: M = 55.35 kN/m, $b = \frac{2}{3}d$

The beam section will be designed as a balanced section. Balanced M.R. = 0.913 bd^2 = M

$$0.913 \times \frac{2}{3}d \cdot d^2 = 55.35 \times 10^6$$

$$d^3 = \frac{55.35 \times 10^6 \times 3}{0.913 \times 2}$$

$$d = 449.69 \ mm,$$

$$b = \frac{2}{3}(449.69) = 299.79 \ mm$$

$$A_{st} = 0.44\% \ bd = \frac{0.44}{100}(299.79)(449.69)$$

$$= 593.175 \ mm^2$$

Provide b = 300 mm, d = 450 mm

Provide 3 bars of 16 mm of (603 mm²)

Example 17.7 A singly-reinforced concrete beam is 300 mm wide and 450mm deep to the centre of the tensile reinforcement which consists of 4 bars of 16 mm diameter. If the safe stresses in concrete and steel are 7 N/mm² and 230 N/mm² respectively, find the moment of resistance of the section. Take m = 13.33.

Solution.

$$b = 300 \ mm, \ d = 450 \ mm$$

$$= 4 \times 201 = 804 \ mm^2$$

Depth of actual Neutral axis

$$\frac{m\sigma_{cbc}}{\sigma_{st}} = \frac{n_c}{d - n_c}, \qquad \frac{13.33 \times 7}{230} = \frac{n_c}{450 - n_c}$$

Therefore n_c = 129.9 mm

Depth of actual Neutral axis,

Taking moments about the neutral axis,

$$300\frac{n^2}{2} = 13.33 \times 804 (450 - n)$$

$$n^2 + 71.45\,n - 32151.96 = 0$$

$$n = 147.1 mm \qquad But\ n_c = 129.9\ mm$$

Since n>n$_c$ the section is over reinforced

Therefore Concrete attains its safe stress earlier to steel.

Moment of resistance = $bn_c\frac{\sigma_{cbc}}{2}\left(d - \frac{n}{3}\right)$ $= 300 \times 147.1 \times \frac{7}{2}\left(450 - \frac{147.1}{3}\right) Nmm$

$= 61.9313 \times 10^6\ Nmm = 61.9313\ kNm$

Example 17.8 A singly-reinforced concrete beam 350 mm wide and 550mm deep to the centre of the tensile reinforcement is reinforced with 3 bars of 18 mm diameter. Find the moment of resistance of the section. What would be the moment of resistance if the reinforcement is changed to 4 bars of 18 mm diameter. Use M 20 concrete and Fe 415 steel.

Solution. Safe stresses $\sigma_{cbc} = 7\ N/mm^2$, $\sigma_{st} = 230\ N/mm^2$

Modular ratio, $$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = \frac{40}{3}$$

Depth of critical Neutral axis

$$\frac{m\sigma_{cbc}}{\sigma_{st}} = \frac{n_c}{d - n_c}$$

$$\frac{40}{3} \times \frac{7}{230} = \frac{n_c}{550 - n_c} = \frac{28}{69}$$

$$69\,n_c = 28 \times 550 - 28\,n_c$$

$$n_c = \frac{28 \times 550}{97} = 158.8\ mm$$

Case (i) When 3 bars of 18 mm diameter are provided

$$A_{st} = 3 \times 254 = 762\ mm^2$$

Position of actual neutral axis

Taking moments about the neutral axis

Design of Structures

$$350 \frac{n^2}{2} = \frac{40}{3} \times 762 \, (550 - n)$$

$n^2 + 58.057 \, n - 31931.429 = 0$

$n = 152 \text{mm}$ But, $n_c = 158.8$ mm

Since $n < n_c$ the section is under-reinforced.

Therefore Steel reaches its safe stress earlier to concrete.

Moment of resistance = $A_{st} \, \sigma_{st} \left(d - \frac{n}{3} \right) = 762 \times 230 \left(550 - \frac{152}{2} \right) Nmm$

    = 87.513×10^6 Nmm = 87.513 kNm

Case(ii) When 4 bars of 18 mm diameter are provided

A_{st} = 4 x 254 = 1016 mm²

Taking moments about the neutral axis,

$$350 \frac{n^2}{2} = \frac{40}{3} \times 1016 \, (550 - n)$$

$n^2 + 77.409 n - 42575.24 = 0$

Therefore $n = 171.2$ mm But $n_c = 158.8$ mm

Since, $n > n_c$ the section is over reinforced

Therefore Concrete reaches its safe stress earlier to steel

Moment of resistance $= bn_c \frac{\sigma_{cbc}}{2} \left(d - \frac{n}{3} \right) = 350 \times 171.2 \times \frac{7}{2} \left(550 - \frac{171.2}{3} \right) Nmm$

    = 103.394×10^6 Nmm = 103.394 kNm

Example 17.9 A singly- reinforced concrete beam 300 mm wide has an effective depth of 500 mm, the effective span being 5 m. It is reinforced with 804 mm² of steel. If the beam carries a total load of 16 kN/m on the whole span, determine the stresses produced in concrete and steel. Take m = 13.33.

Solution. Maximum B.M. for the beam = $\frac{16 \times 5^2}{8}$ = 50 kNm

Position of neutral axis

Taking moments about the neutral axis,

Design of Structures

$$\frac{300\, n^2}{2} = 13.33 \times 804\,(500 - n)$$

$$n^2 + 71.4488n - 35724.4 = 0$$

Therefore $\qquad n = 156.63$ mm

Moment of resistance = Bending moment

$$300 \times 156.63 \, \frac{c}{2}\left(500 - \frac{156.63}{3}\right) = 50 \times 10^6$$

$$c = \frac{50 \times 10^6}{150 \times 156.63 \times 447.79} = 4.75 \text{ N/mm}^2$$

Stress in steel $= mc\,\frac{d-n}{n} = \frac{13.33 \times 4.75\,(500 - 156.63)}{156.63}$ N/mm^2

$$= 138.80 \text{ N/mm}^2$$

Example 17.10 A singly-reinforced beam 350mm wide and 550mm deep has an effective span of 6 m and carries an all inclusive load of 20 kN/m. The beam is reinforced with 4 bars of 20 mm diameter at an effective cover of 35 mm. Find the maximum stresses produced in concrete and steel. Take m = 13.33.

Solution. Area of steel $\qquad A_{st} = 4 \times 314 = 1256$ mm^2

Maximum B.M. $\qquad = \frac{20 \times 6^2}{8} = 90$ kNm $= 90 \times 10^6$ Nmm

Position of actual neutral axis

Effective depth $\qquad d = 550 - 35 = 515$ mm

Taking moments about the neutral axis,

$$350\,\frac{n^2}{2} = 13.33 \times 1256\,(515 - n)$$

$$n^2 + 95.67n - 49270.7 = 0$$

Solving, we get $\qquad n = 179.23$ mm

Let the maximum compressive stress reached in concrete be c N/mm^2

Equating M.R. to the B.M.

$$350 \times 179.23 \times \frac{c}{2}\left(515 - \frac{179.23}{3}\right) = 90 \times 10^0$$

$$c = 6.30 \text{ N/mm}^2$$

Stress in steel

$$t = mc\,\frac{d-n}{n} = \frac{13.33 \times 6.30\,(515 - 179.23)}{179.23} = 157.33\,N/mm^2$$

Example 17.11 Find the moment of resistance of a singly reinforced beam section 225 mm wide and 350 mm deep to the centre of the tensile reinforcement if the permissible stresses in concrete and steel are 230 N/mm² and 7N/mm². The reinforcement consists of 4 bars of 20 mm diameter. What maximum uniformly distributed load this beam can safely carry on a span of 8 m? Take m = 13.33

Solution

$$A_{st} = 4 \times \frac{\pi}{4}\,(20)^2 = 1256.6\,mm^2$$

Taking moments about the neutral axis (see Fig. 17.5),
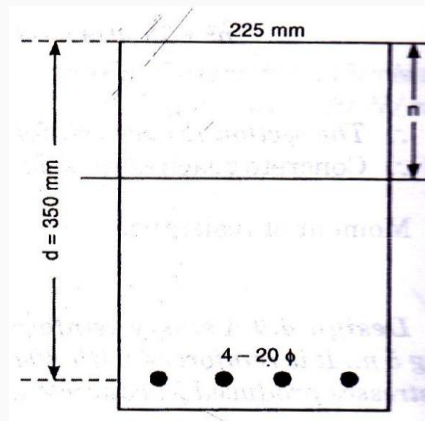
$$\frac{225\,n^2}{2} = 13.33 \times 1256.6(350 - n)$$

Therefore $n^2 + 148.893n - 52112.598 = 0$

Therefore $n = 165.67$

The depth of critical neutral axis is given by

$$\frac{13.33 \times 7}{230} = \frac{n_c}{350 - n_c}$$

Therefore $n_c = 101\,cm$

Since $n > n_c$, the beam section is over reinforced.

Therefore Concrete reaches its permissible stress earlier to steel.

Moment of resistance

$$= bn \frac{\sigma cbc}{2} \left[d - \frac{n}{2} \right]$$

$$= 225 \times 165.67 \times \frac{7}{2} \left(350 - \frac{165.67}{3} \right)$$

= 38458075 Nmm = 38.458 kNm

w = safe uniformly distributed load on the beam

Let maximum bending moment

$$= \frac{wl^2}{8} \quad = \frac{w \times 8^2}{8} = 38.458$$

$$w = 4.807 \; kN/m.$$

LESSON 18. Analysis of Doubly Reinforced Sections

18.1 INTRODUCTION

A beam or slab reinforced with main steel both in tension and compression zones is said to be doubly reinforced. Often due to headroom considerations, architectural or some other such reasons, it is necessary to restrict the dimensions of a beam. The resisting moment of the beam with the limited dimensions, as worked out by the formula, MR=Rbd², may be less than the bending moment the beam may be required to resist. In order that the beam may be safe, it is necessary to reinforce the beam in such a way that it is capable of developing the moment of resistance equal to the external bending moment.

The moment of resistance of the beam can be increased to a certain extent by gradually increasing the area of tensile reinforcement till the permissible stress in concrete is reached. This increase in resisting moment may not be sufficient to serve the purpose and hence the tensile steel will be necessary to be increased further. The further increase in tensile steel will cause a further increase in the compressive stress in concrete, which is not desirable. Hence in order to prevent the compressive stress in concrete from exceeding its safe permissible value, steel must be introduced in the compression zone to take up extra compressive stress. Thus the beam gets doubly reinforced.

A doubly reinforced section is generally provided under the following conditions;

1. When the depth and breadth of the beam are restricted and it has to resist greater bending moment than a singly reinforced beam of that section would do.

2. When the beam is continuous over several supports, the section of the beam at the supports is usually designed as a doubly reinforced section.

3. When the member is subjected to eccentric loading.

4. When the bending moment in the member reverses according to the loading conditions e.g., the wall of an underground R.C.C. storage reservoir, brackets etc.

5. When the member is subjected to shocks, impact or accidental lateral thrust.

18.2 Modular ratio for compression steel

A section reinforced with steel in compression and in tension zone is said to be doubly reinforced. The steel reinforcement provided in the compression zone is thus subjected to compressive stress. We know that when subjected to continuous compressive stress, concrete undergoes creep or plastic deformation ($\delta_{plastic}$) in addition to elastic deformation ($\delta_{elastic}$). In such a situation the value of the modulus of elasticity of concrete (E_c) is given by

$$E_c = \frac{\sigma_{cbc}}{\delta_{elastic} + \delta_{plastic}}$$

which obviously works out to be smaller than

$$E_c = \frac{\sigma_{cbc}}{\delta_{elastic}}$$

Thus the modulus of elasticity of concrete in compression works out to be smaller than that in tension. This calls for use of modified modular ratio for compression zone.

The code accordingly stipulates that modified value of modular ratio (m_c) to be used for compression steel shall be = 1.5 m.

Hence compression stress in compression shall be calculated by multiplying the stress in the surrounding concrete by 1.5.

However, the compressive stress in compression steel thus calculated shall not exceed the permissible value of as given in Table 15.10.

It is observed that although adoption of m_c = 1.5 m has very little effect on the moment of resistance of the section but it leads to higher value of stress in compression steel which results in considerable economy.

18.3 EQUIVALENT AREA OF STEEL IN COMPRESSION

In view of the above, the expression for equivalent area of steel in compression works out to be

$$= m_c A_{sc} - A_{sc}$$
$$= (m_c - 1)A_{sc}$$
$$= (1.5m - 1)A_{sc}$$

18.4. LOCATION OF NEUTRAL AXIS

The location of neutral axis of a doubly reinforced beam can be determined by the following methods.

Method I. This method of finding the neutral axis is adopted when the stresses in concrete and steel are given. Since the stress diagram of a doubly-reinforced beam is identical to that of a singly-reinforced beam, we get

$$\frac{n}{d} = \frac{m \cdot \sigma_{cbc}}{m \cdot \sigma_{cbc} + \sigma_{st}}$$

The value of neutral axis thus obtained is called critical neutral axis. Hence for given stresses, the depth of critical neutral axis (n_c) for doubly-reinforced beam is same as that for a singly reinforced beam.

Method II. This method is based on the assumption that neutral axis of the homogeneous section, always passes through the C.G. of the section. Hence the moment of the transformed areas above and below the neutral axis (N.A.) of the R.C beam (moments being taken about the neutral axis) must be equal.

Therefore, equating the moment of compressive areas about N.A. to the moment of tensile area about N.A., we have

$$\frac{bn^2}{2} + (m_c - 1)A_{sc}(n - d_c) = m \cdot A_{st}(d - n)$$

where = distance of the C.G. of compression steel from the extreme compression edge of the beam.

If the area of tensile and compressive steel and the dimensions of the section are given, methods II should be adopted for finding out the neutral axis of the section.

18.5 MOMENT OF RESISTNCE

The moment of resistance (of a doubly-reinforced beam can be determined by taking moments of compressive forces in concrete and compression steel about the C.G. of the tensile reinforcement

We are given

$$M_r = b.n.\frac{c}{2}\left(d - \frac{n}{3}\right) + (m_c - 1)A_{sc}.c'(d - d_c)$$
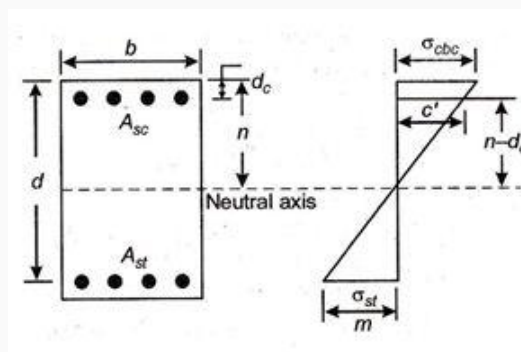
Referring to the stress diagram Fig. 18.1, we find that

$$\frac{c'}{c} = \frac{n - d_c}{n}$$

or

Design of Structures

$$c' = \frac{(n - d_c)}{n} c$$

From the review of the expression for the moment of resistance, it is observed that the moment of resistance of a doubly reinforced beam consists of two components:

(i) $b.n.\frac{c}{2}\left(d - \frac{n}{3}\right)$ i.e., the moment of the compressive force in concrete about the C.G. of the tensile steel. This component thus represents the moment of resistance of a singly-reinforced beam without any compression reinforcement. Let us denote it by M_1.

(ii) $(m_c - 1)A_{sc}.c'(d - d_c)$ i.e., the moment of compressive force in steel is compression about the C.G. of the tensile steel. This component thus represents the moment which the beam can resist in excess of M_1. Let us denote it by M_2.

$$M_r = M_1 + M_2$$

Let

$A_{st1} =$ the area of tensile steel required for the balanced section corresponding to the moment M_1. And

and $A_{st2} =$ the area of additional tensile steel required to develop the moment M_2.

Thus

$$A_{st1} = \frac{M_1}{\sigma_{st} \times Leverarm} = \frac{M_1}{j_1 d \sigma_{st}}$$

and

$$A_{st2} = \frac{M_1}{\sigma_{st} \times Leverarm} = \frac{M_2}{\sigma_{st}\left(d - d_c\right)}$$

It may be noted that the additional tensile steel is actually required to balance the steel in compression. Hence can also be calculated alternatively by taking moment of the equivalent concrete area of compression steel and the moment of the equivalent concrete area of additional tensile steel (about the neutral axis

$$m.A_{st2}\left(d - n\right) = \left(m_c - 1\right)A_{sc}\left(n - d_c\right)$$

or $$A_{st2} = \frac{\left(1.5m - 1\right)A_{sc}\left(n - d_c\right)}{m\left(d - n\right)}$$

Hence total tensile steel required for the section $$A_{st} = A_{st1} + A_{st2}$$

In the expression for moment of resistance the value of (c) stress in concrete to be adopted will depend upon the position of neutral axis of the section. In case of a balanced section, the value of c will be $=\sigma_{cbc}$. The value of c will work out to be less than the permissible stress in concrete σ_{cbc} in the following two cases.

Case (a) In this case the area of the tensile steel provided may be insufficient to balance even the compressive stress in concrete. Thus steel attains its maximum permissible tensile stress first and the concrete will not be subjected to full value of permissible compressive stress. The neutral axis in such a case will be above the critical neutral axis n_c.

Case (b) In this case, the area of compression steel provided may be more than that required to balance the additional tensile reinforcement ($A_{st}2$). Hence the stress in tensile steel will attain its maximum permissible tensile stress first and the neutral axis will lie above the critical neutral axis (n_c).

Hence in all such cases where the actual N.A. lies about the critical axis (i.e. n_c), the concrete will not be subjected to its full permissible stress and the value (c) should be worked out form the following relation

$$\frac{c}{t/m}=\frac{n}{d-n}$$

or

$$c=\frac{t}{m}\frac{n}{\left(d-n \right)}=\frac{\sigma_{st}}{m}.\frac{n}{\left(d-n \right)}$$

18.6 STEEL BEAM THEORY

In the design of doubly reinforced beam by steel beam theory, equal area of tensile and compression steel is provided and the total compressive force is assumed to be resisted by the compression steel. Thus in this method, the concrete is altogether neglected both in compression as well as in tension zone. The area of compression at top and tension steel at the bottom of the beam act like the top and bottom flanges of an imaginary rolled steel joist (R.S.J) the concrete between them acting as an imaginary web of the R.S.J. Thus the lever arm in this case becomes equal to the centre to centre distance between the compression and tension steel.

Hence the moment of resistance of the section in the case is given

$$M_r=\sigma_{st}A_{st}\left(d-d_c \right)$$
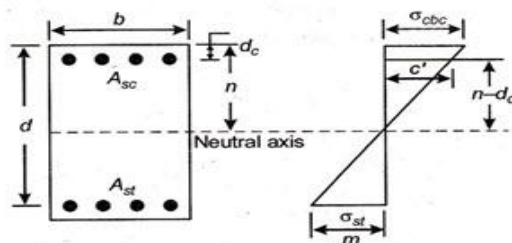
MODULE 10.

LESSON 19. Design of Doubly Reinforced Sections

19.1 INTRODUCTION

Three main types of problem may be put in case of design of doubly reinforced section.

Type I. In this type following information is given in the question itself.

1. The dimensions of the section.

2. The area of reinforcement in tension and compression.

3. The maximum permissible stresses in concrete and steel.

It is required to determine the moment of resistance of the section.

Procedure to solve: The solution of this type of problem involves the steps given below:

1. Find the position of the actual neutral axis of the section by equating the moment of the areas of the concrete and equivalent area of compression steel to the moment of the equivalent concrete area of steel in tension about the neutral axis. This is given by the equation

$$\frac{bn^2}{2} + (m_c - 1)A_{sc}(n - d_c) = m \cdot A_{st}(d - n)$$

2. Find the position of the critical neutral axis (n_c) by the equation

$$\frac{n_c}{d} = \frac{m \cdot \sigma_{cbc}}{m \cdot \sigma_{cbc} + \sigma_{st}}$$

3. If the actual neutral axis lies above the critical neutral axis, the stress in tensile steel attains its maximum permissible value (i.e. t =) first and the corresponding value of stress in concrete at top (c) is given by

$$c = \frac{\sigma_{st}}{m} \cdot \frac{n}{(d - n)}$$

and stress in concrete surrounding steel in compression is given by

$$c' = \frac{(n - d_c)}{n} c$$

Having known the values of c and c', the moment of resistance of the section can be obtained by taking the moments of all the forces about the tensile steel. This is given by the equation.

$$M_r = b.n.\frac{c}{2}\left(d - \frac{n}{3}\right) + (m_c - 1)A_{sc}.c'(d - d_c)$$

It may be noted that the value of (c) in the expression is different from permissible compression stress in concrete i.e.,.

(iv) If the actual neutral axis lies below the critical neutral axis or coincides with it, the stress in concrete attains its maximum permissible value first and hence the moment of resistance of the section is obtained by the equation.

$$M_r = b.n.\frac{\sigma_{cbc}}{2}\left(d - \frac{n}{3}\right) + (1.5m - 1)A_{sc}.c'(d - d_c)$$

Type II. In this type following information is given:

1. The dimensions of the section.

2. Area of reinforcement in tension and compression.

3. The modular ratio and the maximum bending moment to which the section is subjected to.

It is required to find out the stresses developed in concrete and steel.

Procedure to solve: The solution of this type of problem involves the steps given below.

1.Find the position of N.A. by the equation

$$\frac{bn^2}{2} + (m_c - 1)A_{sc}(d - d_c) = m \cdot A_{st}(d - n)$$

...(3.2)

2.Find stress in concrete (c') surrounding compression steel by the equation

$$c' = \frac{c}{n}(n - d_c)$$

...(3.4)

3.To find c, equate the moment of resistance of the doubly reinforced section to the external bending moment (M). This is given by

$$b.n.\frac{c}{2}\left(d - \frac{n}{3}\right) + (m_c - 1)A_{sc}.c'(d - d_c) = M$$

In this equation except c everything is known and hence c can be worked out.

4.Find the stress in steel by adopting the value of c as obtained in step (iii) in the formula.

$$t = cm\left(d - \frac{n}{n}\right)$$

Type III. In this type of problem, the dimensions of the section, the maximum permissible stress in concrete and steel, and the bending moment to which the section is subjected to are given and it is required to design the section.

Procedure to solve: The solution of this type of problem is given in -----------

Example 19.1 Find the moment of resistance of a beam 250 mm x 500 mm in section if it is reinforced with 2 bars of 16mm dia, at top and 4 bars of 22 mm dia. At the bottom each at an effective cover of 38 mm. Safe stresses in the materials are:

$$\sigma_{cbc} = 5 \ N/mm^2$$

$$\sigma_{st} = 140 \ N/mm^2$$

$$m = 19$$

Solution Equating the moments of the area of concrete and equivalent concrete area of compression steel to the moment of the equivalent concrete area of steel in tension about the neutral axis, we get

$$\frac{b.n^2}{2} + (m_c - 1)A_{sc}(n - d_c) = mA_{st}(d - n)$$

From the given data, we have

$$b = 250$$
$$d = 500 - 38 = 462 \ mm$$
$$A_{sc} = 2 \ x \ \frac{\pi}{4} \ x(16)^2 = 402 \ mm^2$$
$$A_{sc} = 4 \ x \ \frac{\pi}{4} \ x(22)^2 = 1521 \ mm^2$$

Subsututing the values in the above equation, we have

$$\frac{250}{5}n^2 + (1.5 \times 19 - 1) \times 402 \times (n - 38) = 19 \times 1521 \ (462 - n)$$

or
$$n^2 + 319.63 - 110170 = 0$$

which gives   n=208.58mm

The value of critical neutral axis can be obtained by the expression

$$\frac{n_c}{d} = \frac{m \cdot \sigma_{cbc}}{m \cdot \sigma_{cbc} + \sigma_{st}} = \frac{19 \ x \ 5}{19x5 + 141} = 0.404$$

$$n_c = 0.404 \times 462 = 186.65 \ mm$$

Since the actual neutral axis lies below the critical N.A., the stress in concrete will reach its maximum permissible value of $\sigma_{cbc} = 5N/mm^2$ first. first.

Hence the stress in concrete surrounding compression steel is given

$$c' = \frac{\sigma_{cbc}}{n}(n - d_c) = \frac{5}{208.58}(208.58 - 38)$$

$$= 4.09 \ N/mm^2$$

The moment of resistance of the beam is given by

$$M_r = \frac{b.n.c}{2}\left(d - \frac{n}{3}\right) + (1.5m - 1)A_{sc}.c'(d - d_c)$$

$$= \frac{250 \times 208.58 \times 5}{2}\left(\frac{462 - 208.58}{3}\right) + (15 \times 19 - 1)402 \times 4.09 \ (462 - 38)$$

$$= 70.33 \ x \ 10^6 \ N.mm = 70.33 \ kNm$$

Example 19.2 A reinforced concrete beam is b mm wide and d mm deep upto the centre of the tensile reinforcement. The beam is doubly reinforced. The tensile reinforcement and the compressive reinforcement are each equal to 1.5%. The compressive steel is placed at an effective cover of 0.1 d from the top face of the beam. Calculate the moment of resistance of the beam. The following data being given:

$$\sigma_{cbc} = 4.09 \ N/mm^2$$
$$\sigma_{st} = 140 \ N/mm^2$$
$$m = 19$$

Solution From the given data:

$$A_{sc} = \frac{1.5}{100}bd$$
$$A_{st} = \frac{1.5}{100}bd$$
$$d_c = 0.1d$$

Equating the moment of equivalent areas about the neutral axis, we get

$$\frac{bn^2}{2} + (m_c - 1)A_{sc}(n - d_c) = mA_{st}(d - n)$$

Or $\qquad \frac{bn^2}{2} + (1.5 \ m - 1)\frac{1.5}{100}bd \ (n - 0.1d) = 19 \times \frac{1.5}{100}bd(d - n)$

Or $\qquad n^2 + 1.395 \ nd - 0.652 \ d^2 = 0$

which gives $\qquad\qquad\qquad n = 0.37 \ d$

Critical neutral axis n_c is given by

$$\frac{n_c}{d} = \frac{m \cdot \sigma_{cbc}}{m \cdot \sigma_{cbc} + \sigma_{st}} = \frac{19 \times 5}{19 \times 5 + 140}$$

or $\qquad\qquad\qquad n_c = 0.404d$

The actual neutral axis lies above critical neutral axis hence the stress in steel reaches its maximum permissible value of 140 N/mm² first.

Hence the stress in concrete and stress in concrete surrounding compression steel shall calculated as under

$$\frac{c}{t/m} = \frac{n}{d-n}$$

or
$$C = \frac{t}{m}\cdot\frac{n}{(d-n)} = \frac{140 \times 0.37d}{19\,(d-0.37)} = 4.33 \; N/mm^2$$

Similarly
$$\frac{c\prime}{t/m} = \frac{(n-d_c)}{d-n}$$

or
$$c' = \frac{t}{m}\cdot\frac{(n-d_c)}{d-n} = \frac{140}{19} \times \frac{(0.37\,d-0.1d)}{(d-0.37d)} = 3.16 \; N/mm^2$$

The moment of resistance of the section is given by

$$M_r = b.n.\frac{c}{2}\left(d-\frac{n}{3}\right) + (m_c-1)A_{sc}.c'(d-d_c)$$
$$= b \times 0.37d \times \frac{4.33}{2}\left(d-\frac{0.37d}{3}\right) + (1.5 \times 19-1) \times \frac{1.5bd}{100} \times 3.16\,(d-0.1d)$$
$$= 0.702\,bd^2 + 1.173\,bd^2 = 1.875\,bd^2 \; Nmm$$

Example 19.3. A doubly reinforced concrete beam is 250 mm wide and 510 mm deep to the centre of tensile steel reinforcement. The compression reinforcement consists of 4 Nos. 18 mm dia. bars placed at an effective cover of 40 mm from the compression edge of the beam. The tensile reinforcement consist of 4 Nos. 20 mm dia. bars. If the beam section is subjected to a bending moment of 85 kNm, calculate the stresses in concrete and tension and compression steel. Adopt m = 11.

Solution: Area of tensile reinforcement

$$A_{sc} = 4 \; x \; \frac{\pi}{4} \; x(22)^2 = 1247 mm^2$$

Area of compression steel

$$A_{st} = 4 \; x \; \frac{\pi}{4} \; x(18)^2 = 1018\,mm^2$$

Equating the moments of the equivalent areas about N.A., we get

$$\frac{bn^2}{2} + (m_c-1)A_{sc}(n-d_c) = mA_{st}(d-n)$$

or
$$\frac{250n^2}{2} + (1.5 \times 11-1)1018(n-40) = 11 \times 1257(510-n)$$

or
$$n^2 + 236.84\,n - 61463 = 0$$

or

which gives   n=156.33 mm.

Let the maximum stress developed in concrete be c. The stress in concrete surrounding compression steel or c' is given by

$$\frac{c'}{c} = \frac{n - d_c}{n}$$

$$c' = n\left(\frac{156.33 - 40}{156.33}\right) = 0.74c$$

Equating the moment of resistance of the beam to the external bending moment, we get

$$M = b.n.\frac{c}{2}\left(d - \frac{n}{3}\right) + (m_c - 1)A_{sc}.c'(d - d_c)$$

$$85 \times 10^6 = \frac{250 \times 156.33 \times c}{2}\left(510 - \frac{156.33}{3}\right)(1.5 \times 11 - 1) \times 1018 \times 0.74c\,(510 - 40)$$

=14.44 x 10⁶ c

$$C = \frac{85 \times 10^6}{14.44 \times 10^6} = 5.89\ N/mm^2$$
or

and
$$c' = 0.74c = 0.74 \times 5.89 = 4.36\ N/mm^2$$
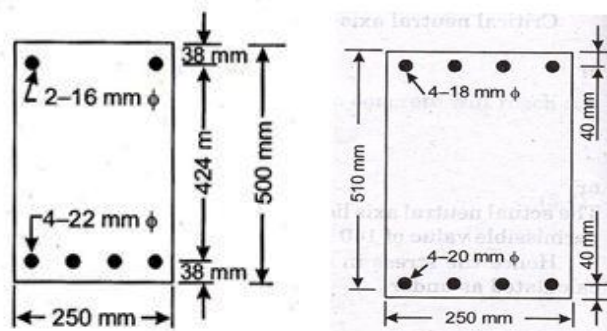
Stress in compression steel $= 1.5\,mc'$

$$= 1.5 \times 11 \times 4.36 = 71.94\ N/mm^2$$

Stress in tensile steel is given by

$$t = \frac{mc\,(d - n)}{n}$$

$$t = \frac{11 \times 5.89(510 - 156.33)}{156.33} = 146.58\ N/mm^2$$

LESSON 20. Theory of T-Beams

20.1 INTRODUCTION

In this type of beam, the R.C.C. floor or roof slab is case monolithic with the beam as shown in Fig. 20.1. The stirrups and the bent up bars of the beam extend into the slab and a portion of the slab acts with the beam for resisting compressive stresses. This results in increasing the moment of resistance of the beam. The slab cast integrally with the beam is called flange of the beam and the part of the beam projecting below the slab or flange is known as rib or web of the beam.

In case of simply a supported beam the bending moment is of sagging nature throughout its length. Hence the slab forming the flange of the T-beam is subjected to compression all along the span and the beam behaves as a T-beam throughout the span. On the other hand in case of a continuous beam the bending moment is of sagging nature at mid-span and it is of hogging nature at the supports as shown in Fig. 20.2. In the span portion the beam top remains under compression between the points of zero bending moment and hence the contribution of flange remains effective within the mid-length (of the beam upto the points of zero bending moment. This length can be assumed to be equal to 0.7 times the effective span and the beam thus behaves as T-beam only for this length. Beyond the points of zero BM and over the supports, the flange of the beam is subjected to tension.

Since the tensile resistance of concrete is totally ignored in our basic assumptions, the availability of slab as flange of the beam in tension zone of the beam is of no use. Hence the T-beam behaves as a rectangular beam over support (refer Fig. 20.3). Since the depth of the beam is fixed by taking advantage of the compressive resistance of concrete in flange of the T-beam, it is obviously less than the depth required for a rectangular beam. Thus the section of the beam at the supports is restricted in its dimensions and is usually designed as doubly reinforced section.

The slab forming the flange of the T-beams may be spanning either transverse to the beam or it may be spanning parallel to it. However, when the slab spans parallel to the beam (i.e. the main reinforcement of the slab is parallel to the beam) adequate reinforcement should be provided transverse to the beam throughout its length upto a distance of on either side of the rib of the beam near the top face of the flange as shown in Fig. 20.4.

As per IS: 456 – 1978, the area or such reinforcement should not be less than 60 per cent of the area of main reinforcement provided at mid-span of the slab.

20.2 DIMENSIONS OF A T-BEAM

Figure 20.5 shows the important dimensions of a T-beam which are as under :

Design of Structures

(1) Thickness of the flange (d_f). This is equal to the overall depth of the slab forming the flange of the T-beam.

(2) Breadth of web (b_w). This is the breadth of the beam projecting below the slab. The breadth of web should be sufficient to accommodate the tensile reinforcement in the beam with suitable spacing between the bars.

(3) Overall depth of beam (D). The overall depth of the beam depends upon the span as well as loading conditions. In case of simply supported beams it may be assumed to be 1/12 to 1/15 of the span. In case of continuous beam, the assumed overall depth may be taken as 1/15 to 1/20 of span for light loads; 1/12 to 1/15 of span for medium loads and 1/10 to 1/12 of span for heavy loads.

(4) Effective width of flange (b_f). It is obvious that the portion of slab (acting as flange) away from the beam web is stressed lesser than the portion immediately above the web. In order to simplify calculations certain width of flange (which normally works out to be less than actual width) is considered to be under uniform stress and hence effective for resisting compression in the beam. This width is termed as effective width of flange. The effective width of flange mainly depends upon the span, breadth of web and the thickness of slab acting as flange.

The code IS : 456-1978 stipulates that the effective width of flange may be taken as under but in no case the effective width of flange shall be greater than the breadth of web plus half the sum of the clear distances to the adjacent beams on either side.

Effective width of flange:

(a) For T-beams

$$b_f=\frac{{l_o}}{6}+{b_w}+6{d_f}$$

(b) For L-beams

$$b_f = \frac{{l_o}}{12} + {b_w} + 3{d_f}$$

(c) For isolated beams, the effective flange width shall be obtained as below but in no case greater than the actual width.

(i) In case of T-beam,

$$b_f = \frac{{l_o}}{12} + {b_w} + 3{d_f}$$

(ii) In case of L-beams (refer Fig. 20.6),

$$b_f=\frac{0.5{l_o}}{\left(\frac{{l_o}}{b}\right)+4}+{b_w}$$

where

effective width of flange

= distance between the points of zero moments in the beam

= breadth of web

= thickness of flange

= actual width of the flange. and

Note. For continuous beams and frames may be assumed as 0.7 times the effective span.

20.3. LOCATION OF NEUTRAL AXIS

The depth of the neutral axis is determined by equating the moment of area of concrete in compression to the moment of equivalent area of steel in tension. Depending upon the thickness of the flange and the bending moment applied to the beam, the neutral axis may lie (i) within the flange or (ii) it may lie outside the flange. The two cases are treated separately.

Case I. Neutral axis within the flange: In this case N.A. may be located exactly in the same manner as in case of singly reinforced sections (refer Fig. 20.7). N.A. in this case is worked out by the expression.

$$(b_f n^2)/2= m.A_s t(d-n)$$

Case II. Neutral axis below the flange (refer Fig. 20.8): If the value of n as obtained from the above equation works out to be more than the thickness of flange it will have to be re-worked out by equating the moments of the compressive and tensile areas about N.A. by the equation

$$b_f d_f (n-d_f /2)+b_w(n - d_f)^2 /2 = m.A_s t(d - n)$$

In this expression the term $b_w\frac{{{{\left({n-{d_f}}\right)}^2}}}{2}$ is normally very small and it is considered desirable to ignore it.

Thus the simplified form of equation becomes $b_f{d_f}\left({n - \frac{{{d_f}}}{2}} \right) = m.{A_{st}}(d - n)$

If however, the maximum stresses in concrete and steel are given, the neutral axis is worked out by use of the relation:

$$\left({\frac{n}{{d-n}}}\right)=\frac{{{\sigma _{cbc}}}}{{\frac{{{\sigma _{st}}}}{m}}}=\frac{{m.{\sigma _{cbc}}}}{{{\sigma _{st}}}}$$

20.4. LEVER ARM OF T-BEAM

If c and c_s be the compressive stress in the top and bottom edges of the flange respectively and be the distance of C.G. of the total compression below the top edge, then from the trapezoidal stress diagram shown in Fig. 20.8, we have

$$\bar y=\left({\frac{{c+2{c_s}}}{{c+{c_s}}}}\right)\frac{{{d_f}}}{3}$$

where $c_s=\frac{{c\left({n-{d_f}}\right)}}{n}=\frac{{{\sigma _{cbc}}.\left({n - {d_f}}\right)}}{n}$

Lever arm of the T-beam $$a=d-\bar y$$

20.5 MOMENT OF RESISTANCE

The moment of resistance of a T-beam is determined by taking the moment of the total compression in the beam about the C.G. of the tensile reinforcement.

The moment of resistance (M_r) = Total compression x lever arm.

(i) If the neutral axis lies in the flange
$$M_r = b_f . n . \frac{c}{2}\left(d - \frac{n}{3}\right)$$

(ii) If the neutral axis lies in the web
$$M_r = b_f . d_f . \frac{c+c_s}{2} + a$$

20.6 MOMENT OF RESISTANCE TAKING COMPRESSION IN WEB INTO ACCOUNT

The moment of resistance of a T-beam shall normally be worked out by use of above equations. If, however, it is specifically desired to consider compression in web as well, the moment of resistance in that case will comprise of the following.

(i) Moment of resistance due to compression in flange. Let it be denoted by M_1 and

(ii) Moment of resistance due to compressive force in web. Let it be denoted by M_2

$$M_r = M_1 + M_2$$

Now, $$M_1 = \rm{Total\,compression\,in\,flange} \times \rm{lever\,arm}$$

$$= b_f . d_f \left(\frac{c+c_s}{2}\right)(d-\bar y)$$

and $$M_2 = \rm{Total\,compression\,in\,web} \times \rm{lever\,arm}$$

$$= b_w \left(n - d_f \right) x \frac{C_s}{2}\left[d - \left(d_f + \frac{n - d_f}{3} \right) \right]$$

$$M_r = b_f . d_f \left(\frac{c +c_s}{2}\right)\left(d-\bar y\right)+b_w\left(n-d_f\right)\times \frac{C_s}{2}\left[d-\left(d_f+\frac{n-d_f}{3}\right)\right]$$

20.7 ECONOMICAL DEPTH OF A T-BEAM

To solve an expression for economical depth of T-beam proceed as under:

Let

$$R_{conc}$$ =cost of concrete per cubic centimeter

$$R_{steel}$$ = cost of steel per cubic centimeter

and r = ratio of the cost of steel to cost of concrete

Design of Structures

$$r=\frac{{R_{steel}}}{{R_{conc}}}$$

The volume of concrete in the flange remains fixed and is not considered in the calculation.

Let

d_t = the cover measured below the centre of the tensile reinforcement in the T-beam

d = effective depth of the beam

d_f = overall depth of the flange

b_w = breadth of the flange

and A_{st} = Area of tensile reinforcement

Consider 1 cm length of the T-beam web.

