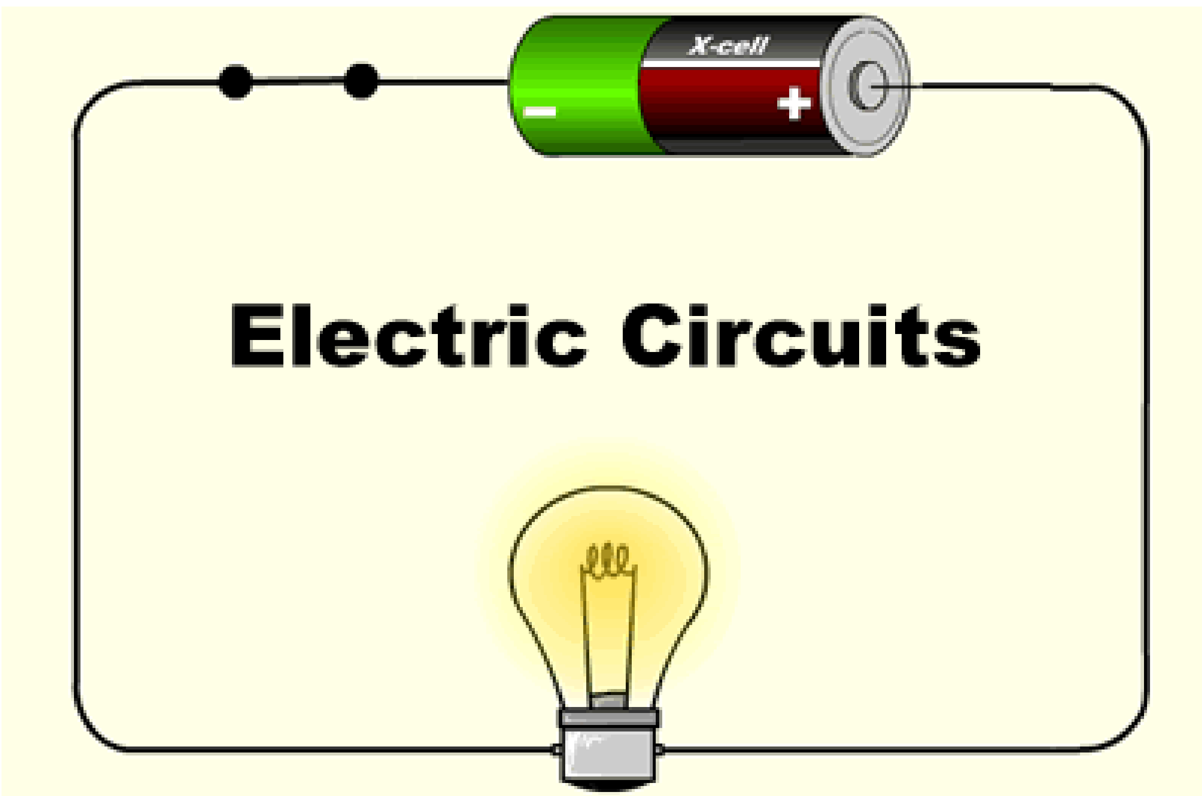
Electrical Circuits



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Electrical Circuits

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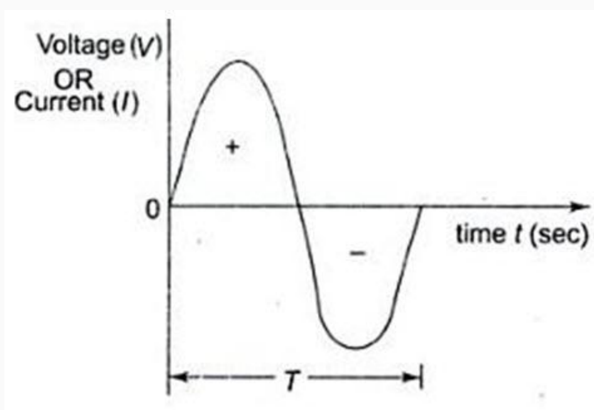
Module 1. Average and effective value of sinusoidal and linear periodic wave forms

LESSON 1. Average and effective value of sinusoidal forms

1. Introduction to Alternating Currents and Voltages

1.1 The Sine Wave

Many a time, alternating voltages and currents are represented by a sinusoidal wave, or simply a sinusoid. It is a very common type of alternating current (ac) and alternating voltage. The sinusoidal wave is generally referred to as a sine wave. Basically an alternating voltage (current) waveform is defined as the voltage (current) that fluctuates with time periodically, with change in polarity and direction. In general, the sine wave is more useful than other waveforms, like pulse, saw tooth, square, etc. There are a number of reasons for this. One of the reasons is that if we take any second order system, the response of this system is a sinusoid. Secondly, any periodic waveform can be written in terms of sinusoidal function according to Fourier there. Another reason is that its derivatives and integrals are also sinusoids. A sinusoidal function is easy to analyze. Lastly, sinusoidal function is easy to generate, and it is more useful in the power industry. The shape of a sinusoidal waveform is shown in Fig.1.1.



The waveform may be either a current waveform, or a voltage waveform. As seen from the Fig. 1.1, the wave changes its magnitude and direction with time. If we start at time $t = 0$, the wave goes to a maximum value and returns to zero, and the 4n decreases to a negative maximum value before returning to zero. The sine wave changes with time in an orderly manner. During the positive portion of voltage, the current flows in one direction; and during the negative portion of voltage, the current flows in the opposite direction. The complete positive and negative portion of the wave is one cycle of the sine wave. Time is designated by t. The time taken for any wave to complete one full cycle is called the period (T). In general, any periodic wave constitutes a number of such cycles. For example, one cycle of a sine wave repeats a number of times as shown in Fig.1.2. Mathematically it can be represented as $f((t) = f(t + T)$ for any t.

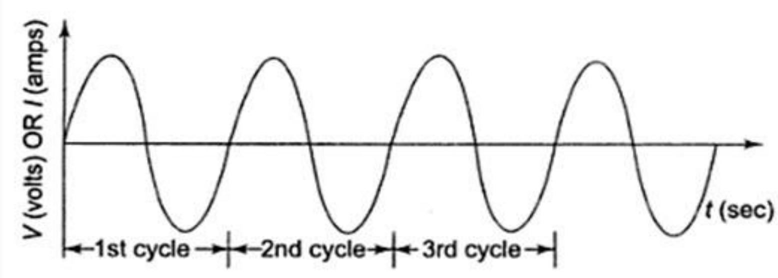


Fig.1.2

The period can be measured in the following different ways (See Fig.1.3).

1. From zero crossing of one cycle to zero crossing of the next cycle.
2. From positive peak of one cycle to positive peak of the next cycle, and
3. From negative peak of one cycle to negative peak of the next cycle.

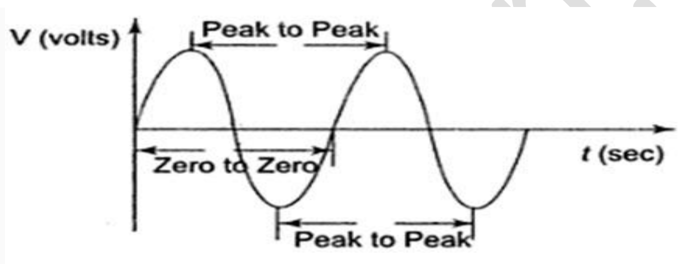


Fig.1.3

The frequency of a wave is defined as the number of cycles that a wave completes in one second.

In Fig.1.4 the sine wave completes three cycle in one second. Frequency is measured in hertz. One hertz is equivalent to one cycle per second, 60 hertz is 60 cycles per second and so on. In Fig.1.4, the frequency denoted by f is 3 Hz, that is three cycles per second. The relation between time period and frequency is given by

$$f={1 \over T}$$

A sine wave with a longer period consists of fewer cycles than one with a shorter period.

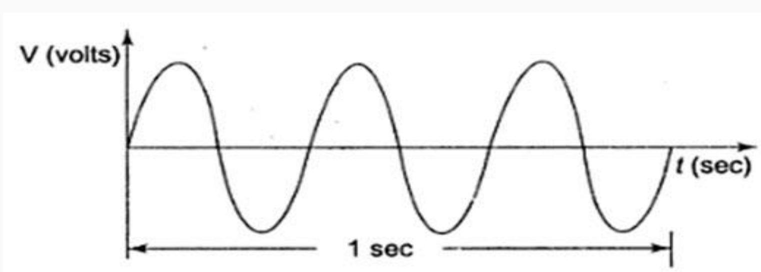


Fig.1.4

1.2. Angular Relation of a Sine Wave

A sine wave can be measured along the *X-axis* on a time base which is frequency-dependent. A sine wave can also be expressed in terms of an angular measurement. This angular measurement is expressed in degrees or radians. A radian is defined as the angular distance measured along the circumference of a circle which is equal to the radius of the circle. One radian is equal to 57.3^0. In a 360^0 revolution, there are 2 \[\pi\] radians. The angular measurement of a sine wave is based on 360^0 or 2\[\pi\] radians for a complete cycle as shown in Figs.1.5 (a) and (b).

A sine wave completes a half cycle in 180^0 or \[\pi\] radian, a quarter cycle in 90^0 or \[\pi\]/2 radians, and so on.

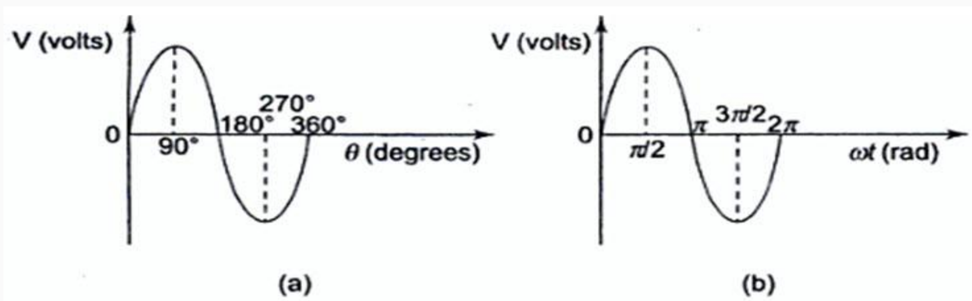


Fig.1.5

Phase of a Sine Wave

The phase of a sine wave is an angular measurement that specifies the position of the sine wave relative to a reference. The wave shown in Fig.1.6 is taken as the reference wave. When the sine wave is shifted left or right with reference to the wave shown in Fig.1.6, there occurs a phase shift. Figure 4.8 shows the phase shifts of a sine wave.

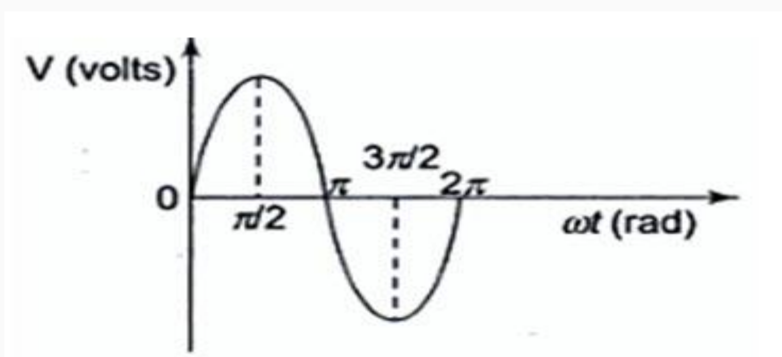


Fig.1.6

In Fig.1.7 (a), the sine wave is shifted to the right by 90^0 (\[\pi\]/2 rad) shown by the dotted lines. There is a phase angle of 90^0 between A and B. Here the waveform B is lagging behind waveform A by 90^0. In other words, the sine wave A is leading the waveform B by 90^0. In Fig.1.7 (b) the sine wave A is lagging behind the waveform B by 90^0. In both cases, the phase difference is 90^0.

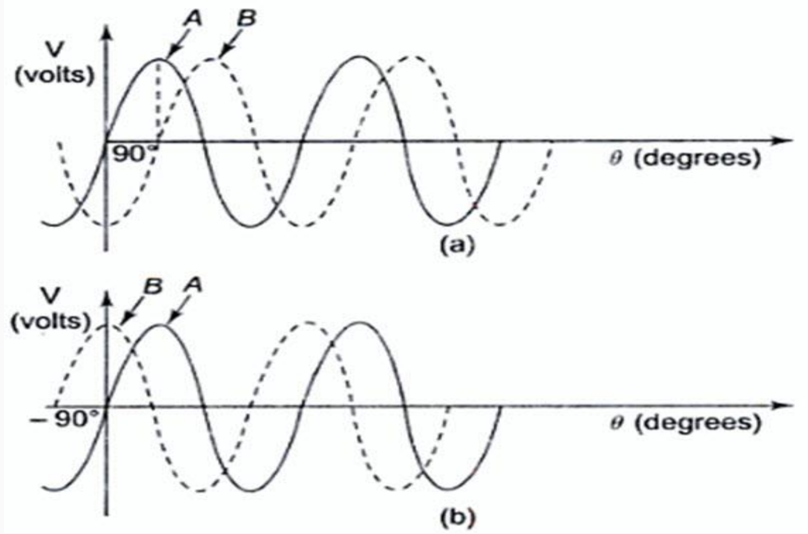


Fig. 1.7

1.3. The sine Wave Equation

A sine wave is graphically represented as shown in Fig. 1.8 (a). The amplitude of a sine wave is represented on vertical axis. The angular measurement (in degrees or radians) is represented on horizontal axis. Amplitude A is the maximum value of the voltage or current on the Y-axis.

In general, the sine wave is represented by the equation

$$V(t) = V_m \sin w_t$$

The above equation states that any point on the sine wave represented by instantaneous value v(t) is equal to the maximum value times the sine of the angular frequency at the point. For example, if a certain sine wave voltage has peak value of 20 V, the instantaneous voltage at a point π/4 radius along the horizontal axis can be calculated has

$$V(t) = V_m \sin w_t$$

\[=20\,\sin \left({{\pi\over 4}} \right)=20 \times 0.707=14.14V\]

When a sine wave is shifted to the left of the reference wave by a certain angle ø, as shown in Fig.1.8 (b), the general expression can be written as

$$v(t) = V_m \sin (wt + ø)$$

When a sine wave is shifted to the right of the reference wave by a certain angle ø, as shown in Fig.1.8 (c), the general expression can be written as

$$v(t) = V_m \sin (wt - ø)$$

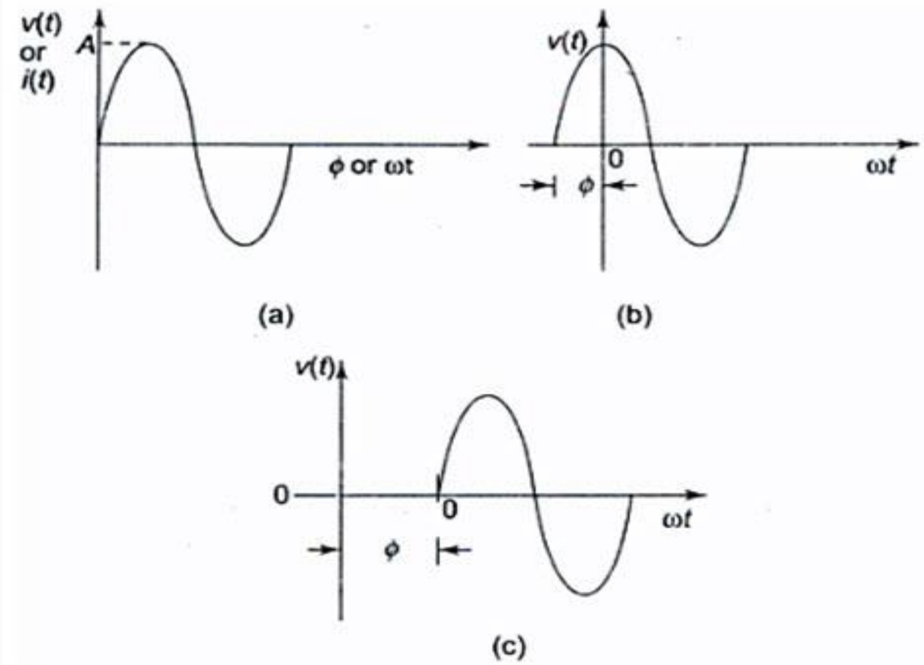


Fig.1.8

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LESSON 2. Linear periodic wave forms

2.1 Voltage and Current Values of A sine Wave

As the magnitude of the waveform is not constant, the waveform can be measured in different ways. These are instantaneous, peak, peak to peak, root mean square *(rms)* and average values.

Instantaneous value

Consider the sine wave shown in Fig.2.1. At any given time, it has some instantaneous value. This value is different at different points along the waveform.

In fig. 2.1 during the positive cycle, the instantaneous values are positive and during the negative cycle, the instantaneous values are negative. In Fig.2.1 shown at time 1 ms, the value is 4.2 V; the value is 10 V at 2.5 ms, - 2V at 6 ms and -10 V at 7.5 and so on.

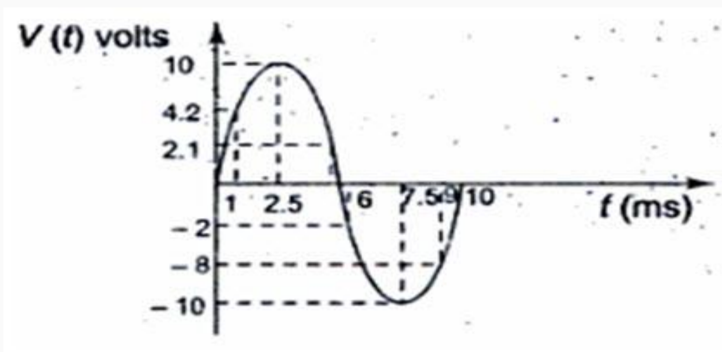


Fig.2.1

Peak value

The peak value of the sine wave is the maximum value of the wave during positive half cycle, or maximum value of wave during negative half cycle. Since the value of these two is equal in magnitude, a sine wave is characterized by a single peak value. The peak value of the sine wave is shown in Fig.2.2; here the peak value of the sine wave is 4 V.