Volume of concrete per cm length of web $= b_w(d-d_f+d_t)$

Cost of concrete in the web per cm length $= b_w(d-d_f+d_t)R_{conc}$

Similarly cost of steel per cm length of web $= A_{st} \times R_{steel}$

$$= r.R_{conc}.A_{st}$$ $$\left[{\frac{{R_{steel}}}{{R_{conc}}} = r} \right]$$

Since $$A_{st}=\frac{M}{{j.d.\sigma _{st}}}$$

Cost of steel per cm length $$= r.R_{conc} \times \frac{M}{{j.d.\sigma _{st}}}$$

Let R be the total cost per cm length of the beam web

$$R =R_{conc}.b_w\left({d-d_f+d_t}\right)+R_{conc}\frac{M}{{j.d.\sigma _{st}}}$$

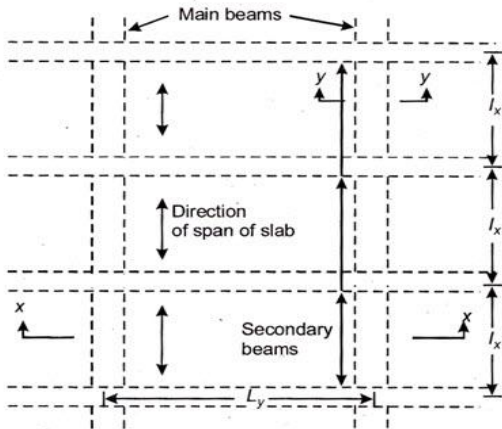$$=R_{conc}.\left[{b_w\left({d-d_f+d_t}\right)+\frac{M}{{j.d.\sigma _{st}}}}\right]$$
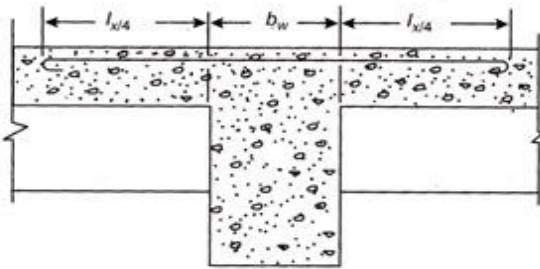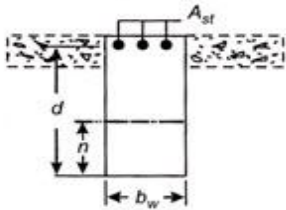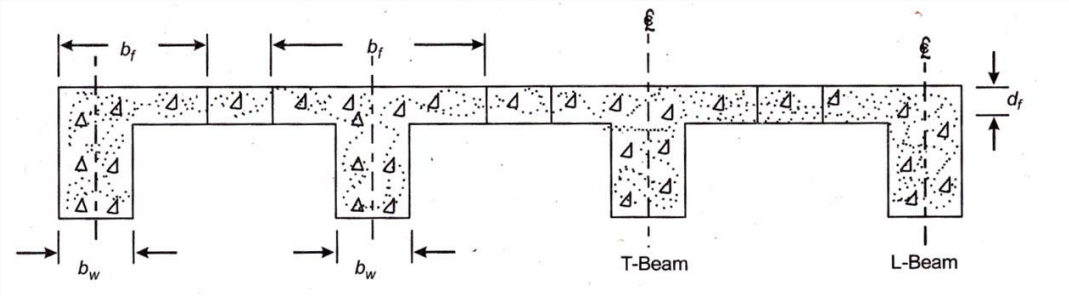
In this expression, b_w, d_f and are fixed and *d* is the only variable. Then for the cost to be minimum $\frac{{d(R)}}{{d(d)}}$ should be equal to 0

$$\frac{{d(R)}}{{d(d)}}=0=R_{conc}\left[{b_w-\frac{{rM}}{{j.d^{2}.\sigma _{st}}}}\right]$$

or $$R_{conc}\left[{b_w-\frac{{rM}}{{j.d^{2}.\sigma _{st}}}}\right]=0$$
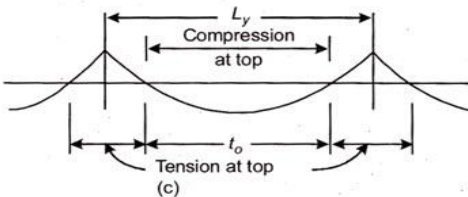
or $$d^2 =\frac{{r.M}}{{j.\sigma _{st}.b_{w.}}}$$

$$or\ d=\sqrt{\frac{r.M}{j.\sigma_{st}.b_{w}}}$$

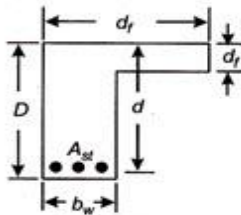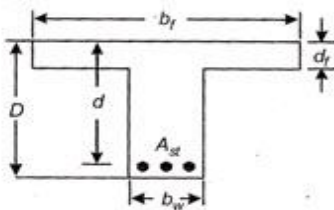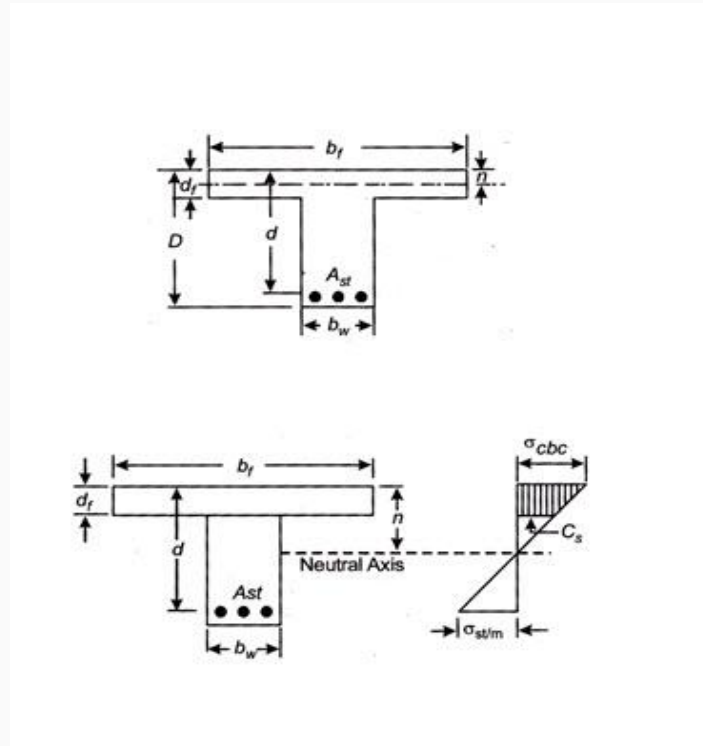

(a) Plan
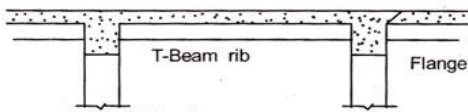
(b) Section x–x

(c)

MODULE 11.

LESSON 21. Design of T- Beams

21.1 INTRODUCTION

Three main types of problem may be put in case of T-beams.

Type I. In this type the following information is given:

(i) The dimension of the section

(ii) Area of steel reinforcement

(iii) The maximum permissible stresses in concrete and steel.

It is required to determine the moment of resistance of the section.

Procedure to solve: The solution of this type of problem involves the steps given below.

(i) Calculate the position of the actual neutral axis by equating the moment of the compressive area of concrete to the moment of equivalent concrete area of steel in tension about the N.A. Assuming that the neutral axis lies within the flange

$$\left[\frac{b_f.n^2}{2}\right]= m.A_{st}\left(d-n\right)$$

If n from this expression works out to be more than the thickness of the flange (d_f, the N.A. will be in the web and it will be necessary to re-work it out by use of expression.

$$b_f.d_f\left(n-\frac{d_f}{2}\right) = m.A_{st}\left(d-n\right)$$

(ii) Find the position of critical neutral axis (n_c) given by

$$\frac{n_{st}}{d}=\frac{m.\sigma_{cbc}}{m.\sigma_{cbc}+\sigma_{st}}$$

(iii) If the actual neutral axis lies above the critical N.A., the section is under-reinforced and steel attains its maximum permissible value of t=σ_{st} first. The corresponding stress in concrete is calculated from the relation.

$$c=\frac{1}{m}.\frac{n}{\left(d - n\right)}=\frac{\sigma_{st}}{m}.\frac{n}{(d - n)}$$

(i) If the neutral axis lies in the flange the moment of resistance of the section is given by

$$M_r = b_f.n.\frac{c}{2}\left(d -\frac{n}{3}\right)$$

If the N.A. lies in the web, the moment of resistance is given by

Design of Structures

$$M_r = b_f . d_f \, \frac{c+c_s}{2} \times a$$

Where $\quad c_r = c\left(\frac{n-d_f}{n}\right)$

And $\quad a = d - \bar{y}$

(v) If the actual N.A. lies below the critical N.A., the section is over-reinforced and concrete attains its maximum permissible stress of $c = \sigma_{cbc}$ first.

The moment of resistance of the section is given by

$$M_r = b_f . d_f (c + c_s)/2$$

(where c =).

Type II. In this type the following information is given:

1. The dimensions of the section.

2. Area of reinforcement

3. The maximum bending moment to which the section is subjected to.

It is required to determine the stresses developed in concrete and steel.

Procedure to solve: The solution of this type of problem involves the steps given below.

(i) Calculate the position of the neutral axis by equating the moment of equivalent areas about the neutral axis. Assuming that N.A. lies in flange, we get

$$\frac{b_f . n^2}{2} = m . A_{st}(d - n).$$

If the value of n, obtained from this equation is more than the thickness of flange d_f, the N.A. falls in web and it will be necessary to re-work it out by use of expression.

$$b_f . d_f \left(a - \frac{d_f}{2}\right) = m . A_{st}\left(d - n\right)$$

(ii) Equate the moment of resistance of the beam to the external bending moment *(M)* and calculate the compressive stress developed in concrete by the relevant equation.

(a) If N.A. lies within flange, we have

$$b_f . n . \frac{c}{2}\left(d - \frac{n}{3}\right) = M$$

(b) If N.A. lies in web, we have

$$b_f . d_f \frac{(c + c_s)}{2}\times a = M$$

Where $\quad c_s = c\left(\frac{n - d_f}{n}\right)$

And $$a = d - \bar{y}$$

(iii) Having calculated c, the stress developed in steel can be worked out by the relationship.

$$t = m.c\left(\frac{d - n}{n}\right)$$

Type III. In this type of problem, the maximum permissible stress in concrete and steel and the bending moment (M) to which the section is subjected to are given and it is required to design the section.

Procedure to solve: The solution of this type of problem involves the steps given in ------------ chapter.

Example 20.1 A T-beam has an effective width of flange as 1750 mm. The thickness of the flange in 150 mm and the beam is reinforced with 35 sq. cm of tensile steel placed at a depth of 500 mm below the top of flange. If the width of web is 300 mm, find the position of the neutral axis of the beam. Take m = 15.

Solution: Assuming that the neutral axis of the beam is situated below the flange and equating the moment of the equivalent areas about N.A., we get

$$b_f.d_f\left(n-d_f\right) = m.A_{st}(d - n)$$

or $$1750 \times 150 \left(n - \frac{150}{2}\right) = 15 \times 3500 \left(500 - n\right)$$

Which Gives $$n = 145.83mm$$

This shows that our assumption that n lies below the flange is wrong and hence the correct value of the actual neutral axis will be given by the expression.

$$\frac{b_f.n^2}{2} = m.A_{st}(d - n)$$

or $$\frac{1750n^2}{2}=15\times 3500(500 - n)$$

or $$n^2+60n-30000=0$$

Which Gives $$n = 145.78$$

Example 20.2 An isolated T-beam simply supported over a span of 6 m has a flange width of 1500 mm. The thickness of the flange is 80 mm and the beam has an effective depth of 500 mm up to the centre of tensile reinforcement which consists of 4 Nos. of 25 mm. dia. bars. Calculate the moment of resistance of the section neglecting compression resistance of the area of web above the neutral axis. The width of web is 250 mm.

Adopt:

Design of Structures

$$\sigma_{cbc} = 5 \, N/mm^2$$
$$\sigma_{st} = 140 \, N/mm^2$$
$$m = 19$$

Solution The effective width of flange for an isolated T-beam is given by

$$b_f = \frac{l_0}{\left(\frac{l_0}{b}\right)+4} + b_w = \frac{6\times1000}{\frac{6\times1000}{1500}+4} + 250 = 1000mm.$$

$$A_{st} = 4\times\frac{\pi}{4}\times\left(25\right)^2 = 1963.5mm^2$$

To find N.A., equating the moments of equivalent areas about N.A. assuming that the neutral axis falls outsides the flange, we get

$$\rm{b_f.d_f (n - d_f/2) = m.A_{st} (d - n)}$$

or $$1000\times80\left(n-\frac{80}{2}\right)=19\times1963.5(500 - n)$$

Which gives $$n = 186.33mm$$

The depth of critical neutral axis is given by

$$\frac{n_c}{d}=\frac{m.\sigma_{cbc}}{m.\sigma_{cbc} + m.\sigma_{st}} = \frac{19 \times 5}{19\times 5 + 140} = 0.404$$

$$n_c = 0.404\times d = 0.404\times500 = 202mm$$

Since the actual N.A. lies above the critical N.A., the stress in steel reaches the maximum permissible value of $t = \sigma_{st} = 140N/mm^2$ first. The corresponding stress in concrete is given by

$$c=\frac{\sigma_{st}}{m}.\frac{n}{d - n}=\frac{140}{19}\times\frac{186.33}{(500-186.33)}= 4.38N/m\mathbf{m^2}$$

and the stress in concrete at the bottom of flange is given by

$$c_s=\frac{c(n - d_f)}{n} =\frac{4.38(186.33 - 80)}{186.33}=2.2N/mm^2$$

$$\bar y = \left(\frac{c + 2c_s}{c + c_s}\right)\frac{d_f}{3}=\left(\frac{4.38+2\times2.50}{4.38 +2.50}\right)\frac{80}{3}=35.36mm$$

Lever arm of the T-beam $$a=d-\bar y=500-36.36=463.64mm$$

The moment of resistance of the T-beam

$$M_r = \rm{Total \, compression \, x \, Lever \, arm} = b_f.d_f\frac{c+c_s}{2}\times a$$

$$M_r=1000x80\left(\frac{4.38 + 2.50}{2}\right)x463.64=127.6x10^6Nmm=127.6kNm$$

Alternatively, the moment of resistance is also given by

$$M_r = \rm{Total\,compression \times Lever\,arm} = \sigma_{st}.A_{st} \times a$$

$$= 140 \times 1963.5 \times 463.64 = 127.6 \times 10^6 Nmm = 127.6 kNm$$

Example 20.3 Solve example after taking the compressive force in web in to account.

Solution. To find the neutral axis: Equating the moment of the equivalent areas about N.A., we get,

$$b_f.d_f.\left(n - \frac{d_f}{2}\right) + \frac{b_w}{2}\left(n - d_f\right)^2 = m.A_{st}(d - n)$$

$$1000 \times 80 \left(n - \frac{80}{2}\right) + \frac{250}{2}(n - 80)^2 = 19 \times 1963.5(500 - n)$$

Which Gives $n = 176.39$

As already Calculated $n_c = 202mm$

Since the actual N.A. lies above the critical N.A. the stress in steel reaches its maximum permissible value of $t = \sigma_{st} = N/m^2$ first. The corresponding stress in concrete at top of flange is given by concrete at the bottom face of flange is given by

$$c_s = \frac{\sigma_{st}}{m}.\frac{n}{d-n} = \frac{140}{19} \times \frac{176.39}{(500-176.39)} = 4.02 N/m^2$$

And the stress in concrete at the bottom face of flange is given by

$$c_s = c.\frac{(n - d_f)}{n} = \frac{4.02 \times (176.39 - 80)}{176.39} = 2.2\, N/mm^2$$

$$\bar{y} = \left(\frac{c + 2c_s}{c + c_s}\right)\frac{d_f}{3} = \left(\frac{4.02 + 2 \times 2.2}{2.02 + 2.2}\right)\frac{80}{3} = 36.10\, mm$$

The moment of resistance of the section (M_r) will be equal to the sum of the following.

(i) Moment of resistance due to compressive force of flange (M_1); and

(ii) Moment of resistance due to compressive force of rib (M_2)

$$M_1 = M_1 + M_2$$

now, $M_1 = b_f.d_f\left(\frac{c + c_s}{2}\right)(d - \bar{y})$

$$= 1000 \times 80 \left(\frac{4.02+2.2}{2} \right) (500 - 36.10) = 115 \times 10^6 \, Nmm$$

$$M_r = b_w (n - d_f) x \frac{c_s}{2} \left[d - \left(d_f + \frac{n - d_f}{3} \right) \right]$$

$$= 250(176.39 - 80) \times \frac{2.2}{2} \left[500 - \left(80 + \frac{176.39 - 80}{3} \right) \right]$$

$$= 10.28 \, x \, 10^6 \, Nmm$$

The moment of resistance of the T-beam

$$M_r = 115 x 10^6 + 10.28 x 10^6 = 125.28 x 10^6 Nmm = 125.28 kNm$$

Example 20.4 The flange of an isolated T-beam is 100 mm thick and 1600 mm wide. Its web is 250mm wide and the effective depth of the beam up to the centre of tensile reinforcement is 600 mm. The tensile reinforcement consists of 4 Nos. 20 mm dia. bars. The beam is simply supported over a span of 7 m. If the beam section is subjected to a bending moment of 150 kNm, calculate the stresses developed in concrete and steel reinforcement. Take m = 19.

Solution The effective width of flange of an isolated T-beam is given

$$b_f = \frac{l_0}{\left(\frac{l_0}{b} \right) + 4} + b_w = \frac{7000}{\frac{7000}{1600} + 4} + 250 = 1086mm.$$

$$A_{st} = 4 \times \frac{\pi}{4} \times \left(20 \right)^2 = 1256 mm^2$$

Equating the moment of equivalent area about the neutral axis (neglecting the compression in web), we get

$$b_f d_f . \left(n - \frac{d_f}{2} \right) = m.A_{st}(d - n)$$

or $$1086 {\rm{x}} 100 {\rm{}} \left(n - \frac{100}{2} \right) = 19 x 1256 (600 - n)$$

which gives $$n = 149mm$$

Let c, be the maximum compressive stress developed in concrete at the top of the flange and be the compressive stress developed at the bottom of the flange.

$$c_s = c.\frac{\left(n - d_f \right)}{n} = c.\frac{\left(149 - 100 \right)}{149} = 0.33c$$

$$\bar y = \left(\frac{c + 2c_s}{c + c_s} \right) \frac{d_f}{3} = \left(\frac{c + 2 \times 0.33c}{c + 0.33c} \right) \frac{100}{3} = 41.6mm$$

The moment of resistance of the section is given by

∴ The moment of resistance of the section is given by

$$M_r = b_f.d_f\frac{(c+c_s)}{2}.(d-\bar{y}) = 1086 \times 100\frac{(c+0.30c)}{2}.(600-41.6)$$

$$= 40.32 \times 10^6 c\ Nmm$$

Given external B. M. = 150 kNm

$$= 150 \times 10^6\ Nmm$$

Equating the moment of resistance of the section to the external B.M., we get
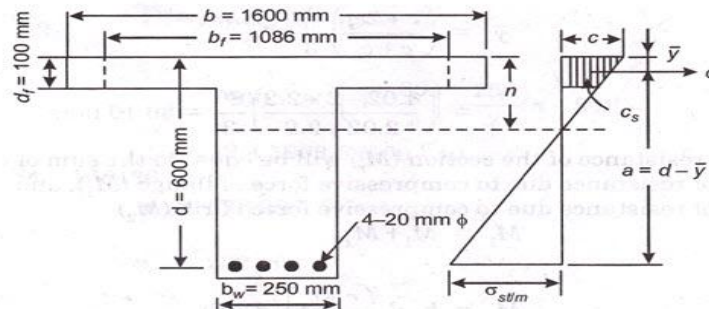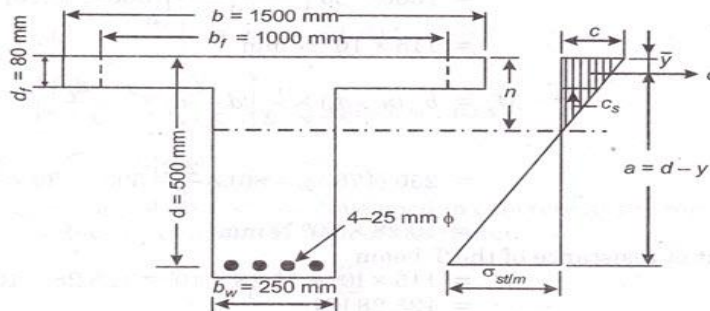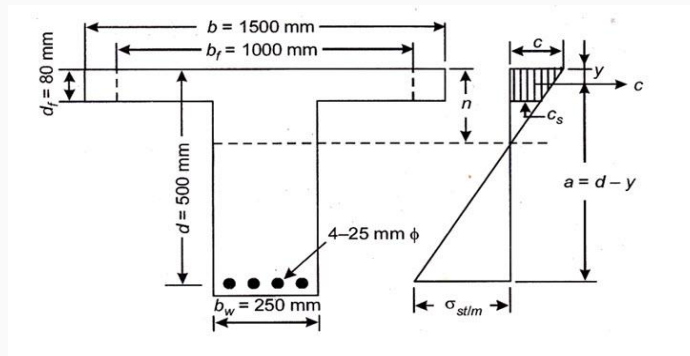
$$40.32\ x\ 10^6 c = 150 \times 10^6$$

∴
$$c = \frac{150 \times 10^6}{40.32 \times 10^6} = 3.72\ N/mm^2$$

The stress in steel reinforcement is given by

$$t = m.c.\frac{(d-n)}{n}$$

$$t = 19\ x\ 3.72\frac{(600-149)}{149}$$

$$= 214\ N/mm^2.$$

LESSON 22. Shear Stress in Beams

22.1 SHEAR STRESSES INDUCED IN HOMOGENEOUS BEAMS

If a beam of homogeneous material is loaded with a concentrated load say W, the shear force at any section of the beam on account of the load would be equal to . If equal resistance to the shear force could be offered throughout the depth of the beam, the shear stress at the section of the beam

$$= \frac{W}{2 \text{ x Area of the section}} = \frac{W}{2BD}$$

would have been and hence the shear force diagram would have been a rectangle indicating uniform shear resistance of the beam from top face to the bottom face.

Actually, the shear stress in a homogenous beam is zero at the top and bottom face of the beam and increases to its maximum value at the neutral axis of the beam i.e., at . Hence, the stress diagram is parabolic as shown in the Fig. 22.1. It can be proved by simple mechanics that the maximum shear stress in the beam,

$$s = \frac{3}{2} \times \frac{Shear\ force}{Area\ of\ cross-section\ of\ the\ beam} = \frac{3}{2} \times \frac{V}{BD}$$

Where   V= maximum shear force in the beam.

22.2 SHEAR STRESS INDUCED IN R.C. BEAMS

In case of reinforced concrete beam, the concrete below the neutral axis in neglected and S.F. is assumed to be resisted by the bond between the steel and the concrete. Hence, the shear stress in a R.C. beam increase from zero at the top face of the beam to its maximum value at the neutral axis and from neutral axis down to the C.G. of the reinforcing bars, it remains uniform as shown in Fig. 22.2.

If V be the total shear force in the beam then, from stress diagram.

V = Area of stress diagram x Breadth of the beam.

Area of the stress diagram consists of two parts.

(I) Area of parabolic part $(hkg) = \frac{2}{3} \times Height \times Base = \frac{2}{3} n \times \tau_s$

(II) Area of rectangle part $(hijk) = \tau_s \, x \, (d - n)$

$$V = \left\{\frac{2}{3}n \times \tau_s + \tau_s(d - n)\right\}b = \left(\frac{2}{3}n\tau_s + \tau_s d - n\tau_s\right)b$$

$$= \left(\tau_s d - \frac{\tau_s n}{3}\right)b = \tau_s b\left(d - \frac{n}{3}\right) = \tau_s bj.d$$

Or
$$\tau_s = \frac{V}{j.d.b} = \frac{V}{ab}$$

Hence, shear stress in an R.C. beam is given by the equation

$$shear\ stress = \frac{Shear\ force}{Lever\ arm\ x\ Breadth\ of\ the\ beam}$$

22.3 NOMINAL SHEAR STRESS

In IS: 456-1978 the equation for shear stress given above has been simplified by dropping the lever arm factor and by changing the term shear stress by the term nominal shear stress. This simplification is reasonable since the nom.inal shear stress represents merely a measure of the average intensity of stress in the beam.

The formula for calculating nominal shear stress in beams or slab of uniform depth specified in the code is
$$\tau_s = \frac{V}{d.b}$$

Where

$\tau_v = nominal\ shear\ stress$

$V = shear\ force\ due\ to\ design\ load$

$b = breadth\ of\ the\ member, which\ for\ flanged\ sections\ shall\ be\ taken\ as\ the$
$\quad breadth\ of the\ web, b_w$

$d = effective\ depth$

22.3.1 Nominal Shear Stress in Case Beams of Varying Depth

Beams of uniform width and varying depths are commonly used in practice. Cantilever beams continuous beam with haunches at support, footings etc. fall under this category. In case of beams of varying depth the nominal shear stress is calculated by the modified equation given below.

$$\tau_v = \frac{V \pm \frac{M}{d}\tan\beta}{bd}$$

Where $\qquad$ τ_v, V, b and d has same meaning as above

$M = bending\ moment\ at\ the\ section$

$\beta = angle\ between\ the\ top\ and\ bottom\ edges\ of\ the\ beam.$

The negative sign in the formula applies when the bending moment M increases numerically in the same direction as the effective depth d increases and the positive sign when the moment decreases numerically in this direction.

22.4 EFFECT OF SHEAR IN R.C. BEAMS

The effect of shear in R.C. beams is to create principal tensile and compression stresses equal in magnitude to the shear stress as obtained by normal shear stress equation given above but acting at 45˚ to the horizontal.

The effect of shear on a block ABCD is shown in Fig. 22.3. It is noted that when the block is subjected to shear stress of intensity, compressive stresses are developed along the diagonal plane BD and tensile stresses are developed along the diagonal plane AC. The intensity of the diagonal compressive or tensile stress being each equal to . Thus if the block is weak in compression, it will fail by the crushing of a material of the block on account of the diagonal compressive stress along diagonal plane BD. On the other hand, if the material of the block is weak in tension, it will have a tendency to split up into two parts along the diagonal plane AC.

22.5 SHEAR FAILURE OF BEAMS WITHOUT SHEAR REINFORCEMENT

As an outcome of rigorous experimental test it has been observed that beams without shear reinforcement can fail in the following ways.

(a) Diagonal tension failure: In this type of failure diagonal cracks appear in the beams (near support) which are inclined nearly at 45˚ to the horizontal as shown in Fig. 22.4. This situation arises when magnitude of shear force is large in relation to bending moment.

(b) Flexural shear failure. In this case the cracks appear normally at 90˚ to the horizontal as shown in Fig. 22.5. This type of failure occurs when bending moment is comparatively large in relation to the shear force.

(c) Diagonal compression failure. This type of failure takes place by crushing of concrete in the compression zone near the load as the diagonal crack formed independently penetrates in that zone as shown in Fig. 22.6.

Shear reinforcement essentially provided to prevent formation of crack and failure of the beam due to shear. To guard against diagonal compression failure highlighted in team (c) above, the code has fixed the upper limit for maximum allowable shear stress in a member.

22.6 SHEAR RESISTANCE OF CONCRETE WITHOUT SHEAR REINFORCEMENT

As a result of extensive studies it has been established that concrete in beam without shear reinforcement is capable of resisting certain amount of shear force. This shear strength or shear resistance of concrete is due to many factors (refer Fig. 22.7) the most important of which are:

(a) Shear force resisted by uncracked compression zone of concrete.

(b) Shear force resisted by vertical component of the force due to aggregate interlock.

(c) Shear force across longitudinal tensile reinforcement in beam (also known as dowel force).

IS: 456-1978 has specified values of permissible shear stress in concrete (which account for the cumulative effect of all the above factors for working out the shear resistance of concrete. The value of (for different grades of concrete and different percentage of longitudinal tensile reinforcement as given in the code are reproduced in Table 22.1

Table 22.1 Permissible shear stress in concrete

	Permissible shear stress in concrete(in N/mm^2 in different grades of concrete					
	M15	M 20	M25	M30	M35	M40
0.25	0.22	0.22	0.23	0.23	0.23	0.23
0.50	0.29	0.30	0.31	0.31	0.31	0.32
0.75	0.34	0.35	0.36	0.37	0.37	0.38
1.00	0.37	0.39	0.40	0.41	0.42	0.42
1.25	0.40	0.42	0.44	0.45	0.45	0.46
1.50	0.42	0.45	0.46	0.48	0.49	0.49
1.75	0.44	0.47	0.49	0.50	0.52	0.52
2.00	0.44	0.49	0.51	0.53	0.54	0.55
2.25	0.44	0.51	0.53	0.55	0.56	0.57
2.50	0.44	0.51	0.55	0.57	0.58	0.60
2.75	0.44	0.51	0.56	0.58	0.60	0.62
3.00 and above	0.44	0.51	0.57	0.60	0.62	0.63

Note : A_s is that area of longitudinal tension reinforcement which continues at least one effective   depth beyond the section being considered except at supports where the full area of   tension reinforcement may be used.

The shear resistance of concrete (V_c) in a beam is worked out by multiplying value of obtained from Table 22.1 with cross-sectional area of the beam i.e., shear force resisted by concrete $V_c = \tau_c . b . d$

22.7 DESIGN SHEAR STRENGTH OF CONCRETE

(i) Permissible shear stress in concrete without shear reinforcement: The permissible shear stress in concrete in beams without shear reinforcement shall be as given Table 22.1

For solid slabs the permissible shear stress in concrete, shall be where K has the value given in Table 22.2

<div align="center">Table 5.2 K value</div>

Overall depth of slab in (mm)	300 or more	275	250	225	200	175	150 or less
K	1.00	1.05	1.10	1.15	1.20	1.25	1.30

Note : The above do not apply to flat slabs.

(ii) Permissible shear stress in concrete with shear reinforcement. When shear reinforcement is provided the nominal shear stress (in beams shall not exceed. Given in Table 22.2.

<div align="center">Table 5.3 Maximum shear stress in beams</div>

Grade of concrete	M 15	M 20	M 25	M 10	M 35	M 35	M 40
	1.6	1.8	1.9	2.2	2.3	2.5	2.5

Maximum shear stress for slabs: For slabs shall not exceed half the value of given in Table 22.2.

22.8 MINIMUM SHEAR REINFORCMENT

It has now been established that in beam without shear reinforcement sudden diagonal tension failure occur without warning. This makes such a member unsafe. It is observed that provision of certain minimum amount of shear reinforcement (even if the shear force developed at the section is less than shear resistance of concrete) has a distinct advantage. Such reinforcement besides resisting part of shear by itself also improves the capacity of concrete compression zone and the longitudinal tensile reinforcement to resist shear. The minimum shear reinforcement specified in the code also caters for any sudden transfer of tensile stress from the web concrete to the shear reinforcement.

As per IS: 456-1978 when the value of the nominal shear (as calculated from equation works out to be less than the permissible shear stress in concrete (, minimum shear reinforcement in the form of stirrups, shall be provided in accordance with the relation

$$\frac{A_{sv}}{b \times S_v} \geq \frac{0.4}{f_y}$$

Where

$A_{sv} = total\ cross-sectional\ area\ of\ stirrup\ legs\ effective\ in\ shear$
$S_v = stirrup\ spacing\ along\ the\ length\ of\ the\ member$
$b = breadth\ of\ the\ beam\ or\ breadth\ of\ the\ web\ of\ flanged\ beam\ and$
$f_y = characteristic\ strength\ of\ stirrup\ reinforcement\ in\ N/mm$
   $which\ shall\ not\ be\ taken\ greater\ than\ 415\ N/mm^2.$

The term characteristic strength is defined as the strength of material below which not more than 5 per cent of the test results are expected to fall. For mild steel reinforcement is taken = 250 N/mm² and for High Yield Strength Deformed bars (HYSD bars) is taken = 415 N/mm².

The above provision need not be applied to members of minor structural importance such as lintels or when the maximum shear stress calculated (is less than half the permissible shear stress in concrete (.

The above equation can be re-arranged as under to obtain an expression giving maximum

c/c spacing of stirrups required for minimum shear reinforcement $\qquad S_v \leq \frac{2.5\ A_{sv}.f_y}{b}$

22.9 MAXIMUM SPACING OF SHEAR REINFORCEMENT

As per IS: 456-1978 the maximum spacing of shear reinforcement measured along the axis of the member shall be as under

(i) For vertical stirrups   0.75d or 450mm whichever is less

(ii) For inclined stirrups at 45˚   d or 450mm whichever is less

Where d is the effective depth of the member.

22.10 DESIGN OF SHEAR REINFORCEMENT

When the shear force-V (or shear stress) at a section exceeds the shear resistance of concrete (or permissible shear stress in concrete-) shear reinforcement have to be provided to prevent formation of cracks or failure of the member. The method of designing shear reinforcement based on the truss analogy is accepted by the code. In this analogy it is assumed that concrete and the shear reinforcement form a lattice-grider truss wherein tension is carried by the longitudinal bars and the shear reinforcement and the concrete carries the thrust in the compression zone and the diagonal thrust across the web.

If be the shear force to be carried by the shear reinforcement, the shear capacity of a section

can be written as $\qquad V = V_c + V_s$

In this expression the values of V (i.e., the shear force due to design loads) and V_c (i.e., shear resistance of concrete =) are known and as such in normal practice the design procedure will

involve the determination of shear reinforcement for shear force = Shear reinforcement can be provided in any of the following forms:

(i) In the form of vertical bars known as stirrups.

(ii) In the form of bent up bars along with the stirrups.

(iii) In the form of inclined stirrups.

The design of different forms of shear reinforcement is described in the following articles.

22.10.1 Design of Vertical Stirrups

Vertical stirrups may consist of 5mm to 12mm diameter bars bent around the tension reinforcement and their free ends taken into the compression zone of the beam. In the compression zone the stirrups are anchored to the longitudinal bars (known as anchor bars) so that the vertical legs may resist tension without slippage. In case of doubly reinforced beams the stirrups are taken around the compression reinforcement and suitably anchored. Depending upon the magnitude of shear force (to be resisted, the vertical stirrups may be one legged, two legged, four legged, six legged and so on. The various form of stirrups are shown in Fig. 22.8.

To derive an expression for shear force resisted by vertical stirrups.

Let

A_{sv} = total cross − sectional area of stirrups legs within a distance s_v

s_v = centre to centre spacing of stirrups

σ_{sv} = permissible tensile stress in shear reinforcement which shall not be taken greater than 230 N/mm^2

d = effective depth

V_s = shear force to be resisted by shear reinforcement

Let us assume that concrete has failed in diagonal tension on account of shear force. Let the diagonal crack be inclined at 45° to the axis of the beam and extend to the full depth of the beam. The horizontal distance up to which the crack extends will therefore be equal to the effective depth (d) of the beam – cover (d′) to the anchor bars.

Since d′ is very small as compared to d we may consider the distance of horizontal extension of crack as d.

From Fig. 22.9 the number of stirrups (n) crossing the diagonal crack $\qquad n = \dfrac{d}{s_v}$

Shear force resisted by stirrup legs passing through one section $\qquad = \sigma_{sv}.A_{sv}$

∴ Total shear force resisted by n, number of stirrups $\qquad V_s = \sigma_{sv}.A_{sv}.n$

Or $\qquad\qquad\qquad V_s = \dfrac{\sigma_{sv}.A_{sv}.d}{V_s}$

The above formula is adopted for the design of vertical stirrups as shear reinforcement. In the formula, the values of , and d are known. We assume suitable diameter and number of legs for the stirrups (and work out the c/c spacing of the stirrups by re-writing the above formula as under

$$S_v = \frac{\sigma_{sv}.A_{sv}.d}{V_s}$$

It should, however, be ensured that:

(1) The area of shear reinforcement (provided is not less than the area of minimum shear reinforcement specified by the code (Ref. Art 22.8).

(2) The centre to centre spacing () does not exceed the maximum limits prescribed in the code. (Ref. Art. 22.9).

22.10.2 Design of Inclined Bars or Inclined Stirrups as Sheer Reinforcement

In a beam some longitudinal bars can be bent up near support where they are no longer needed to resist bending moment. The bars can be bent up at uniform spacing at different cross-

section along the length of the beam or all the bars (which are no longer needed for resisting B.M.) can be kept up at the same cross-section.

The bars thus bent up are helpful in resisting shear. In order to be fully effective in shear the bent up bars are continued beyond the neutral axis in the compression zone for a distance equal to the development length. (For details regarding development length refer Lesson 23).

The expression for shear force resisted by the inclined bars can be derived by considering the truss analogy. Instead of bending up bars some designers prefer to use inclined stirrups. The case of inclined stirrups or bent up bars is identical and as such the following formula will apply to both bent up bars as well as inclined stirrups.

Let

A_{sv} = total cross − sectional area of stirrup legs or bent

  up bars within a distance S_v

s_v = centre to centre spoacing of the stirrups or

  bent up bars along the length of the member

σ_{sv} = permissible tensile stress in shear reinforcement which shall not be

  taken greater than 230 N/mm^2

α = angle between the inclined stirrups or bent up bar and the axis

  of the member

d = effective depth

V_s = shear force to be resisted by shear reinforcement

Design of Structures

Case I. For inclined stirrups or a series of bars bent up at different cross-section.

From the Fig.(22.10) it can be seen that the number of inclined bars or stirrups (n) crossing the diagonal crack

$$= \frac{d + d \cot \alpha}{s_v}$$

or

$$n = \frac{d(1 + \cot\alpha)}{s_v}$$

Vertical component of the force resisted by inclined bars or stirrups passing through section

$$= \sigma_{sv}.A_{sv}\sin \alpha$$

∴ Total shear force resisted by n number of inclined bars or stirrups

$$V_s = \sigma_{sv}.A_{sv}\sin\alpha \times n = \frac{\sigma_{sv}.A_{sv}d}{s_v}(\sin\alpha + \cos\alpha)$$

Case II. For single bar or single group of parallel bars, all bent up at the same cross-section.

As already explained in derivation in Case I above $\quad V_s = \sigma_{sv}.A_{sv}\sin \alpha$

IS: 456-1978 does not permit the shear reinforcement to be entirely provided in the form of bent up bars since there is insufficient evidence to show that such reinforcement is satisfactory. As per code where bent up bars are provided as shear reinforcement their contribution towards shear resistance shall not be taken more than half that of the total shear reinforcement.

In other words the bent up bars can be used only in combination with stirrups, where the stirrups must make up 50% of the total shear reinforcement. In situations where more than one type of shear reinforcement is used to reinforce the same portion in of the beam, the total shear resistance shall be computed as the sum of the resistance for various types separately. The area of the stirrups shall not be less than the minimum specified in Art. 22.8.

22.11 CRITICAL SECTION FOR SHEAR

As per IS: 456-1978 the shear computed at the face of support shall be used in the design of the member at that section except when the reaction in the direction of the applied shear introduces compression into the end region of the member, sections located at a distance less than d from the face of the support may be designed for the same shear as that computed at distance d.

Fig. 22.11 shows examples of cases where the support reaction does not include compression in the end region. In such situation a diagonal shear crack is likely to start at the face of support. Hence the critical section for shear (section X – X) is taken at the face of the support.

In all cases shown Fig. 22.12 the reaction from the beam/slab introduces compression in the end region which has the advantage of displacing the diagonal shear crack away from the face of the support. Hence the code allows the support section to be designed for shear

computed at a distance d away from the support. Thus the critical section for shear may in such case be treated to be located at a distance d from the face of the support.

It is however, proposed to consider critical section for shear at the face of the support in the above referred cases in Fig. 22.12 to simplify design. The following examples have been solved accordingly.

Example 22.1 A reinforced concrete beam 200 mm wide and 450 mm deep to the centre of tensile reinforcement is subjected to shear force of 98 kN at the supports. The area of the tensile steel available at the supports is 0.75 per cent. Design suitable shear reinforcement for the beam. Also calculate the minimum shear reinforcement for the beam. Adopt the following data

(i)   *Grade of concrete* $= M\ 15$

(ii)   *Characteristic strength of stirrup reinforcement* $(f_y) = 250\ N/mm^2$

(iii) $\sigma_{st} = \sigma_{sv} = 140 N/mm^2$

(iv) $m = 19$

Solution The nominal shear stress (τ_c) in the beam is given by the equation

$$\tau_v = \frac{V}{bd}$$

From the example:

The shear force $V = 98 \times 1000\ N$

$b = 200\ mm$

$d = 450\ mm$

$\therefore$ $\tau_v = \frac{98 \times 1000}{200 \times 450} = 1.09\ N/mm^2$

It is given that $p = \frac{100\ A_s}{bd} = 0.75$

From Table 22.1 value of permissible shear stress (τ_c) for M 15 grade of concrete with $p = 0.75$ is given as $\tau_c = 3.4\ N/mm^2$

Also from Table 22.3 value of nominal shear stress $(\tau_{c\ max})$ for M 15 grade of concrete is given as $\tau_{cmax} = 1.6\ N/mm^2$

The calculated value of nominal shear stress (τ_v) is greater than the permissible shear stress in concrete (τ_{cmax}) in concrete. Hence the beam section is safe. However, shear reinforcement will have to be provided for carrying a shear force

$$V_s = \tau_c.b.d = 98 \times 1000 - 0.34 \times 200 \times 450 = 67400\ N$$

Providing shear reinforcement in the form of vertical stirrup and using 12 mm ϕ 2 legged sunups, we get

$$A_{sv} = 2 \times \frac{\pi}{4} \times (12)^2 = 226.2\ mm^2$$

The design shear strength of vertical stirrup is given by the equation $V_s = \frac{S_{sv} \cdot A_{sv} \cdot d}{S_v}$

∴ The centre to centre spacing of the stirrup $S_v = \frac{S_{sv} \cdot A_{sv} \cdot d}{V_{sv}}$

$$= \frac{140 \times 226.2 \times 450}{67400} = 211.4 \; mm = 210 \; mm \; c/c \; (say)$$

Hence provide 12 mm φ 2 legged stirrups @ 210 mm c/c near supports.

The spacing adopted should, however, be not more than max. spacing permissible for shear reinforcement as worked out below.

Minimum shear reinforcement. Centre to centre spacing of two legged stirrups to meet the requirement of minimum shear reinforcement for the beam can be worked out by use of equation.

$$S_v = \frac{2.5 \; A_{sv} \cdot f_v}{b}$$

Using 10 mm φ 2 legged stirrups for minimum shear reinforcement, we get

$$A_{sv} = 2 \times \frac{\pi}{4} \times (10)^2 = 157 \; mm^2$$

∴ The centre to centre spacing of the stirrup

$$S_v = \frac{2.5 \times 157 \times 250}{200} = 490 \; mm.$$

Maximum spacing for shear reinforcement. As per rules the maximum spacing of the stirrups should not exceed 0.75 d or 450 mm whichever is less. In this case

0.75 d = 0.75 x 450 = 337.5 mm = 335 mm (say)

The maximum spacing of stirrups as permissible under rule is less than obtained from requirement of minimum shear reinforcement .

Hence provide 10 mm 2 legged stirrups @ 335 mm c/c.

Example 5.2 A simply supported reinforced concrete beam, 300 mm wide and having an effective depth of 600 mm carries a uniformly distributed load of 35 kN/m (inclusive of its own weight) over a clear span of 6m. Design suitable shear reinforcement for the beam assuming that 0.5% tensile reinforcement is available throughout its length. The following data being given:

(i) Grade of concrete = M 15

(ii) Characteristic strength of stirrup reinforcement (= 250 N/mm²

(iii) $\sigma_{st} = \sigma_{sv} = 140 \ N/mm^2$
(iv) $m = 19$.

Solution The shear force (V), at the end of the beam is given by

$$V = \frac{wL}{2} = 35 \times 1000 \times \frac{6}{2} = 105000 \ N$$

The nominal shear stress (τ_v) is given by the equation

$$\tau_v = \frac{V}{bd} = \frac{105000}{300 \times 600} = 0.58 \ N/mm^2.$$

From Table 22.1 the value of permissible shear stress (τ_s) for M 15 grade of concrete $\frac{100 \ A_s}{bd} = 0.50$ is given as $\tau_c = 0.29 \ N/mm^2$

Also from Table 22.2 the value of maximum permissible shear stress in M 15 grade of concrete is given as $\tau_{cmax} = 1.6 \ N/mm^2$

The calculated value of nominal shear stress is less than . Hence the beam section is O.K.

However, since the value of nominal shear stress is more than the permissible shear stress , shear reinforcement will have to be designed for section near support. The shear stress diagram for the beam is shown in the Fig. 22.13.

To calculated distance x from the centre of the beam, where permissible shear stress (less than(), shear reinforcement will have to be designed for section near support.

The shear stress diagram for the beam is shown in the Fig. 5.13.

To calculate distance x from the centre of the beam, where permissible shear stress is developed. From Fig. 22.13, we have

$$\frac{x}{0.29} = \frac{3}{0.58}$$

or $x = \frac{3}{0.58} \times 0.29 = 1.5 \ m$

Hence the designed shear reinforcement is required in length AC or BD = 3 – 1.5 = 1.5 m from either end. In the remaining length CD, nominal shear reinforcement is to be provided to meet the requirement of minimum shear reinforcement in the beam.

Design of shear reinforcement. Magnitude of shear force (V_s) for which shear reinforcement is to be designed is given by

$$V_s = V - \tau_c b.d$$
$$= 105000 - 0.29 \times 3000 \times 600$$
$$= 52800 \; N.$$

Using 10 mm φ 2 legged stirrups, we get

$$A_{sv} = 2 \times \frac{\pi}{4} \times (10)^2 = 157 mm^2$$

c/c spacing (S_v) of vertical stirrups acting as shear reinforcement is given by

$$S_v = \frac{S_{sv}.A_{sv}.f_y}{b} = \frac{140.157.600}{52800} = 250 \; mm$$

The c/c spacing (s_v) of the 10 mm φ 2 legged nominal stirrups from consideration of minimum shear reinforcement in the beam is given by

$$S_v = \frac{2.5.A_{sv}.f_y}{b} = \frac{2.5 \times 157 \times 250}{300} = 327 \; mm = 325 \; mm \; (say)$$

Maximum spacing of shear reinforcement. The maximum spacing (S_v) of stirrups should not exceed 0.75 d or 450 mm or $S_v = 325 \; mm$ whichever is less

In this case $\qquad 0.75 \, d = 0.75 \times 600 = 450 \; mm$

Hence the centre to centre spacing of the 10 mm 2 legged stirrups is to be varied from 250 mm at ends to 325 mm at a section say z (meters) from the mid span. Let the shear force at that section be = .

From S.F diagram

$$\frac{V_z}{z} = \frac{V}{3}$$

or $\qquad\qquad V_z = \frac{105000}{3} z = 35000 \, z$

Balance of shear force to be resisted by stirrup at that section is obtained from the equation

$$V_s' = V_z - \tau_c.bd$$
$$= 35000z - 0.29 \, x \, 300 \, x \, 600$$
$$= (35000z - 52200)$$

$$S_v' = \frac{\sigma s_v.A_{sv}.d}{V_z'}$$

$$= \frac{140 \times 157 \times 600}{(35000z - 52200)}$$

Since S_v' should not be more than 325 mm

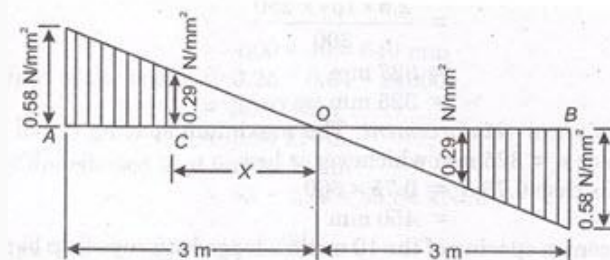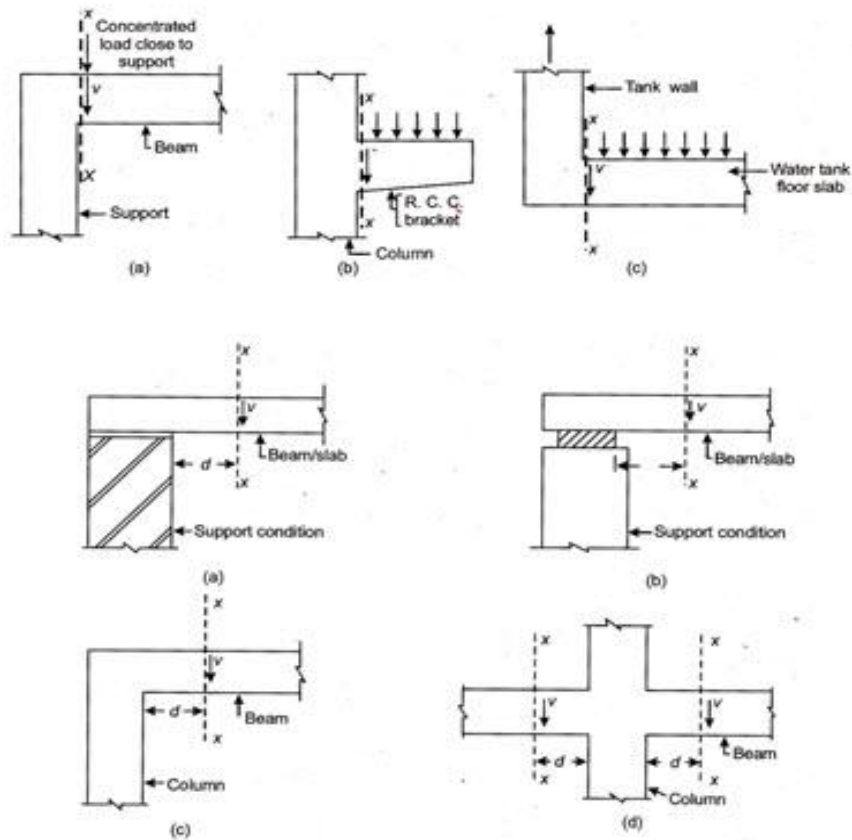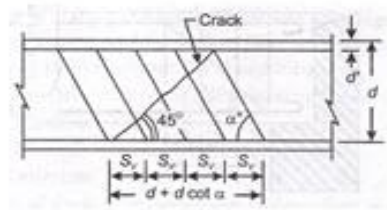$$\therefore \qquad \frac{140 \times 157 \times 600}{35000 \, z - 52200} = 325$$

Which gives $z = 2.65 \; m$

Hence vary the c/c spacing of 10 mm 2 legged stirrups from 250 mm at end to 325 mm c/c at 2.65m from mid span. For the remaining length provide the stirrups at spacing of 325 mm c/c.

(a) Single legged stirrup, (b) and (c) Two legged stirrups. (d) Four legged stirrups, (e) Six legged stirrups.
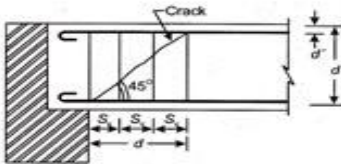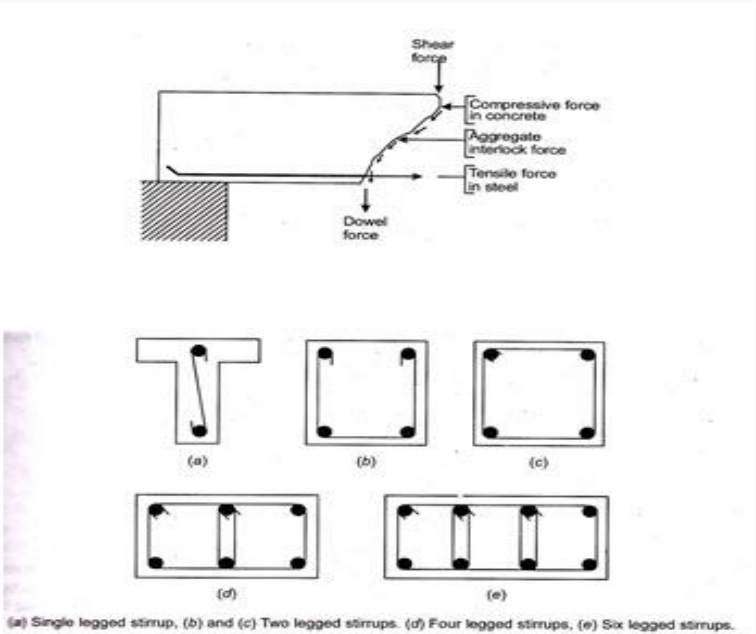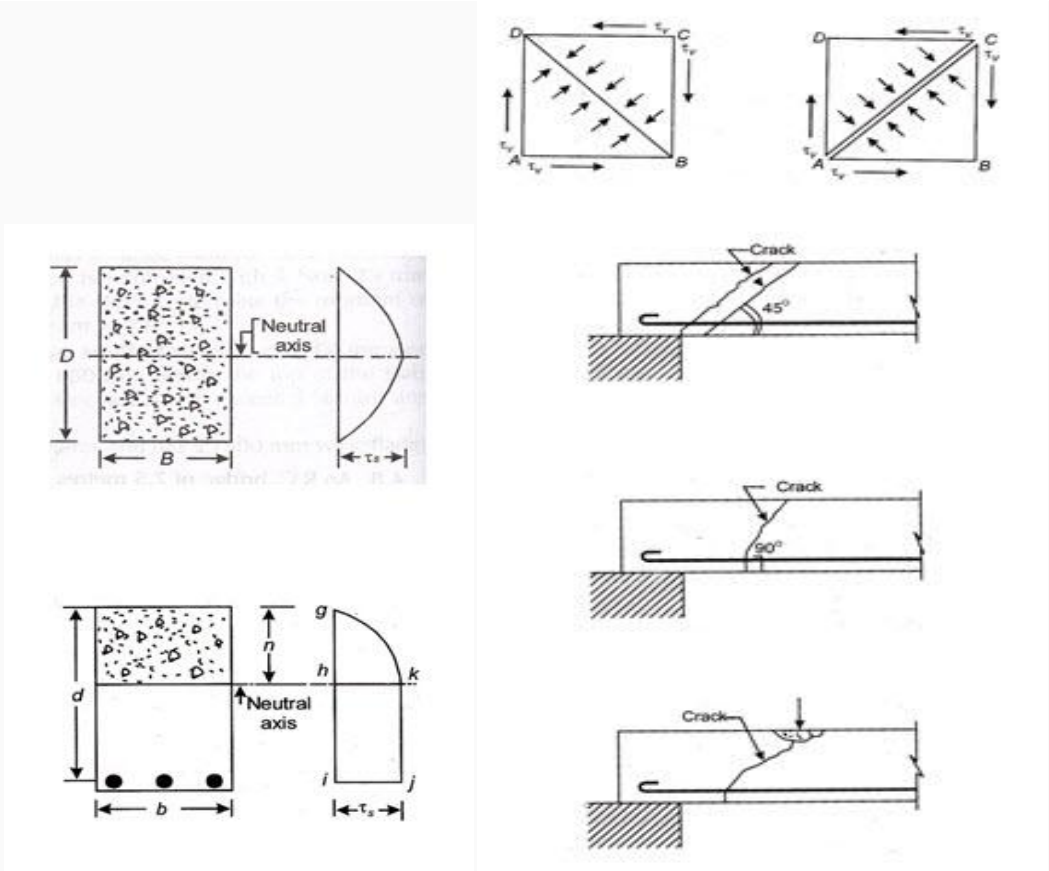
LESSON 23. Bond and Development Length

23.1 INTRODUCTION

The fact which makes it possible to combine steel and concrete is that concrete on setting grips fast the embedded steel rods. Thus when the R.C.C. member is loaded the transference of force between concrete and embedded steel reinforcements takes place only virtue of the grip, adhesion or bond between the two materials. The bond between concrete and steel must be sufficient to make them act jointly. In case the grip between the two materials is not perfect, an R.C. beam when loaded will fill as the steel reinforcement on account of the imperfect bond will slip and will not contribute to resist any stresses developed in the beam. The grip depends upon the mix and quality of the concrete, surface and shape of the bars, the length of embedment and the cover of concrete on steel reinforcement.

To achieve increased bond between steel and concrete the following factors should be kept in view:

(i) Use rich mix of concrete

(ii) The compaction and curing of concrete should be perfect

(iii) Provide adequate cover to steel reinforcement

(iv) Use rough surface steel bars. The bars with smooth or polished surface will not be able to provide adequate frictional resistance for the purpose of perfect grip.

(v) Use deformed or twisted bars.

23.2 DEVELOPMENT LENGTH

The check for satisfying the requirement of permissible bond stress specified in the earlier code has now been replaced by the concept of development length. It is obvious that bar with sufficient embedment in concrete cannot be pulled out. Development length is the minimum length of bar which must be embedded in concrete beyond any section to develop by bond (between the concrete and steel), a force equal to the total tensile force in the bar at that section. Development length is represented by a symbol L_d and it is expressed in terms of the diameter of the bar.

Refer Fig. 23.1. Let a mild steel reinforcing bar of diameter ( ) be embedded in a concrte block. Let T be the pull or tensile force applied to the bar at its free end and Let L_d be the minimum length of embedment of the bar in the concrete block so as to withstand the tensile force without any slippage.

If be the stress developed in the steel reinforcement bar due to the T. The pull T can also be written as

$$T = \text{Area of bar} \times \text{Stress} = \frac{\pi}{4}.\phi^2 x\, \sigma_s$$

The tensile force due to the pull has to be transmitted to the concrete by bond stress in the embedment length L_d. Bond stress is the local longitudinal shear stress per unit of bar surface. Bond stress can also be defined as the shear force per unit of nominal surface area of a reinforcing bar acting parallel to the bar on the interface between the bar and the surrounding concrete. The magnitude of bond stress varies along the length of the bar. Its value will be maximum at lower face of block and minimum at end of bar in concrete. Based on experimental evidence it is seen that value of L_d derived based on average bond stress works out to be safe.

Let be the average bond stress developed in concrete due to the pull in the bar. achieve condition of no slippage of bar and equilibrium

Force developed in bar in concrete = Applied pull

or Surface area of bar $\times L_d \times \tau_{bd} = T$

or $\qquad \pi\phi L_d.\tau_{bd} = \frac{\pi}{4}.\phi^2.\sigma_s$

or $\qquad L_d = \frac{\phi\sigma_s}{4\tau_{bd}}$

Code has specified values of average permissible bond stress for plain bar in tension for different grades of concrete which has been reproduced in Table 23.1 for ready reference.

TABLE 23.1 Permissible bond stress (Average) for plain bars in tension.

Grade of concrete	M 10	M 15	M 20	M 25	M 30	M 35	M 40
Permissible stress in bond (τ_{bd}) $[inN/m{m^2}]$	----	0.6	0.8	0.9	1.0	1.1	1.2

Note 1. The bond stress given above shall be increased by 25 per cent for bars in compression.

Note 2. In the case of deformed bars conforming to IS : 1139 -1966 and IS: 1786 – 1979, the bond stress given in Table 23.1 may be increased by 40%.

23.3 DEVELOPMENT OF STRESS IN REINFORCEMENT

For ensuring full development of the calculated tension or compression in any bar at any section, it is necessary that the bar under consideration should extend on each side of the section by appropriate length which is termed as development length. The formula for calculating development length of bars in tension and compression for mild steel and HYSD bars are given below:

(i) Development length of bars in tension. The development length L_d for bars in tension is given by

Design of Structures

$$L_d = \frac{\phi \sigma_s}{4\tau_{bd}}$$

Where

ϕ = nominal diameter of the bar

σ_s = stress in bar at the section considered at design load

τ_{bd} = design bond stress as given Table 23.1

Note. The development length includes anchorage values of hooks in tension reinforcement.

It may be noted that the above expression for development length is similar to the one for bond length as given in earlier code, except change of symbol in the relationship. Based on the above formula the values of for plain and deformed bars for M 15 grade of concrete can be worked out as under:

(a) Development length for plain m.s. bars in tension :

For plain m.s. bars

(a) Development length for plain m.s. bars in tension :

For plain m.s. bars $\sigma_s = 140\ N/mm^2$

and For M 15 grade of concrete $\tau_{bd} = 0.6\ N/mm^2$

$$\therefore L_d = \frac{140}{4 \times 0.6}\ \phi = 58\ \phi.$$

(b) Similarly the development length for deformed bars in tension will work out to be as under:

For deformed bars $\sigma_s = 230\ N/mm^2$

$$\tau_{bd} = 0.6 + \frac{40}{100} \times 60 = 0.84\ N/mm^2$$

$$\therefore L_d = \frac{230}{4 \times 0.84}\ \phi = 68\ \phi.$$

The development length of mild steel bars and HYSD bars for different grades of concrete are given Table 23.2

TABLE 23.2 Development lengths for M.S and HYSD bars in tension

Grade of concrete	Mild steel bars ($\sigma_{st}=140N/mm^2$)		HYSD bars ($\tau_{st}=230N/mm^2$)	
	τ_{bd} in N/mm^2	L_d	τ_{bd} in N/mm^2	L_d
M 15	0.6	58 ϕ	0.84	68 ϕ
M 20	0.8	44 ϕ	1.12	52 ϕ
M 25	0.9	39 ϕ	1.26	46 ϕ

M 30	1.0	35 ϕ	1.40	41 ϕ
M 35	1.1	32 ϕ	1.54	37 ϕ
M 40	1.2	29 ϕ	1.68	34 ϕ

(ii) Development length for bars in compression. It is simpler to pull a bar out of concrete than to push it inside. The Code accordingly permits 25 percent increase in the value permissible bond stress for bars in compression. Based on the above, the development length for bars in compression is given by

$$L_{d\ compression} = \frac{\sigma_s}{5\tau_{bd}}\phi$$

23.4 ANFHORAGE FOR REINFORCEMENT BARS

The development length of bars as obtained from the above formula can be provided in the form of straight length or it may be partially straight and partially anchored. The anchorage is normally provided in the form of bends and hooks.

(i) Anchoring bars in tension: In case of deformed bars in tension, the development length is provided straight without end anchorage. In case of plain bars ends hooks are normally provided for anchorage.

The anchorage value of bend shall be taken as 4 times the diameter of the bar for each 45° bend subject to a maximum of 16 times the diameter of the bar. The dimensions of a standard hook and a standard 90° bend are shown in Fig. 23.2). The value of k to be adopted depends upon the type of steel. Its value as per code is as under.

	Type of steel	Min. value of k
(i)	Mild steel	2
(ii)	Cold worked steel	4

The anchorage value of a standard hook and a standard bend are taken as 16 and 8 respectively.

Let be the required value of development length for a reinforcing bar and and be the straight lengths required when the end bar anchorage are provided in the form of a semi-circular hook and a right angle bend hook respectively. The value of and in term of can be written as

$$l_1 = L_d - 10\phi$$
$$l_2 = L_d - 8\phi$$

(ii) Anchoring bars in compression: The anchorage length of straight bar in compression shall be equal to the development length of bar in compression as obtained from the formula

$$L_d = \frac{\sigma_s}{5\tau_{bd}} \phi$$

The end of a bar in compression require no special anchorage.

(iii) Anchoring shear reinforcement: The shear reinforcement can be provided in the form of inclined bar as well as stirrups.

(a) In case of inclined bars: The development length shall correspond to development length of bars in tension allowing for hook and bends when provided and measures as under:

(1) In tension zone; from the end of inclined portion of bars.

(2) In compression zone; from the mid-depth of the beam..

(b) In case of stirrups: In case of stirrups complete development length and anchorage shall be deemed to have been provided when

(i) the bar is bent through an angle of at least 90° round a bar of at least its own diameter and is continued beyond the end of the curve for a length of at least eight diameters or

(ii) when the bar is bent through an angle of 135° and is continued beyond the end of the curve for a length of at least six bar diameters or

(iii) when the bar is bent through an angle of 180° and is continued beyond the end of the curve for a length of at least four bar diameters.

23.5 TO DECIDE THE CURTAILMENT OF BARS

A repeated reference has been made in this chapter and in the chapter on shear regarding bent up bars. To find out the point at which the bars at mid-span of a simply supported beam loaded with uniformly distributed load, can be safely curtailed or bent up, proceed as below:

Area of steel at mid span of a beam is obtained by using the formula

$$A_{st} = \frac{M}{j.d.\sigma_{st}}$$

Assuming the beam to be of constant depth, the denominator i.e., j.d. may be taken to be a constant, say K

$$A_{st} = \frac{1}{K}.M$$

$$A_{st} \propto M$$

or

Or the number of bars required at any section of the beam the bending moment at the section.

Let M_c and n_c, represent the bending moment and the number of bars, (provided to resist the bending moment) respectively at the mid span of the beam.

Let M_x and n_x represent the bending moment at any distance x from the mid-span and the number of the bars which can be curtailed. The number of bars left at section x to resist the B.M. of

$$M_x = n_c - n_x$$

Since, $$n_c \propto M_c$$

and $$(n_c - n_x) \propto M_x$$

∴ $$\frac{n_c - n_x}{n_c} = \frac{M_x}{M_c}$$

Or $$1 - \frac{n_x}{n_c} = \frac{M_x}{M_c}$$

Or $$\frac{n_x}{n_c} = \frac{M_c - M_x}{M_c}$$

Now $$M_c = \frac{wl^2}{8}$$

and $$M_c = \frac{wl^2}{8}$$

$$M_x = \frac{wl^2}{8} - \frac{wx^2}{2}$$

∴ $$\frac{M_c - M_x}{M_c} = \frac{\frac{wl^2}{8} - \frac{wl^2}{8} + \frac{wx^2}{2}}{\frac{wl^2}{8}} = \frac{4x^2}{l^2}$$

or $$\frac{n_x}{n_c} = \frac{4x^2}{l^2}$$

∴ $$x = \frac{1}{2}\sqrt{\frac{n_x}{n_c}}$$

This equation gives us a relation between the number of bars which can be curtailed or bent up at any distance x, from the mid span of the beam, so that the beam remains safe from considerations of bending moment.

LESSON 24. Basic Rules for Design of Beams and Slabs

24.1 INTRODUCTION

The details given below are based on the recommendations made in IS: 456-1978.

24.2. EFFECTIVE SPAN

(a) *For simply supported beam and slab:* The effective span of a simply supported beam or slab is taken as the distance between the centre to centre of support or the clear distance between the supports plus the effective depth of the beam of slab whichever is smaller.

(b) *For continuous beam or slab:* In case of a continuous beam or slab, where the width of the support is less than 1/12 the clear span, the effective span shall be worked out by following the rule given in (a) above.

In case the supports are wider than 1/12 of the clear span or 600 mm whichever is less, the effective span shall be taken as under.

(i) For end span with one end fixed and the other continuous or for intermediate spans, the effective span shall be the clear span between supports.

(ii) For end span with one end free and the other continuous, the effective span shall be equal to the clear span plus half the effective depth of the beam or slab or the clear span plus half the width of the discontinuous support, whichever is less.

Note: In case of span with roller or rocker bearings the effective span shall always be the distance between the centres of bearings.

(c) Frames. In the analysis of a continuous frame, effective span shall be taken as the centre to centre distance between the supports.

24.3 CONTROL OF DEFLECTION/DEPTH OF BEAMS AND SLABS

It is necessary to impose a check on the magnitude of deflection in a structural member with a view to ensure that the extent of deflection does not adversely affect the appearance or efficiency of the structure or finishes or partition etc. Control on deflection is also necessary to prevent structural behavior of the member being different from the assumption made in the design. As per Code for beams and slabs, the vertical deflection limits may be assumed to be satisfied, provided that the span to depth ratio are not greater than the values obtained as below.

(a) Basic values of span to effective depth ratios for spans up to 10m.

(i)   Cantilever   7

(ii)   Simply supported   20

(iii)   Continuous   26

(b) For spans above 10m, the values in (a) may be multiplied by 10/span in metres, except for cantilever in which case deflection calculations should be made.

(c) Depending on the area and the type of steel for tension reinforcement the values in (a) or (b) shall be modified as per Fig.24.1.

(d) Depending on the area of compression reinforcement the values of span to depth ratio shall be further modified as per Fig.24.2.

(e) For flanged beams, the values of (a) or (b), be modified as per Fig.24.3 and the reinforcement percentage for use in Fig. 24.1 and Fig. 24.2 should be based on area of section equal to .

Note 1. For slabs spanning in two directions, the shorter of the two spans should be used for calculating the span to effective depth ratios.

Note 2. For two-way slabs of small spans (up to 3.5m) with mild steel reinforcement, the span to overall depth ratios given below may generally be assumed to satisfy vertical deflection limits for loading class upto 3000 N/m².

Simply supported slabs   35

Continuous slabs   40

For high strength deformed bars, of grade Fe 415, the values given above should be multiplied by 0.8.

24.4 SLENDERNESS LIMITS FOR BEAMS

To ensure lateral stability, a simply supported or continuous beam shall be so proportioned that the clear distance between the lateral restraints does not exceed 60 b or $\dfrac{250\,b^2}{d}$ whichever is less, where d is the effective depth of the beam and b, the breadth of the compression face mid-way between the lateral restraints.

For a cantilever, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed 25b or $\dfrac{100\,b^2}{d}$ whichever is less.

24.5 REINFORCEMENT IN BEAMS

24.5.1 Tension Reinforcement

(i) Minimum reinforcement: The minimum area of the tension reinforcement in beams shall not be less than that given by the following expression

$$\frac{A_s}{bd} = \frac{0.35}{f_y}$$

Where

A_s = minimum area of tension reinforcement

b = breadth of the beam or the breadth of the web of $T-beam$

d = effective depth

f_y = characteristics strength of reinforcement in N/mm^2

(ii) Maximum reinforcement: The maximum area of tension reinforcement in a beam shall not exceed 0.04 $b\,D$. Where D is the overall depth of the beam.

24.5.2 Compression Reinforcement

The maximum area of compression reinforcement in a beam shall not exceed 0.04 bD. For effective lateral restraint, the compression reinforcement in beams shall be enclosed by stirrups.

24.5.3 Side Face Reinforcement

Where the depth of the web in a beam exceeds 750 mm, side face reinforcement shall be provided along the two faces. The total area of such reinforcement shall be not less than 0.1 per cent of the web area and shall be distributed equally on the two faces at a spacing not exceeding 300 mm or web thickness whichever is less.

24.5.4 Minimum Area of Shear Reinforcement

Minimum shear reinforcement in the form of stirrups shall be provided that

$$\frac{A_{sv}}{b \times s_v} \geq \frac{0.4}{f_y}$$

where

A_{sv} = total cross sectional area of stirrups legs effective in shear

S_v = stirrups spacing along the length of the member

b = breadth of the beam or breadth of the web (b_w) in case of a flanged beam

f = characteristic strength of the stirrup reinforcement in N/mm^2

which shall not be taken greater than 415 N/mm^2.

24.5.5 Maximum Spacing of Sheer Reinforcement

Maximum spacing of shear reinforcement measured along the axis of the member shall be as under

(i)	For vertical stirrups	0.75d or 450mm whichever is less
(ii)	For inclined stirrups at 45˚	d or 450mm whichever is less

24.6 REINFORCEMENT IN SLABS

24.6.1 Minimum Reinforcement

The area of reinforcement in either direction in slabs should not be less than 0.15 per cent of the total cross-sectional area in case mild steel bars are used as reinforcement. In case of high strength deformed bars of welded wire fabric, this value can be reduced to 0.12 per cent.

24.6.2 Maximum Diameter

The maximum diameter of the reinforcing bar in a slab should not exceed $1/8^{th}$ of the total thickness of the slab.

24.7 CLEAR COVER TO REINFORCEMENT

The clear cover of concrete (excluding plaster or other decorative finish) to reinforcement in different structured members should be as under.

(a) The clear cover for tensile, compressive, shear or any other reinforcement in slab shall not be less than 15 mm or the diameter of the reinforcing bar whichever is more.

(b) The clear cover of longitudinal reinforcing bar in the beam shall not be less than 25 mm or the diameter of the reinforcing bar whichever is more.

(c) The clear cover at each end of reinforcing bar in the beam or slab shall not be less than 25 mm or twice the diameter of such bar whichever is more

(d) The clear cover for a longitudinal reinforcing bar in a column shall not be less than 40 mm or the diameter of the reinforcing bar which is more. However in case of columns having minimum dimensions of 200 mm or less, and whose reinforcing bar diameter does not exceed 12mm, a clear cover of 25 mm can be adopted.

(e) The clear cover for any other reinforcement should not be less than 15 mm or the diameter of the reinforcing bar whichever is more.

(f) In case the surface of concrete of a structural member is exposed to action of harmful chemicals, acids, vapours, saline atmosphere, sulphurous smoke etc. or concrete surface is in contact with earth contaminated with such chemicals, it is necessary to provide increased cover. The increase in cover may be between 15 mm to 50 mm over and above the values of cover specified in (a) to (e) above.

(g) For reinforced concrete members periodically immersed in a sea water, or subjected to sea spray, the cover of concrete shall be 50 mm more than specified in (a) to (e) above.

Note 1. When concrete of grade M 25 and above is used in R.C.C. work, the additional thickness of cover as specified in (f) and (h) above may be reduced to half.

Note 2. In all such cases the cover should not exceed 75 mm.

24.8. SPACING OF REINFORCEMENT

24.8.1 Minimum distance between Individual Bars

(i) The minimum horizontal distance between two parallel main reinforcing bars shall not be less than the diameter of the bar (in case of unequal diameter bars, the diameter of the larger bar is considered) or 5mm more than the nominal maximum size of coarse aggregate used in the concrete, whichever is more.

(ii) In case where it is desired to provide main bars in two or more layers one over the other, the minimum vertical clear distance between any two layers of the bars, shall normally be 15 mm or two-thirds the nominal maximum size of aggregate or the maximum size of the bar whichever is the greatest.

24.8.2 Maximum Distance between Bars in Tension

(i) The pitch of the main tensile bars in R.C. slab should not exceed three times the effective depth of the slab or 450 mm whichever is smaller.

(ii) The pitch of the bars provided to act as distribution bars or bars provided to guard against temperature and shrinkage in an R.C. slab, shall not exceed five times the effective depth of the slab or 450mm, whichever is smaller.

24.9 CURTAILMENT OF TENSION REINFORCEMENT IN FLEXURAL MEMBERS

(a) The main reinforcement in beams and slabs may be curtailed or bent up, beyond the point at which it is no longer required to resist bending. The curtailed reinforcement shall, however, extend beyond that point, for a distance equal to the effective depth of the member or 12 times the bar diameter whichever is greater except at simple supports or end of cantilever. Besides this, certain requirement regarding shear will have to be satisfied as per provision in the relevant clause in the code.

(b) Positive moment reinforcement

(i) At least one-third of the positive moment reinforcement in simply supported member and one-fourth of the positive moment reinforcement in case of continuous member should extend along the same face of the member into support to a length = /3 where = development length of the bar.

(ii) When a flexural member is part of primary lateral load resisting system, the positive reinforcement required to be extended into the support as described in (b) above shall be anchored to develop its design stress in tension at the face of the support.

(iii) At simple supports and at the point of inflection, positive moment tension reinforcement shall be limited to a diameter such that L_d computed for σ_{st} $\left(\text{by the relation } L_d = \frac{\phi \sigma_{st}}{4 \tau_{bd}}\right)$ does not exceed $\frac{M_1}{V} + L_0$

Or
$$L_d \not> \frac{M_1}{V} + L_0$$

$M_1 = $ moment of resistance of the section assuming all reinforcement at the section to be stressed at σ_{st}.

$V = $ shear force at the section due to design load.

$L_0 = $ sum of the anchorage beyond the centre of the support and the equivalent anchorage value of any hook or mechanical anchorage at simple support and at the point of inflection, L_0 is limited to the effective depth of the members or 12ϕ, whichever is greater; and

$\phi = $ diameter of bar.

The value of $\frac{M_1}{V}$ in the above expression may be increased by 30 percent when the ends of the reinforcement are confined by a compressive reaction. In case of situations where the $\frac{M_1}{V} + L_0$ works out to be less than L_d, it is possible to satisfy the requirement in the above equation by using smaller diameter of main reinforcement thereby having a reduced value of L_d. The concept of L_o is explained with the help of Fig. 24.4.

Let $c' = $ side cover to the reinforcing bar.

$x' = $ length of the bar from centre line of the support to the beginning of the hook.

$L_0 = $ sum of anchorage beyond the centre of support and the equivalent anchorage value.

In Fig. 24.4 the blackened portion of the bar shows the standard hook having an anchorage value of 16. In case of standard hook of mild steel reinforcement the anchorage value of the length of the bar between the beginning of the hook and the outer face of the hook can be taken as 3.

Let l_s be the width of the support

∴
$$L_o = x' + 16\phi$$

From Fig. 24.4
$$x' = \frac{l_s}{2} - c' - 3\phi$$

Substituting value of x' in the equation, we get
$$L_o = \left(\frac{l_s}{2} - c' - 3\phi\right) + 16\phi$$
$$= \frac{l_s}{2} - c' + 13\phi$$

In case no hook is provided (as in case of HYSD bars)
$$L_o = \frac{l_s}{2} - x'$$

(c) Negative moment reinforcement. At least one-third of the total reinforcement provided for negative moment at the support shall extend beyond the point of inflection for a distance not less than the effective depth of the member or 12 or one sixteenth of the clear span whichever is greater.

24.10 LAP SPLICE

When it is necessary to provide laps in reinforcing bars the length of lap shall not be less than the following values. The splices should be staggered and as far as possible provided away from sections of maximum stress.

24.10.1 Lap Length for Bars in Flexural Tension

The minimum lap length for bars in flexural tension including anchorage value of hooks shall be greater of the following

(i)   30 x diameter of bar $= 30\phi$

(ii) $L_d = \dfrac{Diameter\ of\ bar\ x\ actual\ tensile\ stress\ in\ bar}{4\ x\ design\ bond\ stress} = \dfrac{\phi\sigma_s}{4\ \tau_{bd}}$

The straight length of lap shall, however, not be less than 15Ø or 20 cm. If Ø be the diameter of plain m.s.round bar; be the actual tensile stress in bar; M 15 be the grade of concrete used (for which design bond stress=0.6 N/mm²), the lap length of bar for case (ii) above will be

$$= \frac{\phi \times 140}{4 \times 0.6} = 58\phi$$

Similarly if high-yield strength deformed bars are used as reinforcement; the actual tensile stress in bar taken as 230 N/mm^2 and the design bond stress for M 15 grade of concrete for HYSD bars as 0.84 N/mm^2 $\left(i.e., 0.6 + \frac{400}{100}0.6\right)$, the lap length for case (ii) above will work out to be

$$= \frac{\phi \times 230}{4 \times 0.84} = 68\phi$$

24.10.2 Lap Length for Bars in Direct Tension

The minimum lap length for bars in direct tension including anchorage value of hooks shall be greater of the following:

(i) 30Ø

(ii) 2L$_d$

24.10.3 Lap Length for Bars in Compresion

The minimum lap length for bars in compression shall be greater of the following

    (i) 24 x diameter of bar = 24 ϕ

or

    (ii)L_d compression $= \dfrac{diameter\ of\ bar\ x\ actual\ compression\ stress\ in\ bar}{5\ x\ design\ bond\ stress} = \dfrac{\phi\sigma_s}{5\ \tau_{bd}}$

24.10.4 Splicing Bars of Different Diameter

When bars of two different diameter are to be spliced, the lap length shall be calculated on the basis of the smaller diameter bar since the force to be transmitted at the slice is governed by the thinner bar.