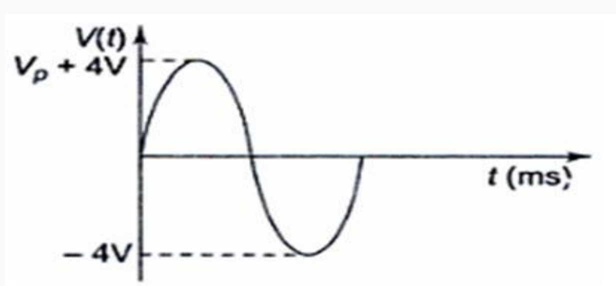


Fig. 2.2

Peak to Peak Value

The peak to peak value of a sine wave is the value from the positive to the negative peak as shown in Fig.2.3. Here the peak to peak value is 8V.

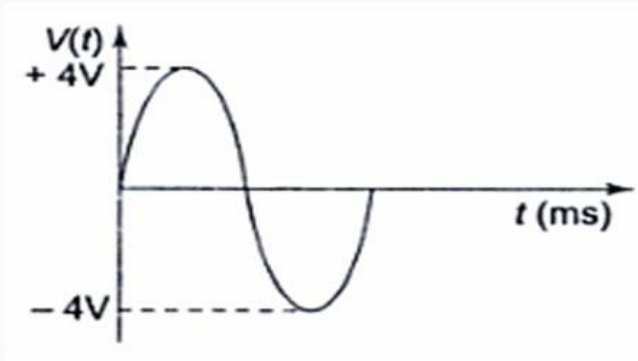


Fig. 2.3

Average Value

In general, the average value of any function *v(t),* with period T is given by

$${v_{av}}={1 \over T}\int\limits_0^T {v\left(t \right)} dt$$

That means that the average value of a curve in the *X-Y* plane is the total area under the complete curve divided by the distance of the curve. The average value of a sine wave over one complete cycle is always zero. So the average value of a sine wave is defined over a half-cycle, and not a full cycle period. The average value of the sine wave is the total area under the half-cycle curve divided by the distance of the curve.

$v(t) = V_p \sin wt$ is given by

$${v_{av}}={1\over\pi}\int\limits_0^\pi{{V_P}Sin\,\,\omega t\,\,d(\omega t)}$$

$$={1 \over \pi }\left[{ - {V_P}Cos\omega t} \right]_0^\pi$$

$$={{2{V_P}} \over \pi }=\,\,0.637\,{V_P}$$

The average value of a sine wave is shown by the dotted line in Fig.2.4

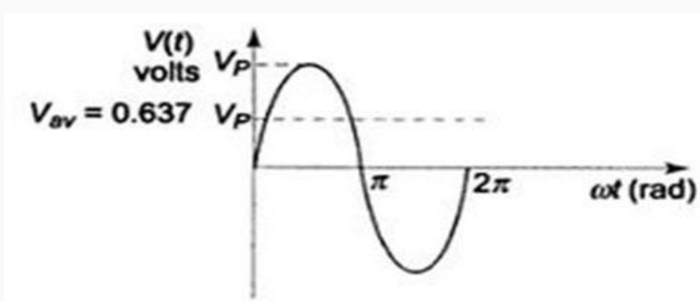


Fig.2.4

Root Mean Square Value or Effective Value

The root mean square *(rms)* value of a sine wave is a measure of the heating effect of the wave. When a resistor is connected across a dc voltage source as shown in Fig. 2.5 (a), a certain amount of heat is produced in the resistor in a given time. A similar resistor is connected across an ac voltage source for the same time as shown in Fig. 2.5 (b). The value of the ac voltage is adjusted such that the same amount of heat is produced in the resistor as in the case of the dc source. The value is called the *rms* value.

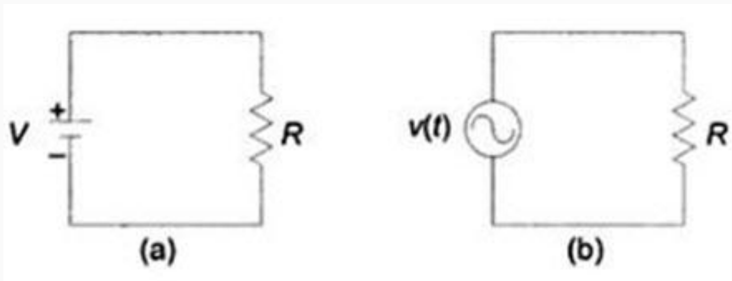


Fig.2.5

That means the *rms* value of a sine wave is equal to the dc voltage that produces the same heating effect. In general, the *rms* value of any function with period *T* has an effective value given by

$${V_{rms}}=\sqrt {{1 \over T}\int\limits_0^T {{v\left({t)} \right)}\limits^{ - - 2} dt} }$$

Consider a function $v(t) = V_p$ Sin wt

The *rms* value, $${V_{rms}}=\sqrt {{1 \over T}\int\limits_0^T {{{\left({{V_p}\,\sin \,\omega t} \right)}^2}\,\,d\left({\omega t} \right)} }$$

$$=\,\sqrt {{1 \over T}\int\limits_0^{2\pi} {V_p^2\,\left[{{{1 - \cos \,2\omega t} \over 2}} \right]d\,\left({\omega t} \right)} }$$

$$=\,{{{V_p}} \over {\sqrt 2 }}=0.707\,\,{V_p}$$

If the function consists of a number of sinusoidal terms, that is

$$v(t)\,={V_0} + \left({{V_{c1}}\cos \,\,\omega t + {V_{c2}}\,\,\cos \,2\,\,\omega t + ...} \right) + \left({{V_{s1}}\,\sin \,\omega t + {V_{s2}}\,\,\sin \,2\,\omega t + ...} \right)$$

$$Vrms=\sqrt {V_0^2 + {1 \over 2}\left({V_{c1}^2 + V_{c2}^2 + ...} \right) + {1 \over 2}\left({V_{s1}^2 + V_{s2}^2 + ...} \right)}$$

Peak Factor

The peak factor of any waveform is defined as the ratio of the peak value of the wave to the *rms* value of the wave.

Peak factor $$={{{V_p}} \over {{V_{rms}}}}$$

Peak factor of the sinusoidal waveform $={{{V_p}} \over {{V_p}/\sqrt 2 }}=\sqrt 2=1.414$

Form Factor

Form factor of a waveform is defined as the ratio of *rms* value to the average value of the wave.

Form factor $={{{V_{rms}}} \over {{V_{av}}}}$

Form factor of a sinusoidal waveform can be found from the above relation.

For the sinusoidal wave, the form factor $={{Vp/\sqrt 2 } \over {0.637\,{V_p}}}=1.11$

2.2. Phase Relation in a Pure Resistor

When a sinusoidal voltage of certain magnitude is applied to a resistor, a certain amount of sine wave current passes through it. We known the relation between *v(t)* and *i(t)* in the case of a resistor. The voltage / current relation in case of a resistor is linear,

i.e. $v(t) = i(t) R$

Consider the function

$i\left(t \right)={I_m}\sin \,\omega t=IM\left[{{I_m}{e^{j\omega t}}} \right]\,or\,{I_m}\angle {0^0}$

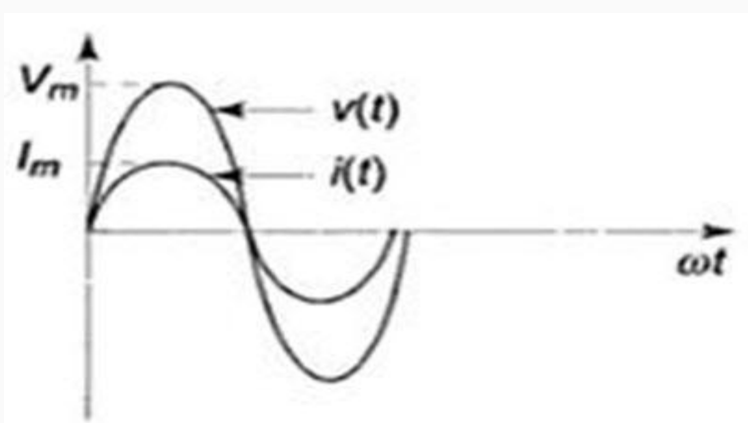
If we substation this in the above equation, we have

$v(t)=I_mR \sin wt = V_m \sin wt$

$IM\left[{{V_m}{e^{j\omega t}}} \right]\,or\,{V_m}\angle {0^0}$

where $V_m = I_m R$

If we draw the waveform for both voltage and current as shown in Fig.2.6. there is no phase difference between these two waveforms. The amplitudes of the waveform may differ according to the value of resistance.



As a result, in pure resistive circuits, the voltages and currents are said to be in phase. Here the term impedance is defined as the ratio of voltage to current function. With ac voltage applied to elements, the ratio of exponential voltage to the corresponding current (impedance) consists of magnitude and phase angles. Since the phase difference is zero in case of a resistor, the phase angle is zero. The impedance in case of resistor consists only of magnitude, i.e.

$$Z={{{V_m}\angle {0^0}} \over {{I_m}\angle {0_0}}}=R$$

2.3. Phase Relation in a Pure Inductor

The voltage current relation in the case of an inductor is given by

$$v\left(t \right)=L{{di} \over {dt}}$$

Consider the function $i\left(t \right)={I_m}\sin \,\omega t=IM\left[{{I_m}{e^{j\omega t}}} \right]\,or\,{I_m}\angle {0^0}$

$$v\left(t \right)=L{d \over {dt}}\left({{I_m}\sin \,\omega t} \right)$$

$$=L\omega {I_m}\,\cos \,\omega t\,{I_m}\,\cos \,\omega t$$

$$v\left(t \right)={V_m}\cos \,\omega t,\,\,or\,\,{V_m}\,\sin \left({\omega t + {{90}^0}} \right)$$

$$=IM\left[{{V_m}{e^{j\left({\omega t + {{90}^0}} \right)}}} \right]\,\,or\,\,{V_m}\angle {90^0}$$

Where ${V_m}=\omega L{I_m}={X_L}{I_m}$

and ${e^{j{{90}^0}}}=j=1\angle {90^0}$

If we draw the waveforms for both, voltage and current, as shown in Fig.2.7, we can observe the phase difference between these two waveforms.

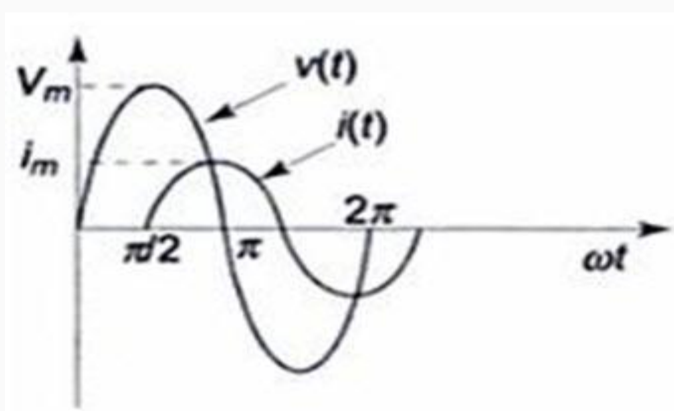


Fig.2.7

As a result, in a pure inductor the voltage and current are out of phase. The current lags behind the voltage by 90^0 in a pure inductor as shown in Fig.2.8.

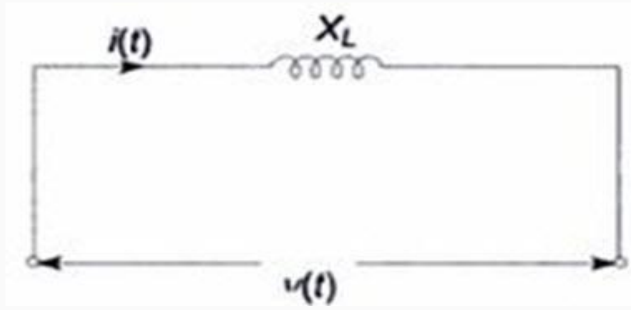


Fig.2.8

The impedance which is the ratio of exponential voltage to the corresponding current, is given by

$$Z={{{V_m}\sin \left({\omega t + {{90}^0}} \right)} \over {{I_m}\sin \omega t}}$$

where $${V_m}=\omega L{I_m}$$

$$={{{I_m}\omega L\sin \left({\omega t + {{90}^0}} \right)} \over {{I_m}\sin \omega t}}$$

$$={{\omega L\,{I_m}\angle {{90}^0}} \over {{I_m}\angle {0^0}}}$$

$$Z=j\omega L=j{X_L}$$

where $X_L = wL$ and is called the inductive reactance.

Hence, a pure inductor has impedance whose value is wL,

2.4. Phase Relation in a Pure Capacitor

The relation between voltage and current is given by

$$v\left(t \right)={1 \over C}\int {i\left(t \right)} dt$$

Consider the function $$i\left(t \right)={I_m}\sin \,\omega t = IM\left[{{I_m}{e^{j\omega t}}} \right]\,or\,{I_m}\angle {0^0}$$

$$v\left(t \right)={1 \over C}\int {{I_m}\;\sin \,\omega t\,d\left(t \right)}$$

$$={1 \over {\omega C}}{I_m}\left[{ - \cos \,\omega t} \right]$$

$$={{{I_m}} \over {\omega C}}\sin \,\left({\omega t - {{90}^0}} \right)$$

$$v\left(t \right)={V_m}\,\sin \,\left({\omega t - {{90}^0}} \right)$$

$$=IM\left[{{\mathop{\rm Im}\nolimits} {e^{j\left({\omega t - {{90}^0}} \right)}}} \right]\,\,or\,\,{V_m}\angle-{90^0}$$

where $${V_m}={{{I_m}} \over {\omega C}}$$

$$={{{V_m}\angle-{{90}^0}} \over{{I_m}\angle {0^0}}}=Z={{ - j}\over{\omega C}}$$

Hence, the impedance is $$Z={{-j}\over{\omega C}}=-j{X_C}$$

where $${X_C}={1\over{\omega C}}$$ and is called the capacitive reactance.

If we draw the waveform for both, voltage and current, as shown in Fig.2.9, there is a phase difference between these two waveforms.

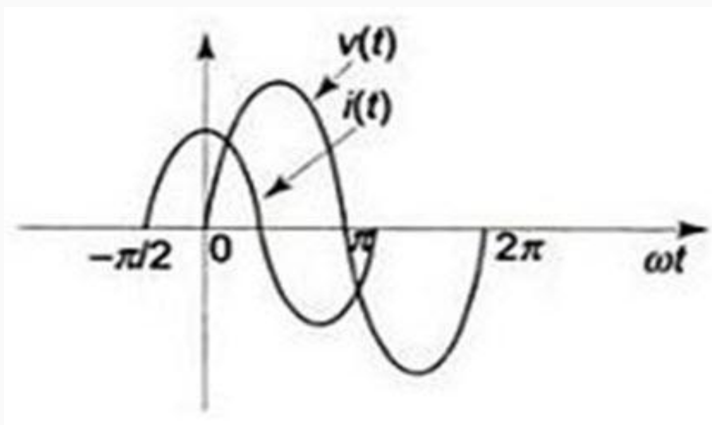


Fig.2.9

As a result, in a pure capacitor, the current leads the voltage by 90^0. The impedance value of a pure capacitor.

$${X_C}={1 \over {\omega C}}$$

Module 2. Independent and dependent sources, loop current and loop equations (Mesh current method)

LESSON 3. Independent Sources

3.1. The Circuit

Simply an electric circuit consists of three parts: (1) energy source, such as battery or generator, (2) the load or sink, such as lamp or motor, and (3) connecting wires as shown in Fig. 3.1. This arrangement represents a simple circuit. A battery is connected to a lamp with two wires. The purpose of the circuit is to transfer energy from source (battery) to the load (lamp). And this is accomplished by the passage of electrons through wires around the circuit.

The current flows through the filament of the lamp, causing it to emit visible light. The current flows through the battery by chemical action. A closed circuit is defined as a circuit in which the current has a complete path to flow. When the current path is broken so that current cannot flow, the circuit is called an open circuit.

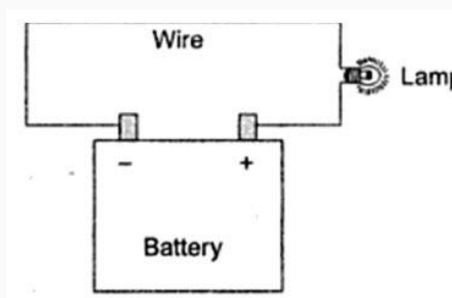


Fig.3.1

More specifically, interconnection of two or more simple circuit elements (viz. voltage sources, resistors, inductors and capacitor) is called an electric network. If a network contains at lest one closed path, it is called an electric circuit. By definition, a simple circuit element is the mathematical model of two terminal electrical devices, and it can be completely characterized by its voltage and current. Evidently then, a physical circuit must provide means for the transfer of energy.

Broadly, network elements may be classified into four groups, viz.

1. Active or passive
2. Unilateral or bilateral
3. Linear or nonlinear
4. Lumped or distributed

3.1.1. Active and Passive

Energy source (voltage or current sources are active elements, capable of delivering power to some external device. Passive elements are those which are capable only of receiving power. Some passive elements like inductors and capacitors are capable of storing a finite amount of energy, and return it later to an external element. More specifically, an active element is capable of delivering an average power greater than zero to some external device over an infinite time interval. For example, ideal sources are active elements. A passive element is defined as one that cannot supply average power that is greater than zero over an infinite time interval. Resistors, capacitors and inductors fall into this category.

3.1.2. Bilateral and Unilateral

In the bilateral element, the voltage-current relation is the same for current flowing in either direction. In contrast, a unilateral element has different relations between voltage and current for the two possible directions of current. Examples of bilateral elements are elements made of high conductivity materials in general. Vacuum diodes, silicon diodes, and metal rectifiers are examples of unilateral elements.

3.1.3. Linear and Nonlinear Elements

An element is said to be linear, if its voltage-current characteristic is all times a straight line through the origin. For example, the current passing through a resistor is proportional to the voltage applied through it, and the relation is expressed as VµI or V=IR. A liner element or network is one which satisfies the principle of superposition, i.e. the principle of homogeneity and additivity. An element which does not satisfy the above principle is called a nonlinear element.

3.1.4. Lumped and Distributed

Lumped elements are those elements which are very small in size and in which simultaneous actions takes place for any given cause at the same instant of time. Typical lumped elements are capacitors, resistors, inductors and transformers. Generally the elements are considered as lumped when their size is very small compared to the wave length of the applied signal. Distributed elements, on the other hand, are those which are not electrically separable for analytical purposes. For example, a transmission line which has distributed resistance, inductance and capacitance along its length may extend for hundreds of miles.