24.11 ANCHORAGE VALUE OF BEND

If a bar in tension has its end bent to a hooked shape, the calculated development length of the bar shall be reduced by a length equal to the anchorage value of the type of hook provided. The anchorage value of standard semi-circular hook, 45˚ bend and standard L-hook is taken as 16Ø , 12Ø and 8Ø respectively of the hooked bar.

For a bar in compression, no hooks need be provided as they deprive the bar of its proper axial end bearing and also tend to cause outward buckling of the bar.

Normally, deformed bars are not provided with end hooks.

24.12 BENDING MOMENT CO-EFFICIENTS FOR BEAMS AND SLABS

The following cases are considered:

(a)   Simply supported members

(b)   Members continuous over two spans

(c)   Members continuous over three or more spans.

24.12. 1 For Simply Supported Members

In case of simply supported beams and slabs, resting on two supports or having only one span and loaded with uniformly distributed load

Max +ve B.M. is given by

$$M = +\frac{wl^2}{8}\ Nm$$

Where

w = {Sum of total dead load + imposed load (fixed) + imposed load (not fixed)} in Newton per metre.

and l = effective span of the member in metres.

24.12.2 For Members Continuous Over Two Equal or Approximately Equal Spans

In case of beams and slabs continuous for two equal or approximately equal spans (the spans are considered approximately equal when they do not differ in length by more than 15% of the longest span) and loaded with uniformly distributed load.

$$\text{Max. +ve B.M. near the centre} = +\frac{wl^2}{10} \; Nm$$

and $$\text{Max. -ve B.M. over interior support} = -\frac{wl^2}{8} \; Nm$$

When a beam or slab is built into a masonry wall at its simply supported ends, it is subjected to partial restraint and should, therefore, the designed to resist a –ve B.M. $= \frac{wl^2}{24}$ at the face of the support.

24.12.3 For Members Continuous Over three of more Approximately Equal Spans

In case of beams and slabs continuous over three or more approximately equal spans (the spans are considered approximately equal when they do not differ in length by more than 15% of the longest span) and loaded with uniformly distributed load, the bending moments at the mid-span and support can be worked out by use of the following formulae as given in IS: 456-1978.

	Near middle of end span	At middle of interior span	At support next of the end support	At other interior supports
Bending Moment due to dead load and imposed load (fixed)	$+\frac{w_d l^2}{12}$	$+\frac{w_d l^2}{24}$	$+\frac{w_d l^2}{10}$	$-\frac{w_d l^2}{12}$
Bending Moment due to imposed load (not fixed)	$+\frac{w_s l^2}{10}$	$+\frac{w_s l^2}{12}$	$+\frac{w_s l^2}{9}$	$+\frac{w_s l^2}{9}$

where

w_d = Total dead load and imposed load (fixed)

w_s = Total imposed load (not fixed)

l = effective span

24.13 SHEAR FORCE CO-EFFICIENTS FOR BEAMS AND SLABS

24.13.1 For Beams and Slabs Simply Supported over Span or Continuous for Two Spans

In case of beams and slab simply supported over one span or continuous for two spans and loaded with uniformly distributed load, the shear force is given by

$$S.F. = \frac{wl}{2}$$

Where

$w = sum\ of\ [total\ dead\ load + imposed\ load\ (fixed) + imposed\ load\ (not\ fixed)]$

$\quad\quad in\ Netwon\ per\ metre$

and $l = span\ of\ the\ member\ in\ metre.$

24.13.2 For Beams and Slabs Continuous over three or more Spans

In case of beams and slabs continuous over three or more spans which do not differ by more than 15% and loaded with uniformly distributed load, the shear force at different supports can be worked out by use of following formulae as given in IS: 456-1978.

	At end support	At support next to the end support		At all other interior supports
Shear force due to dead load and imposed (fixed)	$0.4\ w_d l$	$0.6\ w_d l$	$0.55\ w_d l$	$0.5\ w_d l$
Shear force due to imposed load (not fixed)	$0.45\ w_s l$	$0.6\ w_s l$	$0.6\ w_s l$	$0.6\ w_s l$

$\quad Where\ w_d = total\ dead\ load\ and\ imposed\ load\ (fixed)$

$\quad\quad\quad w_s = total\ imposed\ load\ (not\ fixed)$

24.14 MODULAR RATIO

The value of modular ratio m for any desired grade of concrete can be obtained by the empirical formula

$$m = \frac{280}{3\sigma_{cbc}}$$

Where σ_{cbc} is permissible compressive stress due to bending in concrete in N/mm² .

24.15 UNIT WEIGHT OF PLAIN CONCRETE AND R.C.C.

Based on recommendation in revised code, the unit weight of plain cement concrete and reinforced cement concrete shall be taken as 24000 N/m³ and 25000 N/m³ respectively.

24.16 GENERAL

1. The check for bond stress specified in the earlier code is now replaced by the concept of development length. In order to ensure development of required stresses in reinforcing bar at any section, it is necessary to extend the bar on either side of the section by appropriate development of length.

2. It is desirable to use one type of reinforcing steel bar (either plain bars or deformed bars) in the design or detailing of a member to avoid chances of error while executing the work. The secondary reinforcement like lies and stirrups can however, invariably be of mild steel even when the main reinforcement consists of HYSD bars.
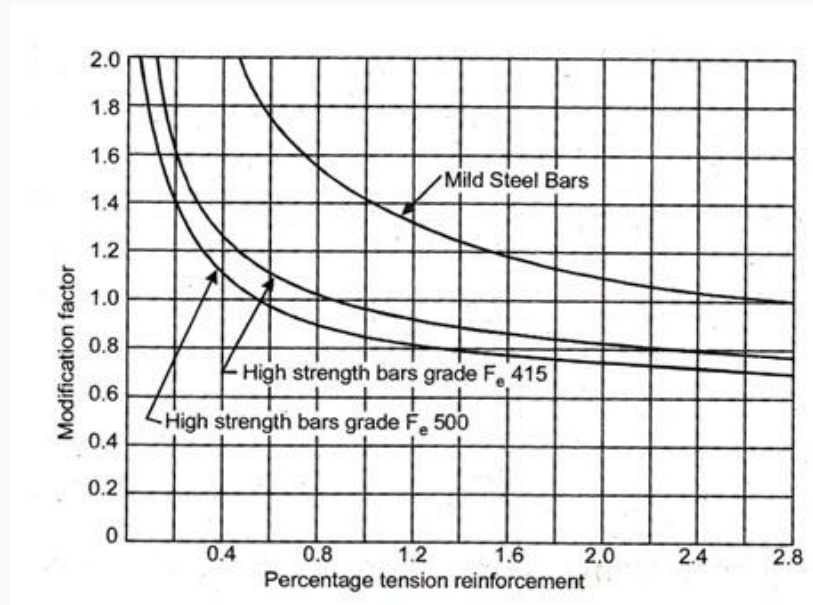
Fig. 24.1 Modification factor for tension reinforcement
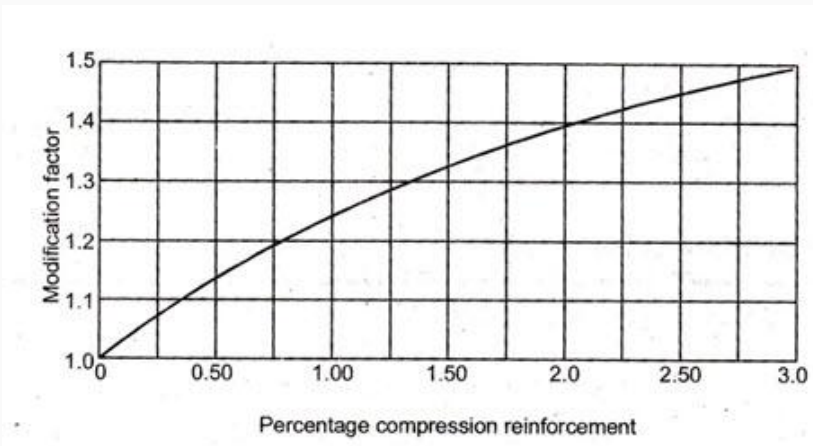
Fig. 24.2 Modification factor for compression reinforcement
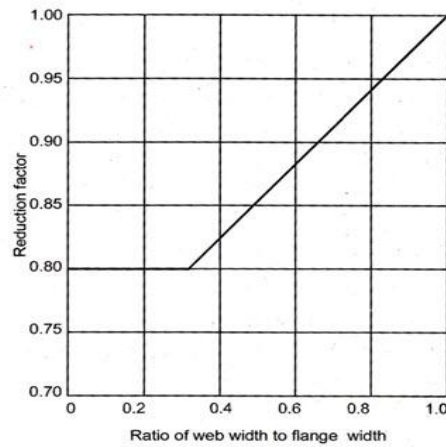
Fig. 24.3 Reduction factor for ratios of span to effective depth for flanged beams
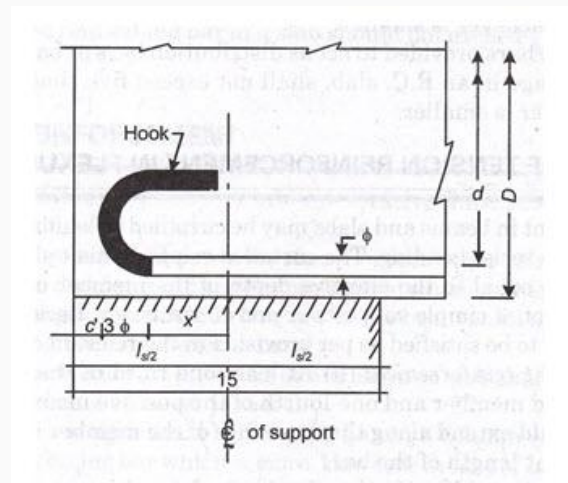
Fig. 24.4 Curtailment of tension reinforcement in flexural members

MODULE 12.

LESSON 25. Design of One Way Slabs

25.1 INTRODUCTION

For small spans say up to 3.75 m in width, which are not subjected to heavy loadings, a simple slab may suffice. When the ratio of the length of a room to its breadth is greater than 2, most of the load is carried by the short span (i.e., the width the room) and as such the slab is designed to span along the width of the room as a one way slab. In case of one-way slab, the main reinforcement of the slab, span along the width of the room while the distribution bars, laid at right angles to the main reinforcement, lie parallel to the length of the room. If the length to breadth ratio of the room is less than 2, the slab is designed as a two-way reinforced slab. In case of two-way slab main reinforcements are provided both along the length as well as along the width of the room. Depending upon the building plan, or the arrangement of main beams and/or secondary beams, the slab may have only one span, or it may be continuous over several supports.

The maximum bending moment for which the slab should be designed varies with the nature of the slab (whether one-way reinforced or two-way reinforced), loading conditions, the number of spans and the end-conditions of the slab. The bending moment which causes tension at the bottom of the slab, usually near the centre of the span, is called positive or sagging bending moment and the bending moment which causes tension at the top of the slab, usually over the supports, is called negative or hogging bending moment. Thus, for positive bending moment the main reinforcement is placed near the bottom face of the slab while for negative bending moment, the main reinforcement is placed near the top face of the slab.

25.2 LOADING ON SLABS

Loading on slabs may be in the form of uniformly distributed loads or concentrated loads or combination of the two. The slab in residential or public building or other similar structures are mostly subjected to uniformly distributed loads only. Slabs in bridges, culverts, or in other similar situations are subjected to the concentrated load on account of vehicles or trains passing over the slab. Thus, the loading on a slab normally consists of:

(a) Live loads.

(b) External dead load. (Finishing and Partitions etc.).

(c) Dead load of the slab itself.

Live loads and external dead loads may be given or these may be calculated with the help of given data, while the dead load due to self-weight of the slab has to be calculated after deciding the probable thickness of the slab vide Basic-Rule at Art. 24.3.

25.3 ARRANGEMENT OF REINFORCEMENTS IN SLABS

A standard bar bending arrangement of the designed reinforcement for a one-way slab and for a continuous slab has been shown in Fig 25.1 and Fig. 25.2 respectively.

25.4. STEPS TO BE FOLLOWED IN THE DESIGN OF ONE-WAY REINFORCED SLAB

Whatever may be the width of the slabs it should always be designed for a width of equal to one metre and the same design should be adopted for the rest of the slab. The design procedure for a uniformly loaded one-way reinforced slab can be divided in the following steps.

(1) Calculate values of the design constants i.e. k (neutral axis factor), j(lever arm factor) and R (moment of resistance factor) from the given stresses in concrete and steel reinforcement.

For

$$\sigma_{cbc} = 5 \text{ N/mm}^2$$
$$\sigma_{st} = 140 \text{ N/mm}^2$$
$$m = 19$$

The value of design constants workout to

k = 0.404

j = 0.865

and R = 0.874

(2) Assume suitable thickness or depth (D) of the slab for working out its self-weight. As a guide for total loads up to *7kN/m²*, assume slab thickness @ 40 mm/m when MS reinforcement (σ_{st} =140 N/) is to be adopted and @ 45 to 50 mm/m when HYSD reinforcement (σ_{st} =230 N/) is to be adopted. For higher values of total load the thickness of the slab to be assumed should be increased suitably.

(3) Calculate effective span for the slab vide basic rule at Art.24.2.

(4) (i) Calculate w_d – total dead load including self wt. of slab, wt. of finishing (flooring, terracing, ceiling plaster etc.) and other fixed type of imposed loads (if any).

(ii) Calculate -total live load and any other not fixed type of imposed load.

Total load for design of slab or $w = w_d + w_s$.

(5) Calculate maximum bending moment (*M*) by the governing formula vide basic rule ar Art. 24.12.

Design of Structures

(6) Calculate effective depth of the slab by considering max. B.M. anywhere in the slab by the formula.

$$d = \sqrt{\frac{M}{R \times b}}$$

Where b = 100mm

(7) Select suitable diameter (Φ) of the main bars and fix the value of overall depth (D) of the slab.

As per rule at Art. 24.6. (ii) it should be ensured that the dia. of bar is not more than th of the total thickness of the slab. If the value of overall depth(D) of the slab works out to be nearly equal to or less than the depth assumed in step(2) proceed further with the design. If on the other hand the value of (D) work out to be appreciably greater than that assumed in step (2) recalculate the self wt. of slab on the basic of revised depth [this should be taken slightly more than that worked out in step (6)] and revise the design.

(8) Calculate area of main reinforcement by the formula $A_{st} = \dfrac{M}{j.d.\sigma_{st}}$

(9) Calculate centre to centre spacing of main bars by the formula $s = \dfrac{1000 \times A_\phi}{A_{st}}$

where

s = Centre to centre spacing of bar in mm
A_ϕ = area of one bar.

Ensure that maximum spacing adopted in design should not exceed $3d$ or 400 mm whichever is smaller.

(10) Calculate area of distribution reinforcement at the rate of 0.15% of the total cross sectional area in case MS bars are used as reinforcement. In case HYSD bars or welded wire fabrics are used, the value can be reduced to 0.12%.

(11) Select suitable diameter of distribution bars and find centre to centre spacing of bars as per step (9) above.

(12) Give check for required effective depth of slab from stiffness/deflection control consideration by following the steps given below.

(i)   Calculate (p) the percentage of tension reinforcement provided $p = \dfrac{100.A_{st}}{bd}$

(ii)   Calculate value of modification factor (M.F.) corresponding to value of (p) calculated in step (i) above from graph in Fig. 24.1.

217   **www.AgriMoon.com**

(iii) Calculate required effective depth (d) from stiffness/deflection control consideration. For

a simply supported slab $$d = \frac{Span}{20 \times modification\ factor}$$

This should work out to be less than the value of effective depth adopted in design.

(13) Give check for shear by following the steps given below.

(i)   Calculate max. shear force(V) from the governing formula vide basic rule at Art. 24.13.

(ii) Calculate nominal shear stress by the formula $$\tau_v = \frac{V}{bd}$$

(iii) Calculate form Table 22.1 value of permissible shear stress (T_c) for the given grade of concrete for percentage of reinforcement provided.

(iv) Obtain value of k from Table 22.2 and work out value of permissible shear stress in slab by the formula $T_c = kT_c$

(v) Obtain the value of maximum permissible shear stress in slab ($\tau_{c\,max}$) from Table 22.3. If $\tau_v < k\tau_c$, the slab is safe in shear and requires no shear reinforcement. If $\tau_v > k\tau_c$, but less than $\frac{1}{2} \times \tau_{c\,max}$, the slab is safe in shear but shear reinforcement need be designed. If $\tau_v > \frac{1}{2} \times \tau_{c\,max}$, the slab is not safe in shear and it should be re-designed by increasing the depth.

(14) Check for development length at supports by the formula $\frac{M_1}{V} + L_o \geq L_d$

Example 25.1 Design a floor slab simply supported over a clear span of 3m. The slab is to be finished with 25 mm thick cement concrete flooring. The superimposed load on the slab is to be 3500 N per square metre. The bearing of the slab on the supporting walls may be taken as 230 mm. Adopt M 15 grade of concrete and mild steel reinforcement.

Solution Design constants. From the given data

$$\sigma_{cbc} = 5 \ N/mm^2$$
$$\sigma_{st} = 140 \frac{N}{mm^2}$$
$$m = 19$$

$\therefore$ The neutral axis factor $\quad k = \frac{n}{d} = \frac{m.\sigma_{cbc}}{m.\sigma_{cbc}+\sigma_{st}} = \frac{19 \times 5}{19 \times 5 + 140} = 0.404$

The lever-arm factor

$$j = 1 - \frac{k}{3} = 1 - \frac{0.404}{3} = 0.865$$

and the co-efficient of resisting moment $R = \frac{1}{2}\sigma_{cbc}.jk = \frac{1}{2} \times 5 \times 0.865 \times 0.404 = 0.874$

Assume overall depth of slab @ 4 cm per metre of span i.e., $4 \times 3 = 12 \ cm = 120 \ mm$

Assuming 10 mm ϕ main bars effective depth (d) $= 120 - (15 + 5) = 100 \ mm$

$\therefore$ Effective span (l) $= 3 \times 1000 + 100 = 3100 \ mm$

Load per sq. metre

(i) Self wt. of 120 mm thick slab $\quad = 0.120 \times 25000 = 3000 \ N$

(ii) Wt. of 25 mm thick flooring $\quad = \frac{25}{1000} \times 24000 = 600 \ N$

(iii) Superimposed load $\qquad\qquad\qquad = 3500 \ N$

$$Total = 7100 \frac{N}{m^2} = 7.1 \ k \ N/m^2$$

Consider a strip of slab 1 m wide.

Load per running metre $= 7.1$ kN

Max. Bending Moment $\qquad\qquad (M) = \frac{wl^2}{8} = \frac{7.1 \times (3.1)^2}{8} = 8.53 \ kNm$

$$= 8.53 \times 10^6 \ Nmm$$

Required effective depth of slab from consideration of max. B.M. is given by

$$d = \sqrt{\frac{M}{R.b}} = \sqrt{\frac{8.53 \times 10^6}{0.874 \times 1000}} = 98.79 \ mm$$

Adopt overall depth of slab (D) $\quad = 120 \ mm$

Available effective depth (d) $\qquad = 100 \ mm$

Area of steel per metre width of slab $A_{st} = \frac{M}{j.d.\sigma_{st}} = \frac{8.53 \times 10^6}{0.865 \times 100 \times 140} = 704.37 \ mm^2$

Using 10 mm ϕ bars, area of one bar $(A_\phi) = \frac{\pi}{4} \times (10)^2 = 78.54 \ mm^2$

$\therefore c/c$ spacing $(s) = \dfrac{1000 \times A_\phi}{A_{st}} = \dfrac{1000 \times 78.54}{704.37} = 111.50 \ say \ 110 \ mm \ c/c$

Which is less than 3 times the effective depth of slab, hence O.K.

Actual A_{st} provided $= \frac{1000 \times 78.54}{110} = 714 mm^2$

Bend up alternate bars at a distance of $\quad \frac{l}{7} = \frac{3100}{7} = 443 \ mm$ from the centre of support

or $443 - \frac{230}{2} = 326 \ say \ 330 mm$ from face of support.

Check for depth of slab from stiffness/deflection consideration

Percentage of tension reinforcement provided $\quad p = \frac{100 \ A_{st}}{bd} = \frac{100 \times 714}{1000 \times 100} = 0.714 \ \%$

Modification factor corresponding to 0.714% tension reinforcement as obtained from graph in Fig. 24.1 is equal to 1.6

$\therefore$ Required effective depth of slab from stiffness/deflection consideration (d)

$$d = \frac{Span}{20 \ x \ modification \ factor} = \frac{3.1 \times 1000}{20 \times 1.6} = 96.88 \ mm$$

Hence effective depth(d) = 100mm adopted in the design is in order.

Distribution reinforcement. The area of minimum reinforcement @0.15% of the total cross-sectional area of concrete

$$A_{sd} = \frac{0.15 \times b \times D}{100} = \frac{0.15 \times 1000 \times 120}{100} = 180 \ mm^2$$

Using 6 mm ϕ distribution bars:

Area of one 6 mm ϕ bar (A_ϕ) $\qquad = \frac{\pi}{4}(6)^2 = 28.3 mm^2$

c/c spacing (s) $\qquad = \frac{1000 \times A_\phi}{A_{st}} = \frac{1000 \times 28.3}{180} = 157.2 \ say \ 150 \ mm$

which is less than 5 times the effective depth of slab, hence O.K.

Check for shear:

Shear force will be maximum at the edge of the support. Its value is given by

$$V = \frac{wL}{2} = \frac{7100 \times 3}{2} = 10650 \ N$$

Nominal shear stress $\qquad \tau_v = \frac{V}{bd} = \frac{10650}{1000 \times 100} = 0.11 \ N/mm^2$

The permissible shear stress(τ_c) for M 15 grade of concrete for $\frac{100 A_{st}}{bd} = 0.704$ % and for slab 150 mm or less in overall depth works out to be $= K \ \tau_c$.

The corresponding values of τ_c and K as obtained form Table 22.1 and Table 22.2 are 0.33 N/mm^2 and 1.3 respectively.

$$\therefore K\tau_c = 1.3 \times 0.33 \ N/mm^2 = 0.43 \ N/mm^2$$

Which is more than τ_v,hence safe.

Check for development length at the support

As per code $\qquad \frac{M_1}{V} + L_o \geq L_d$

Since alternate bars are bent up, area of tensile reinforcement available at support

$$= \frac{1}{2} \times 714 = 357 mm^2$$

Now $\quad M_1 = \sigma_{st}.A_{st}.j.d = 140 \times 357 \times 0.865 \times 100 = 4.32 \times 10^6 \ Nmm$

$\quad V = 10650 \ N$

$\quad L_0 = Sum \ of \ anchorage \ beyond \ the \ centre \ of \ support \ and \ equivalent \ anchorage$
$\quad value \ at \ any \ hook \ if \ required$

The width of support or bearing of slab on supporting walls $\qquad l_s = 230$ mm

Lets the bars be bent in the form of a standard U hook at ends and let the clear end cover for the reinforcement (c') = 25 mm.

$$L_0 = \left(\frac{l_s}{2} - c' - 3\phi\right) + 16\phi = \frac{l_s}{2} - c' + 13\phi = \frac{230}{2} - 25 + 13 \times 10 = 220 \ mm$$

$$\frac{M_1}{V} + L_0 = \frac{4.32 \times 10^6}{10650} + 220 = 626 \ mm$$

$$L_d = \frac{\phi \sigma_{st}}{4\tau_{bd}} = \frac{10 \times 140}{4 \times 0.6} = 583 \ mm$$

Since $\frac{M_1}{V} + L_0 > L_d$, the requirement of development length is satisfied. The designed floor slab is shown in Fig. 25.3.

Example 25.2 Design the floor slab in example using high yield strength deformed bars (HYSD) or tor-steel as main reinforcement instead of mild steel reinforcement. The other data remaining same.

Solution Design constants. From given data

$$\sigma_{cbc} = 5 \ N/mm^2$$
$$\sigma_{st} = 230 \ N/mm^2$$
$$m = 19$$
$$k = \frac{n}{d} = \frac{m.\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = \frac{19 \times 5}{19 \times 5 + 230} = \frac{95}{325} = 0.292$$
$$j = 1 - \frac{k}{3} = 1 - \frac{0.292}{3} = 0.903$$
$$R = \frac{1}{2}\sigma_{cbc}.j.k = \frac{1}{2} \times 5 \times 0.903 \times 0.292 = 0.659$$

Assume overall depth of slab @ 47 mm per metre span when HYSD or Tor steel bars are to be used as main reinforcement.

∴ Assumed $\quad D = 4.7 \times 3 = 14.1 \ cm \ say \ 14 \ cm = 140 \ mm$

Available effective depth assuming 10 mm ϕ bars and 15 mm cover

$$d = 140 - (5 + 15) = 120 \ mm$$

∴ Effective span $\qquad l = 3 \times 1000 + 120 = 3.12 \ m$

Load per sq metre

(i) Self wt. of 140 mm thick slab $\quad = 140 \times 25000 = 3500 \ N$
(ii) Wt. of 25 mm thick flooring $\quad = 600 \ N$
(iii) Super imposed load = 3500 N $\quad = 3500 \ N$

Total $\quad = 7600 \ N/m^2 = 7.6 \ kN/m^2$

Consider a strip of slab 1m wide.

Load per running metre $\qquad = 7.6 \ kN$

Max. bending moment M $\qquad = \frac{wl^2}{8} = \frac{7.6 \times 3.12^2}{8} = 9.25 \ kNm = 9.25 \ x \ 10^6 Nmm$

Required effective depth of slab from consideration of max. B.M. is given by

$$d = \sqrt{\frac{M}{R.b}} = \sqrt{\frac{9.25 \ x \ 10^6}{0.659 \ x \ 1000}} = 118.47 \ mm$$

Hence adopt overall depth of slab (D) = 140 mm

Available effective depth (d) = 140-20 = 120 mm

Area of steel per metre width of slab $A_{st} = \frac{M}{j.d.\sigma_{st}} = \frac{9.25 \times 10^6}{0.903 \times 120 \times 230} = 371.15 mm^2$

c/c spacing of 10 mm ϕ bars ($A_\phi = 78.54 \ mm^2$)

or $\qquad s = \frac{1000 \ x \ A_\phi}{A_{st}} = \frac{1000 \times 78.54}{371} = 211 \ mm \ say \ 210 \ mm \ c/c$

which is less than 3 times the effective depth of slab, hence O.K.

Actual A_{st} provided $= \frac{1000 \times 78.54}{210} = 374 \ mm^2$

Check for effective depth of slab from stiffness/deflection consideration.

Percentage of tension reinforcement provided $\qquad p = \frac{100 A_{st}}{bd} = \frac{100 \times 374}{1000 \times 120} = 0.31 \ \%$

Modification factor corresponding to 0.31% tensile reinforcement (Tor steel 415 grade) obtained from graph in Fig. 24.1 is equal to 1.4

∴ Effective depth of slab from consideration of stiffness/deflection

$$d = \frac{span}{20 \times modificaton \ factor} = \frac{3.12 \times 1000}{20 \times 1.4} = 111 \ mm$$

Hence the effective depth d = 120mm provided in the design is in order.

Distribution reinforcement @ 0.12% of the total cross-sectional area

$$A_{sd} = \frac{0.12 \ bd}{100} = \frac{0.12 \times 1000 \times 140}{100} = 168 \ mm^2$$

Using 6mm φ distribution bars

Area of one 6mm φ bar $(A_\phi) = \frac{\pi}{4}(6)^2 = 28.3 \ mm^2$

c/c spacing(s) $= \frac{1000 \times A_\phi}{A_{st}} = \frac{1000 \times 28.3}{168} = 168.45 \ mm \ say \ 160 \ mm \ c/c$

which is less than 5 times the effective depth of slab, hence O.K.

Check for shear

Shear force will be maximum at the edge of the support. Its value is given by

$$V = \frac{wL}{2} = \frac{7600 \times 3}{2} = 11400 \ N$$

Nominal shear stress $\tau_v = \frac{V}{bd} = \frac{11400}{1000 \times 120} = 0.095 \ N/mm^2$

which is less than $k.\tau_c$ or 1.3 x permissible shear stress for M15 grade of concrete for $\frac{100.A_{st}}{bd} = 0.31 \ \%$

as obtained from Table 22.1, hence safe.

Check for development length at the support

As per code $\frac{M_1}{V} + L_0 \geq L_d$

Let alternate 10mmφ bar be bent up at a distance of $\frac{l}{7} = \frac{3100}{7} = 443 \ mm$

Say 450 mm from support.

∴ A_{st} available at support $= \frac{1}{2} \times 374 = 187 \ mm^2$

$$M_1 = \sigma_{st}.A_{st}.j.d = 230 \times 187 \times 0.903 \times 120 = 4.66 \times 10^6 \ Nm$$
$$V = 11400 \ N$$

$L_0 =$ Sum of anchorage beyond the centre of support and equivalent anchorage value of any hook.

Let the width of the support or bearing of slab on supporting wall $= l_s = 230 \ mm$

Let the bar to be provided with a standard right angle hook at ends and let the clear end cover for the main reinforcement be = 25 mm.

$$L_0 = \left(\frac{l_s}{2} - c' - (4+1)\phi\right) + 12\phi = \frac{230}{2} - 25 + 7 \times 10 = 160 \ mm$$

∴ $\frac{M_1}{V} + L_0 = \frac{4.66 \times 10^6}{11400} + 160 = 569 \ mm$

$$L_0 = \frac{\phi.\sigma_{st}}{4\tau_{bd}} = \frac{10 \times 230}{4 \times (o.6 + 0.4 \times 0.6)}$$

[∵ In case of HYSD bars τ_{bd} can be increased by 40%]

$$= 684 \ mm$$

Since $\frac{M_1}{V} + L_o < L_d$, the requirement of development length are not met and the design needs revision.

Revision. To meet the requirement of development length one of the methods is to use smaller diameter bars so that the value of L_d works out to be smaller.

Let us use 8 mm φ as main reinforcement, c/c spacing of 8 mm φ bars $(A_\phi = 50 \ mm^2)$

$$s = \frac{1000 \times A_\phi}{A_{st}} = \frac{1000 \times 50}{371} = 134 \ mm \ say \ 130 \ mm \ c/c$$

Actual A_{st} provided $= \frac{1000 \times 50}{130} = 385 \ mm^2$

[Let the bars be bent up at a distance of l/7 from support.]

$$M_1 = \sigma_{st}.A_{st}.j.d = 230 \times \frac{1}{2} \times 385 \times 0.903 \times 120 = 4.8 \times 10^6 \ Nmm$$
$$V = 11400 \ N$$
$$L_0 = \frac{230}{2} - 25 + 7 \times 8 = 146 \ mm$$
$$\frac{M_1}{V} + L_o = \frac{4.8 \times 10^6}{11400} + 146 = 567 \ mm$$
$$L_d = \frac{\phi.\sigma_{st}}{4\tau_{bd}} = \frac{8 \times 230}{4 \times (0.6 + 0.4 \times 6)} = 549 \ mm$$

Since $\frac{M_1}{V} + L_o > L_d$, the slab is safe from development length consideration.

Example 25.3 Design a cantilever slab to carry a superimposed load of . The overhang of the slab is 1.5 m. Adopt M 15 grade of concrete and mild steel reinforcement.

Solution Design constant : From the given data

$$\sigma_{cbc} = 5 \ N/mm^2, \ \sigma_{st} = 140 \ N/mm^2, \ m = 19$$
$$k = 0.404, \quad j = 0.865, \quad R = 0.874$$

Load per sq meter : Assume an average thickness of slab = 120 mm

(i) Self wt. of 120 mm thick slab = 0.12 * 25000 = 3000 N

(ii) Superimposed load   = 4000 N

                              Total = 7000 N/m^2 = 7 kN/m^2

Consider a strip of slab 1m inside

Load per running metre (w) = 7kN

Max Bending moment $\quad M = \dfrac{wl^2}{2} = 7 \times \dfrac{1.5^2}{2} = 7.875 \ kNm = 7.875 \times 10^6 \ N \ mm$

Required effective depth of slab from B.M. consideration

$$d = \sqrt{\dfrac{M}{R.b}} = \sqrt{\dfrac{7.875 \times 10^6}{0.874 \times 1000}} = 97.92 \ mm$$

Required effective depth of slab from consideration of stiffness/deflection : From stiffness/deflection consideration, effective depth (d) is given by relation.

$$d = \frac{Span}{7 \times modification\ factor}$$

For a balanced design percentage reinforcement

$$p = \frac{k.\sigma_{cbc}}{2\sigma_{st}} \times 100 = \frac{0.404 \times 5}{2 \times 140} \times 100 = 0.72\%$$

Modification factor corresponding to 0.72% tension reinforcement as obtained from graph in Fig.24.1is equal to 1.6

$$d = \frac{1.5 \times 1000}{7 \times 1.6} = 133.92\ mm\ say\ 134\ mm$$

Using 10 mm φ main bars and clear cover = 15 mm

Overall depth of slab (D) $= 134 + \frac{10}{2} + 15 = 154\ mm\ say\ 150\ mm$

Hence adopt overall depth of slab = 150 mm. The depth of slab may be reduced to 70 mm at free end.

Available effective depth of slab = 150 − 5 − 15 = 130 mm

Area of steel per metre width of slab

$$A_{st} = \frac{M}{j.d.\sigma_{st}} = \frac{7.875 \times 10^6}{0.865 \times 130 \times 140} = 500\ mm^2$$

Using 10mm φ bars, area of one bar $(A_{st} = 78.54\ mm^2)$

c/c spacing (s) $= \frac{78.54 \times 1000}{500} = 157\ mm\ say\ 150\ mm\ c/c$

Distribution reinforcement $= \frac{0.15}{100}\ b.D'$

Where D' is the average depth $= \frac{0.15}{100} \times 1000 \times \left(\frac{150 + 70}{2}\right) = 165\ mm^2$

c/c spacing of 6mm φ bars $(A_\phi = 28\ mm^2)$

$$s = \frac{28 \times 1000}{165} = 170\ mm\ c/c$$

Check for shear :

Max. shear force $\qquad V = wl = 7 \times 1.5 = 10.5\ kN$

$$\tau_v = \frac{V}{bd} = \frac{10.5 \times 1000}{1000 \times 130} = 0.06\ N/mm^2 (safe)$$

Check for development length : The 10 mm main bar must be extended into the support for distance

= L_d = 58.3 Φ = 58.3 * 10 = 583 *mm*

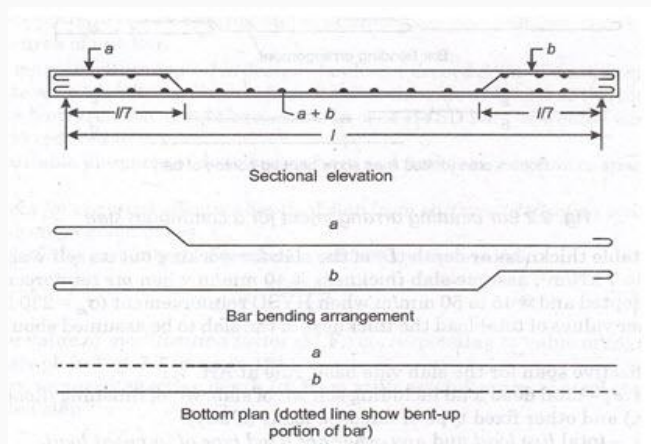The designed floor slab is shown in Fig. 25.4.

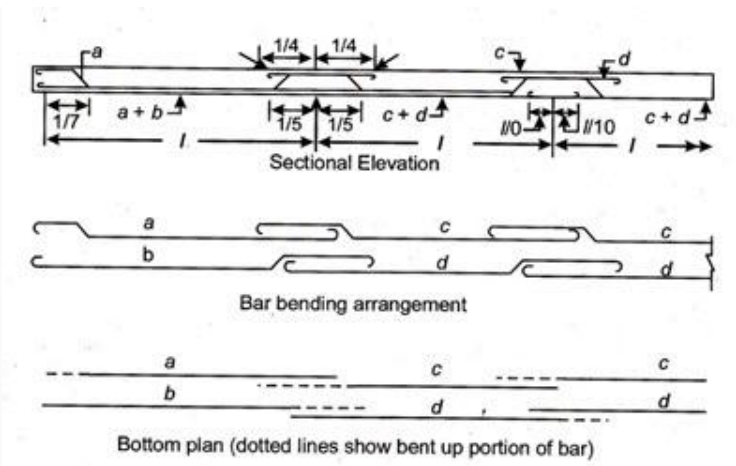Fig. 25.1 Bar bending arrangement for a one-way slab

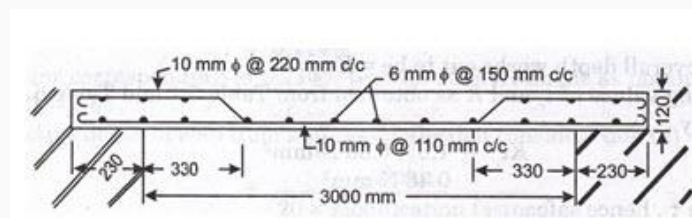Fig. 25.2 Bar bending arrangement for a continuous slab

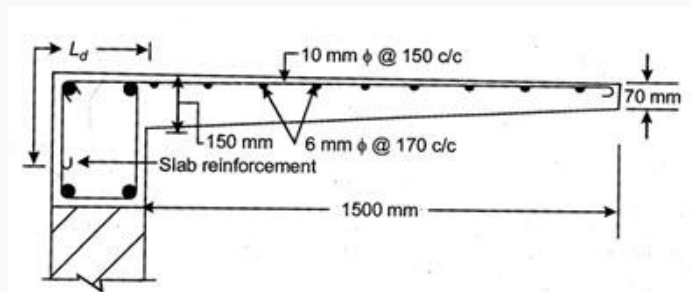Fig. 25.3 Bar bending arrangement (Example 25.1)

Fig. 25.4 Section of slab showing details of reinforcement (Example 25.3)

LESSON 26. Design of Two-way Slabs

26.1 INTRODUCTION

It is seen that one way slab supported on two opposite sides has only one plane of bending and thus the main reinforcements are provided in one direction (i.e., parallel to the plane of bending). The load from the slab in such a case is transferred to two supports. In case the slab is supported along all the four sides, it has a tendency to bend into a dished surface when loaded. Thus at any point the slab is curved in two principal directions or develops bending moment in two directions. Such a slab has to be reinforced at the bottom for tension in two direction perpendicular to each other. The load from the slab in such a case is obviously transferred on all the four supporting sides.

This type of behaviour holds good when the ratio between length and breadth of the slab is less than two. For increased ratio of sides, the slab virtually spans along the shorter side and it is designed as one way slab.

Two way slabs can be divided in following three categories:

(1) Slabs, simply supported along four edges with corners free to lift and loaded uniformly.

(2) Slabs, simply supported along four edges with corners held down and loaded uniformly.

Slabs with edges fixed or continuous and loaded uniformly.

26.2 SLABS SIMPLY SUPPORTED ALONG FOUR EDGES WITH CORNERS FREE TO LIFT AND LOADED UNIFORMLY

These slabs are commonly used as isolated roof slabs for individual rooms in single storeyed buildings. Thus the slabs are laid non-continuous on all the four edges and are not restrained by supporting walls or beams. Since the corners of the slab are not held down, no torsional reinforcements are provided in the slab. The design of such a slab can be carried out by following two methods.

(i) Rankine-Grashoff Theory method, and

(ii) I.S. Code method.

26.2.1 Rankie-Grashoff Theory Method

Let l_x and l_y represent the effective lengths of the slab along the shorter and longer spans respectively as shown in Fig. 26.1. Let w kN/m² be the design load on the slab w_x and w_y be the distribution of w acting on strips parallel to l_x and l_y respectively.

Now according to Rankine – Grashoff Theory, the load (w) is so split up into w_x and w_y that the deflection at the central point O common to both strips must be the same. The statement can be divided into two parts

i.e., $$w_x + w_y = w$$ …(i)

and $$\frac{5 w_x l_x^4}{384EI} = \frac{5 w_y l_y^4}{384EI}$$ …(ii)

Simplifying *(ii)*, we have

$$\frac{w_x}{w_y} = \left(\frac{l_y}{l_x} \right)^4$$

Let $$\frac{l_y}{l_x} = r$$

$$\frac{w_x}{w_y} = r^4 \text{ or } w_x = w_y r^4$$ …(iii)

Since $$w = w_x + w_y$$ therefore $$w_x = w - w_y$$ …(iv)

Equating (iii) and (iv), we have

$$w - w_y = w_y r^4$$ or $$w_y = \frac{1}{1 + r^4} w$$ …(v)

Similarly $$w_x = \frac{r^4}{1 + r^4} w$$ …(vi)

Having evaluated w_x and w_y, the bending moments in the slab can be worked out by the following formulae.

B.M. per metre width along short span (M_x) = $$w_x.\frac{l_x^2}{8}$$

B.M. per metre width along short span (My) = $$w_y.\frac{l_y^2}{8}$$

The bending moments in the short span being larger, govern the depth of a two way slab. The reinforcement parallel to short span (*lx*) should be placed below the reinforcement parallel to long span(*l_y*). From this arrangement of reinforcement it is seen that the effective depth of slab for long span will be smaller.

Design of slab by use of table :

Equation Nos. (v) and (vi) can be written as

$$w_y = k_y.w$$ …(vii)

$$w_x = k_x.w$$ …(viii)

where $$k_y = \frac{1}{1 + r^4} \text{ and } k_x = \frac{r^4}{1 + r^4}$$

It is obvious that the value of k_y and k_x depend upon r and the ratio of length of slab (l_y) to the breadth (l_x) of the slab. The Table 26.1 gives values of k_y and k_x for different values of $$\frac{l_y}{l_x}$$ or r.

TABLE 26.1

$\frac{l_y}{l_x} = r$	k_x	k_y	$\frac{l_y}{l_x} = r$	k_x	k_x
1.00	0.50	0.50	1.40	0.79	0.21
1.05	0.55	0.45	1.50	0.84	0.16
1.10	0.59	0.41	1.60	0.87	0.13
1.15	0.64	0.36	1.75	0.90	0.10
1.20	0.68	0.32	2.00	0.94	0.06
1.25	0.71	0.29	2.50	0.97	0.03
1.30	0.74	0.26	3.00	0.98	0.02

After knowing the value of k_x and k_y for the given ratio of $\frac{l_y}{l_x}$ or r , the bending moments in the slab can be worked out by the following formulae.

B.M. per metre width along short span (M_x) $= w_x.\frac{l_x^2}{8} = k_x.\frac{wl_x^2}{8}$ …(ix)

And B.M. per metre width along long span (My) $= w_y.\frac{l_y^2}{8} = k_y.\frac{wl_y^2}{8}$ …(x)

A review of table indicates that for the ratio of $\frac{l_y}{l_x} = 2,$ the share of total load carried by long span is only 6% and hence it is assumed that such a slab acts almost as a slab spanning in one direction i.e., along the shorter span.

26.2.2 IS Code Method

IS 456-1978 has given a simple method for designing simply supported slabs which do not have adequate provision to resist torsion at corners and to prevent the corners from lifting. In this method the maximum bending moments per unit width of slab are given by the following formulae.

B.M. per unit width along short span $M_x = { \propto _x}.wl_x^2$

B.M. per unit width along long span $M_y = { \propto _y}.wl_x^2$

where $w = total design load per unit area$

$l_x = length of the shorter span$

${ \propto _x} and { \propto _y} are the co - efficients given in Table 26.2.$

TABLE 26.2 Bending moment co-efficients for slabs spanning in two direction at right angles, simply supported on four sides.

$\frac{{l_y}}{{l_x}} = $	1.0	1.1.	1.2	1.3	1.4	1.5	1.75	2.0
${\propto _x}$	0.062	0.074	0.084	0.093	0.099	0.104	0.113	0.118
${\propto _y}$	0.062	0.061	0.059	0.055	0.051	0.046	0.037	0.029

Irrespective of the type of method adopted in design, at least 50% of the tension reinforcement provided at mid-span should be extended to the supports. The remaining 50% should extend to within $0.1{l_x}$ or $0.1{l_y}$ of the support respectively.

26.3 SHEAR FORCE ON THE EDGES OF A TWO-WAY SLAB

The total load of two-way slab loaded with uniformly distributed load and supported on all the four edges is assumed to be transmitted to the supporting walls or edge beams in the manner as shown in Fig. 26.2. The slab is assumed to be divided into two triangles and two trapeziums by lines, AP, DP, BQ and CQ drawn at 45° through the corners ABCD. The load due to the triangular portions of the slab is assumed to be carried by the edge beams or supporting walls, parallel to the width of the slab and that due to the trapezoidal portions of slab is assumed to be carried by the edge beams or walls parallel to length of the slab.

This form of distribution of load holds good both for simply-supported slabs as well as restrained two way slabs.

To find maximum shear force per unit width along short span: From the above it is seen that the total load on edge BC $ = w \times Area of \Delta BQC = w \times \frac{{l_x^2}}{4}$

Average reaction per unit width along BC $ = \frac{{wl_x^2}}{4} \times \frac{l}{{l_x}} = \frac{{w{l_x}}}{4}$

However the maximum reaction per unit width along BC will occur near centre of BC and its value may be taken as $\frac{{w{l_x}}}{3}$. Similarly the shear force per unit width along the long edge AB or CD may be taken as $w{l_x}.\frac{r}{{2 + r}}$

26.4 STEPS TO BE FOLLOWED IN THE DESIGN

The design procedure of two way slabs, simply supported along four edges with corners free to lift and loaded uniformly can be divided into the following steps:

(a) Calculate the effective spans both in respect of short span (${l_x}$) as well as long span (${l_y}$).

(b) Calculate (*w*) i.e., total design load; w=sum of total dead loaded including self wt. of slab, wt. of finishing (flooring, terracing, ceiling plaster etc.) and other fixed type of imposed loads (if any) + total live load and any other not fixed type of imposed load.

(c) Calculate bending moments per unit width along short span and long spans by the governing formulae depending upon the method of design and adopted.

(i) Based on Rankine-Grashoff Theory method

$$M_x = w_x.\frac{l_x^2}{8}$$

$$M_y = w_y.\frac{l_y^2}{8}$$

$$w_x = load per unit run along short span$$

$$w_y = load per unit run along short span$$

(ii) Based on I.S. Code method

$$M_x = \propto_x.wl_x^2$$

$$M_y = \propto_y.wl_x^2$$

$\propto_x$, $\propto_y$ being obtained from Table 26.2.

(d) Calculate the effective depth of the slab by taking bigger of the two values of the bending moments by the formula $d = \sqrt{\frac{M_{max}}{R.b}}$

(e) Calculate the area of steel per metre width along each span by the formulae :

Area of steel per metre width along short span $= \frac{M_x}{j.d.\sigma_{st}}$ sq. mm

Select suitable diameter (ϕ_x) of the bar and find their centre to centre spacing.

Area of steel per metre width along long span $= \frac{M_x}{j.\left(d - \phi_x \right).\sigma_{st}} sq.mm$

Select suitable diameter of bar and find their centre to centre spacing.

(f) Calculate distribution steel at the rate of 0.15% of the gross sectional area of the concrete if plain m.s. bars are used as reinforcement. In case high-yield strength deformed bars (HYSD) are used as reinforcement this % may be taken as 0.12%.

(g) Check for shear

(i) Calculate shear force per unit width along short edge by the formula $V = \frac{w l_x}{3}$

(ii) Calculate shear force per unit width along long edge by the formula $V = w l_x \frac{r}{2 + r}$

where r = $\frac{l_y}{l_x}$

(iii) Calculate shear force per unit width along long edge by the formula $\tau_v = \frac{V}{b.d}$

Further steps relating to check for shear are same as applicable to the design of one-way slab.

(h) Check for development length at supports by the formula $\frac{M_1}{V} + L_o \geq L_d$

Example 26.1 Design an R.C.C. floor slab for a residential building over a single room 4.5 m x 4.5 m clear inside dimensions with 1½ brick walls for support on all the sides. Use M 15 grade of concrete and mild steel reinforcement. Make usual assumptions based on I.S. codes.

Solution: Following assumptions have been made in the design.

(i) That the floor slab is finished with 25 mm thick flooring and the slab is provided with ceiling plaster to present a smooth ceiling finish.

(ii) The live load on the floor is 2000 $N/{m^2}$.

(iii) The slab is simply supported on all the four sides with corners not held down.

(iv) The bearing of the slab on supporting walls is 150 mm.

Fig. 26.3 shows the dimension of room.

Design constants

For M 15 grade of concrete and mild steel reinforcement

$\sigma_{cbc} = 5N/m{m^2}$

$\sigma_{st} = 140\frac{N}{{m{m^2}}} and m = 19$

$k = 0.404; j = 0.865 and R = 0.874$

Let the overall depth of slab = 150 mm

Assuming 10 mm ϕ main bar and clear cover = 15 mm

Effective depth of slab (*d*) = $150 - \frac{{10}}{2} - 15 = 130 mm$

Effective span shall be smaller of the following

(a) c/c of bearing i.e. 4500 + 150 = 4650 mm or

(b) Clear span + (d) = 4500 + 130 = 4630 mm

$l_x = l_y = 4630 mm$

Design of Structures

Load per square metre

(i) Due to self wt. of slab = 0.150 x 25000   =   3750

(ii) Floor finish   = 0.025 x 24000   =   600

(iii) Ceiling plaster (Lump sum)   =   160

(iv) Live load   =   2000

Total   =   6510 = 6.51

$$\frac{{l_x}}{{l_y}} = \frac{4.630}{4.630} = 1$$

Portion of the load carried by short span = Portion of load carried by long span

$$= \frac{6.51}{2} = 3.255 kN/m^2$$

Consider a strip of slab one metre wide

Load per metre (w) $= 3.225 kN/m$

Max. bending moment (M) along each span $= \frac{w l^2}{8} = \frac{3.225 x {4.63}^2}{8} = 8.72 kNm$

$$= 8.72 \times 10^6 Nmm$$

$$d = \sqrt{\frac{M}{R \times 1000}} = \sqrt{\frac{8.72 \times 10^6}{0.874 \times 1000}} = 99.9 mm$$

Required effective depth from stiffness/deflection consideration

$$= \frac{Span}{3.5} = \frac{4.63 \times 1000}{35} = 132 mm$$

Provide overall depth of slab   = 150 mm

and effective depth   = 130 mm

Area of reinforcement along one span (say l_x).

$$A_{s t_x} = \frac{M}{j.d.\sigma_{st}} = \frac{8.72 x 10^6}{0.865 \times 130 \times 140} = 554 m^2$$

c/c spacing of 10 mm ϕ bars $(A_\phi = 78.5 m^2) = \frac{78.5 \times 1000}{554} = 141.7 mm\, say\, 140 mm\, c/c$

Hence provide 10mm ϕ bars @ 140 mm c/c along one span.

Design of Structures

Bend up alternate bar at $$\frac{4630}{7} = 661.4mm$$

Say 660 mm from the centre of bearing.

Area of reinforcement along the span (or l_y) perpendicular to the above span.

$$A_{st}{}_y = \frac{M}{j \times \left(130 - 10 \right) \times 140} = \frac{8.72 \times {10}^6}{0.865 \times 120 \times 140} = 600mm^2$$

c/c spacing of 10 mm ϕ bars $= \frac{78.5 \times 1000}{600} = 131mm \, say \, 130mm \, c/c$

Actual A_{st} provided $= \frac{1000 \times 78.5}{130} = 604mm^2$

Bend up alternate bars at $l/7$ $= 660mm \, from \, the \, centre \, of \, bearing$

Distribution reinforcement $A_{sd} = \frac{0.15 \times b \times D}{100} = \frac{0.15 \times 1000 \times 150}{100} = 255mm^2$

c/c spacing of 6mm ϕ bars $(A_\phi = 28.3mm^2)$

$$= \frac{1000 \times 28.3}{225} = 125mm \, c/c \, say \, 120 \, c/c$$

check for shear:

Max. shear force per metre length on any one of the sides

$$V = \frac{w.l_x}{3} = \frac{6.51 x 4.63}{3} = 10.05kN$$

$$\tau_v = \frac{V}{bd} = \frac{10.05 \times 1000}{1000 \times 130} = 0.078N/mm^2$$

Permissible shear stress τ_c from Table 22.1 will work out to be much more than 0.078 N/mm², hence safe.

Check for development length: As per code $$\frac{M_1}{V} + L_o > L_d$$

Since alternate bars are bent up near support, area of tensile reinforcement available at support

$$= \frac{1}{2} x 604 = 302mm^2$$

Now $$M_1 = \sigma_{st}.A_{st}.j.d = 140 \times 302 \times 0.865 \times 130 = 4.75 \times {10}^6 Nmm$$

$$V = 10.05kN = 10.05 \times 1000N$$

$$L_o = \left(\frac{l_s}{2} - c' - \phi \right) + 16\phi = \frac{l_s}{2} - c' + 13\phi = \frac{150}{2} - 25 + 13 \times 10$$

$$= 180mm$$

Design of Structures

$$\frac{{M_1}}{V} + {L_o} = \frac{4.75 \times {10}^6}{10.05 \times {10}^3} + 180 = 473 + 180 = 653mm$$

$$L_d = \frac{\phi \sigma_{st}}{4 \tau_{bd}} = \frac{10 \times 140}{4 \times 0.6} = 583mm$$

Since $\frac{{M_1}}{V} + {L_o} > {L_d}$ hence safe. Fig. 26.4 shows the section at X-X.
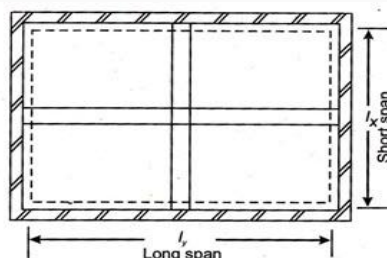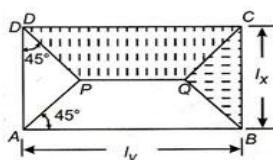

Fig. 26.1 Two way slab
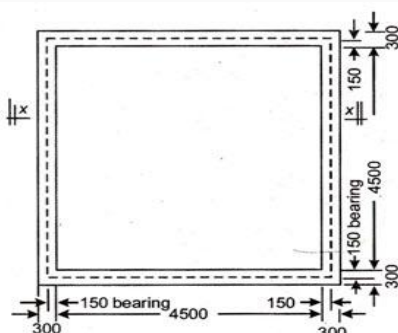

Fig. 26.2 Load taken by supporting wall or edge beam
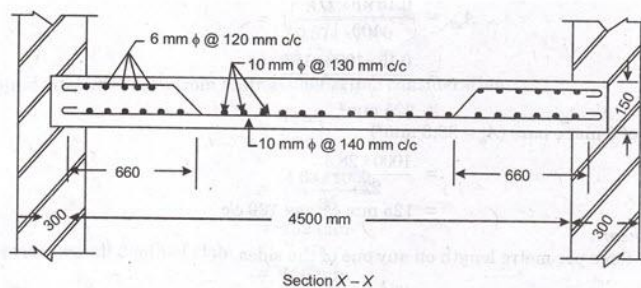

Fig. 26.3 plan of room (Example 26.1)

Fig. 26.4 Section at X-X (Example 26.1)

LESSON 27. Design of Lintel with Sunshade

27.1 INTRODUCTION

A lintel may be defined as a beam provided over an opening for door, window, cupboard etc., in a wall. It serves to bridge the gap of the opening and it permits the construction of wall above. The magnitude of load (due to structure above) carried by the lintel depends upon the location of the opening in the wall and the height of the wall above the opening. Normally the spans for door or windows are small and the height of wall between the lintel and floor/roof slab is adequate and the lintels may be subjected to only the load due to the wall above. However when the height of the wall above the lintel is inadequate, it may also be subjected to the loads due to floor or roof slab besides the weight of masonry above. The computation of load for the design of lintel can be broadly divided in the following five categories.

Case 1. When the length of wall on either side of the opening is more than half the effective span of the lintel.

Case 2. When the length of wall on one side of the opening is less than half the effective span of the lintel.

Case 3. When the length of wall on both side of the opening is less than half the effective span of lintel.

Case 4. When the wall above the lintel has openings.

Case 5. When a slab transfers load to lintel.

Case 1. When the length of wall on either side of the opening is more than half the effective span of the lintel:

In this case it is assumed that because of the arch action developed within the masonry above the opening, only the weight of masonry contained in an equilateral triangle (having base length equal to the effective span l) is transferred on the lintel. This is shown in Fig. 27.1. The load due to weight of masonry and the slab etc. outside the equilateral triangle or dispersion triangle are assumed to be dispersed to the walls on either side of the opening and are not transferred to the lintel.

Case 2. When the length of wall on one side of the opening is less than half the effective span of the lintel.

When the location of the opening is close to an end will such that the length of wall on one side of the opening is less than half the effective span of the lintel, the load transferred to the lintel is taken equal to the weight of masonry contained in a rectangle of height equal to the effective span (l). This shown in Fig. 27.2.

Case 3. When the length of wall on both sides of the opening is less than half the effective span of lintel.

When both ends of the opening are close to end walls, so that the length of wall on each side of opening is less than half the effective span of the lintel, the load transferred to the lintel is taken equal to the weight of masonry contained in a rectangle having width equal to the effective span and height equal to the full height (H) of the wall. This is shown in Fig. 27.3.

Case 4. When the wall above the lintel has openings.

If the wall above the lintel has openings for ventilators etc. and such openings are so located as to intersect the sides of the dispersion triangle, in such cases, the load transferred to the lintel is taken equal to the weight of the masonry contained in the area formed by drawing the dispersion lines at 60° from the top edges of the openings. This is shown in Fig. 27.4.

Case 5. When a slab transfers load to lintel.

If the wall above the lintel supports a floor or roof slab at a height more than the height of the dispersion triangle, the load transferred on the lintel shall be worked out as per details in Case 1. If the level of the floor/roof slab is such that distance between the top of lintel and the slab works out to be less than the height of the dispersion triangle, in such a case, the load transferred to the lintel from the structure above the opening will comprise of the following.

(i) The load (W_1) equal to the weight of masonry contained in the rectangle of height equal to the height of the slab above the lintel and base width equal to the effective span of lintel.

(ii) The load (W_2) equal to design load of slab

(iii) The load (W_3) equal to the weight of masonry contained in the equilateral triangle (with length = l) above the floor slab.

This is shown in Fig. 27.5.

Example 27.1 Design a reinforced concrete lintel over a show case window opening 2.3 metre wide. The window is to be centrally located in 230 mm thick brick wall. The height of the brick work above the lintel may be taken as 2m. A 600 mm wide sun shade is required to be cast monolithic with the lintel. Design the sun shade as well. Adopt M 15 grade of concrete and mild steel reinforcement.

Solution Design constant :

For $$\sigma_{cbc} = 5N/m{m^2}$$

$$m = 19$$

and $$\sigma_{st} = 140N/m{m^2}$$

Therefore, $$k = 0.404$$, $$j = 0.865$$, $$R = 0.874$$

Design of Structures

In order to calculate the load transferred by the sun shade to the lintel, it is necessary to design the sun shade first.

1. Design of sun shade : Assume that thickness of sun shade slab to be = 70 mm.

Loads per metre run consist of :

(i) Due to self weight   = 0.070 x 25000   = 1750 N

(ii) Live load etc.   = 1000 N

                    Total   2750 N/m

Max. B.M. = $\frac{{w{l^2}}}{2} = 2750x\frac{{{{0.6}^2}}}{2}Nm = 0.495x{10^6}Nmm$

$$d = \sqrt {\frac{{0.495x{{10}^6}}}{{0.874x1000}}}= 23.79mm$$

Effective depth(d) from stiffness/deflection consideration:

For a balanced design, percentage reinforcement is given by

$$p = \frac{{k.{\sigma _{cbc}}}}{{2{\sigma _{st}}}} \times 100 = \frac{{0.404 \times 5}}{{2x140}} \times 100 = 0.72$$

Modification factor corresponding to 0.72% reinforcement as obtained from graph in Fig. 24.1 is equal to 1.6. Therefore required effective depth of the cantilever sun shade slab from stiffness/deflection consideration is given by

$$d = \frac{L}{{7 \times Modificationfactor}} = \frac{{600}}{{7 \times 1.6}} = 53.57mm$$

Assuming 6 mm ϕ main bars, overall depth

$$D = 53.57 + 15 + \frac{6}{2} = 71.57say72mm$$

Available effective depth $= 72 - 15 - \frac{6}{2} = 54mm$

$${A_{st}} = \frac{M}{{j.d.{\sigma _{st}}}} = \frac{{0.495 \times {{10}^6}}}{{0.865 \times 54 \times 140}} = 75.69m{m^2}$$

Distribution reinforcement: Magnitude of minimum reinforcement or distribution reinforcement anywhere in the slab is given by

$${A_{sd}} = \frac{{0.15 \times b \times D}}{{100}}$$

Assuming the slab to be tapered to 50 mm at free end.

Average overall depth of slab $= \frac{{72 + 50}}{2} = 61mm$

$${A_{sd}} = \frac{{0.15 \times 1000 \times 61}}{{100}} = 91.5m{m^2}$$

Design of Structures

c/c spacing of 6 mm ϕ bars

$$\left[{A_\phi} = \frac{\pi}{4}{{\left(6 \right)}^2} = 28m{m^2} \right]$$

$$= \frac{1000}{91.5} \times 28$$

$$= 306mmc/c$$

However as per rule

c/c spacing of 6 mm main bars will be restricted to

$$= 3 \times d = 3 \times 54 = 162mmsay150mmc/c$$

and c/c spacing of 6 ϕ mm distribution bar $= 5 \times d = 5 \times 54 = 270mmsay200mmc/c$

2. Design of lintel. Assume the size of the lintel as 230 mm wide x 230 mm deep.

Let the bearing of the lintel on the end supports = 200 mm

Therefore,effective span = 2.3 + 0.20 = 2.5 m

Design loads for Lintel:

(a) To calculate load due to triangular portion of the brickwork.

Height of triangle having a base length of 2.5m and base angle of 60° each

$$= \sqrt 3 \times \frac{2.5}{2} = 2.165m$$

Load due to triangular portion of the brickwork.

Or $$W = \frac{1}{2}x2.5x2.165x0.23x19200 = 11951N$$

(b) Uniformly distributed load (w) for design consist of the following:

(i) Due to self weight of lintel $= 0.23 \times 0.23 \times 25000$ $= 1323\ N$

(ii) Due to sun shade slab $= \left\{\frac{72+50}{2}\right\} \times \frac{1}{1000} \times 0.6 \times 25000$ $= 915\ N$

Total $\quad\quad$ 2238 N

Say 2240 N/m

Max. Bending moment $$M = \frac{Wl}{6} + \frac{w{l^2}}{8}$$

$$= \frac{11951 \times 2.5}{6} + \frac{2240 \times {2.5}^2}{8}$$

$$= 4980 + 1750 = 6730Nm = 6.73x{10^6}Nmm$$

Required effective depth $$d = \sqrt {\frac{M}{R.b}}= \sqrt {\frac{6.73 \times {10}^6}{0.874 \times 230}}= 182mm$$

Design of Structures

Assuming 10 ϕ mm main bars, and clear cover of 20 mm, overall depth

$$D = {\text{}}182{\text{}} + {\text{}}20{\text{}} + {\text{}}\frac{{10}}{2} = 207mm\,say\,200mm$$

Available effective depth $= 200 - \left({20 + \frac{{10}}{2}} \right) = 175mm$

Area of steel required $A_{st} = \frac{M}{{j.d.{\sigma _{st}}}} = \frac{{6.73 \times {{10}^6}}}{{0.865 \times 175 \times 140}} = 317m{m^2}$

No. of 10 mm ϕ bars out of 4 bars are bent up near support at

$$\left[{A_\phi } = \frac{\pi }{4}{{\left({10} \right)}^2} = 79m{m^2} \right]$$

$$= \frac{{317}}{{79}} = 4Nos.$$

Minimum reinforcement: Minimum reinforcement (A_s) required for the lintel is given by the relation $\frac{{{A_s}}}{{bd}} = \frac{{0.85}}{{{f_y}}}$

Taking $\,{f_y} = 250N/m{m^2}$

$${A_s} = \frac{{0.85 \times 230 \times 175}}{{250}} = 137m{m^2}$$

Since ${A_{st}} > {A_s}$, hence safe.

Check for depth from stiffness/deflection consideration:

$$p = \frac{{100{A_{st}}}}{{bd}} = \frac{{100 \times 4 \times 79}}{{230 \times 175}} = 0.79$$

Modification factor corresponding to 0.79% tensile reinforcement as obtained from graph in Fig 24.1 is equal to 1.56

Required effective depth of lintel from stiffness/deflection considerations

$$d = \frac{L}{{20 \times Modification\,factor}}= \frac{{2.3 \times 1000}}{{20 \times 1.56}} = 74mm$$

Which is less than 175 mm, hence safe.

Adopted depth is O.K.

Check for shear : Shear force (V) will be maximum at the edge of the wall support. Its value is given by

$$V = \frac{W}{2} + \frac{{wL}}{2} = \frac{{11951}}{2} + \frac{{2240 \times 2.3}}{2}$$

$$= 5975.5 + 2576 = 8551.5N = 8552N\,say$$

Nominal shear stress ${\tau _v} = \frac{V}{{bd}}$

$${\tau _v} = \frac{{8552}}{{230 \times 175}} = 0.21N/m{m^2}$$

Assuming 2 Nos. 10mm ϕ bars out of 4 bars are bent up near support at

$$\frac{l}{7} = \frac{2.5}{7} = 0.357m$$ say 360 mm from supports

Area of tensile reinforcement available near support $= 2 \times 79mm^2 = 158mm^2$

$$p = \frac{100A_s}{bd} = \frac{100 \times 158}{230 \times 175} = 0.39$$

From Table 22.1 value of permissible shear stress (τ_c) for M 15 grade of concrete and even 0.25% reinforcement is $= 0.22N/mm^2$, which is more than τ_v $(0.21N/mm^2)$ and hence no shear reinforcement need be designed. Thus only minimum shear reinforcement will be provided.

Minimum shear reinforcement : Centre to centre spacing of vertical stirrups to meet the requirements of minimum shear reinforcement is given by

$$S_v = \frac{2.5 A_{sv}.f_y}{b}$$

Using 6mm ϕ 2 legged vertical stirrups

$$A_{sv} = 2 \times \frac{\pi}{4}\left(6 \right)^2 = 57mm^2$$

$$S_v = \frac{2.5 \times 57 \times 250}{230} = 155mm$$

Maximum spacing of shear reinforcement: Maximum c/c spacing of stirrups should not exceed least of following values

$$\left(i \right)0.75d = 0.75 \times 175 = 125mm$$

$$\left(ii \right)450mm$$

$$\left(iii \right)s_v = 155mm$$

Hence provide 6 mm ϕ 2 legged stirrups @ 125 mm c/c throughout the length of the beam. Provided 2-10mm ϕ anchor bars.

Check for development length at supports:

$$\frac{M_1}{V} + L_0 \geq L_d$$

Since 2 – 10 mm ϕ bars are available at supports as tensile reinforcement

$$A_{st} = 2 \times 79mm^2$$

$$M_1 = \sigma_{st}.A_{st}.j.d$$

$$= 140 \times 2 \times 79 \times 0.865 \times 175$$

$$= 3.348 \times 10^6 Nmm$$

$$V = 8552N$$

$${L_0} = \frac{{l_s}}{2} - c' + 13\phi$$

$$= \frac{200}{2} - 25 + 13 \times 10$$

$$= 205mm$$

$$\frac{{M_1}}{V} + {L_0} = \frac{3.348 \times {10}^6}{8552} + 205$$

$$= 392 + 205 = 597mm$$

$${L_d} = \frac{\phi {\text{\sigma }}_{\text{st}}}{\text{}}{4{\tau _{bd}}}$$

$$= \frac{10{\text{x}}140{\text{}}}{4x0.6}$$

$$= 583mm$$

Since $\frac{{M_1}}{V} + {L_0} = {L_d}$, hence O.K.
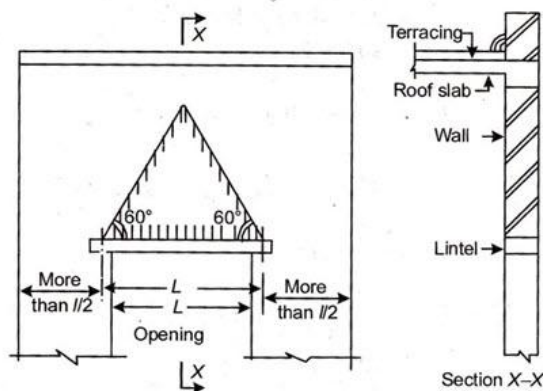
Fig. 27.1 Computation of load - Case 1
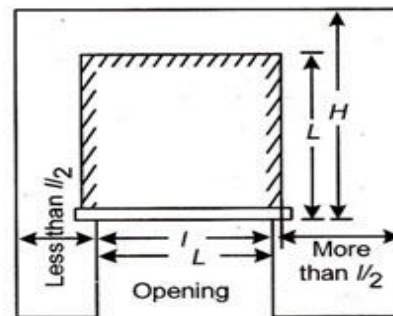
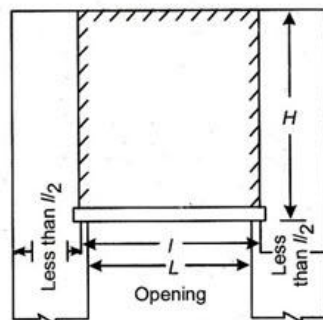
Fig. 27.2 Computation of load - Case 2
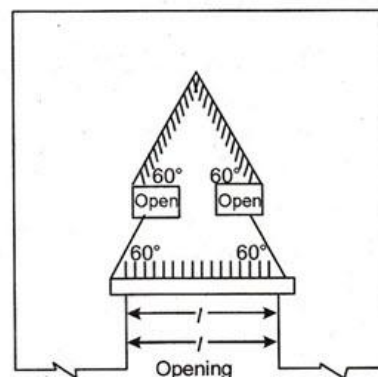

Fig. 27.3 Computation of load - Case 3

Fig. 27.4 Computation of load - Case 4

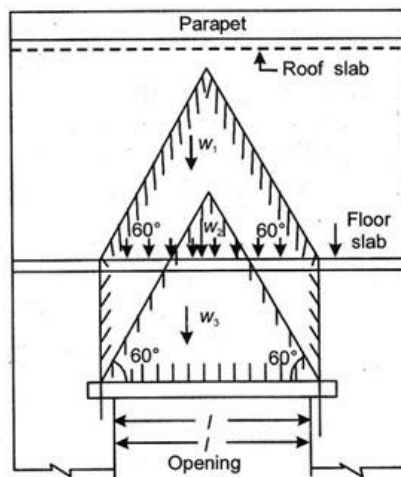Fig. 27.5 Computation of load - Case 5
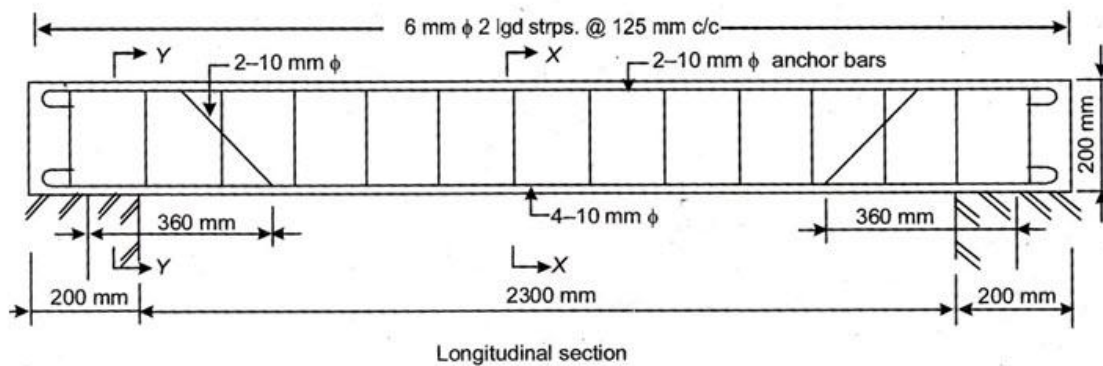

Longitudinal section

Fig. 27.6 Lintel L/s (Example 27.1)

LESSON 28. Axially Loaded RCC Columns

28.1 INTRODUCTION

A reinforced concrete column is said to be subjected to an axial load when the line of the resultant thrust of loads supported by the column is coincident with the line of C.G. of the column in the longitudinal direction. Depending upon the architectural requirements and the loads to be supported, R.C. Columns may be cast in various shapes i.e., square, rectangular, hexagonal, octagonal or circular. Columns of ell-shape or tee-shape are also sometimes used in multi-storeyed buildings. The longitudinal bars in columns help to bear the load in combination with the concrete. These bars are uniformly spaced along the perimeter of the columns as near the surface as permissible. The longitudinal bars are held in position by transverse reinforcement, or lateral binders. The binders prevent displacement of the longitudinal bars during concreting operation and also check the tendency of their buckling outwards under loads.

The transverse reinforcement or binders are of two types. Type (1) consists of separate small diameter steel binder bent around the longitudinal bars. The diameter, centre to centre spacing and the arrangement of the separate binder, depends upon the number and diameter of longitudinal bars and the size of the column. In the second type, reinforcing bar forming the tie, is wound round the longitudinal bars in the form of a closely spaced continuous helix and is termed as spiral or helical reinforcement. The helical reinforcement in addition to rendering support to longitudinal bars against buckling and displacement, also act to confine the concrete within it in the form of a core thereby increasing the load carrying capacity of the column.

Different arrangement of separate binders and helical reinforcement are shown in Fig. 28.1. The load carrying capacity of a column depends upon number of variables. The following points should be kept in view while designing a column to effect saving in cost.

1. Column with separate lateral ties works out to be cheaper than columns with spiral reinforcement.

2. Axially loaded columns with a low percentage of steel works out to be more economical per tonne of load supported than columns with a higher percentage of steel.

3. The richer the concrete, the more economical is the design.

28.2 TYPES OF COLUMNS

Columns can be broadly divided into the following three categories:

(i) Columns reinforced with longitudinal steel and lateral ties or binders.

(ii) Columns reinforced with longitudinal steel and closely spaced spirals.

(iii) Composite columns in which steel or cast iron structural member is encased in a concrete column of the type (i) or (ii) referred above.

(iv) Concrete filled steel pipe columns.

Out of the above types, columns reinforced with longitudinal steel and lateral ties or spirals are most common in use. The columns of type (iii) are recommended when the loads to be carried are extremely heavy and the dimensions of the columns are to be restricted from architectural considerations. Columns of type (iv) are used where loads to be carried are light and it is essential to provide smallest possible diameter of the column from aesthetic reasons.

28.3 EFFECTIVE LENGTH OF A COLUMN

From practical considerations the actual length (L) of a column is taken as the clear distance between the floor and the lower extremity of the capital, drop panel or slab whichever is smaller while in a beam and slab construction the actual length (L) is taken as the clear distance between the floor and the underside of the shallower beam framing into the column in each direction at the next higher floor level.

The effective length of a column depends upon the conditions of its end. For the purpose of design, only the effective length of a column is considered. Table 28.1 gives the effective length (l_{ef}) of a column is terms of unsupported length (l) for various end conditions.

TABLE 28.1 Effective length of compression members (as per IS : 456-1978)

	Degree of end restraint of compression member	Theoretical value of effective length	Recommended value of effective length
1.	Effectively held in position and restrained against rotation at both ends.	0.50 l	0.65 l
2.	Effectively held is position at both ends, restrained against rotation at one end.	0.70 l	0.80 l
3.	Effectively held in position at both ends but not restrained against rotation	1.00 l	1.00 l
4.	Effectively held in position and restrained against rotation at one end and at the other restrained against rotation but not held in position.	1.00 l	1.20 l
5.	Effectively held in position and restrained against rotation at one end and at the other partially restrained against rotation but not held in position.	---	1.50 l
6.	Effectively held in position at one end but not restrained against rotation and at the other end restrained against rotation but not held in position.	2.00 l	2.00 l

7.	Effectively held in position and restrained against rotation as one end but not held in position nor restrained against rotation at the other end.	2.00 *l*	2.00 *l*

Note. *l* is the unsupported length of compression member.

28.4 LONG AND SHORT COLUMNS

In general columns may be divided in two different categories namely (i) short columns and (ii) long columns. A column is considered to be short when the ratio of its effective length to its least lateral dimensions does not exceed 12. If the ratio of the effective length to its least lateral dimension exceeds 12, the column is considered to be a long column.

Since a long slender column buckles more easily, the ratio between the column's effective length and its least lateral dimension have definite relation with the load carrying capacity of the column. On account of its buckling tendency a long column has less strength than a short column of the same sectional area and hence can carry lesser loads as compared to short column.

Thus in long columns the maximum permissible stresses in concrete and steel are reduced by multiplying the respective stresses by a reduction coefficient C_r given by the formula

$$C_r = 1.25 - \frac{l_{ef}}{48b} \qquad \qquad \text{... (i)}$$

where

$C_r = reduction\ coefficient$
$l_{ef} = effective\ length\ of\ column$
$b = least\ lateral\ dimension\ of\ column$

Note. In case of columns having helical binders, (where permissible load is based on the area of concrete core) the least lateral dimension should be taken as the diameter of the concrete core.

Hence, the safe load that a long column can carry is obtained by multiplying the value of load which a short column of the same sectional area can carry by the reduction coefficient C_r . For more exact calculations, the maximum permissible stresses in a reinforced concrete column or part thereof having a ratio of effective column length to least lateral radius of gyration above 40 shall not exceed those which result from the multiplication of the appropriate maximum permissible stress in concrete and steel reinforcement by the co-efficient given by the formula

$$C_r = 1.25 - \frac{l_{ef}}{160.i_{min}}$$

where i_{min} is the least radius of gyration.