3.2 Independent sources

The source which does not depends on other voltages or currents in the network for their value. These are represented by a circle with a polarity of voltage or direction of current indicated inside

Energy Sources

According to their terminal voltage-current characteristics, electrical energy sources are categorized into ideal voltage sources and ideal current sources. Further they can be divided into independent and dependent sources.

An ideal voltage source is a two-terminal element in which the voltage v_s is completely independent of the current i_s through its terminals. The representation of ideal constant voltage source is shown in Fig.3.2 (a).

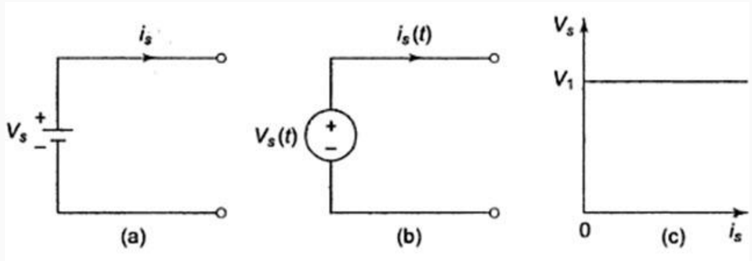


Fig. 3.2

If we observe the v=I characteristics for an ideal voltage source as shown in Fig.3.2 (c) at any time, the value of the terminal voltage v_s is constant with respect to the value of current i_s. Whenever v_s = 0, the voltage source is the same as that of a short circuit. Voltage sources need not have constant magnitude; in many cases the specified voltage may be time-dependent like a sinusoidal waveform. This may be represented as shown in Fig.3.2 (b). In many practical voltage sources, the internal resistance is represented in series with the source as shown in Fig.3.3 (a). In this, the voltage across the terminals falls as the current through it increases, as shown in Fig. 3.3 (b).

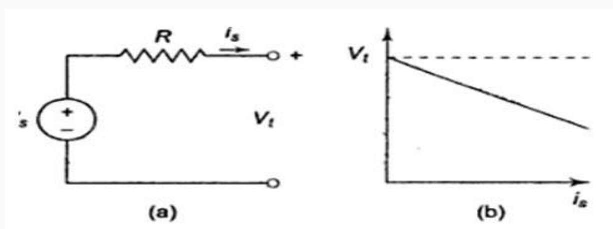


Fig. 3.3

The terminal voltage v_t depends on the source current as shown in Fig. 3.3(b), where $v_t = v_s - I$, R.

An ideal constant current source is a two-terminal element in which the current i_s completely independent of the voltage v_s across its terminals. Like voltage sources we can have current sources of constant magnitude i_s or sources whose current varies with time $i_s(t)$. The representation of an ideal current source is shown in Fig.3.4 (a).

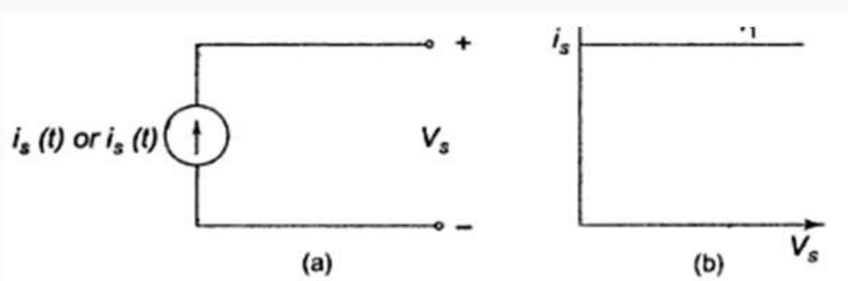


Fig.3.4

If we observe the $v - i$ characteristics for an ideal current source as shown in Fig.3.4 (b), at any time the value of the current i_s is constant with respect to the voltage across it. In many practical current sources, the resistance is in parallel with a source as shown in Fig. 3.5 (a). In this the magnitude of the current falls as the voltage across its terminals increases. Its terminal $v - i$ characteristic is shown in Fig. 3.5 (b). The terminal current is given by $i_t - i_s - (v_s/R)$, where R is the internal resistance of the ideal current source.

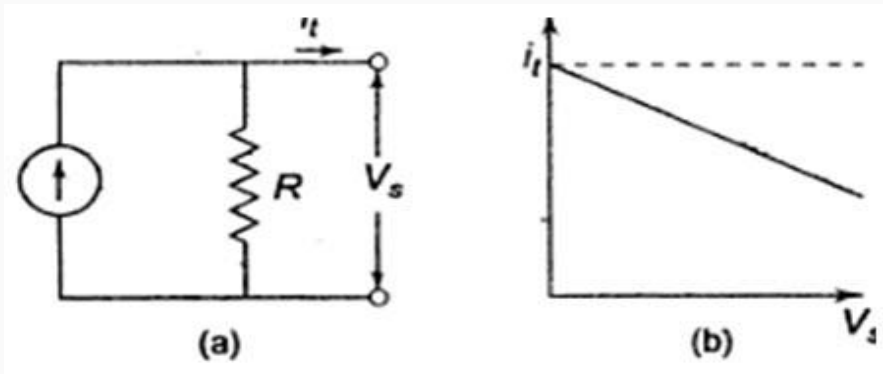


Fig.3.5

The two types of ideal sources we have discussed are independent sources for which voltage and current are independent and are not affected by other parts of the circuit. In the case of dependent sources, the source voltage or current is not fixed, but is dependent on the voltage or current existing at some other location in the circuit.

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LESSON 4. Dependent sources

4.1 Dependent or controlled sources are of the following types

(i) Voltage controlled voltage source (VCVS)

(ii) Current controlled voltage source (CCVS)

(iii) Voltage controlled current source (VCCS)

(iv) Current controlled current source (CCCS)

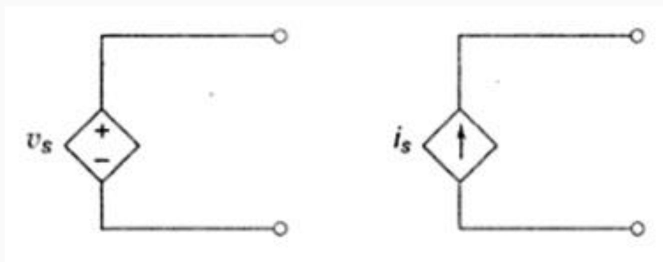

Fig. 4.1

These are represented in a circuit diagram by the symbol shown in Fig.4.1. These types sources mainly occur in the analysis of equivalent circuits of transistors.

4.2. Kirchhoff's Voltage Law

Kirchhoff's voltage law states that the algebraic sum of all branch voltages around any closed path in a circuit is always zero at all instants of time. When the current passes through a resistor, there is a loss of energy and therefore, a voltage drop. In any element, the current always flows from higher potential to lower potential. Consider to circuit in Fig.4.2. It is customary to take the direction of current *I* as indicated in the figure, i.e. it leaves the positive terminal of the voltage source and enters into the negative terminal.

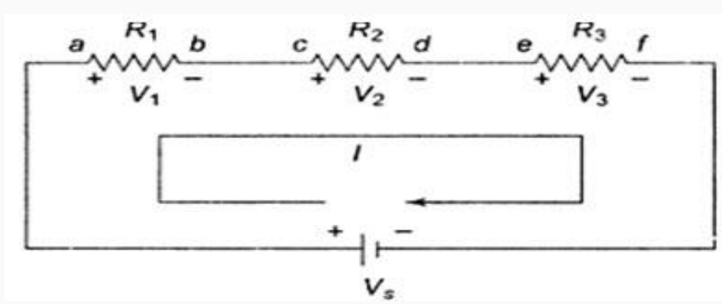

Fig. 4.2

As the current passes through the circuit, the sum of the voltage drop around the loop is equal to the total voltage in that loop. Here the polarities are attributed to the resistors to indicate that the voltages at points a, c and e are more than the voltages at *b*, *d* and *f*, respectively, as the current passes from *a* to *f*.

$$V_s = V_1 + V_2 + V_3$$

Consider the problem of finding out the current supplied by the source V in the circuit shown in Fig. 4.3.

Our first step is to assume the reference current direction and to indicate the polarities for different elements. (See Fig.4.4).

By using Ohm's law, we find the voltage across each resistors as follows.

$$V_{R1} = IR_1,\ V_{R2} = IR_2,\ V_{R3} = IR_3$$

where V_{R1}, V_{R2} and V_{R3} are the voltages across R_1, R_2 and R_3, respectively. Finally, by applying Kirchhoff's law, we can form the equation.

$$V = V_{R1} + V_{R2} + V_{R3}$$

$$V = IR_1 + IR_2 + IR_3$$

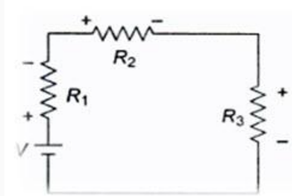

Fig.4.3

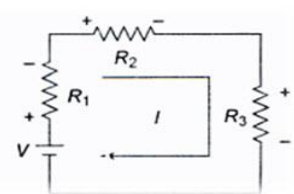

Fig. 4.4

From the above equation the current delivered by the source is given by

$$I = {V \over {{R_1} + {R_2} + {R_3}}}$$

4.3. Voltage Division

The series circuit acts as a voltage divider. Since the same current flows through each resistor, the voltage drops are proportional to the values of resistors. Using this principle, different voltages can be obtained from a single source, called a voltage divider. For example, the voltage across a 40 W resistor is twice that of 20 W in a series circuit shown in Fig. 4.5

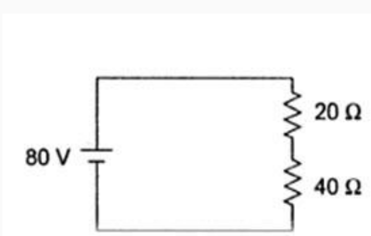

Fig.4.5

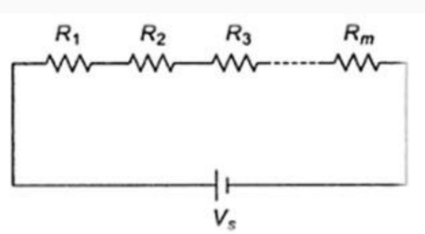

Fig.4.6

In general, if the circuit consists of a number of series resistors, the total current is given by the total voltage divided by equivalent resistance. This is shown in Fig. 4.6.

The current in the circuit is given by $I = V_s/(R_1+R_2+\ldots+R_m)$. The voltage across any resistor is noting but the current passing through it, multiplied by that particular resistor.

Therefore, $V_{R1} = IR_1$

$V_{R2} = IR_2$

$V_{R3} = IR_3$

$V_{Rm} = IR_m$

or

$${V_{Rm}}={{{V_s}\left({{R_m}} \right)} \over {{R_1} + {R_2} + ... + {R_m}}}$$

From the above equation, we can say that the voltage drop across any resistor or a combination of resistors, in a series circuit is equal to the ratio of that resistance value to the total resistance, multiplied by the source voltage, i.e.

$${V_m}={{{R_m}} \over {{R_T}}}{V_s}$$

where V_m is the voltage across mth resistor, R_m is the resistance across which the voltage is to be determined and R_T is the total series resistance.

4.4. Power in a Series Circuit

The total power supplied by the source in any series resistive circuit is equal to the sum of the powers in each resistor in series, i.e.

$P_s = P_1+P_2+P_3+\ldots.+P_m$

Where m is the number of resistors in series, P_s is the total power supplied by source and P_m is the power in the last resistor in series. The total power in the series circuit is the total voltage applied to a circuit, multiplied by the total current. Expressed mathematically,

$${V_m}={{{R_m}} \over {{R_T}}}{V_s}$$

where V_s is the total voltage applied, R_T is the total resistance, and I is the total current.

4.5. Kirchhoff's Current Law

Kirchhoff's current law states that the sum of the currents entering into any node is equal to the sum of the currents leaving that node. The node may be an interconnection of two or more branches. In any parallel circuit, the node is a junction point of two or more branches. The total current entering into a node is equal to the current leaving that node. For example, consider the circuit shown in Fig.4.7, which contains two nodes A and B.

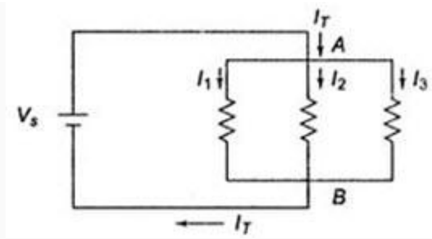

Fig.4.7

The total current I_1, I_2 and I_3. These currents flow out of node A. According to Kirchhoff's current law, the current into node A is equal to the total current out of node A: that is, $I_T = I_1+I_2+I_3$. If we consider node B, all three currents I_1, I_2, I_3 are entering B, and the total current I_T is leaving node B, Kirchhoff's current law formula at this node is therefore the same as at node A.

$$I_1+I_2+I_3 = I_T$$

In general, sum of the currents entering any point or node or junction equal to sum of the currents leaving from that point or node or junction as shown in Fig.4.8.

$$I_1 + I_2 + I_4 + I_7 = I_3 + I_5 + I_6$$

If all of the terms on the right side are brought over to the left side, their signs change to negative and a zero is left on the right side, i.e.

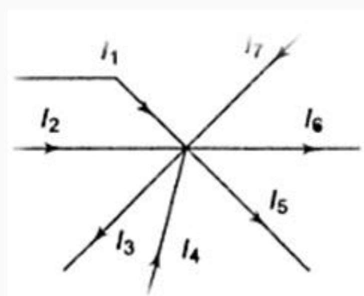

Fig.4.8

$$I_1 + I_2 + I_4 + I_7 - I_3 - I_5 - I_6 = 0$$

This means that the algebraic sum of all the currents meeting at a junction is equal to zero.

4.6. Parallel Resistance

When the circuit is connected in parallel, the total resistance of the circuit decreases as the number of resistors connected in parallel increases. If we consider m parallel branches in a circuit as shown in Fig.4.8, the current equation is

$$I_T = I_1 + I_2 + \dots + I_m$$

The same voltage is applied across each resistor. By applying Ohm's law, the current in each branch is given by

$$ {I_1}={{{V_s}} \over {{R_1}}},\,{I_2} = {{{V_s}} \over {{R_2}}},.....{I_m}={{{V_s}} \over {{R_m}}} $$

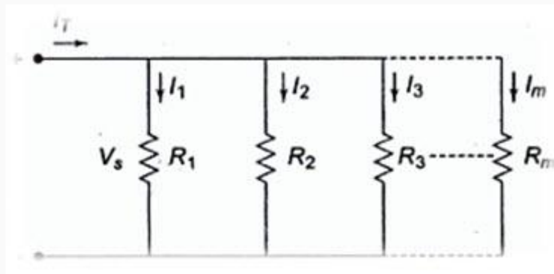

Fig. 4.9

According to Kirchhoff's current law,

$$I_T = I_1 + I_2 + I_3 + \ldots + I_m$$

$$ {{{V_s}} \over {{R_T}}} = {{{V_s}} \over {{R_1}}} + {{{V_s}} \over {{R_2}}} + {{{V_s}} \over {{R_3}}} + ... + {{{V_s}} \over {{R_m}}} $$

From the above equation, we have

$$ {1 \over {{R_T}}}={1 \over {{R_1}}} + {1 \over {{R_2}}} + ... + {{{V_s}} \over {{R_m}}} $$

4.7. Current Division

In a parallel circuit, the current divides in all branches. Thus, a parallel circuit acts as a current divider. The total current entering into the parallel branches is divided into the braches currents according to the resistance values. The branch having higher resistance allows lesser current, and the branch with lower resistance allows more current. Let us find the current division in the parallel circuit shown in Fig. 4.10.

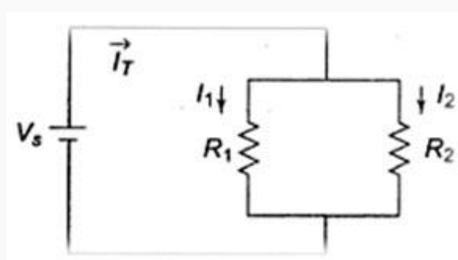

Fig. 4.10

The voltage applied across each resister is V_s. The current passing through each resistors is given by

$$ {I_1}={{{V_S}} \over {{R_1}}},\,{I_2}={{{V_s}} \over {{R_2}}} $$

If *R1* is the total resistance, which is given by $R_1R_2/ (R_1+R_2)$, *Total current* $${I_T}={{{V_S}} \over {{R_T}}}={{{V_s}} \over {{R_1}{R_2}}}\left({{R_1} + {R_2}} \right)$$

or $${I_T}={{{I_1}{R_1}} \over {{R_1}{R_2}}}\left({{R_1} + {R_2}} \right)\,\sin ce\,\,\,{V_s}\,=\,{I_1}{R_1}$$

$${I_1}={I_T}={{{R_2}} \over {{R_1} + {R_2}}}$$

Similarly, $${I_2}={I_T}.{{{R_1}} \over {{R_1} + {R_2}}}$$

From the above equations, we can conclude that the current in any branch is equal to the ratio of the opposite branch resistance to the total resistance value, multiplied by the total current in the circuit. In general, if the circuit consists of m branches, the current in any branch can be determined by

$${I_i}={{{R_T}} \over {{R_i} + {R_T}}}{I_T}$$

where I_i represents the current in the ith branch

R_i is the resistance in the ith branch

R_T is the total parallel resistance to the ith branch and

I_T is the total current entering the circuit.

4.8. Power in a Parallel Circuit

The total power supplied by the source in any parallel resistive circuit is equal to the sum of the powers in each resistor in parallel , i.e.

$$P_S = P_1 + P_2 + P_3 + \ldots + P_m$$

where m is the number of resistors in parallel, P_S is the total power and P_m is the power in the last resistor.

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LESSON 5. Loop current and loop equations (Mesh current method)

5.1 Mesh Analysis

Mesh and nodal analysis are two basic important techniques used in finding solutions for a network. The suitability of either mesh or nodal analysis to a particular problem depends mainly on the number of voltage sources or current sources. If a network has a large number of voltage sources, it is useful to use mesh analysis; as this analysis requires that all the sources in a circuit be voltage sources. Therefore, if there are any current sources in a circuit they are to be converted into equivalent voltage sources, if, on the other hand, the network has more current sources nodal analysis is more useful.

Mesh analysis is applicable only for planar networks. For non-planner circuits mesh analysis is not applicable. A circuit is said to be planar, if it can be drawn on a plane surface without crossovers. A non-planar circuit cannot be drawn on a plane surface without a crossover.