28.5 PERMISSIBLE STRESSES IN R.C. COLUMNS

As a result of experiments all the codes recommend reduction of stresses in concrete in direct compression as well as steel reinforcement in R.C. columns. As per revised IS : 456 – 1978, the permissible stresses for various grades of concrete and for various type of steel reinforcement to be considered in the design of column are reproduced below.

(a) Permissible stress in concrete:

	Permissible stress in compression	
	(Bending) σ_{cbc}	(Direct) σ_{st}
M 10	3.0 N/m^2	2.5 N/m^2
M 15	3.0 N/m^2	2.5 N/m^2
M 20	7.0 N/m^2	5.0 N/m^2
M 25	8.5 N/m^2	6.0 N/m^2
M 30	10.0 N/m^2	8.0 N/m^2

(b) Permissible stress in steel reinforcement : For column bars in compression (σ_{sc})

$$= 130 N/m^2 \, for \, MS \, bars$$

$$= 190 N/m^2 \, for \, HYSD \, bars$$

28.6 LOAD CARRYING CAPACITY OF DIFFERENT TYPES OF SHORT COLUMNS

The safe axial load carrying capacity of different types of short columns shall be as given below.

28.6.1 Short Columns and Pedestals with Lateral Ties

The permissible axial load (P) on a short column or pedestal reinforced with longitudinal bars and lateral ties is given by the equation $P = {\sigma_{CC}}.{A_c} + {\sigma_{sc}}.{A_{sc}}$

Where

σ_{CC} = permissible stress in concrete in direct compression

A_s = cross − sectional area of concrete excluding any finishing material and reinforcing steel

σ_{sc} = permissible compressive stress for column bars

A_{sc} = cross − sectional area of longitudinal steel.

28.6.2 Short Columns with Helical Reinforcement

Fig. 28.2 shows the short column with helical reinforcement. The permissible load for columns with helical reinforcement shall be 1.05 times the permissible load for similar member with lateral ties or rings. This provision can be made applicable only if the ratio of volume of helical reinforcement to the volume of core is not less than

$$0.36\left(\frac{{A_g}}{{A_c}} - 1 \right)\frac{{f_{ck}}}{{f_y}}$$

where

A_g = gross area of the section

A_c

= area of the core of the helically reinforced column measured to the outside diameter of the helix

f_{ck} = characteristic compressive strength of the concrete

f_y = characteristic strength of the helical reinforcement but not excedding 415 N/mm²

28.6.3 Composite Columns

The allowable axial load P on a composite column consisting of structural steel or cast-iron column thoroughly encased in concrete reinforced with both longitudinal and spiral reinforcement shall not exceed that given by the following formula

$$P = {\sigma _{CC}}.{A_c} + {\sigma _{sc}}.{A_{sc}} + {\sigma _{mc}}.{A_m}$$

Where

σ_{cc} = permissible stress in concrete in direct compression

A_c = net area of concrete section, which in equal to the gross area of concrete section; $A_{sc} - A_m$

σ_{sc} = permissible compressive stress for column bars

A_{sc} = cross − sectional area of longitudinal bar reinforcement

σ_{mc} = allowable unit stress in metal core not to exceed 125 N/mm²

for a steel core or 70 N/mm² for a cast iron core

A_m = the cross − sectional area of the steel or cast iron core

28.7 BASIC RULES FOR THE DESIGN OF COLUMNS (AS PER IS : 456-1978)

Longitudinal reinforcement:

(i) The cross-sectional area of longitudal reinforcement in a column shall not be less than 0.8% and not more than 6% of the gross cross-sectional area of the column. In places where bars from a column below have to be lapped with those in the column to be provided above, the maximum percentage of steel should preferably not exceed 4%.

The object of fixing the upper limit of 6% is to avoid such a concentration of steel as would create problems in placing and consolidation of concrete. In normal case, the designer should attempt to restrict the percentage of steel in a column to 4%.

(ii) The minimum number of longitudinal bars provided in a column shall be four in rectangular columns and six in circular columns.

(iii) A column having helical binders must have at least six bars of longitudinal reinforcement within the helical reinforcement.

(iv) The minimum diameter of the longitudinal bars shall not be less than 12 mm and the maximum diameter should preferably not exceed 50 mm.

(v) The minimum cover to the outside of longitudinal bars shall be 40 mm or the diameter of the bar whichever is more. In case where the minimum dimension of a column does not exceed 20 cm and the diameter of the longitudinal bars does not exceed 12 mm, the cover of 25 mm may be used.

(vi) Where it is necessary to splice the longitudinal reinforcement, the bars shall over-lap for a distance of not less than 24 times the diameter of the smallest bar.

(vii) If on account of architectural considerations or otherwise a column has a large cross-sectional area than that required to support the load, the minimum percentage of steel shall be based upon the area of concrete required to resist the direct stress (i.e., the cross-sectional area of column required as per design) and not upon the actual area.

(viii) In the case of pedestals in which the steel reinforcement is not taken in to account in strength calculations, nominal longitudinal reinforcement not less than 0.15% of the cross-sectional area shall be provided.

Note. Pedestal is a compression member, the effective length of which does not exceed three the least lateral dimension.

Transverse reinforcement: To safeguard the longitudinal reinforcement against buckling, the transverse reinforcement may be provided either in the form of ties or helical reinforcement (spiral).

(i) The minimum diameter of the lateral ties or helical reinforcement (spiral) shall not be less than ¼th of the diameter of the largest longitudinal bars and in no case less than 5 mm.

(ii) The maximum diameter of the ties should preferably be not more than 12 mm.

(iii) The pitch of the ties should not be more than the least of the following distances.

(a) The least lateral dimension of the column.

(b) 16 times the smallest diameter of the longitudinal reinforcement bar to be tied.

(c) 48 times the diameter of lateral tie or transverse reinforcement.

In cases where the column is assumed to take increased load on account of the continuous helical binding or spiral reinforcement, the following requirement in respect of the pitch of the helical reinforcement should be strictly followed.

(iv) The pitch of the helical turns should not be more than the least of the following distances:

(a) $\frac{1}{6}$th of the core diameter upto centre of helix.

(b) 75 mm.

(v) The least spacing of the lateral ties may be 150mm and for the spirals the minimum pitch shall be 25 mm or 3 times the diameter of the helical reinforcement member whichever is greater.

Fig. 28.3 and Fig. 28.4 shows the typical details of column splices.

28.8 ARRANGEMENT OF TRANSVERSE REINFORCEMENT

As per IS : 456-1978, the arrangement of transverse reinforcement in a compression member shall be as under:

1. If the longitudinal bars are not spaced more than 75 mm on either side, transverse reinforcement need only to go round corner and alternate bars for the purpose of providing effective lateral supports as shown in Fig. 28.5 (a).

2. If the longitudinal bars spaced at a distance of not exceeding 48 times the diameter of the tie effectively tied in two directions, additional longitudinal bars in between these bars need to be tied in one direction by open ties as shown in Fig. 28.5 (b).

3. When the longitudinal reinforcing bars in a compression member are placed in more than one row, effective lateral support to the longitudinal bars in the inner rows may be assumed to have been provided if:

(i) Transverse reinforcement is provided for the outer-most row.

(ii) No bar of the inner row is close to the nearest compression face than three times the diameter of the largest bar in the inner row as shown in Fig. 28.5 (c) and (d).

28.9 STEPS TO BE FOLLOWED IN THE DESIGN

Fig. 28.6 shows the arrangement of ties for different numbers of column bars. The various steps involved in the design of a column, with independent or separate links, are given below:

(a) Find the load the column is required to carry. Add the self-weight of column to get the total load at the column base.

(b) Decide the grade of concrete and hence the stress in concrete to be adopted in the design.

(c) Depending upon the load, assume, suitable area of reinforcement $(A_{sc}) = 1$ to 2% of gross area (A) of column. Determine approximate area (A) of the column by the formula

$$P = {\sigma _{cc}}\left({A - {A_{sc}}} \right) + {\sigma _{sc}}{A_{sc}}$$

If A_{sc} assumed = 1%$A = 0.01A$

249

Then

(d) Having found the value of A from the above equation, find out the least dimension (b) of the column. In case, it is desired to have a square column, b= $\sqrt A$. While if a circular column of ϕ (b) is desired

$$b = \sqrt {\frac{{4A}}{\pi }}$$

(e) Find the effective length (l_{ef}) , of the column from the given end conditions. If $\frac{{l_{ef}}}{b} < 12$ it becomes a case of short column. In such a case drop the steps (f) and (g) given below and then proceed.

(f) If $\frac{l}{b} > 12$ $= 12$ or it becomes a case of long column. In such a case find the reduction co-efficient C_r given by $C_r = 1.25 - \frac{{l_{ef}}}{{48b}}$

(g) Calculate the load P' for which an equivalent short column should be designed.

$$P' = \frac{P}{{C_r}}$$

Now substitute this value of load, in step (c) and calculate the final area of column and hence determine the final size of column.

(h) Calculate area of longitudinal reinforcement and choose suitable diameter of longitudinal reinforcement.

(i) Find the diameter of the bar to be used as ties, and find pitch of ties in accordance with rules.

Example 28.1

(a) A reinforced concrete column is 400 mm x 400 mm in size and has an effective length of 4500 mm. The column is reinforced with 8 Nos of 20 mm ϕ bars and the grade of concrete used in the work is M 15. Find the magnitude of safe load that such a column can carry.

(b) What will be the magnitude of safe load if the effective length of the column is increased to 8000 mm.

Solution (a) In this case $l_{ef} = 4500mm$

$$b = 400mm$$

The ratio $\frac{{l_{ef}}}{b} = \frac{{4500}}{{400}} = 11.25$

Since it is less than 12, the column is to be treated as a short column.

Load carrying capacity of a short column is given by $\text{P} = {}\sigma_{cc}.A_c + \sigma_{sc}.A_{sc}$

Design of Structures

From M 15 grade of concrete $(\sigma_{cc}) = 4\frac{N}{m{m^2}}, \sigma_{sc} = 130N/m{m^2}$

$$A_{sc} = 8.\frac{\pi}{4}\left(20 \right)^2 = 2513m{m^2}$$

$$A = 400 \times 400 = 160000m{m^2}$$

$$A_c = A - A_{sc} = 160000 - 2513 = 157487m{m^2}$$

Substituting the values in formula above, we get

$$P = \left(4 \times 157487 + 130 \times 2513 \right) = 956638N = 956.6kN$$

(b) In this case $l_{ef} = 8000mm$

The ratio $\frac{l_{ef}}{b} = \frac{8000}{400} = 20$

which is greater than 12. Hence the column is to be treated as a long column.

Reduction factor, $C_r = 1.25 - \frac{l_{ef}}{48b} = 1.25 - \frac{8000}{48 \times 400} = 0.833$

Load carrying capacity of the long column

$$= Load carryig capacity of the short column x C_r$$

$$= 956638 \times 0.833 = 797198N = 797.2kN$$

Example 28.2 Design a short R.C. column required to carry an axial load of 1500 kN. Use M 20 grade of concrete and mild steel reinforcement.

Solution The permissible stress for concrete in direct compression (σ_{cc}) for M 20 grade of concrete $= 5N/m{m^2}$

Let the area of longitudinal reinforcement (A_{sc})

$= 1\%$ of cross-sectional area (A) of the column $= 0.01A$ mm²

The load carrying capacity of a short column is given by

$$P = \sigma_{CC}.A_c + \sigma_{sc}.A_{sc} = \sigma_{CC}\left(A - 0.01A \right) + \sigma_{sc} \times 0.01A$$

$$1500 \times 10^3 = 5\left(A - 0.01A \right) + 130 \times 0.01A = 4.95A + 1.3A = 6.25A$$

or $A = \frac{1500x10^3}{6.25} = 240000m{m^2}$

One side (b) of a square column $= \sqrt A$

$$b = \sqrt{240000} = 489mm say 490mm$$

Hence provide a square column of size 490 mm x 490 mm

Area of longitudinal reinforcement $= 0.01 \times 240000 = 2400 mm^2$

Using 20 mm ϕ bars $\left(A_\phi = \frac{\pi}{4} \times {\left(20 \right)}^2 = 314 mm^2 \right)$

No. of bars required = $\frac{2400}{314} = 7.64\ say\ 8\ Nos$

Design of ties : The diameter of the ties should not be less than 1/4 the diameter of the largest longitudinal bar subject to a minimum of 5 mm.

Hence adopt dia. of ties = 5mm

The *c/c* spacing of the ties should be least of the following:

(i) least lateral dimension of column = 490mm

(ii) 16 x ϕ of longitudinal bar = 16 x 20 = 320mm

(iii) 48 x ϕ of tie = 48 x 5 = 240mm

Hence provide 5mm ϕ @ 240 mm c/c as shown in the Fig. 28.7.

Example 28.3 A 400 mm x 400 mm column 12000 mm long is restrained at both ends and is required to carry an axial load of 900 kN. Design the column using M 20 grade of concrete and mild steel reinforcement.

Solution: For M 20 grade of concrete

$(\sigma_{cc}) = 5 \frac{N}{mm^2}, \sigma_{sc} = 130 N/mm^2$

Effective length of the column $l_{ef} = 0.65 \times 12000 = 7800 mm$

The ratio $\frac{l_{ef}}{b} = \frac{7800}{400} = 19.5 > 12$

Hence the column is a long column.

Reduction factor, $C_r = 1.25 - \frac{l_{ef}}{48b} = 1.25 - \frac{7800}{48 \times 400} = 0.844$

Design load for a short column $= \frac{P}{C_r} = \frac{900}{0.844} = 1066 kN$

Hence the column can now be designed as a short column for a design load of 1066 kN.

$P = \sigma_{CC}.\left(A - A_{sc} \right) + \sigma_{sc}.A_{sc}$

$1066 \times 10^3 = 5\left(400 \times 400 - A_{sc} \right) + 130 \times A_{sc}$

$A_{sc} = 2128 mm^2$

Provide a combination of 20 mm ϕ and 18 mm ϕ bars so as to have 8 bars giving

$$A_{sc} = 2128mm^2 approx$$

Area of 20 mm ϕ bar $\left(A_\phi = \frac{\pi}{4} \times {\left(20 \right)}^2 = 314mm^2 \right)$

Area of 18 mm ϕ bar $\left(A_\phi = \frac{\pi}{4} \times {\left(18 \right)}^2 = 254mm^2 \right)$

Area provided by 4 - 20 mm ϕ + 4 - 18 mm ϕ bars $= 4 \times 314 + 4 \times 254 = 2272mm^2$

Hence provided by 4 - 20 mm ϕ + 4 - 18 mm ϕ bars

Design of ties : The diameter of the ties should not be less than 1/4 the diameter of the largest longitudinal bar or 5 mm whichever is more.

Hence adopt dia. of ties = 5mm

The *c/c* spacing of the ties should be least of the following:

(i) Least lateral dimension of column = 400 mm

(ii) 16 x ϕ of longitudinal bar $= 16 \times 18 = 288mm$

(iii) 48 x ϕ of tie $= 48 \times 5 = 240mm$

Hence provide 5 mm ϕ ties @ 240 mm c/c as shown in the Fig. 28.8.

Example 28.4 Design a short circular R.C. column to carry an axial load of 388 kN. The column is to be provided with circular lateral ties. Adopt M 20 grade of concrete and mild steel reinforcement.

Solution The load carrying capacity of a short column is given by

$$P = {\sigma_{CC}}.{A_c} + {\sigma_{sc}}.{A_{sc}}$$

Let the area of longitudinal reinforcement $({A_{sc}})$

= 2% of cross-sectional area (A) of the column = 0.02A mm²

$${A_c} = A - {A_{sc}} = A - 0.02A$$

$$P = {\sigma_{CC}}.\left(A - {A_{sc}} \right) + {\sigma_{sc}}.{A_{sc}} = {\sigma_{CC}}.\left(A - 0.02A \right) + {\sigma_{sc}}.0.02A$$

$$388 \times 10^3 = 5\left(A - 0.02A \right) + 130 \times 0.02A = 4.9A + 2.6A = 7.5A$$

or $A = \frac{388 \times {10}^3}{7.5} = 51733mm^2$

Design of Structures

Let D be the diameter of the column

$$A = \frac{\pi}{4}{D^2} = 51733$$ or   D=257 mm say 260 mm

Hence adopt diameter of the column   = 260 mm

Area of longitudinal reinforcement   = 0.02 x 51733 = 1035 mm²

Using 16 mm ϕ bar $$\left({A_\phi} = \frac{\pi}{4} \times {{\left({16} \right)}^2} = 201m{m^2}} \right)$$

No. of bars required $$= \frac{{1035}}{{201}} = 5.2 \, say \, 6 \, Nos.$$

Hence provide 6 nos. of 16 mm ϕ bars.

Design of ties : The diameter of the ties should not be less than ¼ the diameter of the largest longitudinal bar subject to a minimum of 5 mm.

In this case ¼ x 16 = 4 mm, hence provide 5mm ϕ circular ties or rings.

The *c/c* spacing of the ties should be least of the following:

(i) Least lateral dimension of column = 260 mm

(ii) 16 x ϕ of longitudinal bar   = 16 x 16 = 256 mm

(iii) 48 x ϕ of tie   = 48 x 5 = 240 mm

Hence provide 5 mm circular ties or ring @ 240 mm c/c as shown in the Fig. 28.9.
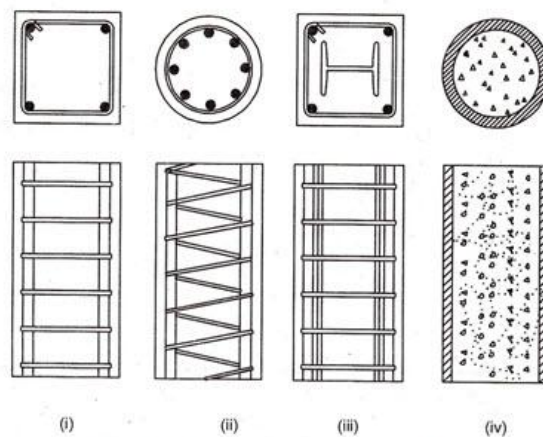
Fig. 28.1 Different types of R.C.C. columns
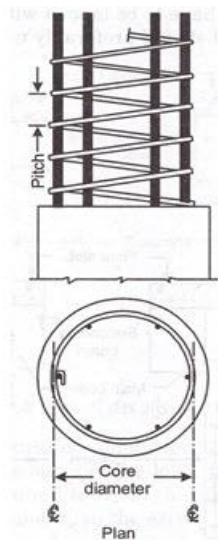
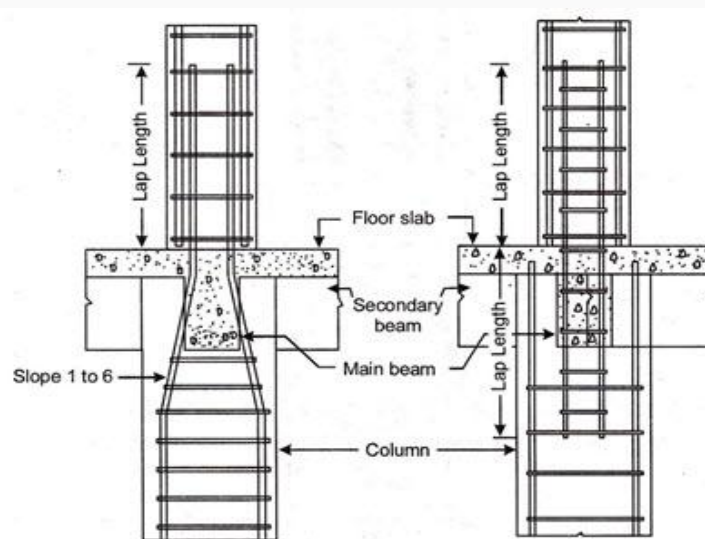Fig. 28.2 Short column with helical reinforcement

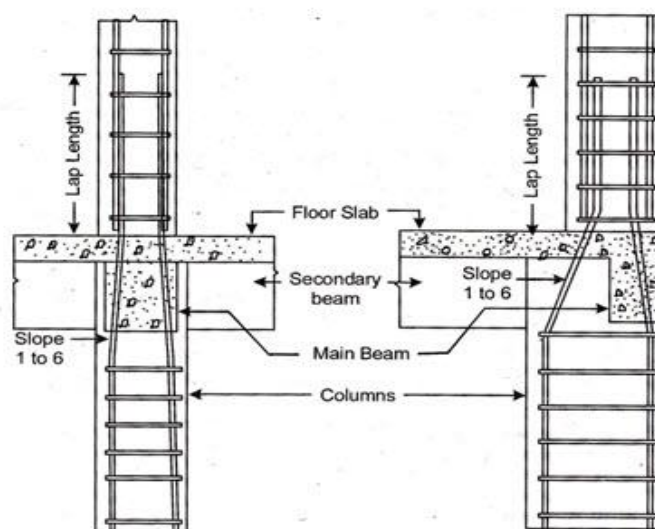Fig. 28.3 Typical details of column splices

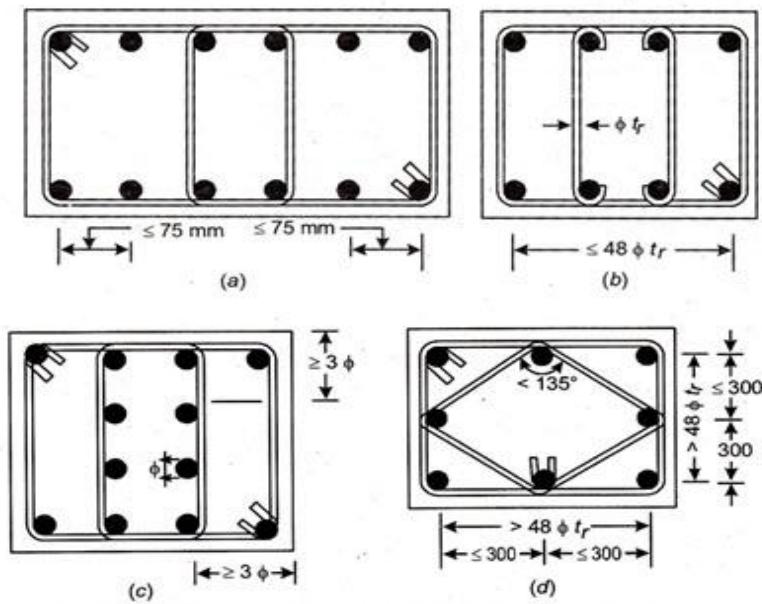Fig. 28.4 Typical details of column splices

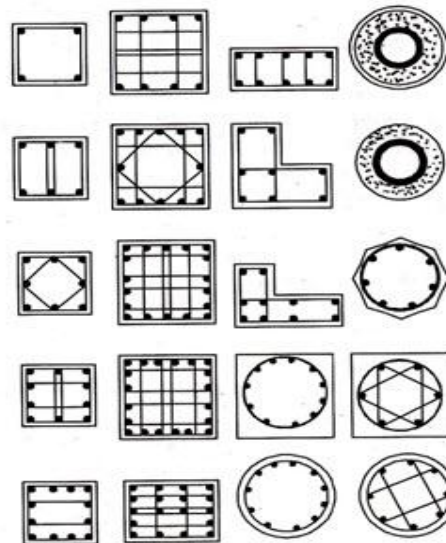Fig. 28.5 Arrangement of transverse reinforcement
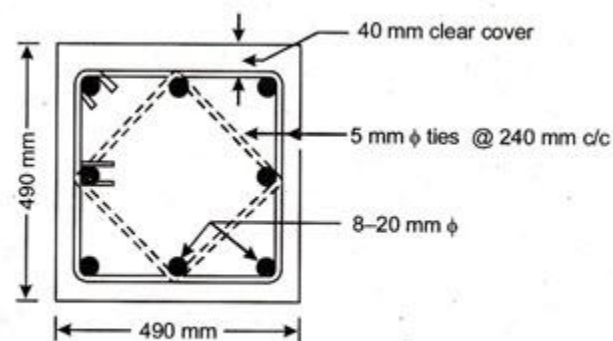

Fig. 28.6 Arrangement of ties
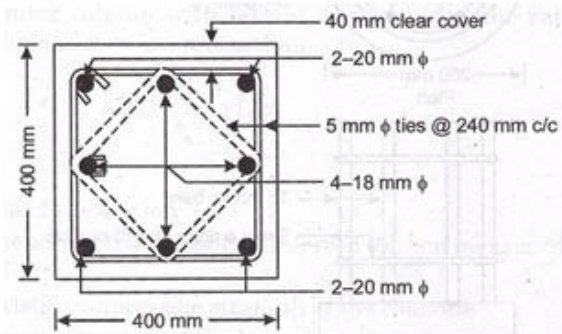

Fig. 28.7 R.C.C square column (Example 28.2)

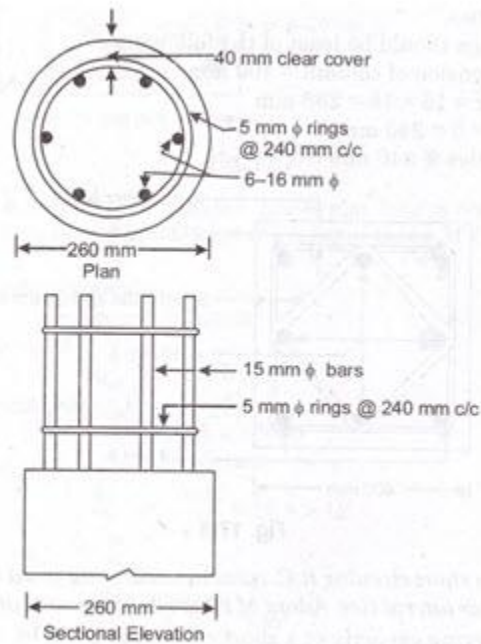Fig. 28.8 R.C.C square column (Example 28.3)

Fig. 28.9 R.C.C circular column (Example 28.4)

LESSON 29. Design of RCC footing for Wall

29.1 INTRODUCTION

Foundation is that part of a structure which transfers the load of the structure to soil on which it rests. This term includes the portion of the structure below ground level (also known as sub-structure) which provides a base for the structure above the ground (also known as super-structure) as well as the extra provisions made to transmit the loads on the structure including its self wt. to the soil below.

It is often misunderstood that the foundation is provided to support the load of the structure. In fact, it is a media to transmit the load of the structure to the sub-soil. The objectives of foundation are:

(i) To distribute the weight of the structure over larger area so as to avoid over-loading of the soil beneath.

(ii) To load the sub – structure evenly and thus prevent unequal settlement.

(iii) To provide a level surface for building operations.

(iv) To take the sub-structure deep into the ground and thus increase its stability preventing overturning.

29.2 TYPES OF FOUNDATIONS

Foundations can be broadly classified into two types:

(i) Deep foundations, and

(ii) Shallow foundations.

(i) Deep foundations: When the foundations are placed considerably below the lowest part of the super-structure it is termed as deep foundations. Pile foundations, pier foundation, well foundation, cassions etc. fall in the category of deep foundation.

(ii) Shallow foundations: When the foundation is placed immediately beneath the lowest part of the super-structure it is termed as shallow foundation. Shallow foundations can be broadly divided in the following groups:

(1) Spread footings

(2) Combined footings

(3) Mat or raft foundation.

29.2.1 Spread Footings

As the name suggest, in case of spread footings the base of the member (a column or a wall) transmitting the load is made wider so as to distribute the load over a larger area. A footing that supports a single column is known as isolated column footing. In case of a wall, the footing has to be a continuous one and hence it is known as wall footing or a continuous footing. Fig. 29.1 (a) to (e) and Fig. 29.2 (a) and (b) show different types of spread footings.

It is seen that square footing works out to be economical for square and circular columns. Under rectangular column, rectangular footings are considered to be more appropriate. In case the load on column is not large or the size of footing works out to be small requiring small depth of footing it is desirable to keep the thickness of footing uniform. In case the depth of the footing works out to be more, it is common practice to gradually reduce the depth of the footings towards the edges to achieve economy. The footing in such a case is termed as sloped footings.

29.2.2 Combined Footings

A common footing provided for two or more columns in a row is known as combined footing.

29.2.3 Mat or Raft Foundation

It is a large combined footing provided for several columns in two or more rows.

Wall footings (Spread footing) have been described in this lesson.

29.3 DESIGN CRITERIA

The basic requirements for the design of shallow foundations are:

(i) The area of the footing should be such that the maximum pressure on the soil does not exceed the safe bearing capacity of the soil.

(ii) Almost all soils get compressed under load and certain amount of settlement is bound to occur. It is necessary to ensure that the total settlement remain within permissible limits.

(iii) The foundation should be provided in such a manner that the structure does not get tilted under load. If the C.G. of the load does not coincide with the C.G. of the footing, the bearing pressure will not be uniform. In such a case there will be higher pressure on the edge of footing nearer to the C.G. of the load which will cause greater settlement of soil at the edge and this can result in tilting of foundation. This can be avoided by providing the footing area in such a manner that the C.G. of the load coincides with the C.G. of the footing.

(iv) The depth at which the foundation should be located depends on the character of the sub-soil and the magnitude of load on the structure. However the foundation must be carried below:

Design of Structures

(a) The depth of frost penetration in region of temperate climate.

(b) The depth at which high volume change in soil due to moisture fluctuation do not cause any adverse effect.

(c) Below the depth of unconsolidated material like muck, garbage dumps, and similar type of made up ground.

(d) The minimum depth prescribed by the local authoricy.

As per IS : 1080- the minimum depth of foundation should not be less than 500 mm. However if good rock is available at smaller depth, only removal of soil may be sufficient for placement of footing.

29.4 DEPTH OF FOUNDATION

For all important buildings it is necessary to get the soil investigation of the site carried out by specialist agency. The test report should contain details regarding the type of sub-soil strata at various depths, depth of water table, and recommendation regarding the bearing capacity of soil at different depths. For normal buildings the depth of foundation below ground level is commonly calculated by the Rankine's formula.

According to Rankine's formula the minimum depth of foundation is given thus:

$$D_f = \frac{P_o}{\gamma} \left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2$$

Where
D_f = minimum depth of foundation in metres
P_0 = bearing capacity of the soil in kN/m^3
γ = density of soil or the unit weight of soil in kN/m^3
ϕ = the angle of repose of the soil.

29.5 PRESSURE DISTRIBUTION UNDER FOOTINGS

The theory of elasticity analysis as well as the actual observations indicates that the pressure distribution under symmetrically loaded footings is not uniform. The actual stress distribution depends upon the nature of subsoil strata and the rigidity of the footings.

When a rigid footing is placed on loose cohesion-less soil, due to the load transmitted by the footing the soil grains at the edges having no lateral restraint displace laterally and in the centre the soil remain relatively confined. The pressure distribution is such a case is as shown in Fig. 29.3 (a).

On the other hand in case of rigid footing on cohesive soils, the load transmitted by the footing causes very large pressure at the edges and the parabola pressure distribution under the footing in such a case is as shown in Fig. 29.3 (b).

However to simplify the analysis the pressure distribution beneath the footings is assumed to be linear as shown in Fig. 29.3 (c). The design based on this assumption compare fairly well

with results of actual studies made in respect of pressure under existing foundations and hence linear pressure distribution is considered to be acceptable.

29.6 ANALYSIS AND DESIGN OF FOOTINGS

The analysis and design of footings can be broadly divided in the following steps.

(a) Determination of the area of footing.

(b) Determination of bending moments and shears at critical section and fixing the depth of footing.

(c) Determination of the area of reinforcement.

(d) Check for development length at critical section.

The area of the footing is worked out based on the load on the member including self wt. of footing and the bearing capacity of the soil. The calculations for bending moment, shear force, development length etc. are made based on provision in IS code. The various recommendations made in IS: 456-1978 for design of footing are given below.

1. General. (i) Footings shall be designed to sustain the applied loads, moments and forces and the induced reactions and to ensure that any settlement which may occur will be as nearly uniform as possible and the safe bearing capacity of the soil is not exceed.

(ii) Thickness at the edge of footing: In reinforced and plain concrete footings, thickness at the edges shall be not less than 150 mm for footings on the soils, nor less than 300 mm above the tops of piles for footings on piles.

2. Moments and forces. (i) In the case of footings on piles, computation for moments and shears may be based on the assumption that the reaction from any pile is concentrated at the centre of the pile.

(ii) For the purpose of computing stresses in footing which support a round or octagonal concrete column or pedestal, the face of the column or pedestal shall be taken as the side of a square inscribed within the perimeter of the round or octagonal column or pedestal.

3. Bending moment. (i) The bending moment at any section shall be determined by passing through the section a vertical plane which extends completely across the footing and computing the moments of the forces acting over the entire area of the footing on one side of the said plane.

(ii) The greatest bending moment to be used in the design of an isolated concrete footing which supports a column, pedestal or walls shall be the moment computed in the manner prescribed in Art. 3(i) at sections located as follows:

(a) At the face of the column, pedestal or wall for footings supporting a concrete column, pedestal or wall.

(b) Half way between the centre line and the edge of the wall, for footing under masonry walls.

(c) Half way between the face of the column or pedestal and the edges of the gusseted base for footings under gusseted bases.

4. Shear and bond. (i) The shear strength of footings is governed by the more severe of the following two conditions.

(a) The footing acting essentially as a wide beam, with a potential diagonal crack extending in a plane across the entire width; the critical section for the condition shall be assumed as a vertical section located from the face of the column pedestal or wall at a distance equal to the effective depth of the footing in case of footing on soils, and a distance equal to the half the effective depth of footing for footings on piles.

(b) Two-way action of the footing with potential diagonal cracking along the surface of truncated cone or pyramid around the concentrated load, in this case the footing shall be designed for shear in accordance with appropriate provision specified.

(ii) The critical section for checking the development length in a footing shall be assumed at the same plane as those described for bending moment in Art. 3 and also at all other vertical planes where abrupt changes of section occur. If the reinforcement is curtailed, the anchorage requirement shall be checked in accordance with provision.

5. Tensile reinforcement. The total reinforcement at any section shall provide a moment of resistance at least equal to the bending moment on the section calculated in accordance with Art. 3.

(i) In one-way reinforced footing the reinforcement shall be distributed uniformly across the full width of the footing.

(ii) In two-way reinforced square footing the reinforcement extending in each direction shall be distributed uniformly across the full width of the footing.

(iii) In two-way reinforced rectangular footing, the reinforcement in the long direction shall be distributed uniformly across the full width of the footing. For reinforcement in the short direction, a central band equal to the width of the footing shall be marked along the length of the footing and portion of the reinforcement determined in accordance with equation given below shall be uniformly distributed across the central band:

$$\frac{Reinforcement\ in\ central\ band\ width}{Total\ reinforcment\ in\ short\ direction} = \frac{2}{\beta + 1}$$

where β is the ratio of the long side to the short side of the footing. The remainder of the reinforcement shall be uniformly distributed in the outer portions of the footing.

6. Transfer of load at the base of column. The compressive stress in concrete at the base of a column or pedestal shall be considered as being transferred by bearing to the top of the supporting pedestal or footing. The bearing pressure on the loaded area shall not exceed the permissible bearing stress in direct compression multiplied by a value = $\sqrt{\dfrac{A_1}{A_2}}$ but not greater than 2.

Where A_1 supporting area for bearing of footing, which in sloped or stepped footing may be taken as the area of the lower base of the largest frustum of a pyramid or cone contained wholly within the footing and having for its upper base, the area actually loaded and having side slope of one vertical to two horizontal

A_2 = Loaded area at the column base

For working stress method of design the permissible bearing stress (σ_{cbc}) on full area of concrete shall be taken as 0.25 f_{ck} .

Hence the permissible bearing stress in concrete (σ_{cbr}) = 0.25 f_{ck}

The actual bearing pressure or bearing stress $= \dfrac{w}{a \times b}$

It has to be ensured that $\dfrac{w}{a \times b}$ should not exceed , (a) and (b) being the dimensions of the column.

(i) Where the permissible bearing stress on the concrete in the supporting or supported member would be exceeded, reinforcement shall be provided for developing the excess force, either by extending the longitudinal bars into the supporting member of by dowels.

(ii) Where transfer of force is accomplished by reinforcement, the development length of the reinforcement shall be sufficient to transfer the compression or tension to the supporting member.

(iii) Extended longitudinal reinforcement or dowels of at least 0.5 per cent of cross-sectional area of the supported column or pedestal and a minimum of four bars shall be provided. Where dowels are used their diameter shall not exceed the diameter of the column bars by more than 3 mm.

(iv) Column bars of diameter larger than 36 mm, in compression only can be dowelled at the footings with bars of smaller size of the necessary area. The dowel shall extend into the column, a distance equal to the development length of the column bar and into the footing, a distance equal to the development length of the dowel.

29.7 DESIGN OF MASONRY WALL FOOTING

Walls can be of masonry or concrete. The loading on the wall is considered to be uniform at the foundation level. The footing for the wall being continuous and subjected to uniform

pressure all along its length as shown in Fig. 29.4. Only one metre wide strip of footing slab is designed and the same design is made applicable to the remaining length of the footing.

Let   B = width of footing in metre

b = width of wall in metre

p_0 = safe bearing capacity of soil in kN/

W = Load from wall in kN/m and

W_1 = weight of the footing in kN/m

∴ The width of footing for the wall is given by $B = \frac{w + w_1}{P_0}$

Net upward soil pressure (p) on the footing is given by $p = \frac{W}{B \times 1} = \frac{W}{B}$

The intensity of pressure (p_1) at the base of masonry is given by $p_1 = \frac{W}{b \times 1} = \frac{W}{b}$

The footing is designed as a cantilever projecting out by a distance of $\frac{B-b}{2}$ from the face of wall and having a fixed support length of $\frac{b}{2}$ on either side of the centre line of the wall. As per code, for masonry walls, the section for maximum B.M. is considered to be located midway between centre of the wall and the edge of the wall. Maximum B.M. per metre length of the wall is given by

$$M = p \times \left(\frac{B-b}{2} + \frac{b}{4}\right) \times \frac{1}{2}\left(\frac{B-b}{2} + \frac{b}{4}\right) - p_1 \times \frac{b}{4} \times \frac{b}{4} \times \frac{1}{2}$$

$$= p \cdot \frac{1}{2}\left[\frac{B-b}{2} + \frac{b}{4}\right]^2 - p_1 \times \frac{1}{2} \cdot \frac{b^2}{16}$$

$$= \frac{W}{b} \cdot \frac{1}{2}\left[\frac{2B-b}{2}\right]^2 - \frac{W}{b} \cdot \frac{1}{2} \cdot \frac{b^2}{16}$$

On further simplification, we get

$$M = \frac{W}{8b}(B - b)\left(B - \frac{b}{4}\right) kNm = \frac{p}{8}(B - b)\left(B - \frac{b}{4}\right) kNm$$

The effective depth of the footing is given by $d = \sqrt{\frac{M}{R \times 1000}}$

As per code, the minimum thickness of the footing at the edge should not be less than 150 mm.

The area of tensile reinforcement is given by $A_{st} = \frac{M}{jd \cdot \sigma_{st}}$

In addition to tensile reinforcement it is necessary to provide longitudinal reinforcement in the footing.

Area of longitudinal reinforcement = 0.15% of sectional area of footing for mild steel reinforcement and 0.12% of sectional area of footing for HYSD bars.

Check for shear: The critical section for shear is considered to be located at a distance of effective depth from the face of the wall.

Check for development length: It should be ensured that the length of the bars provided as tensile reinforcement is not less than beyond the critical section for max. B.M.

Example 29.1 Design a R.C.C. footing for a 300 mm thick brick wall carrying a load of 120 kN per metre length of the wall. The safe bearing capacity of soil is 90 kN/m². Use M 15 grade of concrete and using HYSD reinforcement.

Solution Design constants :

For
$$\sigma_{cbc} = 5 \ N/mm^2, \ m = 19$$
and
$$\sigma_{st} = 230 \ N/mm^2$$
$$k = 0.292, j = 0.903, R = 0.659$$

Load per metre consist of the following:

    (i) Load carried by wall (W)   = 120 kN

    (ii) Self wt. of footing @ 10% of (W) assumed   = 12 kN

                                   Total   = 132 kN

Width of footing or (B) $= \frac{132}{p_o} = \frac{132}{90} = 1.467 \ m$ say 1.5 m

Net upward pressure (p) $= \frac{120}{1.5} = 80 \ kN/m^2$

    Max. B.M. : The section for max. B.M. is considered to be located midway between centre of wall and the edge of the wall. Its value is given by

$$M = \frac{p}{8}(B - b)\left(B - \frac{b}{4}\right)$$
$$= \frac{80}{8}(1.5 - 0.3)\left(1.5 - \frac{0.3}{4}\right) = 17.1 \ kNm = 17.1 \times 10^6 \ Nmm$$

    Required effective depth of footing is given by

$$d = \sqrt{\frac{M}{R \times 1000}} = \sqrt{\frac{17.1 \times 10^6}{0.659 \times 1000}} = 161 \ mm$$

Using 10mmɸ HYSD bars and a clear cover of 50 mm

    Overall depth of slab $= 161 + \frac{10}{2} + 50 = 216 \ mm$ say 220 mm

∴ Available effective depth $d = 220 - 50 - 5 = 165 \ mm$

Area of tensile reinforcement $A_{st} = \frac{M}{j.d.\sigma_{st}} = \frac{17.1 \times 10^6}{0.903 \times 165 \times 230} = 499 \ mm^2$

c/c spacing using 10mm ɸ HYSD bars $(A_\phi = \frac{\pi}{4}(10)^2 = 78.5 \ mm^2)$
$$= \frac{78.5 \times 1000}{499} = 157 \ mm$$ say 150 mm c/c

$$A_{st} \ provided = \frac{78.5 \times 1000}{150} = 523 \ mm^2$$

    Percentage reinforcement $= \frac{100 \times 523}{1000 \times 165} = 0.32$

Longitudinal reinforcement:

@ 0.12% of the area of concrete $= \frac{0.12}{100} \times 1000 \times 220 = 264 \ mm^2$

c/c spacing using 8mm ɸ bars $(A_\phi = \frac{\pi}{4}(8)^2 = 50.26 \ mm^2)$
$$= \frac{50.26 \times 1000}{264} = 190.4 \ mm$$ say 190 mm c/c

Check for shear : Critical section for shear lies at a distance of d from the face of the wall.

Shear force at the critical section

or $V = 80 \times (0.6 - 0.165) = 34.8 \ kN = 34.8 \times 10^3 \ N$

Nominal shear stress, $\tau_v = \frac{34.8 \times 10^3}{1000 \times 165} = 0.21 \ N/mm^2$

which is less than value of τ_c corresponding to even 0.25% reinforcement, hence safe.

Check for development length: $L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$

In the case of HYSD bar, the value of τ_{bd} should be increased by 40%.

∴ $L_d = \frac{\phi \times 230}{4\left(0.6 + \frac{40}{100} \times 0.6\right)} = 68.45 \ \phi = 68.45 \times 10 = 684 \ mm$

Providing a side cover of 50 mm the available straight length of bar beyond critical section for bending is

$$= \frac{1}{2}(B - b) - 50 = \frac{1}{2}[1500 - 300] - 50 = 550 \ mm$$

Since it works out to be less than L_d, it will be necessary to bend the bar at 90° up at the edge for a length of 6ɸ i.e., $6 \times 12 = 72 \ mm$

Thus total length available to meet the requirement of

$$L_d = \text{Straight length of bar} + \text{Anchorage value of } 90° \text{ band}$$
$$= 550 \; mm + 12 \times 12 = 694 \; mm > L_d, hence \; safe$$

The design diagram is shown in Fig. 29.5.

(a) Plan   (b) Plan   (c) Plan
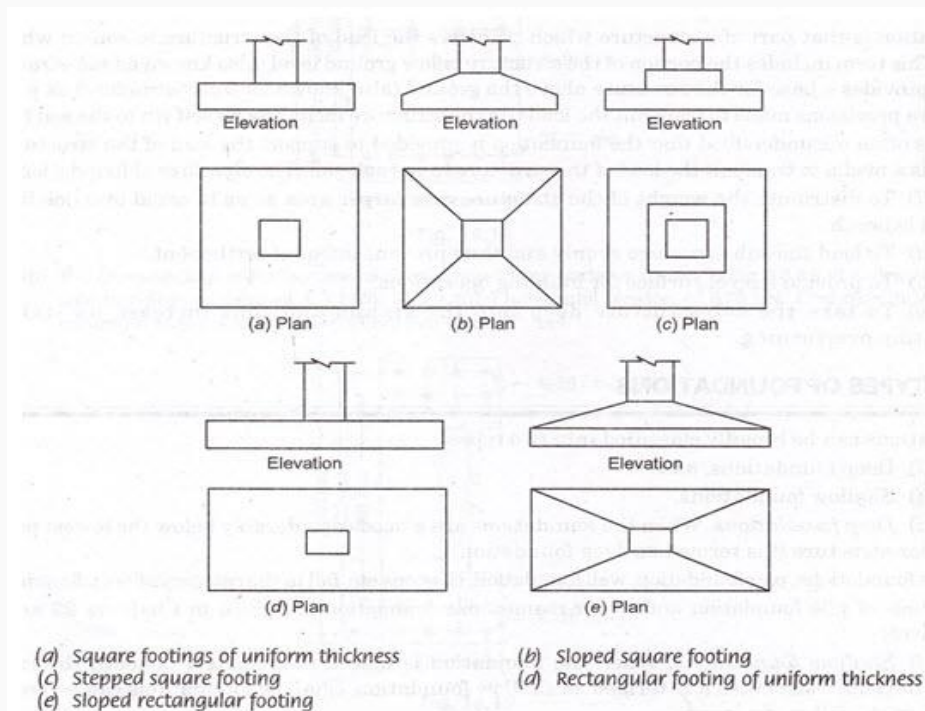
(d) Plan   (e) Plan

(a) Square footings of uniform thickness   (b) Sloped square footing
(c) Stepped square footing   (d) Rectangular footing of uniform thickness
(e) Sloped rectangular footing

Fig. 29.1 Types of spread footings

(a) Plan   (b) Plan

(a) Wall footing
(b) Circular footing

Fig. 29.2 Spread footings

(a) Cohesion less or sandy soil   (b) Cohesive less or clayey soil   (c) Uniform pressure (assumed condition)
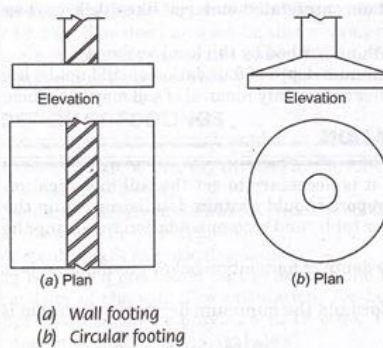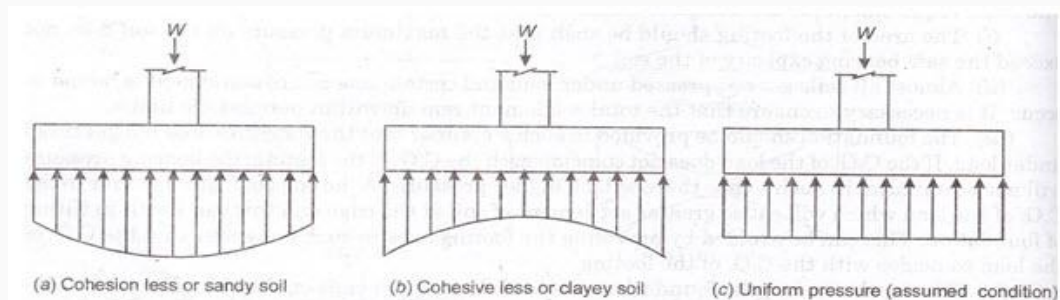
Fig. 29.3 Pressure distribution under footings
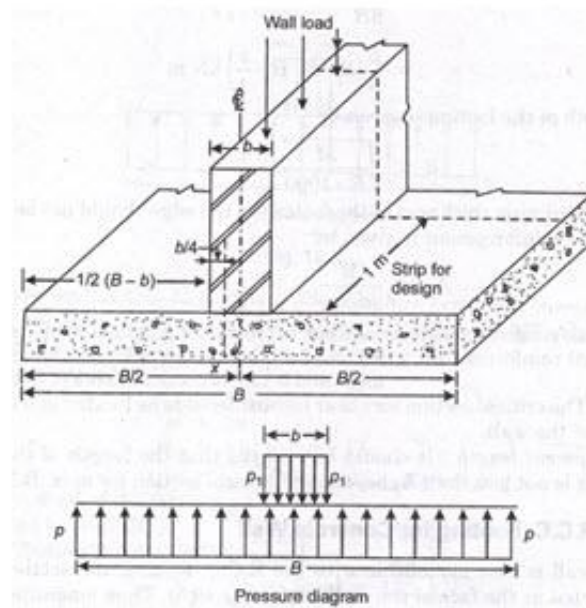
Fig. 29.4 RCC footing for masonry wall
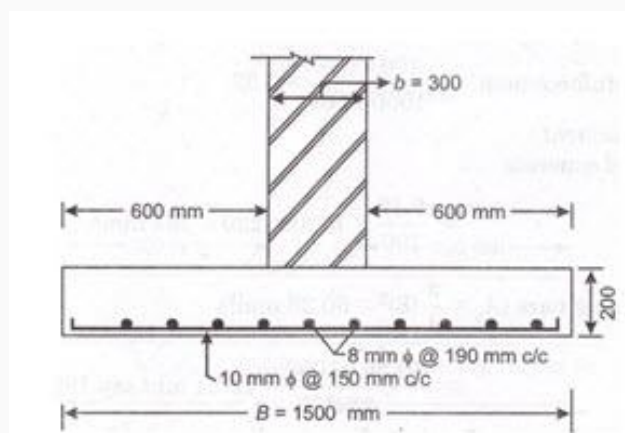
Fig. 29.5 RCC footing for wall (Example 29.1)

LESSON 30. Design of Isolated Column Footing

30.1 INTRODUCTION

The load from an isolated column may be distributed on the bearing strata by providing a square, rectangular or circular flooring. The footing may be in the form of a flat slab of uniform thickness; it may be stepped or it may be sloped at the edges. The isolated column footing can therefore be subdivided into the following categories.

1. Square footing of uniform thickness

2. Rectangular footing of uniform thickness

3. Square sloped footing

4. Rectangular sloped footing

5. Circular footing.

Design of different categories of isolated footings has been dealt separately in the following articles.

30.2 SQUARE FOOTING OF UNIFORM THICKNESS

Let 'b' be the one side of the square column and B be the one side of the square footing.

Let $W = weight\ on\ column\ including\ its\ self\ wt.\ in\ kN$

$W_1 = weight\ of\ the\ footing\ and\ soil\ over\ it.\ This\ is\ normally\ assumed$
$\quad\quad to\ be\ equal\ to\ 10\%\ of\ W\ for\ preliminary\ calculations$

$P_o = safe\ bearing\ capacity\ of\ the\ soil\ in\ kN/m^2$

$p = net\ upward\ pressure\ on\ the\ footing.$

Area required for the footing $= B \times B = \frac{{W + {W_1}}}{{{p_0}}}$

or $B = \sqrt {\frac{{W + {W_1}}}{{{p_0}}}}$

and the net upward pressure on the footing $p = \frac{W}{{{B^2}}}$

(a) Bending moment : Critical section for max. B.M. is taken at the face of the column or pedestal i.e., section X-X.

The value of max. B.M. at section X –X is given by

M= moment of the forces over the entire area on one side of the plane

$$= p \times B \times \left({\frac{{B - b}}{2}} \right) \times \left({\frac{{B - b}}{4}} \right) = p.\frac{B}{8}{\left({B - b} \right)^2}$$

(b) To fix depth of the footing: The effective depth of the footing shall be greater of the following .

1. Depth from consideration of max. B.M.: This is given by formula $$d = \sqrt{\frac{M}{RxB}}$$

2. Depth from consideration of shear: The effective depth obtained from B.M. consideration is to be checked for adequacy in shear. As per IS: 456-1978, the footing slab is required to be checked for following two types of shear.

    (i) Check for one way shear

    (ii) Check for two way shear or punching shear

(i) Check for one way shear: Critical for one way shear is considered at at a distance 'd' from the face of the column or pedestal as shown in Fig. 30.1.

The magnitude of shear force at the critical section is given by

$$V = p \times B \times \left[{\frac{1}{2}\left({B - b} \right) - d} \right]$$

Nominal shear stress, $${\tau _v} = \frac{V}{{B \times d}}$$

The value of $${\tau _v}$$ should work out to be less than or equal to $$k.{\tau _c}.$$ For working out $${\tau _c}$$ it is assumed that the section of the footing is a balanced one having percentage of reinforcement

$$p = \frac{{k.{\sigma _{cbc}}}}{{2{\sigma _{st}}}}x100$$

For M 15 grade of concrete and mild steel reinforcement $${\tau _c}$$ works out to be $$= 0.33N/m{m^2}$$. (From Table 22.1). Similarly from Table 22.2, it can be seen that the value of k for slabs 300 mm or more in thickness =1. Hence the value of $$k.{\tau _c}$$ in such a case works out to $$1 \times 0.33N/m{m^2} = 0.33N/m{m^2}$$. In case the value of $${\tau _v}$$ should work out to be more than $$k.{\tau _c}$$ it becomes necessary to revise the depth. The revised depth of the footing can be obtained by equating

$${\tau _v} = k.{\tau _c}$$

or $$\frac{V}{{B \times d}} = k.{\tau _c}$$

or $$d = \frac{V}{{B \times k.{\tau _c}}}$$

(ii) Check for two way shear: The critical section for two-way shear (also known as punching shear) is considered at a distance d/2 from the periphery of the face of the column or pedestal, as shown in Fig. 30.2

The magnitude of shear force V' at the critical section is given by

$$V' = p\left[{{B^2} - {{\left({b + d} \right)}^2}} \right]$$

Nominal shear stress at the critical section $$\tau {'_v} = \frac{{V'}}{{{b_0} \times d}}$$

where $b_0 = \textit{periphery of the critical section} = 4(b + d)$

The value of $\tau {'_v}$ should work out to be less than ${k_s}x\tau {'_c}$

where ${k_s} = \left({0.5 + {\beta _c}} \right)$ but not greater than 1, ${\beta _c}$ being the ratio of short side to long side of the column

and $$\tau {'_c} = 0.16\sqrt {{f_{ck}}}$$

In normal case ${k_s}$ will works out to be more than 1 and as such, its value is restricted to1. The value of $\tau {'_c}$ for M 15 grade of concrete = 0.16 $\sqrt {15}$ = 0.62 N/mm².

In case the value of $\tau {'_v}$ works out to be more than ${k_s}.\tau {'_c}$ it becomes necessary to revise the depth. The revised depth of the footing can be obtained by equating $$\tau {'_v} = k.\tau {'_c}$$

or $$\frac{V}{{{b_0} \times d}} = {k_s}.\tau {'_c}$$

or $$d = \frac{{V'}}{{{b_0} \times {k_s}.{\tau _c}'}}$$

Adopt higher of the above values of "d" in design.

Since the reinforcing bars are provided by placing bars at right angles to each other in the form of a mesh, the overall depth (D) of the footing is fixed as explained below.

Let the dia. of the reinforcing bar = ϕ and the clear cover for the bottom layer of bars = 50 mm. Since the value of "d" found above is applicable to top layer of bars, overall depth (D) of the footing is given by

$$D = \left({d + \phi+ {\text{}}\frac{1}{2}\phi+ 50} \right){\text{mm}}$$

(a) Area of reinforcement : Area of reinforcement in each direction is given by

$${A_{st}} = \frac{M}{{j.d.{\sigma _{st}}}}$$

The reinforcement is uniformly distributed over the entire width of the footing in each direction.

(b) Check for development length: The development length is checked at a section along the face of the column i.e., the section for maximum B.M.

Example 30.1 Design a square footing of uniform thickness for an axially load column of 500 mm x 500 mm in size transmitting a load of 600 kN. The safe bearing capacity of soil is 150 kN/sq m. Use M 20 grade of concrete and HYSD reinforcement.

Design of Structures

Solution Design constants.

For HYSD $\sigma_{st} = 230 N/mm^2$

For M 20 grade of concrete $\sigma_{cbc} = 7 N/mm^2$

$$m = \frac{280}{3.\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33$$

$$k = \frac{m.\sigma_{cbc}}{m.\sigma_{cbc} + \sigma_{st}} = \frac{13.33 \times 7}{13.33 \times 7 + 230} = 0.288$$

$$j = 1 - \frac{0.288}{3} = 0.904$$

and $R = \frac{1}{2}\sigma_{cbc}.j.k = \frac{1}{2} \times 7 \times 0.904 \times 0.288 = 0.91$

Load on the column (W) = 600 kN
Self wt. of footing and soil cover it (W_1)
Assume @ 10% of (W) = 60 kN

Total 660 kN

Safe bearing capacity of soil = p_o = 150 kN/sq.m.

Required area of footing (A) $= \frac{660}{150} = 4.4 sq.m$

Let B, be one side of the square footing,

$B \times B = A = 4.4 sq.m$ or $B = \sqrt{4.4} = 2.1 m$

Adopt footing of size 2.1 m x 2.1 m.

Net upward pressure (p) : $= \frac{W}{BxB} = \frac{600}{2.1 \times 2.1} = 136 kN/m^2$

(a) Bending moment: Critical section for bending moment is considered at the face of the column i.e., section X-X as shown in the Fig. 30.3. The magnitude of B.M. is given by

$$M = P.\frac{B}{8}\left(B - b \right)^2$$

$$= 136 \times \frac{2.1}{8}\left(2.1 - 0.5 \right)^2$$

$$= 91.4 kNm = 91.4 \times 10^6 Nmm.$$

(b) To fix depth of footing

1. Depth from consideration of max. B.M. : This is given by

$$d = \sqrt{\frac{M}{R.B}} = \sqrt{\frac{91.4 \times 10^6}{0.91 \times 2100}} = 219 mm$$

2. Depth from consideration of shear: Adequacy of the above depth is to be checked from consideration of shear.

(i) Check for one-way shear: The critical section for one-way shear is considered at a distance 'd' from the face of the column. Refer Fig. 30.4.

The magnitude of shear force V at the critical section is given by

$$V = p \times B \times \left[\frac{1}{2}\left(B - b \right) - d \right]$$

$$= 136 \times 2.1\left[\frac{1}{2}\left(2.1 - 0.5 \right) - 0.219 \right] = 166kN$$

$$\tau_v = \frac{V}{B \times d} = \frac{166 \times {10}^3}{2.1 \times 1000 \times 219} = 0.36N/m{m^2}$$

This should be less than or equal to $k\tau_c$

For a balanced section percentage of reinforcement

$$= \frac{k.\sigma_{cbc}}{2 \times \sigma_{st}} \times 100 = \frac{0.228 \times 7 \times 100}{2 \times 230} = 0.44\ \%$$

Corresponding value of τ_c as obtained from Table 22.1 for M 20 grade of concrete

$$\tau_c = 0.22 + \frac{\left(0.30 - 0.22 \right)}{0.25}\left(0.44 - 0.250 \right) = 0.28N/m{m^2}$$

$$k = 1$$

$$kx\tau_c = 1 \times 0.28\frac{N}{m{m^2}} = 0.28N/m{m^2}$$

Since $\tau_v > k\tau_c$, it will be necessary to revise the depth

Revision. In the limiting case $\tau_v = k.\tau_c$

To find the value of "d" so that

$$\tau_v = k.\tau_c$$

$$\tau_v = \frac{V}{bd}$$

or $$\tau_v = \frac{166 \times {10}^2}{2.1 \times 1000 \times d} = k.\tau_v = 0.28$$

$$d = \frac{166 \times {10}^2}{2.1 \times 1000 \times 0.28} = 280mm$$

Adopt revised value of d = 280 mm for check for two-way shear.

(ii) Check for two-way shear: The critical section for two-way shear or punching shear is considered at a distance of d/2 from face of column. Refer Fig. 30.5.

The net S.F. at the critical section is given by

$$V = p\left[{B^2} - {{\left({b + d} \right)}^2} \right]$$

$$= 136\left[{{\left({21} \right)}^2} - {{\left({0.5 + 0.280} \right)}^2} \right] = 517kN$$

Nominal shear stress at the critical section $${\tau _v} = \frac{{V'}}{{{b_o} \times d}}$$

$${b_o} = 4\left({b + d} \right) = 4\left({0.5 + 0.280} \right) = 3.128m = 3.128 \times {10^3}mm$$

$$d = 0.280mm$$

$${\tau _v} = \frac{{V'}}{{{b_o} \times d}} = \frac{{517 \times {{10}^3}}}{{3.128 \times {{10}^3} \times 280}} = 0.592N/m{m^2}$$

This should be less than   or $$= {k_s}.{\tau _c}$$

$$= \left({0.5 + {\beta _c}} \right) \times 0.16\sqrt {{f_{ck}}}$$

$$= 1 \times 0.16\sqrt {20}= 0.715N/m{m^2}$$

Since $${\tau _v} < {k_s}{\tau _c}$$, hence safe.

Thus the effective depth of footing (i.e. d = 280 mm) is governed by requirement of one-way shear. Overall depth, assuming 12 mm ϕ bars (placed one over the other in two directions at right angles) and clear cover of 50 mm

$$D = 280 + 12 + \frac{{12}}{2} + 50 = 348mmsay350mm$$

Available effective depth for bottom layer of bars = 350 - 50 - 12/2 = 294 mm

and "d" for top layer = 294 -12 = 282 mm.

(c) Area of steel reinforcement in each direction is given by

$${A_{st}} = \frac{M}{{j.d.{\sigma _{st}}}} = \frac{{91.4 \times {{10}^6}}}{{0.904 \times 282 \times 230}} = 1560m{m^2}$$

Area of one 12 mm ϕ bars (Aϕ) = $$\frac{\pi }{4}.{\left({12} \right)^2} = 113m{m^2}$$

No. of bars required $$= \frac{{1548}}{{113}} = 13.7say14Nos$$

Hence provide 14 Nos 12 mm ϕ (HYSD) bars uniformly spaced in the width of 2.1 m in each direction at right angles to each other.

Design of Structures

(d) Check for development length:

$$L_d = \frac{\phi \sigma_s}{4\tau_{bd}} = \frac{\phi \times 230}{4\left(0.8 + \frac{40}{1000} \times 0.8\right)} = 51.3\phi = 51.3 \times 12 = 616\, say\, 620mm$$

Providing side cover of 60 mm length of the bar available beyond critical section for B.M.

$$= \frac{1}{2}\left(B - b\right) - 60 = \frac{1}{2}\left(2100 - 500\right) - 60 = 740mm$$

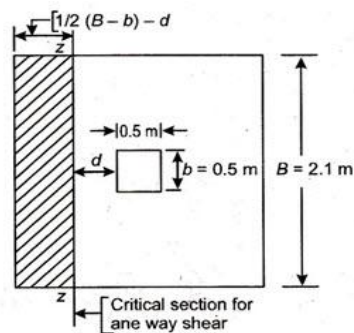Which is more than L_d, hence safe. Fig. 30.6 shows the arrangement of reinforcement.

Fig. 30.1 Critical section for B.M

Fig. 30.2 Critical section for one-way and two-way shear

Fig. 30.3 Critical section for bending moment
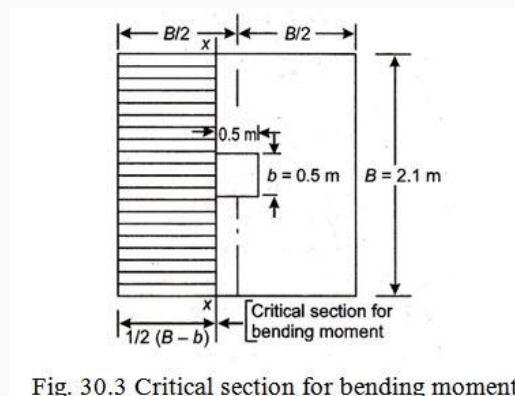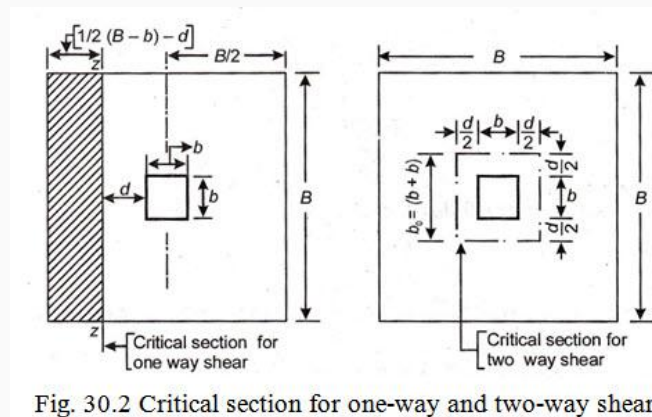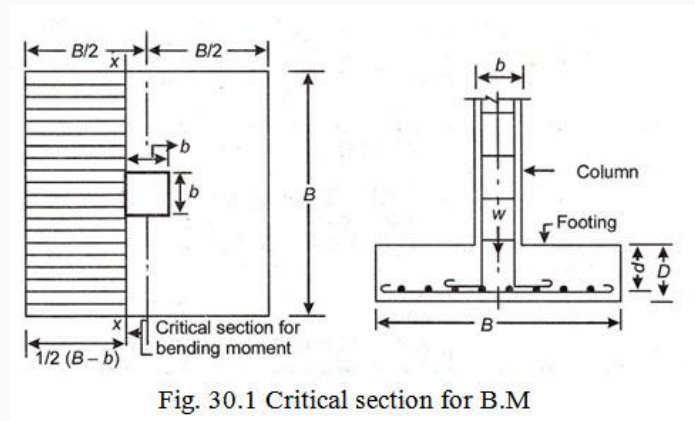
Fig. 30.4 Critical section for one-way shear
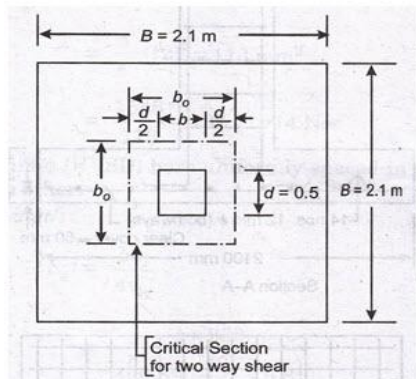
Fig. 30.5 Critical section for two-way shear
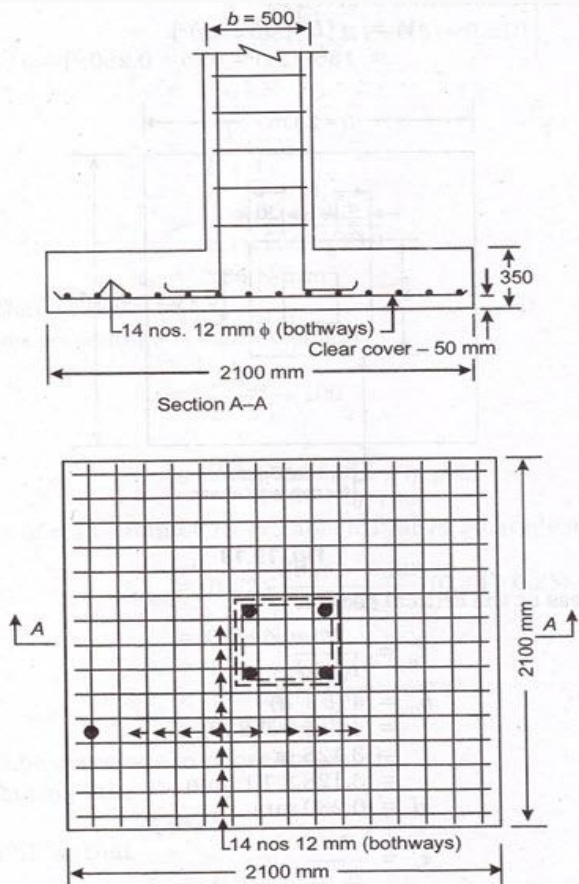
Fig. 30.6 Plan showing the details of reinforcement

LESSON 31. Cantilever Retaining Walls

31.1 INTRODUCTION

Retaining walls are structures constructed for the purpose of retaining earth or other materials like coal, ore, water etc. It may also be defined as a wall provided to maintain ground at two different levels. Provisions of retaining walls become necessary in the construction of hill roads, embankments, bridge abutment, basement in buildings, water reservoir, in preventive measures against soil erosion, in landscaping etc. The material retained by the wall is generally known as backfill. The backfill may be horizontal i.e., levelled with the top of wall or it may be inclined at certain angle to the top. The inclined fill is also known as surcharge. Besides loads due to retained material, the retaining wall may also be subjected to surcharged load (due to automobile, rail road etc.) acting directly on the wall as well as on the backfill. The retaining wall should be stable enough to resist all type of forces acting on it.

31.2 TYPES OF RETAINING WALLS

Based on the method of achieving stability, retaining walls are classified into the following types:

(i) Gravity walls

(ii) Cantilever retaining walls

(iii) Counterfort retaining walls

(iv) Buttressed walls.

31.2.1 Gravity Walls

These walls are constructed in brick masonry, stone masonry or plain cement concrete and it is shown in Fig. 31.1. The wall is so proportioned that the dead weight of the wall provides required stability against the thrust exerted by the backfill including surcharge (if any). The size of the wall is so kept that there is no tensile stress developed at any section of the wall under any condition of loading.

31.2.2 Cantilever Retaining Walls

These are R.C.C. walls made in the form of an invented T as shown in Fig. 31.2. This type of wall proves to be economical for moderate heights say 6 to 7m. The wall consists of three components, (i) the stem, (ii) the toe, and (iii) the heel. Each of these components are designed as a cantilever. The stability of the wall is partially provided by the weight of earth on the heel. Sometimes the cantilever wall is constructed in the form of L. In this case wall has only two components i.e., (i) the stem and (ii) the heel, each being designed as

cantilevers. Many a times to increase the resistance of the wall to sliding it becomes necessary to provide a vertical projection known as "key" below the base of the wall. This type of wall with a key is shown in Fig.31.2 (b).

31.2.3. Counterfort Retaining Walls

When the height of the retaining wall to be provided exceeds 6 to 7m, counterfort retaining wall prove to be economical. In this type of wall the base slab as well as the stem of the wall span horizontally as continuous slabs between vertical brackets known as counterforts as shown in Fig. 31.3. The counterforts are provided behind the wall (on the backfill side) and are subjected to tensile forces. The spacing of the counterforts may vary from $\frac{1}{3}$ to $\frac{1}{2}$ of the height of wall. The more the height of the wall, the closer should be the spacing of counterforts.

31.2.4 Buttressed Walls

This buttressed wall is identical to a counterfort retaining wall with the main difference that the vertical brackets are provided in front of the wall (on face opposite to the face retaining back fill) as shown in Fig. 31.4. The brackets in this case are known as buttresses and by virtue of their location they are subjected to compressive forces.

31.3 DETERMINATION OF EARTH PRESSURE

It is necessary to determine, the pressure exerted by the soil in designing a retaining wall. The pressure mainly depends upon the type of backfill material and the height of wall. Out of the number of theories evolved, Rankine's theory is predominantly used in calculating the soil pressure. The Rankine's theory of earth pressure is based on the assumption that the retained soil is dry, cohesionless and that there is no friction between the soil and the wall. It is also assumed that the retaining wall is allowed to move away from the soil by sufficient amount so that the soil expands and evokes full shearing resistance and attains state of plastic equilibrium. The pressure thus developed is termed as soil earth pressure.

Application of Rankine's theory for the following cases has been dealt in this lesson.

1. Wall retaining dry and levelled backfill.

2. Wall retaining submerged backfill.

3. Wall retaining partly submerged backfill.

4. Wall with backfill levelled and subjected to uniform surcharge.

5. Wall retaining backfill in slope.

Case 1. Wall retaining dry and levelled backfill : Refer Fig. 31.5 for the case of retaining wall where the backfill to be retained is dry or moist and it is levelled with the top surface of the wall. As per Rankine's theory, the intensity of active earth pressure per unit vertical area of the wall at any depth h below the top of the wall is given by the relation

$$p_a = \text{\gamma}.h.\frac{1 - \sin\phi}{1 + \sin\phi}$$

where

p_a = the intensity of active earth pressure

γ = unit weight of soil

ϕ = angle of internal friction of the soil

The above relation is further simplified as $$p_a = {\gamma}.h.k_\alpha$$

where $$k_\alpha = \frac{1 - \sin\phi}{1 + \sin\phi}$$ and it termed as coefficient of internal friction.

Hence the intensity of active soil pressure at the base of the wall where h = H is given by

$$p_a = g.H.k_\alpha$$

Rankine's theory assumes that the distribution of pressure along the height of the wall is triangular and the centre of pressure lies at $$\frac{1}{3}$$ [rd] the height of wall from the base.

∴ Total active soil pressure $$(p_a)$$ = Area of the pressure triangle

$$= \frac{1}{2}{\gamma}.H.k_\alpha \times H$$

or $$p_a = \frac{1}{2}{\gamma}{\text{H}}^2 k_\alpha$$ acting at *H/3* from the base.

Case 2. Wall retaining submerged backfill: If the water table is such that the retained soil remain fully submerged, in such a case the saturated soil results in increasing the weight of the backfill, decreases the angle of repose of the soil which ultimately amounts to increase in pressure on the wall. Refer Fig. 31.6. The lateral pressure exerted by the submerged soil is considered to comprise of the following two components.

(i) Lateral pressure due to submerged soil

(ii) Lateral pressure due to hydrostatic pressure

Due to buoyancy the weight of submerged soil will be less and its repose will also be much less. Hence the intensity of earth pressure at any depth *'h'* below the tap of the wall is given by

$$p_a = {\gamma}h.k'_a + a{\gamma_w}h$$

where,

γ = reduced wt. of soil due to submerged conditions

γ_w = reduced wt. of water

k'_a = coefficient of internal friction based on reduced value of ϕ.

The intensity of pressure at the base of the wall where h = H is given by

$$p_a = \gamma H.k'_a + \gamma_w H$$

Hence total lateral earth pressure at the base of wall is given by

$$P_a = \text{Area of pressure triangle in Fig.31.6}(b)$$
$$+ \text{Area of pressure in Fig.31.6}(c)$$
$$= \frac{1}{2}\gamma.H^2.k'_a + \frac{1}{2}\gamma_w H^2$$

Case 3. Wall retaining partly submerged backfill: In case the water table does not rise up to full height of retaining wall, this will result in a situation where the soil is partially submerged and partly dry.

Refer Fig. 31.7. Let the backfill be moist or dry up to a depth h_1 below the top of wall and let the backfill below this depth i.e., be fully submerged. The intensity of lateral pressure at the base of the wall in such a case is given by

$$p_a = \gamma h_1 k'_a + \gamma^1.h_2.k_a^{\wedge'} + \gamma_w.h_2$$

Case 4. Wall with backfill levelled and subjected to uniform surcharge: Fig. 31.8 shows a retaining wall having backfill leveled with the top of the wall and subjected to uniformly distributed surcharged load. Let the intensity of surcharge load per unit area be w. The lateral pressure imposed by this load does not vary with the height and is uniform.

Its value being = $w.k_a$

Hence total intensity of internal pressure at any depth 'h' is $p_a = \gamma.h.k_a + w k_a$

The pressure intensity at the base of the wall here h = H is given by

$$p_a = \gamma.H.k_a + w k_a$$

Alternatively, the uniform surcharge load can also converted into equivalent additional fictitious height (h_e) of the back fill. The height (h_e) can be obtained by the relationship

$$w.k_a = \gamma.h_e.k_a$$

or $$h_e = \frac{w}{\gamma}$$

Hence intensity of pressure at the base of the wall considering the height of backfill = (H + h_e) will be

$$p_a = \gamma\left(H_2 + h_e \right).k_a$$

This is shown in pressure diagram in Fig. 31.8 (c).

Case 5. Wall retaining back-fill in slope : Fig. 31.9 shows a retaining wall having backfill in slope. Let the slope of surcharged backfill to the horizontal be α . The angle is also known as angle of surcharge. As per Rankine's theory, the intensity of pressure at any depth h in case of a wall retaining soil surcharged at an angle α is given by

$$p_a = \gamma h.\cos\alpha \times \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}$$

Its value at the base of the wall here h=H is given by $p_a = \gamma H.\cos\alpha \times \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}$

It is also assumed that pressure distribution is triangular, the pressure acts parallel to the inclined surface of the backfill and the centre of resultant pressure lies at $\frac{1}{3}$ rd the height or $H/3$ above the base.

Hence total pressure $P_A = Areaofthepressuretriangle$

$$= \frac{1}{2} \times \gamma H^2\cos\alpha.\frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}$$

31.4. PASSIVE EARTH PRESSURE

If the retaining wall is allowed to move towards the back fill, it will compress the soil and the pressure thus exerted is known as passive pressure.

The intensity of passive earth pressure at any depth 'h' in a retaining wall is given by

$$P_p = \gamma h.\frac{1 + \sin\phi}{1 - \sin\phi} = \gamma.h.k_p$$

Where

$$k_p = coefficientofpassiveearthpressure, itsvaluebeing = \frac{1 + \sin\phi}{1 - \sin\phi}$$

The pressure distribution is triangular.

Hence total passive pressure $P_p = \text{Areaofpressuretriangular} = \frac{1}{2} \times \gamma h^2 \times k_p$ acting at $h/3$ from the base.

31.5 CONDITIONS FOR STABILITY OF RETAINING WALL

To avoid failure of the retaining wall it is necessary that the following requirements are satisfied.

(a) It should not over turn

(b) It should not slide

(c) The max. pressure at toe should not exceed the safe bearing capacity of soil.

Before proceeding with the structural design it is necessary to ensure that the preliminary dimensions assumed for the various components of the wall will render it safe against above referred types of failures. If the requirements of stability are not satisfied, its dimension should be revised. Refer Fig. 31.10.

(a)Check against overturning: The lateral pressure due to the backfill and surcharge (if any) tends to overturn the retaining wall about its toe. The overturning moment is stabilized by the weight of wall and the weight of the soil above the heel slab (the weight of soil over the toe is neglected). The retaining wall is considered safe against overturning when the total stabilizing or resisting moment is at least 100% greater than the overturning moment.

Let $\quad M_o = Sum\ of\ overturning\ moment\ about\ toe$

$\qquad M_R = Sum\ of\ stabilizing\ or\ resisting\ moment\ about\ toe.$

$\qquad F.S = Factor\ of\ safety$

$\therefore \qquad F.S. = \dfrac{Sum\ of\ resisting\ moment}{Sum\ of\ overturning\ moment} = \dfrac{M_R}{M_o} = 2$

(b) Check against sliding. The horizontal component of all lateral pressures tends to slide the wall along its base. The sliding tendency is resisted by the frictional resistance between the base of the wall and the soil underneath. It is a common practice to neglect the passive resistance of the soil in front of the toe of the wall in this check. To meet the requirements of stability, the force of resistance should be 50% more than the sliding force.