Figure 5.1 (a) is a planar circuit. Figure 5.1 (b) is a non-planar circuit and Fig. 5.1 (c) is a planar circuit which looks like a non-planar circuit. It has already been discussed that a loop is a closed path. A mesh is defined as a loop which does not contain any other loops within it. To apply mesh analysis, our first step is to check whether the circuit is planar or not and the second is to select mesh current. Finally, writing Kirchhoff's voltage law equations in terms of unknowns and solving them leads to the final solution.

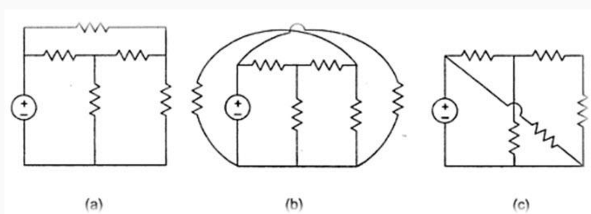

Fig. 5.1

Observation of the Fig.5.2 indicates that there are two loops *abefa*, and *bcdeb* in the network. Let us assume loop currents I_1 and I_2 with directions as indicated in the figure. Considering the loop *abefa* alone, we observe that current I_1 is passing through R_1, and $(I_1 - I_2)$ is passing through R_2. By applying Kirchhoff's voltage law, we can write

$$V_S = I_1R_1 + R_2(I_1 - I_2)$$

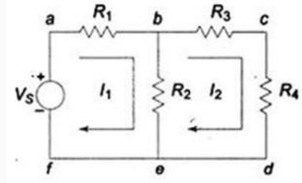

Fig.5.2

Similarly, if we consider the second mesh *bcdeb*, the current I_2 is passing through R_3 and R_4, and $(I_2 - I_1)$ is passing through R_2. By applying Kirchhoff's voltage law around the second mesh, we have

$$R_2(I_2 - I_1) + R_3 I_2 + R_4 I_2 = 0$$

By rearranging the above equations, the corresponding mesh current equations are

$$I_1(R_1 + R_2) - I_2 R_2 = V_S$$

$$- I_1R_2 + (R_2 + R_3 + R_4)I_2 = 0$$

By solving the above equations, we can find the currents I_1 and I_2. If we observe Fig. 5.2, the circuit consists of five branches and four nodes, including the reference node. The number of mesh currents is equal to the number of mesh equations.

And the number of equations = branches – (nodes -1). In Fig. 5.2, the required number of mesh currents would be 5 – (4 – 1) = 2.

5.2 Mesh Equations by Inspection Method

The mesh equations for a general planar network can be written by inspection without going through the detailed steps. Consider a three mesh networks as shown in Fig.5.3.

The loop equations are

$I_1R_1 + R_2(I_1 - I_2) = V_1$ \[..\left({5.1} \right)\]

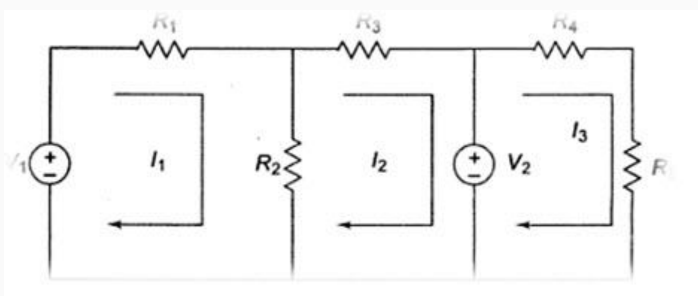

Fig. 5.3

$R_2(I_2 - I_1) = I_2R_3 = - V_2$ \[..\left({5.2} \right)\]

$R_4I_3 + R_5I_3 = V_2$ \[..\left({5.3} \right)\]

Reordering the above equations, we have

$(R_1 + R_2)I_1 - R_2I_2 =- V_1$ \[..\left({5.4} \right)\]

$-R_2I_1 + R_2 + R_3) I_2 = - V_2$ \[..\left({5.5} \right)\]

$(R_4 + R_5)I_3 = V_2$ \[..\left({5.6} \right)\]

The general mesh equations for three mesh resistive network can be written as

$$R_{11}I_1 \pm R_{12}I_2 \pm R_{13}I_3 = V_a \quad \ldots\ldots\ldots (5.7)$$

$$\pm R_{21}I_1 + R_{22}I_2 \pm R_{23}I_3 = V_b \quad \ldots\ldots\ldots (5.8)$$

$$\pm R_{31}I_1 + R_{32}I_2 \pm R_{33}I_3 = V_c \quad \ldots\ldots\ldots (5.9)$$

By comparing the Eqs. 5.4, 5.5 and 5.6 with Eqs. 5.7, 5.8 and 5.9 respectively, the following observations can be taken into account.

1. The self-resistance in each mesh.
2. The mutual resistances between all pairs of meshes and
3. The algebraic sum of the voltages in each mesh.

The self-resistance of loop *1*, $R_{11} = R_1 + R_2$, is the sum of the resistances through which I_1 passes. The mutual resistance of loop *1*, $R_{12} = -R_2$, is the sum of the resistances common to loop currents I_1 and I_2. If the directions of the currents passing through the common resistance are the same, the mutual resistance will have a positive sign; and if the directions of the currents passing through the common resistance are opposite then the mutual resistance will have a negative sign.

$V_a = V_1$ is voltage which drives loop one. Here, the positive sign is used if the direction of the current is the same as the direction of the source. If the current direction is opposite to the direction of the source, then the negative sign is used.

Similarly, $R_{22} = (R_2 + R_3)$ and $R_{33} = R_4 + R_5$ are the self-resistances of loops two and three, respectively. The mutual resistances $R_{13} = 0$, $R_{21} = -R_2$, $R_{23} = 0$, $R_{31} = 0$, $R_{32} = 0$ are the sums of the resistances common to the mesh currents indicated in their subscripts.

$V_b = -V_2$, $V_c = V_2$ are the sum of the voltages driving their respective loops.

5.3. Supermesh Analysis

Suppose any of the branches in the network has a current source, then it is slightly difficult to apply mesh analysis straight forward because first we should assume an unknown voltage across the current source, writing mesh equations as before, and then relate the source current to the assigned mesh currents. This is generally a difficult approach. One way to overcome this difficulty is by applying the supermesh technique. Here we have to choose the kind of supermesh. A supermesh is constituted by two adjacent loops that have a common current source. As an example, consider the network shown in Fig.5.4.

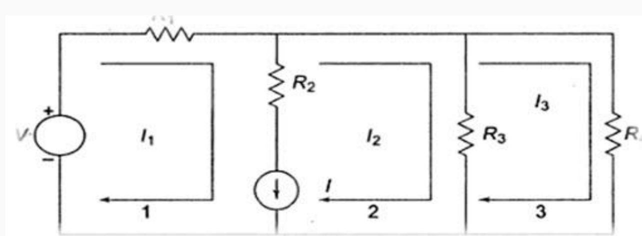

Fig. 5.4

Here, the current source I is in the common boundary for the two meshes 1 and 2. This current source creates a supermesh, which is nothing but a combination of meshes 1 and 2.

$$R_1I_1 + R_3(I_2 - I_3) = V$$

or $$R_1I_1 + R_3I_2 - R_4I_3 = V$$

Considering mesh 3, we have

$$R_3(I_3 - I_2) + R_4I_3 = 0$$

Finally, the current I from current source is equal to the difference between two mesh currents, i.e..

$$I_1 = I_2 = I$$

We have, thus, formed three mesh equations which we can solve for the three unknown currents in the network.

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Module 3. Node voltage and node equations (Nodal voltage method)

LESSON 6. Node voltage and node equation (Nodal voltage method)

6.1. Nodal Analysis

In the previous chapter we discussed simple circuits containing only two nodes, including the reference node. In general, in a N node circuit, one of the nodes is choosen as reference or datum node, then it is possible to write N – 1 nodal equations by assuming N – 1 node voltages. For example, a 10 node circuit requires nine unknown voltages and nine equations. Each node in a circuit can be assigned a number or a letter. The node voltage is the voltage of a given node with respect to one particular node, called the reference node, which we assume at zero potential. In the circuit shown in Fig.6.1, node 3 is assumed as the reference node. The voltage at node 1 is the voltage at that node with respect to node 3. Similarly, the voltage at node 2 is the voltage at that node with respect to node 3. Applying Kirchhoff's current law at node 1; the current entering is equal to the current leaving. (See Fig. 6.2).

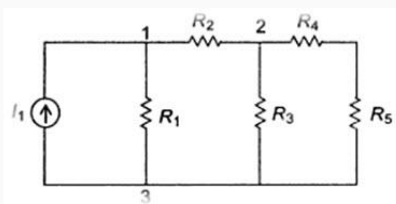

Fig. 6.1

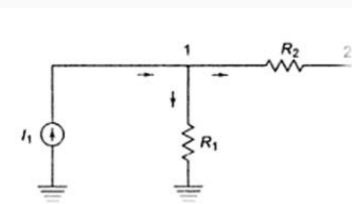

Fig. 6.2

$${I_1}={{{V_1}} \over {{R_1}}} + {{{V_1} - {V_2}} \over {{R_2}}}$$

where V_1 and V_2 are the voltages at node 1 and 2, respectively. Similarly, at node 2, the current entering is equal to the current leaving as shown in Fig. 6.3.

$${{{V_2} - {V_1}} \over {{R_2}}} + {{{V_2}} \over {{R_3}}} + {{{V_2}} \over {{R_4} + {R_5}}}=0$$

Rearranging the above equations, we have

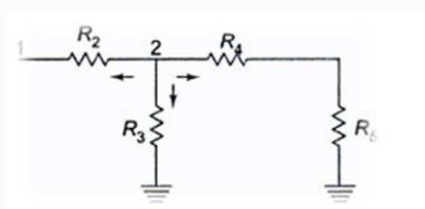

Fig. 6.3

$${V_1}\left[{{1 \over {{R_1}}} + {1 \over {{R_2}}}} \right] - {V_2}\left[{{1 \over {{R_2}}}} \right]={I_1}$$

$$-{V_1}\left[{{1 \over {{R_2}}}} \right] + {V_2}\left[{{1 \over {{R_2}}} + {1 \over {{R_3}}} + {1 \over {{R_4} + {R_5}}}} \right]=\,0$$

6.2. Nodal Equations by Inspection Method

The nodal equations for a general planar network can also be written by inspections, without going through the detailed steps. Consider a three node resistive network,, including the reference node, as shown in Fig.6.4.

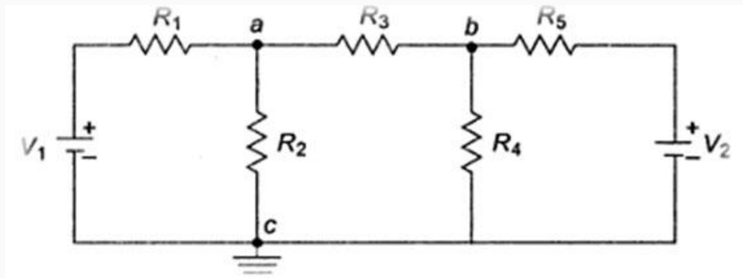

Fig.6.4

In Fig.6.4, the points *a* and*b* are the actual nodes and *c* is the reference node. Now consider the nodes *a* and*b* separately as shown in Fig. 6.5 (a) and (b).

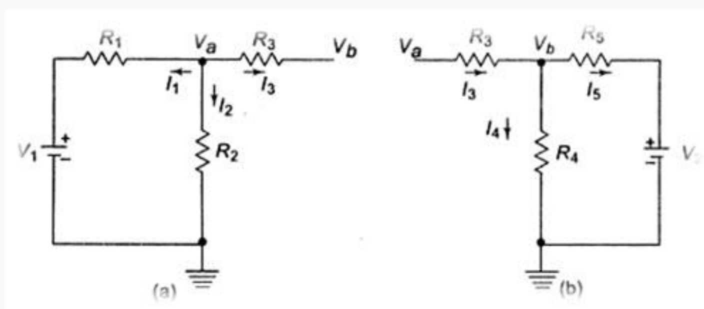

Fig.6.5

In Fig.6.5 (a), according to Kirchhoff's current law, we have

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} = 0 \qquad (6.1)$$

In Fig.6.5 (b), if we apply Kirchhoff's current law, we get

$$I_4 + I_5 = I_3$$

$$\frac{V_b - V_a}{R_3} + \frac{V_b}{R_4} + \frac{V_b - V_2}{R_5} = 0 \qquad (6.2)$$

Rearranging the above equations, we get

$$\left({{1 \over {{R_1}}} + {1 \over {{R_2}}} + {1 \over {{R_3}}}} \right){V_a} - \left({{1 \over {{R_3}}}} \right){V_b}=\left({{1 \over {{R_1}}}} \right){V_1}\ldots\ldots\ldots\ldots\ldots\ldots\left({6.3} \right)$$

$$-\left({{1 \over {{R_3}}}} \right){V_b} + \left({{1 \over {{R_3}}} + {1 \over {{R_4}}} + {1 \over {{R_5}}}} \right){V_b}=\left({{{{V_2}} \over {{R_5}}}} \right)\ldots\ldots\ldots\ldots\ldots\ldots\left({6.4} \right)$$

In general, the above equations can be written as

$${G_{aa}}\,\,\,{V_a} + \,{G_{ab}}\,\,{V_b}={I_1}\ldots\ldots\ldots\ldots\ldots\ldots\left({6.5} \right)$$

$${G_{ba}}\,\,\,{V_a} + \,{G_{bb}}\,\,{V_b}={I_2}\ldots\ldots\ldots\ldots\ldots\ldots\left({6.6} \right)$$

By comparing Eqs.6.3, 6.4 and Eqs.6.5, 6.6 we have the self-conductance at node $a,\,{G_{aa\,}}=\,\left({1/{R_1} + 1/{R_2} + 1/{R_3}} \right)$ is the sum of the conductance connected to node a. Similarly, $\,{G_{bb\,}}=\,\left({1/{R_3} + 1/{R_4} + 1/{R_5}} \right)$, is the sum of the conductance connected to node b. $\,{G_{ab\,}}=\,\left({ - 1/{R_3}} \right)$ is the sum of the mutual conductance connected to node a and node b. Here the entire mutual conductance has negative signs. Similarly, $\,{G_{ba\,}}=\,\left({ - 1/{R_3}} \right)$ is also a mutual conductance connected between nodes b and a. I_1 and I_2 are the sum of the source currents at node a and node b, respectively. The current which drives into the node has positive sign, while the current that drives away from the node has negative sign.

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LESSON 7. Node Analysis

7.1. Supernode Analysis

Suppose any of the branches in the network has a voltage source, then it is slightly, difficult to apply nodal analysis. One way to overcome this difficulty is to apply the supernode technique. In this method, the two adjacent nodes that are connected by a voltage source are reduced to a single node and then the equations are formed by applying Kirchhoff's current law as usual. This is explained with the help of Fig. 7.1.

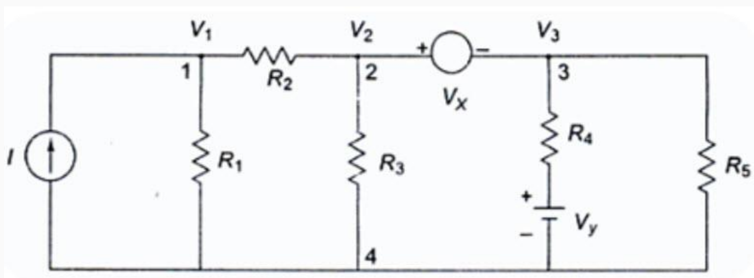

Fig. 7.1

It is clear from Fig. 7.1, that node 4 is the reference node. Applying Kirchhoff's current law at node 1, we get

$$I={{{V_1}} \over {{R_1}}} + {{{V_1} - {V_2}} \over {{R_2}}}$$

Due to the presence of voltage source V_x in between nodes 2 and 3, it is slightly difficult to find out the current. The supernode technique can be conveniently applied in this case.

Accordingly, we can write the combined equation for nodes 2 and 3 as under.

$${{{V_2} - {V_1}} \over {{R_2}}} + {{{V_2}} \over {{R_3}}} + {{{V_3} - {V_y}} \over {{R_4}}} + {{{V_3}} \over {{R_5}}}=0$$

The other equation is

$$V_2 - V_3 = V_x$$

From the above three equations, we can find the three unknown voltages.

7.2. Source Transformation Technique

In solving networks to find solutions one may have to deal with energy source. It has already been discussed in Chapter 1 that basically, energy sources are either voltage sources or current sources. Sometimes it is necessary to convert a voltage source to a current source and vice-versa. Any practical voltage source consists of an ideal voltage source in series with an internal resistance. Similarly, a practical current source consists of an ideal current source in parallel with an internal resistance as shown in Fig. 7.2. R_v and R_i represent the internal resistance of the voltage source $V_{s'}$ and current source $I_{s'}$ respectively.

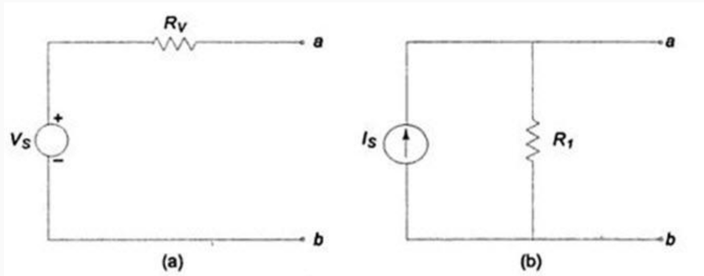

Fig. 7.2

Any source, be it a current source or a voltage source, drives current through its load resistance, and the magnitude of the current depend on the value of the load resistance. Figure 7.3 represents a practical voltage source and a practical current source connected to the same load resistance $R_{L'}$

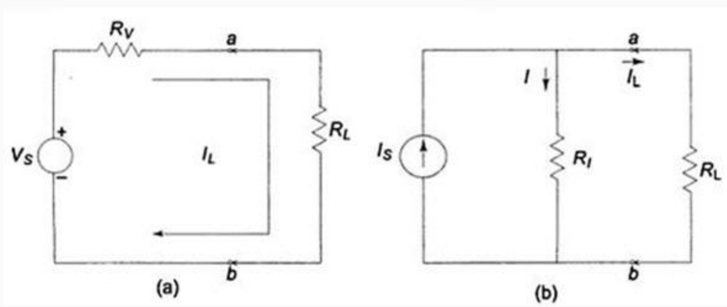

Fig. 7.3

From Fig. 7.3 (a), the load voltage can be calculated by using Kirchhoff's voltage law as

$V_{ab} = V_s - I_L R_v$

The open circuit voltage $V_{OC} = V_s$

The short circuit current $$I_{SC} = \frac{V_s}{R_v}$$

From Fig. 7.3 (b)

$$I_L = I_s - I = I_s - \frac{V_{ab}}{R_1}$$

The open circuit voltage $V_{OC} = I_s R_1$

The short circuit current $I_{SC} = I_S$

The above two sources are said to be equal, if they produce equal amounts of current and voltage when they are connected to identical load resistances. Therefore, by equating the open circuit voltages and short circuit currents of the above two sources we obtain.

$V_{OC} = I_s R_1 = V_s$

$$I_{SC} = I_s = {{V_s} \over {R_v}}$$

It follows that $R_1 = R_v = R_s$ \ $V_s = I_s R_s$

Where R_s is the internal resistance of the voltage or current source. Therefore, any practical voltage source, having an ideal voltage V_s and internal series resistance R_s can be replaced by a current source $I_s = V_s / R_s$ in parallel with an internal resistance R_s. The reverse transformation is also possible. Thus, a practical current source in parallel with an internal resistance R_s can be replaced by a voltage source $V_s = I_s R_s$ in series with an internal resistance R_s.

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Module 4. Network theorems Thevenin' s, Norton' s, Superposition

LESSON 8. Network theorems:Superposition

8. Introduction to network theorems

Concept of a theorem: a relatively simple rule used to solve a problem, derived from a more intensive analysis using fundamental rules of mathematics. At least hypothetically, any problem in math can be solved just by using the simple rules of arithmetic (in fact, this is how modern digital computers carry out the most complex mathematical calculations: by repeating many cycles of additions and subtractions!), but human beings aren't as consistent or as fast as a digital computer. We need "shortcut" methods in order to avoid procedural errors. In electric network analysis, the fundamental rules are Ohm's Law and Kirchhoff 's Laws. While these humble laws may be applied to analyze just about any circuit configuration (even if we have to resort to complex algebra to handle multiple unknowns), there are some "shortcut" methods of analysis to make the math easier for the average human. As with any theorem of geometry or algebra, these network theorems are derived from fundamental rules. In this chapter, I'm not going to delve into the formal proofs of any of these theorems. If you doubt their validity, you can always empirically test them by setting up example circuits and calculating values using the "old" (simultaneous equation) methods versus the "new" theorems, to see if the answers coincide. They always should!

8.1 Superposition Theorem

Superposition theorem is one of those strokes of genius that takes a complex subject and simplifies it in a way that makes perfect sense. A theorem like Millman's certainly works well, but it is not quite obvious *why* it works so well. Superposition, on the other hand, is obvious. The strategy used in the Superposition Theorem is to eliminate all but one source of power within a network at a time, using series/parallel analysis to determine voltage drops (and/or currents) within the modified network for each power source separately. Then, once voltage drops and/or currents have been determined for each power source working separately, the values are all "superimposed" on top of each other (added algebraically) to find the actual voltage drops/currents with all sources active. Let's look at our example circuit again and apply Superposition Theorem to it:

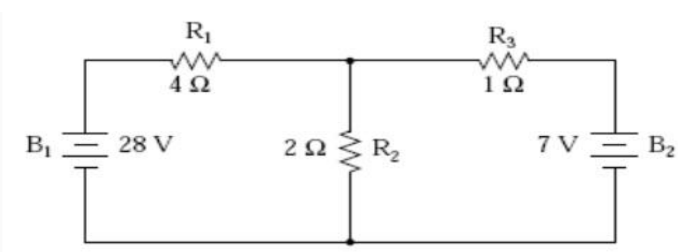

Fig.8.1

Since we have two sources of power in this circuit, we will have to calculate two sets of values for voltage drops and/or currents, one for the circuit with only the 28 volt battery in effect and one for the circuit with only the 7 volt battery in effect:

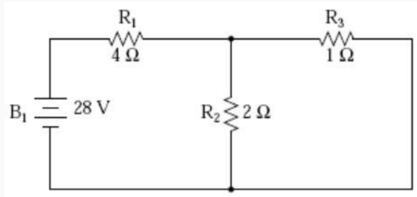

Fig. 8.2

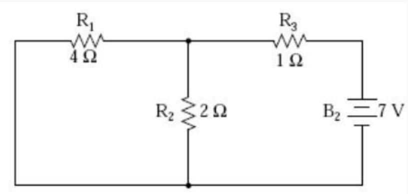

Fig. 8.3

When re-drawing the circuit for series/parallel analysis with one source, all other voltage sources are replaced by wires (shorts), and all current sources with open circuits (breaks).Since we only have voltage sources (batteries) in our example circuit, we will replace every inactive source during analysis with a wire. Analyzing the circuit with only the 28 volt battery, we obtain the following values for voltage and current:

	R1+R2// R3					
	R1	R2	R3	R2// R3	Total	
E	24	4	4	4	28	Volts
I	6	2	4	6	6	Amps
R	4	2	4	0.667	4.667	Ohms

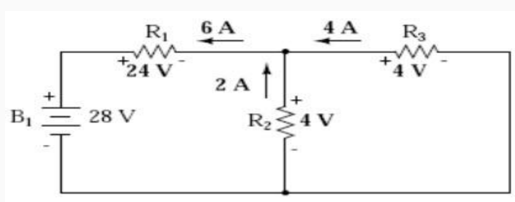

Fig. 8.4

Analyzing the circuit with only the 7 volt battery, we obtain another set of values for voltage and current:

	R3+R1// R2					
	R1	R2	R3	R1// R2	Total	
E	4	4	3	4	7	Volts
I	1	2	3	3	3	Amps
R	4	2	1	1.333	2.333	Ohms

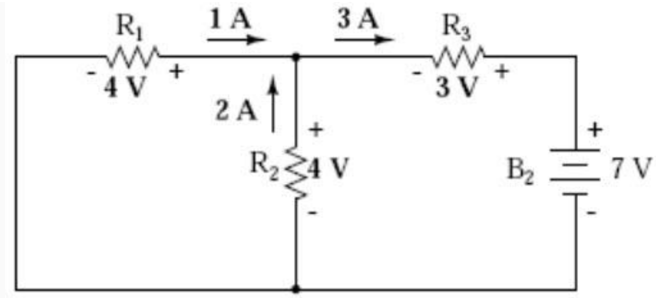

Fig. 8.5

When superimposing these values of voltage and current, we have to be very careful to consider polarity (voltage drop) and direction (electron flow), as the values have to be added algebraically.

With 28 V battery	*With 7 V battery*	*With both batteries*
24 V (+ −) E_{R1}	4 V (− +) E_{R1}	E_{R1} **20 V** (+ −) *24 V - 4 V = 20 V*
E_{R2} 4 V (+ −)	E_{R2} 4 V (+ −)	E_{R2} **8 V** (+ −) *4 V + 4 V = 8 V*
4 V (+ −) E_{R3}	3 V (− +) E_{R3}	E_{R3} **1 V** (+ −) *4 V - 3 V = 1 V*

Fig. 8.6

Applying these superimposed voltage figures to the circuit, the end result looks something like this:

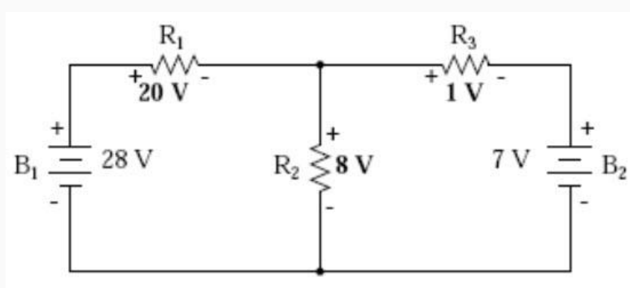

Fig. 8.7

Currents add up algebraically as well, and can either be superimposed as done with the resistor voltage drops, or simply calculated from the final voltage drops and respective resistances (I=E/R). Either way, the answers will be the same. Here I will show the superposition method applied to current:

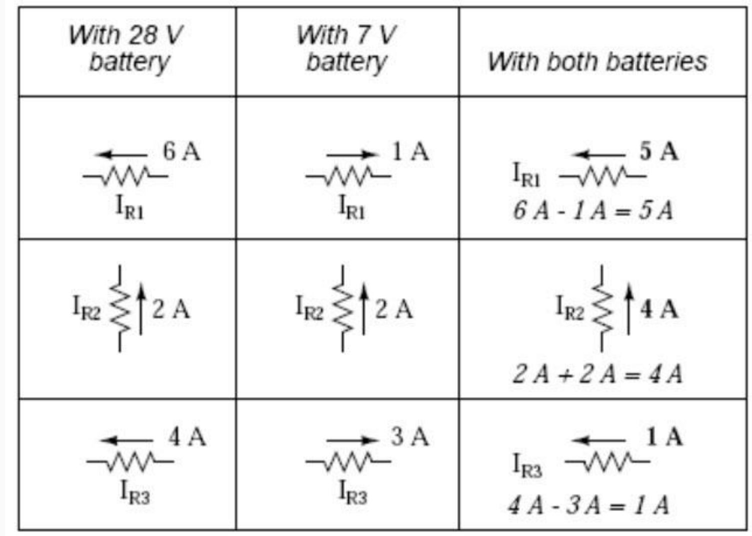

Fig. 8.8

Once again applying these superimposed figures to our circuit:

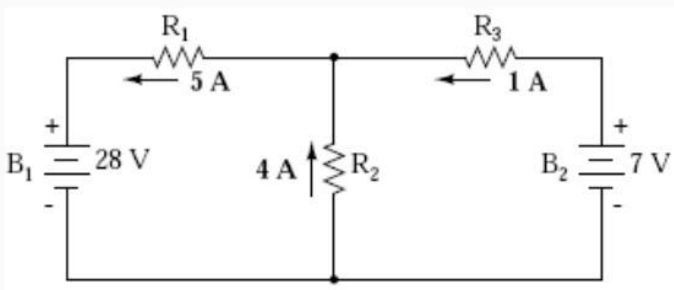

Fig. 8.9

Quite simple and elegant, don't you think? It must be noted, though, that the Superposition Theorem works only for circuits that are reducible to series/parallel combinations for each of the power sources at a time (thus, this theorem is useless for analyzing an unbalanced bridge circuit), and it only works where the underlying equations are linear (no mathematical powers or roots). The requisite of linearity means that Superposition Theorem is only applicable for determining voltage and current, not power!!! Power dissipations, being nonlinear functions,

do not algebraically add to an accurate total when only one source is considered at a time. The need for linearity also means this Theorem cannot be applied in circuits where the resistance of a component changes with voltage or current. Hence, networks containing components like lamps (incandescent or gas-discharge) or varistors could not be analyzed. Another prerequisite for Superposition Theorem is that all components must be "bilateral," meaning that they behave the same with electrons flowing either direction through them. Resistors have no polarity-specific behavior, and so the circuits we've been studying so far all meet this criterion. The Superposition Theorem finds use in the study of alternating current (AC) circuits, and semiconductor (amplifier) circuits, where sometimes AC is often mixed (superimposed) with DC. Because AC voltage and current equations (Ohm's Law) are linear just like DC, we can use Superposition to analyze the circuit with just the DC power source, then just the AC power source, combining the results to tell what will happen with both AC and DC sources in effect. For now, though, Superposition will suffice as a break from having to do simultaneous equations to analyze a circuit.

REVIEW:

The Superposition Theorem states that a circuit can be analyzed with only one source of power at a time, the corresponding component voltages and currents algebraically added to find out what they'll do with all power sources in effect.

To negate all but one power source for analysis, replace any source of voltage (batteries) with a wire; replace any current source with an open (break).

LESSON 9. Network theorems Thevenin's and Norton's

9.1 Thevenin's Theorem

Thevenin's Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a single voltage source and series resistance connected to a load. The qualification of "linear" is identical to that found in the Superposition Theorem, where all the underlying equations must be linear (no exponents or roots). If we're dealing with passive components (such as resistors, and later, inductors and capacitors), this is true. However, there are some components (especially certain gas-discharge and semiconductor components) which are nonlinear: that is, their opposition to current *changes* with voltage and/or current. As such, we would call circuits containing these types of components, nonlinear circuits.

Thevenin's Theorem is especially useful in analyzing power systems and other circuits where one particular resistor in the circuit (called the "load" resistor) is subject to change, and re-calculation of the circuit is necessary with each trial value of load resistance, to determine voltage across it and current through it. Let's take another look at our example circuit:

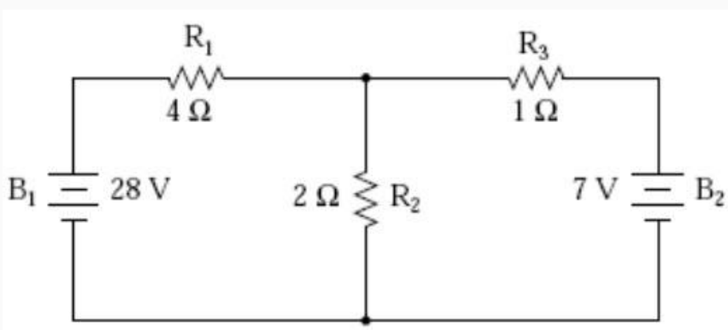

Fig. 9.1

Let's suppose that we decide to designate R2 as the "load" resistor in this circuit. We already have four methods of analysis at our disposal (Branch Current, Mesh Current, Millman's Theorem, and Superposition Theorem) to use in determining voltage across R2 and current through R2, but each of these methods are time-consuming. Imagine repeating any of these methods over and over again to find what would happen if the load resistance changed (changing load resistance is *very* common in power systems, as multiple loads get switched on and off as needed. the total resistance of their parallel connections changing depending on how many are connected at a time). This could potentially involve a *lot* of work! Thevenin's Theorem makes this easy by temporarily removing the load resistance from the original circuit and reducing what's left to an equivalent circuit composed of a single voltage source and series resistance. The load resistance can then be re-connected to this "Thevenin equivalent circuit" and calculations carried out as if the whole network were nothing but a simple series circuit:

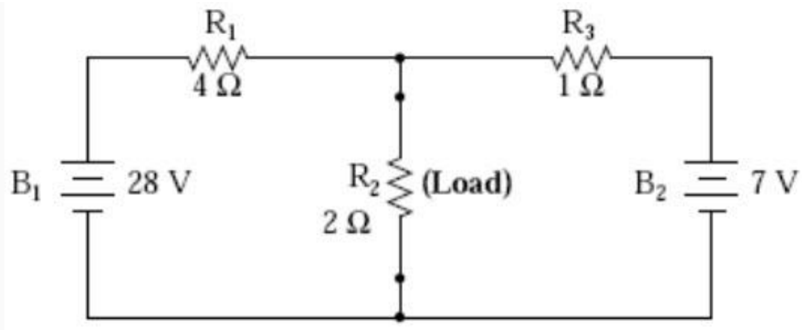

Fig. 9.2

After Thevenin conversion

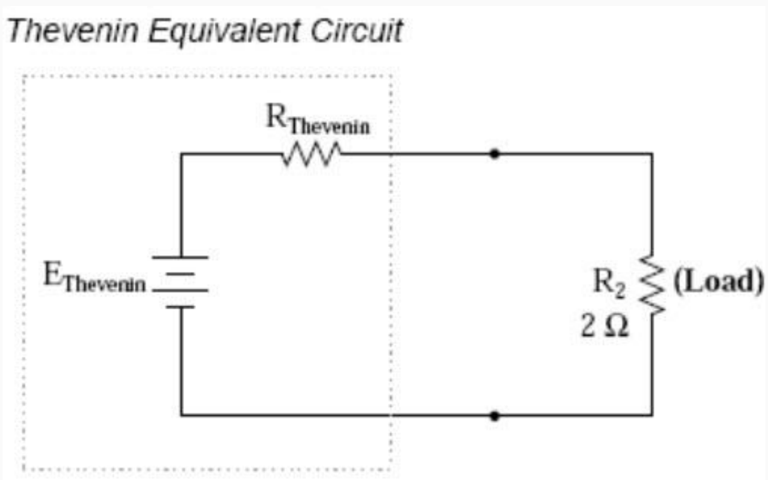

Fig. 9.3

The "Thevenin Equivalent Circuit" is the electrical equivalent of B1, R1, R3, and B2 as seen from the two points where our load resistor (R2) connects. The Thevenin equivalent circuit, if correctly derived, will behave exactly the same as the original circuit formed by B1, R1, R3, and B2. In other words, the load resistor (R2) voltage and current should be exactly the same for the same value of load resistance in the two circuits. The load resistor R2 cannot "tell the difference" between the original network of B1, R1, R3, and B2, and the Thevenin equivalent circuit of $E_{Thevenin}$, and $R_{Thevenin}$, provided that the values for $E_{Thevenin}$ and $R_{Thevenin}$ have been calculated correctly. The advantage in performing the "Thevenin conversion" to the simpler circuit, of course, is that it makes load voltage and load current so much easier to solve than in the original network. Calculating the equivalent Thevenin source voltage and series resistance is actually quite easy. First, the chosen load resistor is removed from the original circuit, replaced with a break (open circuit):

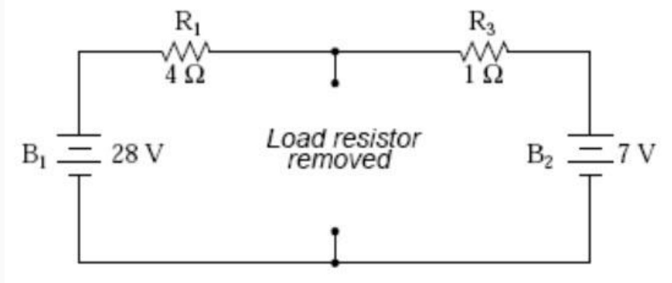

Fig. 9.4

Next, the voltage between the two points where the load resistor used to be attached is determined. Use whatever analysis methods are at your disposal to do this. In this case, the original circuit with the load resistor removed is nothing more than a simple series circuit with opposing batteries, and so we can determine the voltage across the open load terminals by applying the rules of series circuits, Ohm's Law, and Kirchhoff 's Voltage Law:

	R_1	R_2	Total	
E	16.8	4.2	21	Volts
I	4.2	4.2	4.2	Amps
R	4	1	5	Ohms

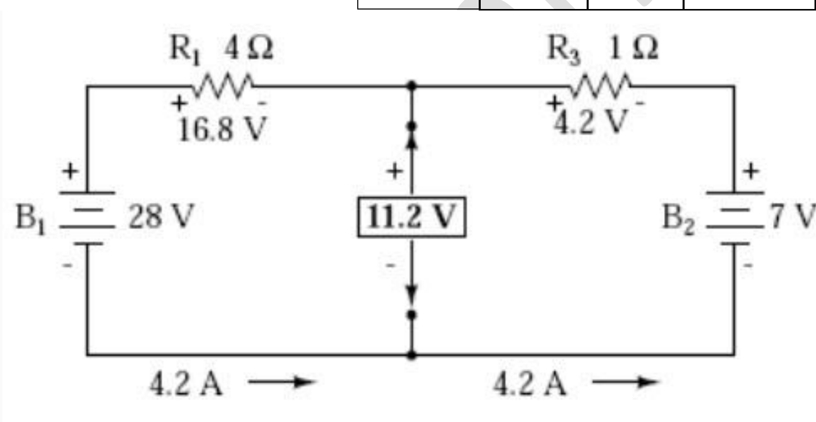

Fig. 9.5

The voltage between the two load connection points can be figured from the one of the battery's voltage and one of the resistor's voltage drops, and comes out to 11.2 volts. This is our "Thevenin voltage" (EThevenin) in the equivalent circuit:

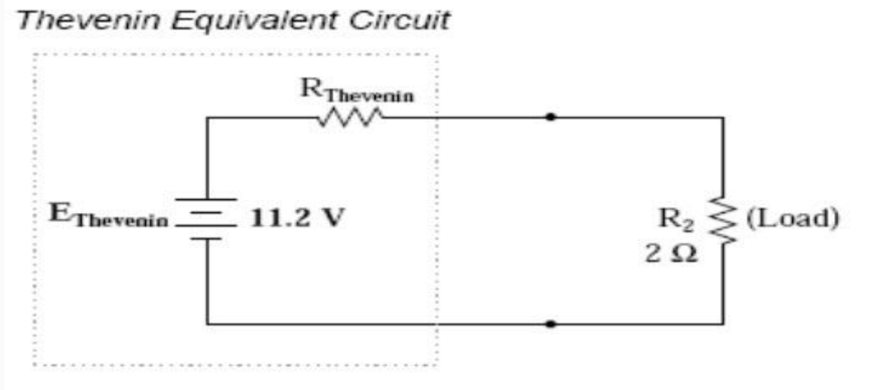

Fig. 9.6

To find the Thevenin series resistance for our equivalent circuit, we need to take the original circuit (with the load resistor still removed), remove the power sources (in the same style as we did with the Superposition Theorem: voltage sources replaced with wires and current sources replaced with breaks), and figure the resistance from one load terminal to the other:

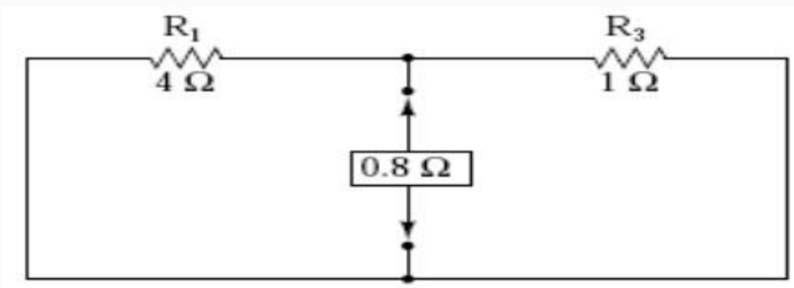

Fig. 9.7

With the removal of the two batteries, the total resistance measured at this location is equal to R1 and R3 in parallel: 0.8. This is our "Thevenin resistance" ($R_{Thevenin}$) for the equivalent circuit:

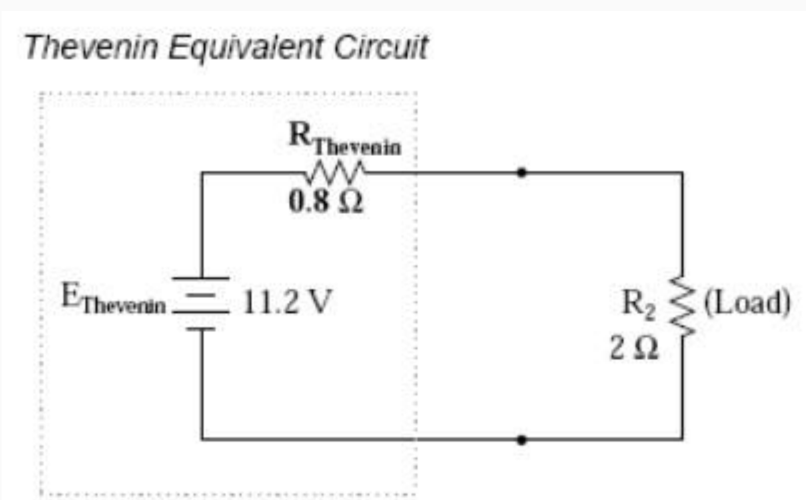

Fig.9.8

With the load resistor (2) attached between the connection points, we can determine voltage across it and current through it as though the whole network were nothing more than a simple series circuit:

	$R_{Thevenin}$	R_{Load}	Total	
E	3.2	8	11.2	Volts
I	4	4	4	Amps
R	0.8	2	2.8	Ohms

Notice that the voltage and current figures for R2 (8 volts, 4 amps) are identical to those found using other methods of analysis. Also notice that the voltage and current figures for the Thevenin series resistance and the Thevenin source (total) do not apply to any component in the original, complex circuit. Thevenin's Theorem is only useful for determining what happens to a *single* resistor in a network: the load. The advantage, of course, is that you can quickly determine what would happen to that single resistor if it were of a value other than 2 without having to go through a lot of analysis again. Just plug in that other value for the load resistor into the Thevenin equivalent circuit and a little bit of series circuit calculation will give you the result.

REVIEW:

Thevenin's Theorem is a way to reduce a network to an equivalent circuit composed of a single voltage source, series resistance, and series load. Steps to follow for Thevenin's Theorem:

- Find the Thevenin source voltage by removing the load resistor from the original circuit and calculating voltage across the open connection points where the load resistor used to be.
- Find the Thevenin resistance by removing all power sources in the original circuit (voltage sources shorted and current sources open) and calculating total resistance between the open connection points.
- Draw the Thevenin equivalent circuit, with the Thevenin voltage source in series with the Thevenin resistance. The load resistor re-attaches between the two open points of the equivalent circuit.
- Analyze voltage and current for the load resistor following the rules for series circuits.

Norton's Theorem

Norton's Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a single current source and parallel resistance connected to a load. Just as with Thevenin's Theorem, the qualification of "linear" is identical to that found in the Superposition Theorem: all underlying equations must be linear (no exponents or roots).

Contrasting our original example circuit (fig.9.9) against the Norton equivalent: it looks something like this:

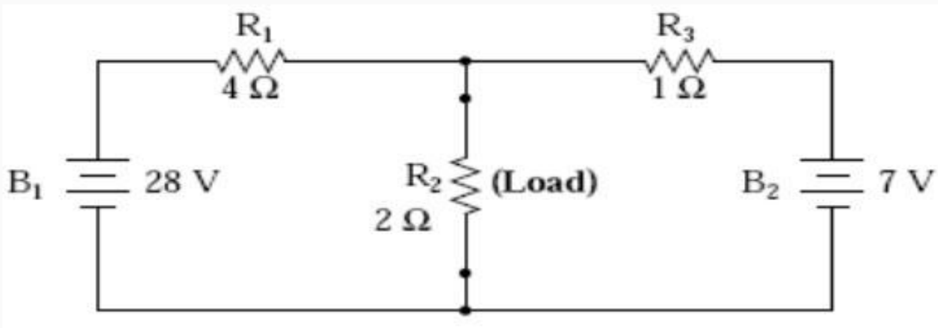

Fig. 9.9

. . . after Norton conversion (fig.9.10) . . .

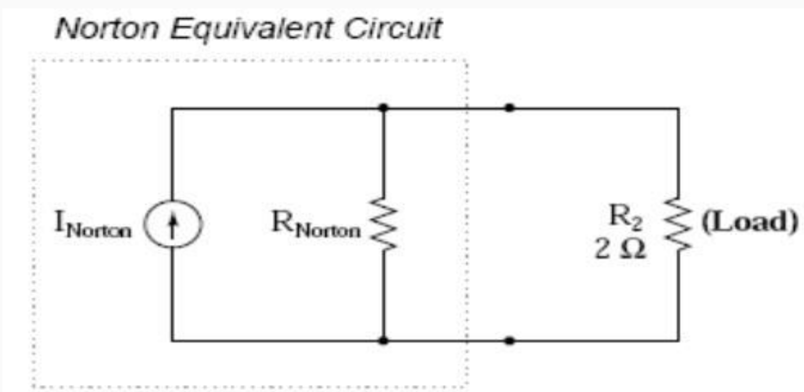

Fig. 9.10

Remember that a current source is a component whose job is to provide a constant amount of current, outputting as much or as little voltage necessary to maintain that constant current. As with Thevenin's Theorem, everything in the original circuit except the load resistance has been reduced to an equivalent circuit that is simpler to analyze. Also similar to Thevenin's Theorem are the steps used in Norton's Theorem to calculate the Norton source current (I_{Norton}) and Norton resistance (R_{Norton}). As before, the first step is to identify the load resistance and remove it from the original circuit (fig.9.11).

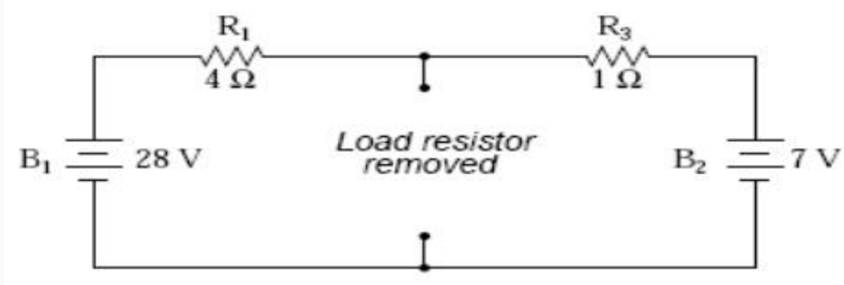

Fig. 9.11

Then, to find the Norton current (for the current source in the Norton equivalent circuit), place a direct wire (short) connection between the load points and determine the resultant current. Note that this step is exactly opposite the respective step in Thevenin's Theorem, where we replaced the load resistor with a break (open circuit fig.9.12).

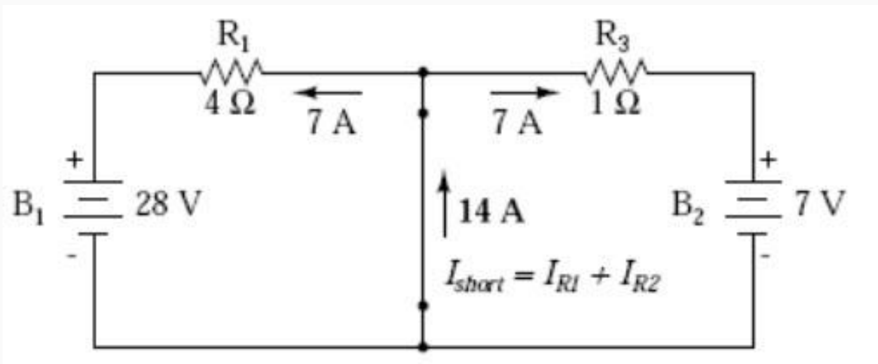

Fig. 9.12

With zero voltage dropped between the load resistor connection points, the current through R1 is strictly a function of B1's voltage and R1's resistance: 7 amps (I=E/R). Likewise, the current through R3 is now strictly a function of B2's voltage and R3's resistance: 7 amps (I=E/R). The total current through the short between the load connection points is the sum of these two currents: 7 amps + 7 amps = 14 amps. This figure of 14 amps becomes the Norton source current (I_{Norton}) in our equivalent circuit (fig.9.13).

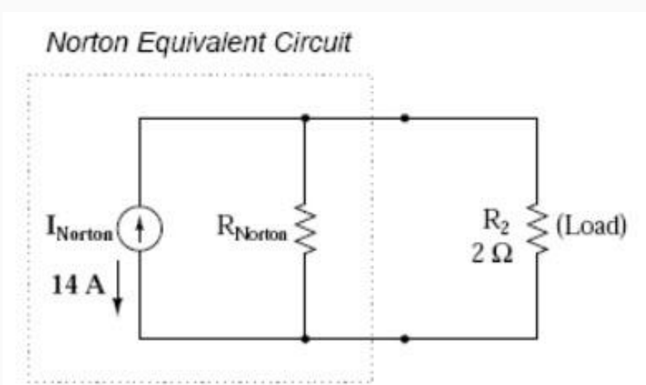

Fig. 9.13

Remember, the arrow notation for a current source points in the direction opposite that of electron flow. Again, apologies for the confusion. For better or for worse, this is standard electronic symbol notation. Blame Mr. Franklin again! To calculate the Norton resistance

(R_{Norton}), we do the exact same thing as we did for calculating Thevenin resistance ($R_{Thevenin}$): take the original circuit (with the load resistor still removed), remove the power sources (in the same style as we did with the Superposition Theorem: voltage sources replaced with wires and current sources replaced with breaks), and figure total resistance from one load connection point to the other (fig.9.14).

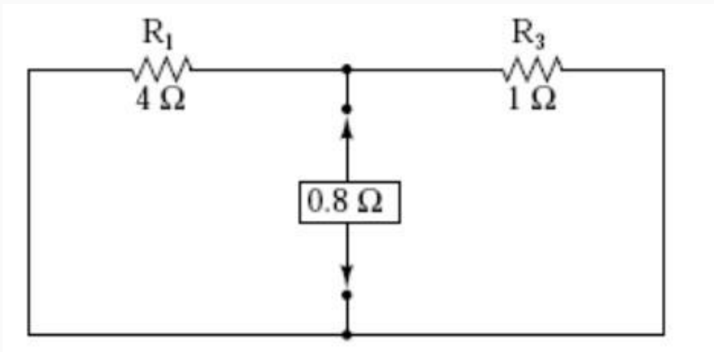

Fig. 9.14

Now our Norton equivalent circuit looks like this (fig.9.15)

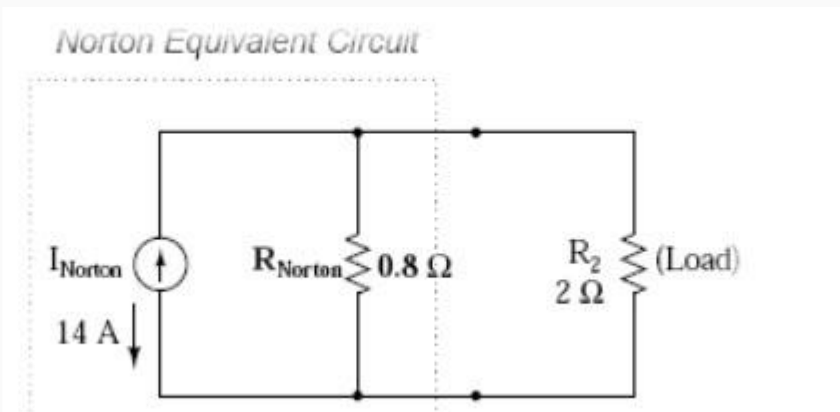

Fig. 9.15

If we re-connect our original load resistance of 2, we can analyze the Norton circuit as a simple parallel arrangement:

	R_{Nortan}	R_{Load}	Total	
E	8	8	8	Volts
I	10	4	14	Amps
R	0.8	2	571.43m	Ohms

As with the Thevenin equivalent circuit, the only useful information from this analysis is the voltage and current values for R2; the rest of the information is irrelevant to the original circuit. However, the same advantages seen with Thevenin's Theorem apply to Norton's as well: if we wish to analyze load resistor voltage and current over several different values of load resistance, we can use the Norton equivalent circuit again and again, applying nothing more complex than simple parallel circuit analysis to determine what's happening with each trial load.

REVIEW:

Norton's Theorem is a way to reduce a network to an equivalent circuit composed of a single current source, parallel resistance, and parallel load. Steps to follow for Norton's Theorem:

- Find the Norton source current by removing the load resistor from the original circuit and calculating current through a short (wire) jumping across the open connection points where the load resistor used to be.
- Find the Norton resistance by removing all power sources in the original circuit (voltage sources shorted and current sources open) and calculating total resistance between the open connection points.
- Draw the Norton equivalent circuit, with the Norton current source in parallel with the Norton resistance. The load resistor re-attaches between the two open points of the equivalent circuit.
- Analyze voltage and current for the load resistor following the rules for parallel circuit

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Module 5. Reciprocity and Maximum power transfer

LESSON 10. Maximum Power Transfer Theorem

10. Maximum Power Transfer Theorem

The Maximum Power Transfer Theorem is not so much a means of analysis as it is an aid to system design. Simply stated, the maximum amount of power will be dissipated by a load resistance when that load resistance is equal to the Thevenin/Norton resistance of the network

supplying the power. If the load resistance is lower or higher than the Thevenin/Norton resistance of the source network, its dissipated power will be less than maximum. This is essentially what is aimed for in radio transmitter design, where the antenna or transmission line “impedance” is matched to final power amplifier “impedance” for maximum radio frequency power output. Impedance, the overall opposition to AC and DC current, is very similar to resistance, and must be equal between source and load for the greatest amount of power to be transferred to the load. A load impedance that is too high will result in low power output. A load impedance that is too low will not only result in low power output, but possibly overheating of the amplifier due to the power dissipated in its internal (Thevenin or Norton) impedance.

Taking our Thevenin equivalent example circuit, the Maximum Power Transfer Theorem

tells us that the load resistance resulting in greatest power dissipation is equal in value to the Thevenin resistance (in this case, 0.8):

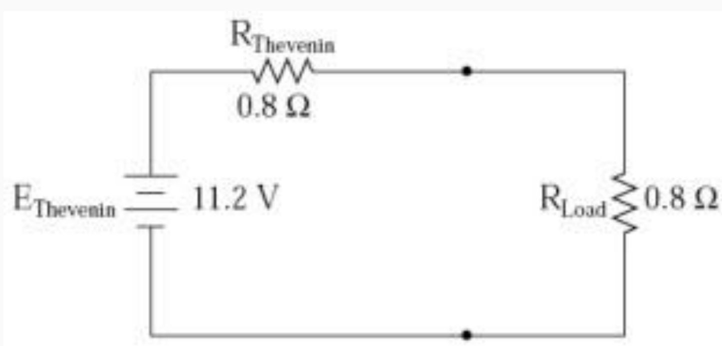

Fig. 10.1

With this value of load resistance, the dissipated power will be 39.2 watts:

	$R_{Thevenin}$	R_{Load}	Total	
E	5.6	5.6	11.2	Volts
I	7	7	7	Amps
R	0.8	0.8	1.6	Ohms
P	39.2	39.2	78.4	Watts

If we were to try a lower value for the load resistance (0.5 instead of 0.8, for example), our power dissipated by the load resistance would decrease:

	$R_{Thevenin}$	R_{Load}	Total	
E	6.892	4.308	11.2	Volts
I	8.615	8.615	8.615	Amps
R	0.8	0.5	1.3	Ohms
P	59.38	37.11	96.49	Watts

Power dissipation increased for both the Thevenin resistance and the total circuit, but it decreased for the load resistor. Likewise, if we increase the load resistance (1.1 instead of 0.8, for example), power dissipation will also be less than it was at 0.8 exactly:

	$R_{Thevenin}$	R_{Load}	Total	
E	4.716	6.484	11.2	Volts
I	5.895	5.895	5.895	Amps
R	0.8	1.1	1.9	Ohms
P	27.80	38.22	66.02	Watts

If you were designing a circuit for maximum power dissipation at the load resistance, this theorem would be very useful. Having reduced a network down to a Thevenin voltage and resistance (or Norton current and resistance), you simply set the load resistance equal to that Thevenin or Norton equivalent (or vice versa) to ensure maximum power dissipation at the load. Practical applications of this might include radio transmitter final amplifier stage design (seeking to maximize power delivered to the antenna or transmission line), a *grid tied inverter* loading a solar array, or electric vehicle design (seeking to maximize power delivered to drive motor).

10.1 The Maximum Power Transfer Theorem is not:

Maximum power transfer does not coincide with maximum efficiency. Application of The Maximum Power Transfer theorem to AC power distribution will not result in maximum or even high efficiency. The goal of high efficiency is more important for AC power distribution, which dictates relatively low generator impedance compared to load impedance. Similar to AC power distribution, high fidelity audio amplifiers are designed for relatively low output impedance and relatively high speaker load impedance. As a ratio, "output impedance": "load impedance" is known as *damping factor*, typically in the range of 100 to 1000. [5] [6]

Maximum power transfer does not coincide with the goal of lowest noise. For example, the low-level radio frequency amplifier between the antenna and a radio receiver is often designed for lowest possible noise. This often requires a mismatch of the amplifier input impedance to the antenna as compared with that dictated by the maximum power transfer theorem.

REVIEW:

The Maximum Power Transfer Theorem states that the maximum amount of power will be dissipated by a load resistance if it is equal to the Thevenin or Norton resistance of the network supplying power.

The Maximum Power Transfer Theorem does **not** satisfy the goal of maximum efficiency.

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LESSON 11. Reciprocity Theorem

The circuit in the figure is a concrete example of reciprocity. The reader should solve the circuit, and determine the values of the current I in the two cases, which will be equal (0.35294 A). If E is reversed, then the direction of I is reversed, so the direction does not matter so long as both E and I are reversed at the same time.

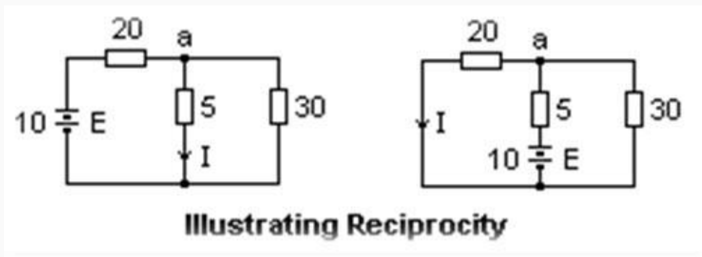

Fig. 11.1

A non-bilateral element, such as a rectifying diode, destroys reciprocity, as the circuit at the left shows. Here it is obvious that I = 0 on the right, while I ≠ 0 on the left. Even if the polarity of E is reversed so that the diode is forward-biased on the right, the currents are not reciprocal. On the left, I will be 0.21294 A, while on the right it will be 0.32824 A, as the reader can show by solving the circuits.

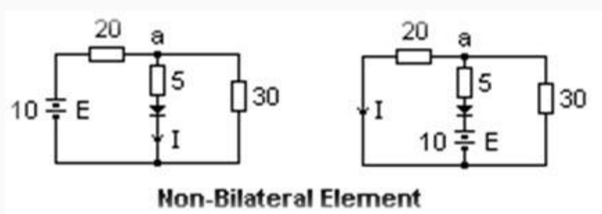

Fig. 11.2

A nonlinear element also destroys reciprocity. The circuit at the right includes a resistor whose voltage drop is proportional to the square of the current, $V = 10i^2$. In the circuit at the left, I = 0.3798 A, while on the right I = 0.3397 A. The inclusion of controlled sources or active elements may also destroy reciprocity.

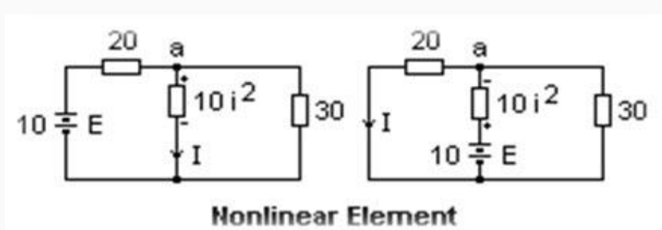

Fig. 11.3

11.1. Two-Port Networks

Consideration of reciprocity leads naturally to two-port networks. These are networks with four terminals considered in two pairs as ports at which connections are made. The emf E in the reciprocity theorem is considered to be connected to one port, say port 1, while the current is at port 2, assumed to be short-circuited. The ports result from breaking into two of the branches of the network. One terminal of each port is denoted by (+) to specify the polarity of the voltage applied at the port, and currents are positive when they enter the (+) terminal. The fundamental variables are V_1, I_1, V_2 and I_2. Any two of these variables are functions of the remaining two. For certain networks, some of the four choices are not admissible. In most cases, the variables appearing in the models are variations from DC bias conditions, not the DC variables themselves.

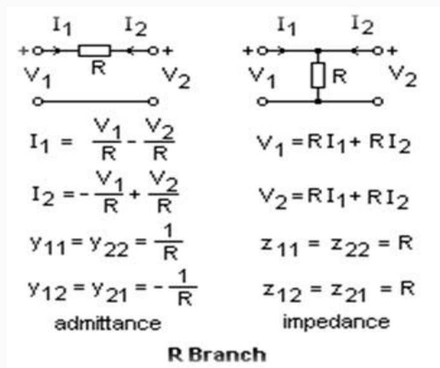

Fig. 11.4

A single resistor forms two two-ports, depending on whether it is in series or shunt. For the series resistor, it is normal to take the dependent variables as I_1 and I_2, and the independent variables V_1 and V_2. The coefficients are called the admittance parameters, since admittance is the ratio of current to voltage. If the resistor is connected in shunt, the natural independent variables are I_1 and I_2, while V_1 and V_2 are the dependent variables. The coefficients in this case are the impedance parameters, since impedance is the ratio of voltage to current. In both cases, we see that the off-diagonal or transfer coefficients are equal.

Consider the admittance model, where the coefficients are not necessarily equal to the simple ones for a single resistor. If an emf E is applied at the input, $E = V_1$, and the output is shorted, $V_2 = 0$, we see that $I_2 = y_{21}E$. If we apply the same voltage E to the output, and short the input, we have $I_1 = y_{12}E$. Therefore, the currents are equal, and the network is reciprocal. Reciprocity is the result of the equality of the transfer admittances. A similar result can be obtained for the impedance model, where reciprocity holds if $z_{12} = z_{21}$, as the reader can easily show. For our simple example of a single resistor, these conditions hold.

We note that there is no impedance model for the series resistor, and no admittance model for the shunt resistor. In general, it is possible to transform from impedance to admittance and vice versa, but the determinant of the coefficients is in the denominator of the transformation formula, and for a single resistor this determinant is zero. If Δ is the determinant of the admittance coefficients, then $z_{11} = y_{22}/\Delta$, $z_{22} = y_{11}/\Delta$, $z_{12} = y_{21}/\Delta$ and $z_{21} = y_{12}/\Delta$. Similar formulas hold for the inverse transformation. A full list of transformations is given in the first Reference. The conclusion is that if $y_{12} = y_{21}$, then $z_{21} = z_{12}$ as well. This, of course, is a general result. The particular way of modeling a network does not affect the property of reciprocity.

A popular model, especially for active elements, is the hybrid model in which the independent variables are V_2 and I_1. The equations for the series resistor are $V_1 = RI_1 + V_2$ and $I_2 = -I_1$. The hybrid parameters for this case are $h_{11} = R$, $h_{22} = 0$, $h_{12} = -h_{21} = 1$. Note that the off-diagonal elements are of opposite sign in this case, but of the same magnitude. This is the mark of reciprocity in the hybrid model. As an exercise, the reader may show that for the shunt resistor $h_{11} = 0$, $h_{22} = 1/R$, and the transfer coefficients are the same as those for the series resistor.

There is a fourth model in which the output variables I_1 and V_1 are expressed in terms of the input variables V_2 and I_2: $V_1 = AV_2 - BI_2$, and $I_1 = CV_2 - DI_2$. Note the change in sign of I_2, which is conventional so it will be in the same direction as I_1 for an identity transformation A = D = 1, B = C = 0. If networks are cascaded, these matrices simply multiply. In this model, reciprocity is expressed by AD - CB = 1. This can be proved by expressing A, B, C and D in terms of the impedance parameters. $A = z_{11}/z_{21}$, $B = z_{11}z_{22}/z_{21} - z_{12}$, $C = 1/z_{21}$ and $D = z_{22}/z_{21}$. The result is $AD - CB = z_{12}/z_{21} = 1$ if the network is reciprocal. It is also clear that any cascade of reciprocal networks is also reciprocal, since the determinant of the product of matrices is the product of the determinants.

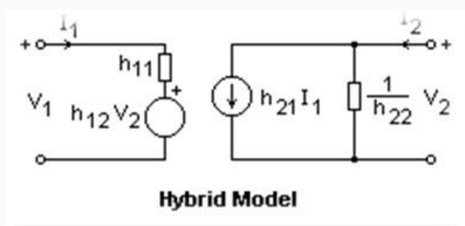

Fig. 11.5

Any model can be expressed in terms of controlled sources and impedances. This is shown at the right for they hybrid model. The h_{11} is represented by an impedance, h_{22} by an admittance. h_{12} and h_{21} are the reverse and forward transfer ratios, which are pure numbers. A current source appears in the output, while a voltage source appears in the input. This is the model often used to represent a transistor, in which h_{21} is usually denoted h_{fe} or β, the base to collector current gain. The input is between base and emitter, the output between collector and emitter. Since h_{12} is much smaller than h_{21}, the transistor is not reciprocal. A good approximate model results if $h_{11} = h_{21}\,(25\Omega/I_c)$, $h_{12} = h_{22} = 0$. I_c is the DC collector current. A typical value for h_{21} is 100. For accurate work, the hybrid coefficients can be expressed as functions of the bias conditions.

A vacuum tube can be represented by an admittance model, in which the plate current I_p is a joint function of the grid voltage V_g and the plate voltage V_p. The input port is connected to grid and cathode, the output port to plate and cathode. As long as the grid is biased negatively with respect to the cathode, $I_g = 0$, and two of the admittance parameters are zero. The remaining equation can be written $I_p = g_m V_g + V_p/r_p$, where $y_{12} = g_m$ is the transconductance and $1/y_{22} = r_p$ is the plate resistance. When I_p is held constant, $-g_m r_p V_g = V_p$, which defines the amplification factor $\mu = g_m r_p$. The same model can be used for an FET. We note that none of these models are reciprocal, which is worth remembering. These models are extremely useful in circuit design.

11.2. Proof of the Reciprocity Theorem

We wish to show that in a network of linear, bilinear elements, that is, in one constructed of ordinary impedances, that if when a voltage V is inserted in one loop the current I in another loop due to the insertion of this voltage is the same as the current at the first position due to the insertion of a voltage V in the second loop, or that the network is reciprocal. We shall do this by explicitly calculating the currents in the two cases, and observing that they are equal.

Consider the network as made up of N independent loops, of which loop 1 contains the input port, and loop 2 the output port. Let there be no emf's in the network. If there are, they can be taken care of by superposition, and we can set them all to zero for our purposes. The loop currents are I_k, k = 1 to N. If emf V is at port 1 and port 2 is shorted, then Kirchhoff's laws give $\Sigma z_{1k}I_k = V$, and $\Sigma z_{jk}I_k = 0$, j = 2 to N. The current I_2 can then be expressed in terms of determinants as $I_2 = -V \det A/\Delta$, where the determinant in the numerator has been expanded by minors of the second column, which is all zeros except for V in the first position. A is the matrix of the z's with the first row and second column taken away. Δ is the determinant of the matrix of the z_{jk}. Write out this solution in detail if it is not clear from this description.

Now connect V in port 2, and short port 1. The solution by determinants for I_1 is $-V \det B/\Delta$, where B is the matrix of the z's with the second row and first column taken away. If we compare matrices A and B, we see that they are transposes of each other, provided that $z_{jk} = z_{kj}$. However, the off-diagonal z's are just the mutual impedances of the current loops (impedances common to a pair of loops), and do not depend on the order of the subscripts. (They are not the coefficients of an impedance model here, simply actual impedances.) Since the determinant of the transpose of a matrix is equal to the determinant of the matrix, the two solutions are the same, and $I_1 = I_2$. This proves the reciprocity theorem for networks made from linear, bilateral elements, Q.E.D.

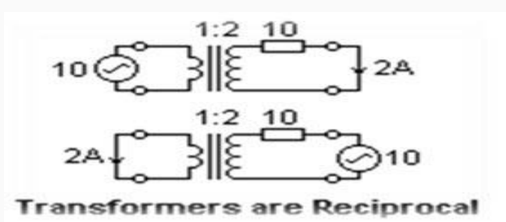

Fig. 11.6

We have proved the reciprocity theorem for a relatively limited class of networks, but it is possible to extend the theorem more widely. The figure at the right shows that reciprocity holds for ideal transformers at least. Since real transformers may be modeled by networks containing only impedances and ideal transformers, real transformers must also be reciprocal, and this extends the utility of the theorem further. Reciprocity crops up in many unexpected places, such as in electromagnetic fields and microwaves.

Module 6. Star- Delta conversion solution of DC circuit by Network theorems

LESSON 12. Star- Delta conversion solution of DC circuit by Network theorems

12.1. Star Delta Transformation

In the complicated networks involving large number of resistances/ Kirchhoff's Laws give us complex set of simultaneous equations. It is time consuming to solve such set of simultaneous equations involving large number of unknowns. In such a case application of Star-Delta or Delta-Star transformation, considerably reduces the complexity of the network and brings the network into a very simple form. This reduces the number of unknowns and hence network can be analyzed very quickly for the required result. These transformations allow us to replace three star connected resistances of the network, by equivalent delta connected resistances, without affecting currents in other branches and vice-versa.

Standard 3-phase circuits or networks take on two major forms with names that represent the way in which the resistances are connected, a Star connected network which has the symbol of the letter, Y (wye) and a Delta connected network which has the symbol of a triangle, Δ (delta). If a 3-phase, 3-wire supply or even a 3-phase load is connected in one type of configuration, it can be easily transformed or changed it into an equivalent configuration of the other type by using either the Star Delta Transformation or Delta Star Transformation process.

A resistive network consisting of three impedances can be connected together to form a T or "Tee" configuration but the network can also be redrawn to form a Star or Y type network as shown below.

12.2. T-connected and Equivalent Star Network

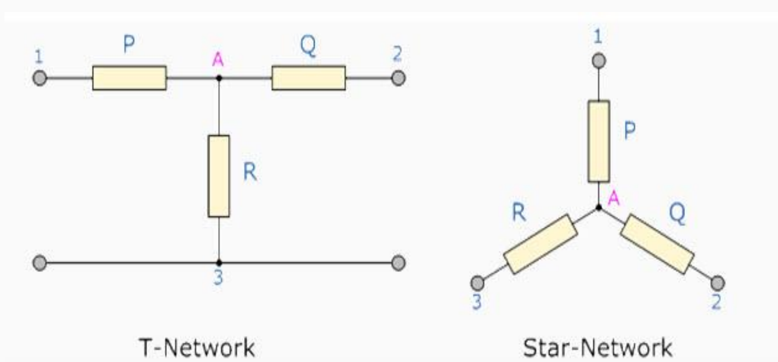

Fig. 12.1

As we have already seen, we can redraw the T resistor network to produce an equivalent Star or Y type network. But we can also convert a Pi or π type resistor network into an equivalent Delta or Δ type network as shown below.

12.3. Pi-connected and Equivalent Delta Network.

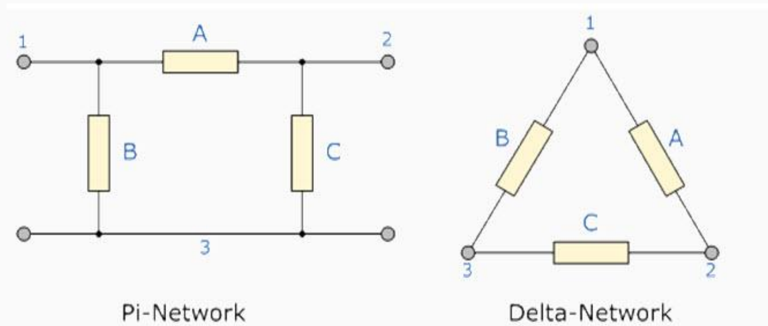

Fig. 12.2

Having now defined exactly what is a Star and Delta connected network it is possible to transform the Y into an equivalent Δ circuit and also to convert a Δ into an equivalent Y circuit using a the transformation process. This process allows us to produce a mathematical relationship between the various resistors giving us a Star Delta Transformation as well as a Delta Star Transformation.

These transformations allow us to change the three connected resistances by their equivalents measured between the terminals 1-2, 1-3 or 2-3 for either a star or delta connected circuit. However, the resulting networks are only equivalent for voltages and currents external to the star or delta networks, as internally the voltages and currents are different but each network will consume the same amount of power and have the same power factor to each other.

12.4. Delta Star Transformation

To convert a delta network to an equivalent star network we need to derive a transformation formula for equating the various resistors to each other between the various terminals. Consider the circuit below

12.4.1. Delta to Star Network.

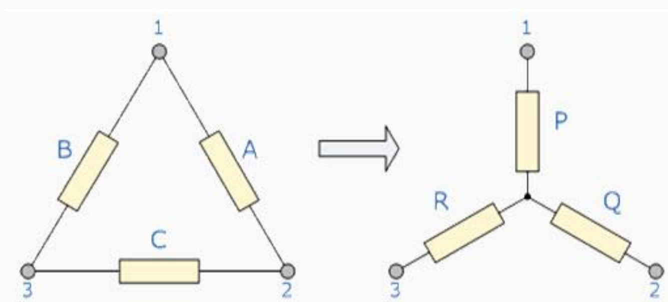

Fig. 12.3

Compare the resistances between terminals 1 and 2.

$$P-Q=A \text{ in parallel with}\left({B - C} \right)$$

$$P - Q={{A\left({B + C}\right)} \over {A + B + C}}..............\left({EQ1} \right)$$

Resistance between the terminals 2 and 3.

$$Q - R=C\,\text{in parallel with}\left({A - B} \right)$$

$$Q - R={{C\left({A + B} \right)} \over {A + B + C}}........................\left({EQ2}\right)$$

Resistance between the terminals 1 and 3.

$$P - R=B\,\text{in parallel with}\left({A - C} \right)$$

$$P - R={{B\left({A + C} \right)} \over {A + B + C}}......................\left({EQ3} \right)$$

This now gives us three equations and taking equation 3 from equation 2 gives:

$$EQ3 - EQ2=\left({P - R} \right) - \left({Q + R} \right)$$

$$P + R={{B\left({A + C} \right)} \over {A + B + C}} + Q - R={{C\left({A + B} \right)}\over {A + B + C}}$$

$$P - Q={{BA - CB} \over {A + B + C}} - {{CA + CB} \over {A + B + C}}$$

$$P - Q={{BA - CA} \over {A + B + C}}$$

Then, re-writing Equation 1 will give us:

$$P + Q={{AB - AC} \over {A + B + C}}$$

Adding together equation 1 and the result above of equation 3 minus equation 2 gives:

$$\left({P - Q} \right) + \left({P + Q} \right)={{BA - CA} \over {A + B + C}} + {{AB - AC} \over {A + B + C}}$$

$$=2P={{2AB} \over {A + B + C}}$$

From which gives us the final equation for resistor P as:

$$P={{AB} \over {A + B + C}}$$

Then to summarize a little the above maths, we can now say that resistor P in a Star network can be found as Equation 1 plus (Equation 3 minus Equation 2) or Eq1 + (Eq3 - Eq2).

Similarly, to find resistor Q in a star network, is equation 2 plus the result of equation 1 minus equation 3 or Eq2 + (Eq1 - Eq3) and this gives us the transformation of Q as:

$$Q={{AC} \over {A + B + C}}$$

and again, to find resistor R in a Star network, is equation 3 plus the result of equation 2 minus equation 1 or Eq3 + (Eq2 - Eq1) and this gives us the transformation of R as:

$$R={{BC} \over {A + B + C}}$$

When converting a delta network into a star network the denominators of all of the transformation formulas are the same: A + B + C, and which is the sum of ALL the delta resistances. Then to convert any delta connected network to an equivalent star network we can summarized the above transformation equations as:

$$P={{AB} \over {A + B + C}}$$

$$Q={{AC} \over {A + B + C}}$$

$$R={{BC} \over {A + B + C}}$$

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LESSON 13. Delta to Star Transformations solution of DC circuit by Network theorems

13.1. Delta to Star Transformations Equations are

$$P={{AB} \over {A + B + C}}$$

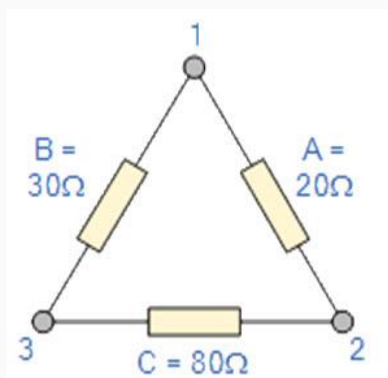
$$Q={{AC} \over {A + B + C}}$$

$$R={{BC} \over {A + B + C}}$$

If the three resistors in the delta network are all equal in value then the resultant resistors in the equivalent star network will be equal to one third the value of the delta resistors, giving each branch in the star network as: $R_{STAR} = 1/3R_{DELTA}$

Example No1

Convert the following Delta Resistive Network into an equivalent Star Network.



$$Q={{AC} \over {A + B + C}}$$

$$Q={{20x80} \over {20 + 30 + 80}}$$

$$Q=12.31\Omega$$

$$P={{AB} \over {A + B + C}}$$

$$P={{20x30} \over {20 + 30 + 80}}$$

$$P=4.61\Omega$$

$$R={{BC} \over {A + B + C}}$$

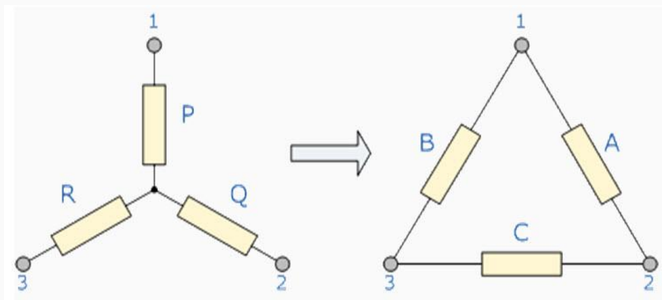
$$R={{30x80} \over {20 + 30 + 80}}$$

$$R=18.46\Omega$$

13.2. Star Delta Transformation

We have seen above that when converting from a delta network to an equivalent star network that the resistor connected to one terminal is the product of the two delta resistances connected to the same terminal, for example resistor P is the product of resistors A and B connected to terminal 1. By rewriting the previous formulas a little we can also find the transformation formulas for converting a resistive star network to an equivalent delta network giving us a way of producing a star delta transformation as shown below.

13.2.1. Star to Delta Network.



13.1

The value of the resistor on any one side of the delta, Δ network is the sum of all the two-product combinations of resistors in the star network divide by the star resistor located "directly opposite" the delta resistor being found. For example, resistor A is given as:

$$A={{PQ + QR + RP} \over R}$$

with respect to terminal 3 and resisor B is given as:

$$B={{PQ + QR + RP} \over Q}$$

with respect to terminal 2 with resistor C given as:

$$C={{PQ + QR + RP} \over P}$$

with respect to terminal 1.

By dividing out each equation by the value of the denominator we end up with three separate transformation formulas that can be used to convert any Delta resistive network into an equivalent star network as given below.

13.2.2. Star Delta Transformation Equations are

$$A={{PQ} \over R} + Q + P$$

$$B={{RP} \over Q} + P + R$$

$$C={{QR} \over P} + Q + R$$

Star Delta Transformations allow us to convert one circuit type of circuit connection to another in order for us to easily analyze a circuit and one final point about converting a star resistive network to an equivalent delta network. If all the resistors in the star network are all equal in value then the resultant resistors in the equivalent delta network will be three times the value of the star resistors and equal, giving: $R_{DELTA} = 3R_{STAR}$

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Module 7. Sinusoidal steady state response of circuits

LESSON 14. Sinusoidal steady state response of circuit

14.1. Steady State and Transient Response

A circuit having constant sources is said to be in steady state if the currents and voltages do not change with time. Thus, circuits with currents and voltages having constant amplitude and constant frequency sinusoidal functions are also considered to be in a steady state. That means that the amplitude or frequency of a sinusoid never changes in a steady state circuit.

In a network containing energy storage elements, with change in excitation, the currents and voltages change from one state to other state. The behavior of the voltage or current when it is changed from one state to another is called the transient state. The time taken for the circuit to change from one steady state to another steady state is called the transient time. The application of KVL and KCL to circuits containing energy storage elements results in differential, rather than algebraic, equations. When we consider a circuit containing storage elements which are independent of the sources, the response depends upon the nature of the circuit and is called the natural response. Storage elements deliver their energy to the resistances. Hence the response changes with time, gets saturated after some time, and is referred to as the transient response. When we consider sources acting on a circuit, the response depends on the nature of the source or sources. This response is called forced response. In other words, the complete response of a circuit consists of two parts: the forced response and the transient response. When we consider a differential equation, the complete solution consists of two parts: the complementary function and the particular solution. The complementary function dies out after short interval, and is referred to as the transient response or source free response. The particular solution is the steady state response, or the forced response. The first step in finding the complete solution of a circuit is to form a differential equation for the circuit. By obtaining the differential equation, several methods can be used to find out the complete solution.

14.2. DC Response of an R-L Circuit

Consider a circuit consisting of a resistance and inductance as shown in Fig.14.1. The inductor in the circuit is initially uncharged and is in series with the resistor. When the switch S is closed, we can find the complete solution for the current. Application of Kirchhoff's voltage law to the circuit results in the following differential equation.

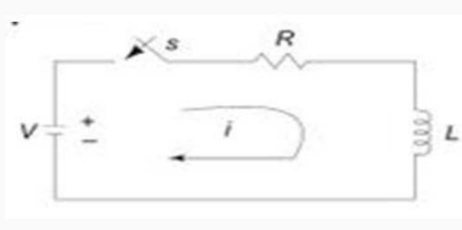


Fig.14.1

$$V=Ri + L{{di} \over {dt}}\ldots\ldots\left({14.1} \right)$$

$$or\,\,{{di} \over {dt}} + {R \over L}i={V \over L}\ldots\ldots\left({14.2} \right)$$

In the above equation, the current t is the solution to be found and V is the applied constant voltage. The voltage V is applied to the circuit only when the switch S closed. The above equation is a linear differential equation of first order. Comparing it with a non-homogeneous differential equation.

$${{dx} \over {dt}} + Px = K\ldots\ldots\left({14.3} \right)$$

whose solution is

$$x={e^{ - pt}}\,\,f\,K{e^{ + Pt}}\,dt + c{e^{ - Pt}}\ldots\ldots\left({14.4} \right)$$

where c is an arbitrary constant. In a similar way, we can write the current equations as

$$i=c{e^{ - \left({R/L} \right)t}} + {e^{ - \left({R/L} \right)t}}\,\int {{V \over L}{e^{\left({R/L} \right)t}}\,dt}$$

$$i=c{e^{ - \left({R/L} \right)t}}\, + {V \over R}\ldots\ldots\left({14.5} \right)$$

To determine the value of c in Eq.14.5, we use the initial conditions. In the circuit shown in Fig.14.1, the switch S is closed at t=0. At $t=0^-$, i.e. just before closing the switch S, the current in the inductor is zero. Since the inductor does not allow sudden changes in currents, at $t=0^+$ just after the switch is closed, the current remains zero.

Thus at t = 0, I 0

Substituting the above condition in Eq.14.5, we have

$$0=c + {V \over R}$$

Hence $$c=-{V \over R}$$

Substituting the value of c in Eq.14.5, we get

$$i={V \over R} - {V \over R}\exp \left({ - {R \over L}t} \right)$$

$$i={V \over R}\,\left({1 - \exp \left({ - {R \over L}t} \right)} \right)\ldots\ldots\left({14.6} \right)$$

Equation 14.6 consists of two parts, the steady state part V/R, and the transient part $(V/R)e^{-(R/L)t}$. When switch S is closed, the response reaches a steady state value after a time interval as shown in Fig.14.2.

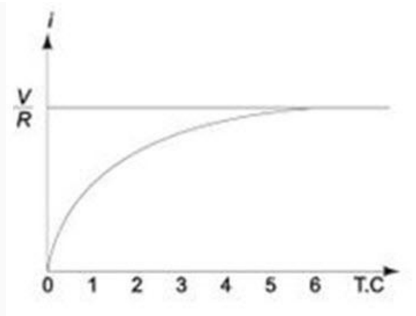


Fig.14.2

Here the transition period is defined as the time taken for the current to reach its final or steady state value from its initial value. In the transient part of the solution, the quantity L/R is important in describing the curve since L/R is the time required for the current to reach from its initial value of zero to the final value V/R. The time constant of a function $${V \over R}e{\,^{ - \left({{R \over L}t} \right)}}$$ is the time at which the exponent of e is unity, where e is the base of the natural logarithms. The term L/R is called the time constant and is denoted by t

$$\tau={L \over R}\,\,\sec$$

The transient part of the solution is

$$i=-{V \over R}\,\exp \,\left({ - {R \over L}t} \right) = {V \over R}{e^{ - t/\tau }}$$

At one TC, i.e. at one time constant, the transient term reaches 36.8 percent of its initial value.

$$i\left(\tau\right)=-{V \over R}{e^{ - t/\tau }}=-{V \over R}{e^{ - 1}}=-0.368{V \over R}$$

Similarly,

$$i\left({2\tau } \right)=-{V \over R}{e^{ - 2}}=-0.135{V \over R}$$

$$i\left({3\tau } \right)=-{V \over R}{e^{ - 3}}=-0.0498{V \over R}$$

$$i\left({5\tau } \right)=-{V \over R}{e^{ - 5}}=-0.0067{V \over R}$$

After 5 TC, the transient part reaches more than 99 percent of its final value. In Fig. 14.1, we can find out the voltages and powers across each element by using the current.

Voltage across the resistor is

$${\nu _R}=Ri=R \times {V \over R}\left[{1 - \exp \left({ - {R \over L}t} \right)} \right]$$

$${\nu _R}=V\left[{1 - \exp \left({ - {R \over L}t} \right)} \right]$$

Similarly, the voltage across the inductance is

$${\nu _L}=L{{di} \over {dt}}$$

$$=L{V \over R} \times {R \over L}\exp \left({ - {R \over L}t} \right)=V\,\,\exp \,\left({ - {R \over L}t} \right)$$

The responses are shown in Fig.14.3.

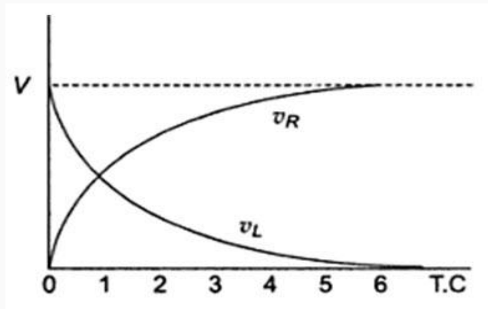


Fig.14.3

Power in the resistor is

$${P_R}=\nu _R^i=V\left({1 - \exp \left({ - {R \over L}t} \right)} \right)\,\left({1 - \exp \left({ - {R \over L}t} \right)} \right){V \over R}$$

$$={{{V^2}} \over R}\left({1 - 2\exp \left({ - {R \over L}t} \right) + \,\exp \left({ - {{2R} \over L}t} \right)} \right)$$

Power in the inductor is

$${P_L}={\nu _L}i = V\;\;\exp \,\left({ - {R \over L}t} \right) \times {V \over R}\left({1 - \exp \left({ - {R \over L}t} \right)} \right)$$

$$={{{V^2}} \over R}\left({\exp \left({ - {R \over L}t} \right) - \,\exp \left({ - {{2R} \over L}t} \right)} \right)$$

The responses are shown in Fig.14.4

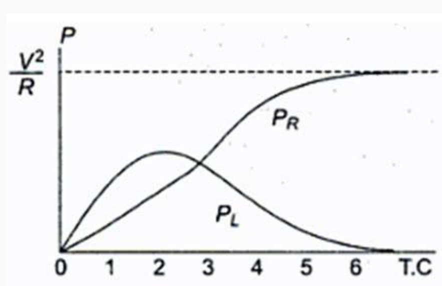


Fig.14.4

14.3 DC Response of an R-C Circuit

Consider a circuit consisting of resistance and capacitance as shown in Fig.14.5. The capacitor in the circuit is initially uncharged, and is in series with a resistor. When the switch S is closed at t=0, we can determine the complete solution for the current. Application of the Kirchhoff's voltage law to the circuit results in the following differential equation.

$$V=Ri + {1 \over C}\int {i\,\,dt} \quad ....\left({14.7} \right)$$

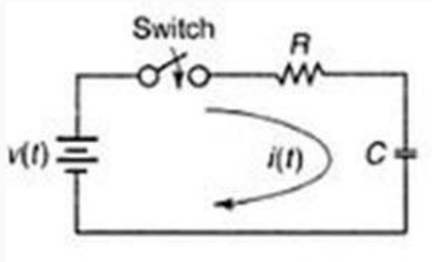


Fig.14.5

By differentiating the above equation, we get

$$0=R{{di} \over {dt}} + {i \over C} \quad ....\left({14.8} \right)$$

or $${{di} \over {dt}} + {1 \over {RC}}i=0 \quad ....\left({14.9} \right)$$

Equation 14.9 is a linear differential equation with only the complementary function. The particular solution for the above equation is zero. The solution for this type of differential equation is

$$i = ce^{-t/RC} \quad ....\left({14.10} \right)$$

Here, to find the value of c, we use the initial conditions.

In the circuit shown in Fig.14.6, switch S is closed at $t = 0$. Since the capacitor never allows sudden changes in voltage, it will act as a short circuit at $t=0^+$. So, the current in the circuit at $t=0^+$ V/R

$$At\,t = 0,\,the\,\,current\,i={V \over R}$$

Substituting this current in Eq. 14.10, we get

$${V \over R}=c$$

The current equation becomes

$$i={V \over R}{e^{ - t/RC}} \quad ....\left({14.11} \right)$$

When switch S is closed, the response decays with time as shown in Fig.14.6.

In the solution, the quantity RC is the time constant, and is denoted by t,

where t = RC sec

After 5 TC, the curve reaches 99 percent of its final value. In fig. 14.5, we can find out the voltage across each element by using the current equation.

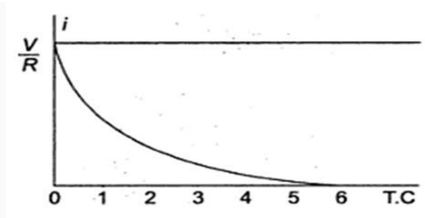


Fig.14.6

Voltage across the resistor is

$${\nu _R}=Ri=R \times {V \over R}{e^{ - \left({1/RC} \right)t}};\,\nu R = V{e^{ - t/RC}}$$

Similarly, voltage across