Let $\Sigma_w = sum\ of\ vertical\ loads$

$\qquad \mu = co-efficient\ of\ friction\ between\ base\ concrete\ and\ soil$

$\qquad P = total\ horizontal\ force\ tending\ to\ slide\ the\ wall$

$\therefore \qquad$ Total force opposing sliding $= \mu \times \Sigma W$

$\qquad$ Factor of safety against sliding $= \dfrac{\mu \times \Sigma_w}{P} = 1.5$

If the factor of safety against sliding works out to be less than 1.5, a key may be provided under the base slab. The passive pressure developed by the key resists sliding and raises the factor of safety to required limit.

(c)Check against maximum pressure at toe:

Let $\Sigma_w = sum\ of\ vertical\ loads$

$\qquad b = width\ of\ base\ of\ wall$

$\qquad e = eccentricity\ or\ the\ distance\ between\ the\ middle\ point\ of\ base\ and\ the\ point$

$\qquad\qquad of\ intersection\ of\ the\ resultant\ on\ the\ base$

$\qquad p_1 = intensity\ of\ soil\ pressure\ at\ the\ toe$

$\qquad p_2 = intensity\ of\ soil\ pressure\ at\ the\ heel.$

The maximum and minimum pressure are given by

$$p_1 = \frac{\Sigma W}{b}\left(1 + \frac{6e}{b} \right)$$

$$p_2 = \frac{\Sigma W}{b}\left(1 - \frac{6e}{b} \right)$$

To meet the requirements of stability p_1 should not exceed the safe bearing capacity of soil. In addition, it should be ensured that no tension is developed at the base i.e., the value of p_2 should not be negative. To meet this requirement the resultant of the sum of all vertical forces and the horizontal active pressure should cut the base of the wall within the middle third.

31.6 DEPTH OF FOUNDATION

The foundations of the retaining wall should be placed at such a depth where soil of required bearing capacity is available. However, the minimum depth of foundation should not be less then that given by Rankine's formula according to which

$$D_f = \frac{P_o}{\gamma}\left(\frac{1 - \sin\phi}{1 + \sin\phi} \right)^2$$

Where

$D_f = minimum\ depth\ of\ foundation\ in\ metres$

$P_0 = bearing\ capacity\ of\ soil\ in\ kN/m^2$

$\gamma = unit\ weight\ of\ soil\ in\ kN/m^3$

$\phi = angle\ of\ repose\ of\ soil.$

31.7 PRELIMINARY DIMENSIONS OF CANTILEVER RETAINING WALL

Prior to start of structural design of a retaining wall it is necessary to adopt some tentative dimensions for different components of the wall. Based on these dimensions the wall is checked for stability (checked for overturning, sliding and maximum pressure at toe) and in case the stability requirements are satisfied, structural design of different wall components is taken up.

Commonly adopted proportions for wall components are as under:

(i) The stem. Depending upon the height *(H)* of the wall the top width of stem can vary between 200 mm to 300 mm.The bottom width may vary between H/15 to H/10 or it can be decided based on bending moment consideration.

(ii) Base width(b). The base width *(b)* of the retaining wall vary between 0.4 *H* to 0.65 *H*.

For surcharged walls (b) may vary between 0.55 *H* to 0.75 *H*.

(iii) Toe projection. This may be around b/3. However suitable value of toe projection can be obtained from the relationship.

$$Toe\ projection = b \times \beta$$

$$where\ \beta = 1 - \frac{P_o}{2.2 \gamma H}$$

$$P_o = bearing\ capacity\ of\ soil$$

$$\gamma = unit\ weight\ of\ back\ fill$$

(iv)Thickness of base slab:

$$\frac{H}{15} to \frac{H}{10}$$
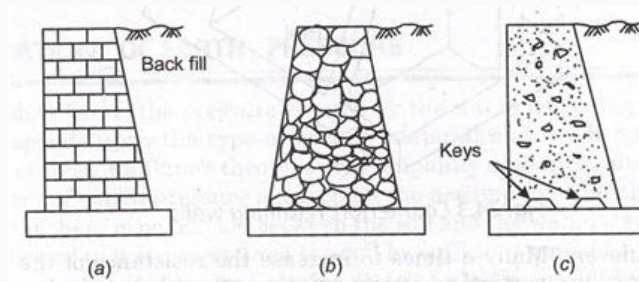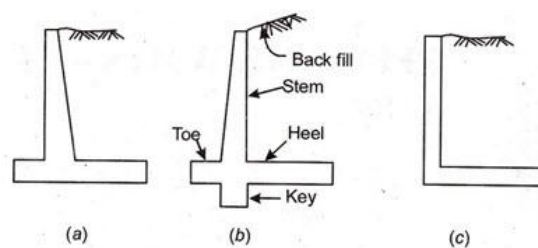

Fig. 31.1 Gravity walls

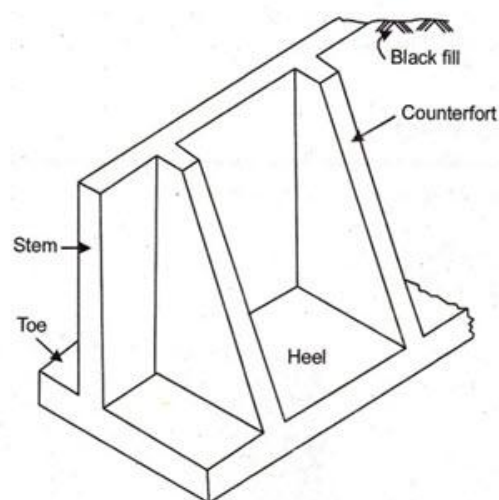Fig. 31.2 Cantilever retaining walls
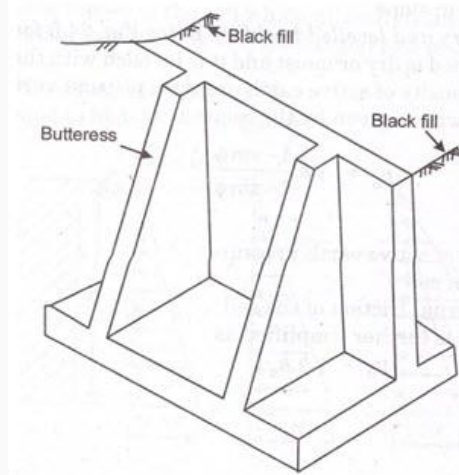

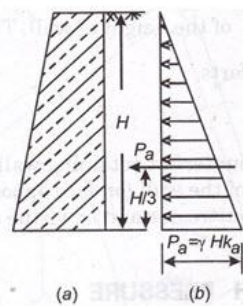Fig. 31.3 Counterfort retaining walls

Fig. 31.4 Buttressed wall
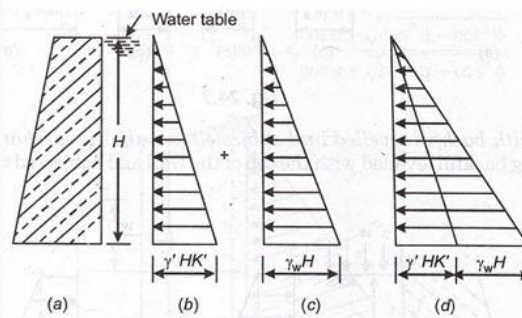

Fig. 31.5 Wall retaining dry and levelled backfill
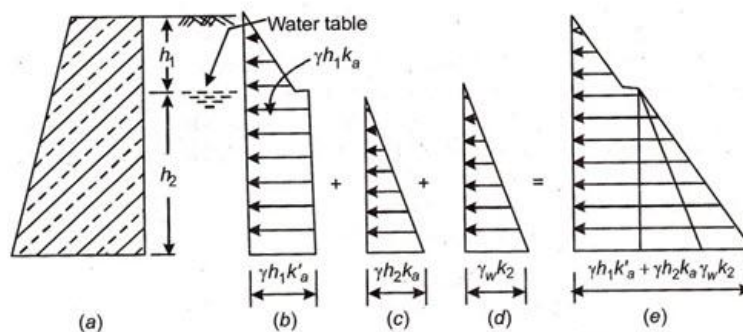

Fig. 31.6 Wall retaining submerged backfill

Fig. 31.7 Wall retaining partly submerged backfill
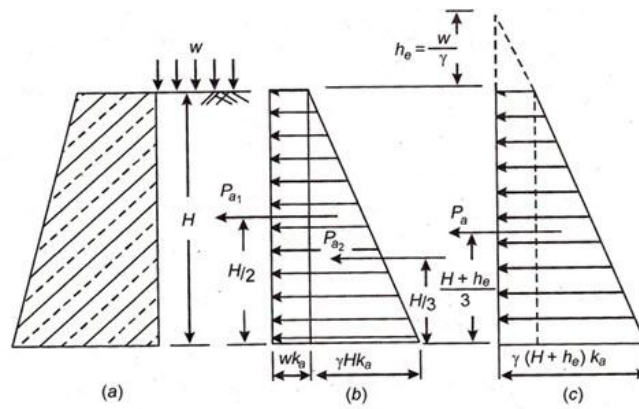
Fig. 31.8 Wall with backfill levelled
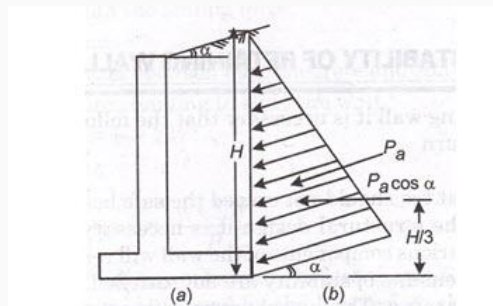
Fig. 31.9 Wall retaining backfill in slope
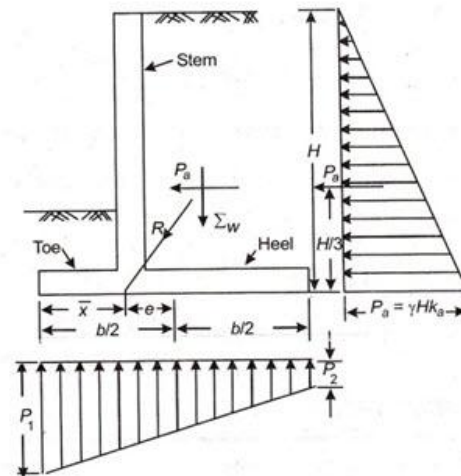
Fig. 31.10 Forces acting on retaining wall

LESSON 32. Design of RCC Cantilever Retaining Walls

32.1 INTRODUCTION

The different components of the wall are treated as cantilever slab uniformly loaded in the direction of the length of the wall. Hence the wall is designed for one metre length and the same design holds good for the remaining length of the wall. Shear reinforcement is normally not provided in a retaining wall. In case the shear stress at the critical section for any/all of the components of wall works out to be more than permissible shear stress, the thickness of the component should be increased to bring the shear stress within permissible limit. The main reinforcement in stem, toe and heel must be extended in to the base slab, beyond the respective critical section for bending by a distance = L_d . The salient aspects to be considered in the design of each part of the wall are summarized below.

(1) Design of stem: The stem is designed as a vertical cantilever slab fixed at the base. It is subjected to lateral soil pressure. The distribution of pressure along the height of the wall is triangular. Refer the Fig. 32.1. The maximum bending moment occur at junction G. The total lateral pressure acts at $H_1/3$ and it value $= \frac{1}{2}.{\text{\gamma}}H_1^2.{k_a}.$

The magnitude of maximum B.M. at G $= \frac{1}{2}.{\text{\gamma}}H_1^2.{k_a}. \times \frac{{H_1}}{3} = \frac{{{\text{\gamma}}H_1^3}}{6}.{k_a}$

The main reinforcement is placed near the back face of the wall. The reinforcement can be curtailed towards the top of the stem since the bending moment varies as H_1^3 . Distribution reinforcement is provided @ 0.15% of the average area of cross-section. Temperature reinforcement is provided near the front face in the form of vertical and horizontal reinforcement equal in area to distribution reinforcement. The critical section for shear is considered at G. The shear stress at G should be less than permissible shear stress. The stem reinforcement must be extended by a distance = L_d in the base slab to meet the requirement of development length.

(2) Design of toe slab: The toe slab is designed as a cantilever slab fixed at the front face of the stem i.e., at B. The toe is acted upon by two forces (i) large upward soil reaction and (ii) downward force due to weight of toe slab. Refer the Fig. 32.2.

The net pressure being upwards, it tends to bend the toe upwards so that tension develops on the bottom face. Hence main reinforcement in the toe slab is placed near the bottom face of the slab.

Critical section for shear considered at a distance of effective depth from face of stem B. The main reinforcement must be extended beyond B by a distance = L_d to meet the requirements of development length.

(3) Design of heel slab: This is designed as a cantilever slab fixed at back face of stem at G. The heel slab is subjected to large downward forces to weight of column of earth above the heel slab and self weight of heel slab and small upward force due to upward soil reaction. The net pressure being downwards it tends to bend the slab downwards so that tension develops on top face. Hence main reinforcement in the heel slab is placed near the top face of the slab. Critical section for shear is considered at G. The main reinforcement must be extended beyond G by a distance = L_d , to meet requirement of development length. Fig. 32.3 shows the requirements for development length for main reinforcement for retaining wall components at junction of stem, heel and toe slab.

32.2 DRAINAGE OF RETAINING WALL

In case the wall is designed for dry or moist backfill conditions, it is necessary to make adequate drainage arrangement to discharge the rain water that will percolates within the backfill soil. If this is not done, the wet soil will impose large lateral soil pressure which can endanger the stability of the wall. Drainage of wall is achieved providing weep holes in the stem at suitable intervals say 2 m c/c by vertically and horizontally. The lowest weep hole is kept 300 mm above the ground level on toe side. In order to prevent blockage of the weep holes, a 450 mm thick layer of some filter media (stone chips, gravel, or similar granular material) is placed between the wall right from footing up to the top of stem.

32.3 STEPS TO BE FOLLOWED IN THE DESIGN OF A CANTILEVER RETAINING WALL

(i) Calculate the depth of foundations from Rankine's formula (if required) and fix total height of the wall.

(ii) Select tentative proportions for the different components of the wall and fix the thickness for the stem from consideration of max. B.M. at its junction with base slab.

(iii) Calculate all vertical and horizontal forces and check the stability of the wall against overturning. The factor of safety should not be less than 2.0.

(iv) Check the stability of the wall against sliding. The factor of safety should not be less than 1.5. Provide key under base slab if necessary.

(v) Check that the max. soil pressure at toe does not exceed the safe bearing capacity of soil.

(vi) Calculate max. B.M. for different wall components based on detail given in Art. 32.1and design the toe and heel slab first. Since the thickness of the toe/heel slab influences the height of stem, the design of the stem should be taken up subsequently. Check the section for shear and development length. Shear reinforcement is not normally used in the wall. The thickness of the components, should be increased to make it safe in shear.

Example 32.1 Design a cantilever type of R.C.C. retaining wall to retain leveled earthen embankment 3m high above ground level. Density of earth is 16000 N/m³ and its angle of repose is 30°. The safe bearing capacity of soil at a depth of 1m below ground level is 100 kN/m² . The coefficient of friction between soil and concrete may be taken as 0.55. Use M 15 grade of concrete and mild steel reinforcement in the design.

Solution : Design constants:

For M 15 grade of concrete $$\sigma_{cbc} = \frac{5N}{m{m^2}}$$; $$m = 19$$

For mild steel reinforcement $$\sigma_{st} = 140N/m{m^2}$$

$$k = 0.404, j = 0.865 \text{ and } R = 0.874$$

Co-efficient of active earth pressure

$$k_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \text{sin}{30}^{\text{o}}}{1 + \text{sin}{30}^{\text{o}}} = \frac{1}{3}$$

Depth of foundation: From Rankine's formula, minimum depth of foundation $$(D_f)$$ is given by

$$D_f = \frac{p_0}{\text{\gamma}}{\left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)^2} = \frac{110 \times {10}^3}{16000}{\left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)^2}$$

$$= \frac{110 \times {10}^3}{16000} \times {\left(\frac{1}{3} \right)^2} = 0.76m$$

Let us adopt minimum depth of foundation = 1m to ensure that the top of base slab of the wall will remains below the ground.

Total height of (H) of the wall including base slab $$= 3 + 1 = 4m$$

Preliminary dimensions of the wall components:

(i) Width of base slab (b): This may vary between 0.4 H to 0.65 H.

Let us adopt a base width $$= 0.55{\text{}}H$$

$$b = 0.55 \times 4 = 2.2m$$

(ii) Toe projection: Adopt toe projection $$= b \times \beta$$

where $$\beta = 1 - \frac{{P_o}}{2.2\text{\gamma}H} = 1 - \frac{100 \times {10}^3}{2.2 \times 1600 \times 4} = 0.29$$

$$b \times \beta = 2.2 \times 0.29 = 0.63m \text{ say } 0.6m$$

(iii) Thickness of base slab: This normally varies from $$\frac{H}{15}$$ to $$\frac{H}{10}$$.

Let us adopt thickness of base slab $$= \frac{H}{14} = \frac{4}{14} = 0.28m$$

(iv) Thickness of stem: This can be fixed from consideration of max. bending moment.

Clear height of stem wall $$= 4 - 0.28 = 3.72m$$

Consider one metre length of retaining wall

Active earth pressure (*P*) at any depth to '*h*' is given by

$$P = \frac{1}{2}.w.{h^2}.\frac{1 - {\text{sin}}\phi }{1 + {\text{sin}}\phi } = \frac{1}{2}w{h^2}{k_a}$$

For $h = 3.72m$

$$P = \frac{1}{2} \times 16000 \times {3.72^2} \times \frac{1}{3} = 36900N$$

acting at $\frac{3.72}{3}$ m from heel slab top.

Max. BM. at G $M = 36700 \times \frac{3.72}{3}$

$$= 45760Nm = 45760 \times {10^3}Nmm$$

$$d = \sqrt {\frac{45760 \times {{10}^3}}{0.874 \times {{10}^3}}}= 229mm$$

Since the stem wall is in contact with earth

Provide clear cover $= 40mm$

Assuming 16 mm ϕ bars, adopt overall thickness of stem

$$= 229 + 40 + \frac{16}{2} = 227mm\,say\,280mm$$

Available $d = 280 - 40 - 8 = 232mm$

Check for shear : ${\tau _v} = \frac{V}{bd} = \frac{36900}{1000x232}$

$$= 0.16N/m{m^2} < {\tau _b}$$

$$< 0.22N/m{m^2}$$, hence OK.

The height of the stem is comparatively small and as such it will not be advantageous to reduce the thickness at top. The saving made in the quantity of concrete due to tapering of stem will be offset by the increased cost of providing inclined shuttering. As such the thickness of stem will be maintained as 280 mm uniformly throughout its height. Since the thickness of toe and slab influences the height of stem, the final stem thickness and its design should be taken after the design of toe and heel slab. The preliminary dimensions of different components of the wall are shown in Fig. 32.4.

Check for stability of the wall: Before proceeding with the design further, it is necessary to ascertain the stability of the wall with the preliminary dimensions of the base width.

Design of Structures

Following checks are made in this connection:

(a) Check against overturning

(b) Check against sliding

(c) Check against max. pressure at toe

Consider one metre length of the retaining wall.

(a) Check against overturning : The various loads and their moments about the toe slab(at D) are tabulated as under.

S.No	Description of load	Magnitude of load in Newton (N)	Dist. of C.G D in metres (m)	Moment about D in (Nm)
1.	Wt. of stem wall (or W_1)	0.28 x 3.72 x 25000 = 26040	0.6 + 0.28/2 = 0.74	19270
2.	Wt. of base slab (or W_2)	2.2 x 0.28 x 25000 = 15400	2.2 / 2 = 1.10	16940
3.	Wt. of earthfill over heel slab(or W_3)	1.32 x 372 x 16000 = 78570	1.32 / 2 + 0.28 + 0.60 = 1.54	121000
		$\Sigma W = 120010$ N		MR = 157210

Total horizontal earth pressure on the full height of the retaining wall tending to overturn the wall

$$P = \frac{1}{2}w{H^2}\frac{{1 - {\text{sin}}\phi }}{{1 + {\text{sin}}\phi }} = \frac{1}{2} \times 16000 \times {4^2} \times \frac{1}{3} = 42667N$$

acting at $\frac{4}{3}$ from D

Total overturning moment $${M_0} = 42667 \times \frac{4}{3} = 56890Nm$$

Total stabilizing moment $${M_R} = 157210Nm$$

Factor of safety against overturning = $= \frac{157210}{56890} = 2.76 > 2,$, hence safe.

(b) Check against stiding. Total horizontal force tending to slide the wall

$$P = \frac{1}{2}w{H^2}{k_a} = 42667N$$

Total force opposing sliding $= \mu \times \Sigma W = 0.55 \times 120010 = 66010N$

Factor of safety against sliding $= \dfrac{\mu \times \Sigma W}{P} = \dfrac{66010}{42667} = 1.55 > 1.5,$, hence safe.

(c) Check against max. pressure at toe. Net moment or algebraic sum of moments about *D*

$$= M_R - M_o = 157210 - 56890 = 100320 Nm$$

Let $\bar x$ be the distance from toe (D) at which the resultant reaction acts

$$\bar x = \frac{100320}{120010} = 0.836\text{m}$$

Equation (e) $\quad = \dfrac{b}{2} - \bar x = \dfrac{2.2}{2} - 0.836$

$= 0.264\,m < \dfrac{2.22}{6},$, hence O.K.

Max. pressure at toe (*D*) and minimum pressure in heel (*E*) is given by the formula

$$p = \frac{\Sigma W}{b}\left(1 \pm \frac{6e}{b} \right)$$

P_{max} at toe $\left(D \right)$ $\quad = \dfrac{\Sigma W}{b}\left(1 + \dfrac{6.e}{b} \right) = \dfrac{120010}{2.2}\left(1 + \dfrac{6 \times 0.264}{2.2} \right)$

$= \dfrac{\Sigma W}{b}\left(1 + \dfrac{6.e}{b} \right) = \dfrac{120010}{2.2}\left(1 + \dfrac{6 \times 0.264}{2.2} \right)$, hence safe.

P_{max} at heel $\left(E \right)$ $\quad = \dfrac{\Sigma W}{b}\left(1 - \dfrac{6.e}{2.2} \right) = \dfrac{120010}{2.2}\left(1 - \dfrac{6 \times 0.264}{2.2} \right)$

$$15274 N/m^2$$

The intensity of pressure distribution between toe (D) and heel (E) being linear, pressure at junction of stem with toe slab

$$= 15274 + \left(\frac{93823 - 15274}{2.2} \right) \times \left(2.2 - 0.6 \right)$$

$$= 72400 N/m^2$$

and pressure at the junction of stem with heel slab

$$= 15274 + \left(\frac{93823 - 15274}{2.2} \right) \times 1.32$$

$$= 62403 N/m^2$$

Design of Structures

The pressure distributions at various points are given in Fig. 32.5. Since the wall with assumed base width is found to be safe from stability consideration. We can now go ahead with the design of different components of the wall i.e., toe slab, heel slab and the stem.

(1) Design of toe slab : Neglecting the weight of soil on the toe slab, the forces acting on toe slab are :

(i) Upward soil reaction varying from 93823 N/m² at (D) to 72400 N/m² at (B).

(ii) Downward pressure due to self weight of toe slab = 0.28 x 25000 = 7000 N/m²

Net upward pressure under (D)   = 93823 - 7000 = 86823 N/m²

and Net upward pressure under (B)   = 72400 - 7000 = 65400 N/m²

Refer Fig. 32.6.

Consider 1 m length of wall:

Max. B.M. at a vertical section through B = Area of pressure trapezium $DD_1B'B_1$ x   distance of c.g. of the trap, from $B'B_1$

$$= 1 \times \left(\frac{86823 + 65400}{2} \right) \times 0.6 \times \left(\frac{65400 + 2 \times 86823}{65400 + 86823} \right) \times \frac{0.6}{3} = 14343 Nm$$

$$d = \sqrt{\frac{14343 \times 10^3}{0.874 \times 10^3}} = 128mm$$

However adopt same overall depth toe slab as for stem i.e., 280mm at B. Reduce thickness to 200 mm at the edge C. Provide 50 mm clear cover.

Assuming 16mm ϕ bars, effective depth of toe slab $= 280 - 50 - \frac{16}{2} = 222mm$

$$A_{st} = \frac{M}{j.d.\sigma_{st}} = \frac{14343 \times 10^3}{0.865 \times 222 \times 140} = 533 mm^2$$

Spacing of 16mm ϕ bars (A_{ϕ} = 201mm²) $= \frac{201 \times 1000}{533} = 377mm \, say \, 370mm$

However, situations where toe projections as well as the thickness of the base slabe are less, the requirement of area of reinforcement for the toe slab can be adequately met by extending the main reinforcement of the stem in to the slab. Such extension of stem bars becomes essential to meet the requirement of development length of the stem reinforcement as well. As can be seen from the design of stem, 16mm ϕ bars @ 120 mm c/c are available to meet the requirement of toe reinforcement

Distribution reinforcement:

$$= \frac{0.15}{100} \times \left(\frac{280 + 200}{2} \right) \times 1000 = 360 mm^2$$

Design of Structures

Spacing of 10mm ϕ bars $((A_{st}) = 79m{m^2})$ $\frac{79 \times 1000}{360} = 219mm\,say\,200mm\,c/c$

Check for shear:

The critical section for shear (section Z – Z') is considered at a distance "d" from the front face of the stem wall. Refer Fig. 32.7.

From the geometry of net pressure diagram, pressure ordinate at section Z-Z' is given by

$$= 65400 + \left({\frac{86823 - 65400}{0.6}} \right)x0.222 = 73327N/{m^2}$$

S.F. at section Z-Z' = Area of pressure trap. $DD_1Z'Z_1$

$$= \frac{1}{2}\left({86823 + 73323} \right) \times 0.378 = 30268N$$

$${\tau _v} = \frac{V}{bd}$$

Assuming the thickness of toe slab to be reduced to 200 mm at edges, effective depth of slab at section Z-Z'

$$= 200 + \left({\frac{280 - 200}{0.6}} \right) \times 0.378 = 192.4mm$$

$${\tau _v} = \frac{30268}{1000 \times 192.4} = 0.16N/m{m^2} < 0.22N/m{m^2}$$, hence safe.

2. Design of heel slab: From the pressure diagram shown in Fig. 32.5, it can be seen that the upward pressure on the heel slab varies from 62403 N/m² at G to 15274 N/m² at E. In addition to the upward pressure the heel slab is also subjected to the following uniform downward pressure.

(i) Downward pressure due to self wt. of heel slab = 0.28 x 1.32 x 25000 = 9240N

Acting at $\frac{1.32}{2} = 0.66m$ from G.

(ii)Downward pressure due to self wt. of earthfill on heel slab

= 1.32 x 3.72 x 16000 = 78566N

acting at 0.66 m from G.

Total upward soil pressure   = Area of pressure trap. $EE_1G'G_1$

$$= \frac{1}{2}\left({62403 + 15274} \right)x1.32 = 51267N$$

acting at $\left({\frac{62403 + 2x15274}{62403 + 15274}} \right) \times \frac{1.32}{3}$ = 0.526 m from G.

Net B.M. at G   = B.M. due to downward pressure - B.M. due to upward pressure

Design of Structures

= 9240 x 0.66 + 78566 x 0.66 - 51267 x 0.526

= 6098 + 51854 - 26966

= 30986 x 10^3 Nmm

$$d = \sqrt{\frac{30986 \times {10}^3}{0.874 \times {10}^3}} = 188mm$$

However adopt same overall depth as far stem i.e., 280mm at G. Reduce, the thickness to 200 mm at the edges. Provide 50 mm clear cover.

Assuming 16mm ϕ bars, available effective cover = $280 - 50 - \frac{16}{2} = 222mm$

$$A_{st} = \frac{30986 \times {10}^3}{0.865 \times 222 \times 140} = 1153mm^2$$

Spacing of 16mm ϕ bars ($A_{\phi} = 201mm^2$) $= \frac{201}{1153} \times 1000 = 174mm$ say $170mm c/c$

Distribution reinforcement :

Same as far toe slab i.e., 10 mm ϕ bars @ 200 mm c/c

The main reinforcement of the heel should be carried into the toe through a distance equal to

$$L_d = \frac{\phi {\sigma}_{st}}{4\tau_{bd}} = \frac{\phi \times 140}{4 \times 0.6} = 58\phi = 58 \times 16 = 930mm \text{ to the left of G.}$$

Check for shear:

Max. S.F. in heel slab occurs at G. Its value being given by:

V= Total downward pressure - Total upward pressure

= 9240 + 78566 - 51267 = 36532 N

$$\tau_v = \frac{V}{bd} = \frac{36539}{222 x 1000} = 0.16 N/mm^2$$

< minimum value of τ_c , hence safe.

3. Design of stem slab: Since the base slab is 280 mm thick, clear height (H_1) of the stem above base slab $= 4.00 - 0.28 = 3.72m$

Max. B.M. at *G* $= \frac{wh^3}{6}.\frac{1 -- \sin\phi}{1 + \sin\phi} = \frac{16000 \times {3.72}^3}{6} \times \frac{1}{3} = 45759 Nm$

$= 45759 \times {10^3} Nm$

$D = 280$ and $d = 280 - 40 - \frac{16}{2} = 232mm$

$$A_{st} = \frac{45759 \times {10}^3}{0.865 \times 232 \times 140} = 1628mm^2$$

Spacing, using 16 mm ϕ bars $\left({A_\phi } = 201mm^2 \right)$ $= \frac{201}{1628} \times 1000 = 123mm\,say\,120mm$

Hence provide 16 mm ϕ bars @ 120 mm c/c

Extend the bars for a distance of $L_d = 58\phi = 58 \times 16 = 930\,\text{mm}$

beyond G into the bottom face of toe slab.

Beyond the distance of 930 mm alternate bars could be curtailed in the region of toe. However, since the toe projection is only 600 mm, it is proposed not to curtail the bars and continuous 16 mm ϕ bars @ 120 mm c/c in the full length of toe.

Curailment of reinforcement in stem wall :

Since the stem is of uniform thickness. Max. B.M. will get reduced to nearly half at a distance of 0.80 x 3.72 = 2.976 say 2.9m below top

BM. at 2.9 in below top $= \frac{16000 \times {2.9}^3}{6} \times \frac{1}{3} = 21679Nm$

Since this is less than $\frac{1}{2}$ of max. B.M. at G, alternate bars can be curtailed at a depth of 2.9m below top. However as per rules the bars to be curtailed shall extend by a distance equal to 12 x dia. of bars i.e., 12 x 16 = 192 mm or effective depth of the slab i.e., 232 mm whenever is more.

Hence alternate bars shall actually be curtailed at 2.9 – 0.23 = 2.67 m from top. The spacing of bars beyond this point will be 2 120 = 240 mm c/c.

Distribution reinforcement:

$= \frac{0.15}{100} \times 280 \times 100 = 420mm^2$

Area of reinforcement on each face $= 420 \times \frac{1}{2} = 210mm^2$

c/c spacing of 10mm ϕ bars $= \frac{79 \times 1000}{210} = 376mm\,say\,370mm\,c/c$

Hence provide 10mm ϕ bars @ 370 mm c/c both ways at the outer face (exposed) of the stem wall as temperature reinforcement and 10mm ϕ bars @ 370 mm c/c as horizontal reinforcement on inner face of wall.

Check for shear:

Max. S.F. occur at section through G its value being given by

$$V = \frac{w{h^2}}{2}.\frac{1 - \sin\phi }{1 + \sin\phi } = \frac{16000 \times {3.72}^2}{2} \times \frac{1}{3} = 36902N$$

$$\tau_v = \frac{36902}{1000 \times 232} = 0.159 < minimum\ permissible\ value\ of\ \tau_c,$$
Hence safe.

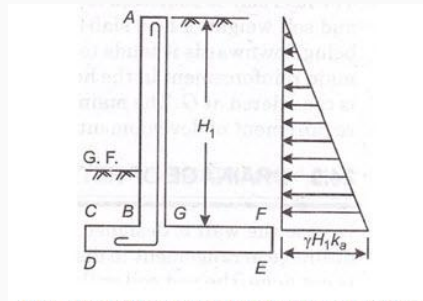The design diagram is shown in Fig. 32.8.

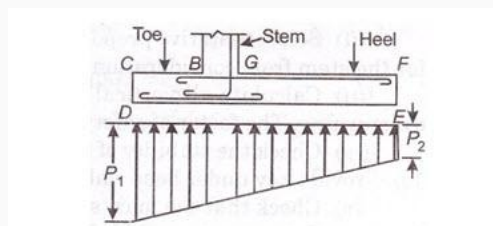Fig. 32.1 Pressure distribution on stem

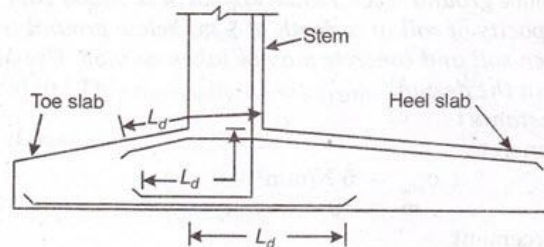Fig. 32.2 Pressure distribution on base slab

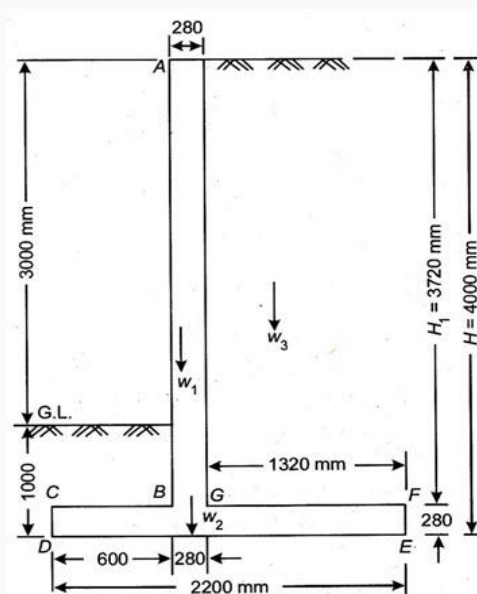Fig. 32.3 Development length of reinforcement for stem, heel and toe
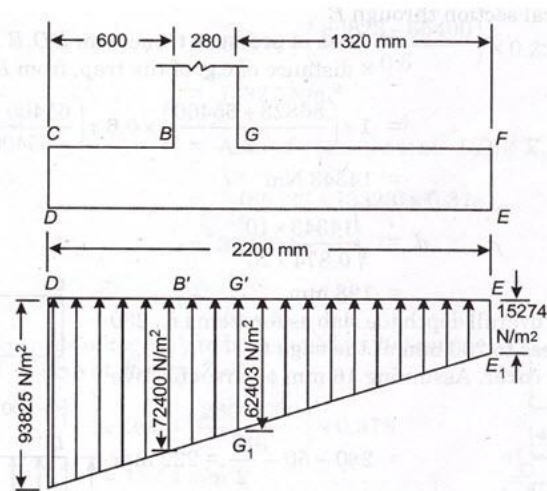

Fig. 32.4 Cantilever retaining wall

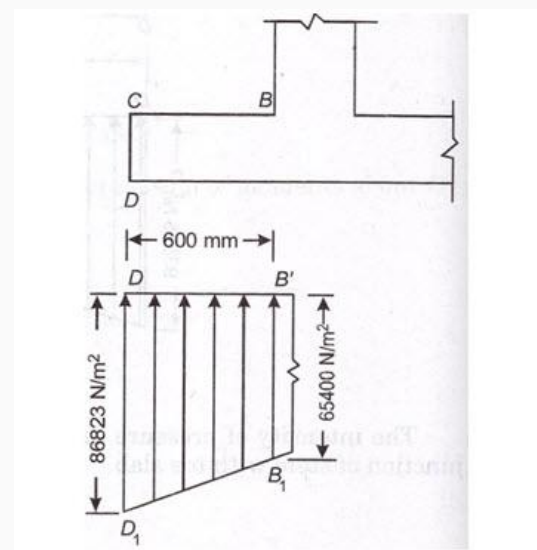Fig. 32.5 Pressure distribution below base slab
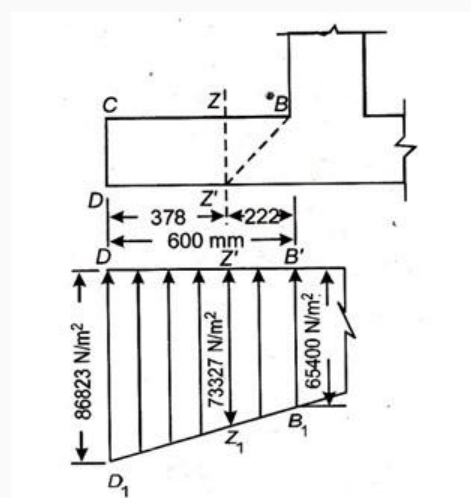

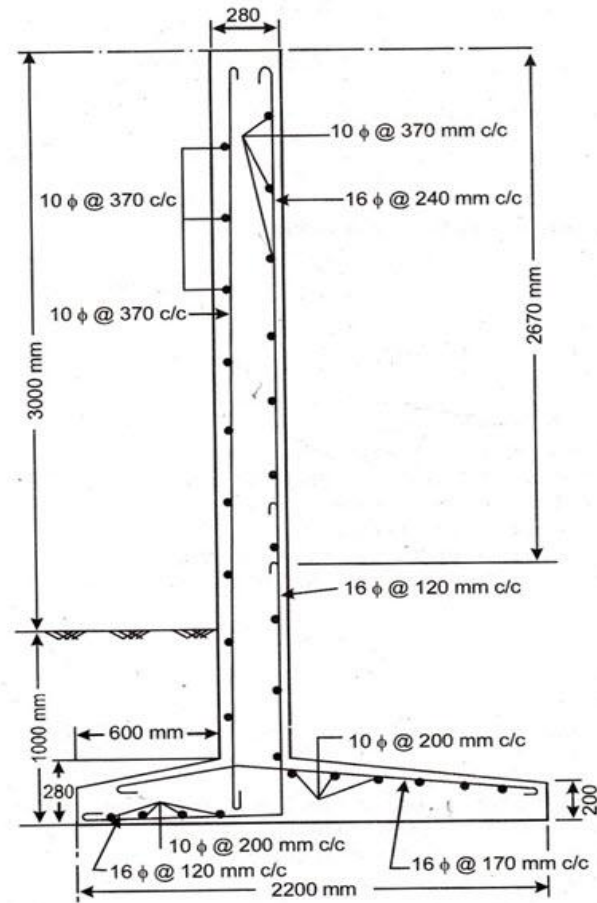Fig. 32.6 Pressure distribution below toe

Fig. 32.7 Critical section for shear

Fig. 32.8 Cross section of cantilever retaining wall with reinforcements

****** ☺ ******

This Book Download From e-course of ICAR

Visit for Other Agriculture books, News, Recruitment, Information, and Events at

WWW.AGRIMOON.COM

Give Feedback & Suggestion at
info@agrimoon.com

Send a Massage for daily Update of Agriculture on WhatsApp

+91-7900 900 676

DISCLAIMER:

The information on this website does not warrant or assume any legal liability or responsibility for the accuracy, completeness or usefulness of the courseware contents.

The contents are provided free for noncommercial purpose such as teaching, training, research, extension and self learning.

****** ☺ ******

Connect With Us: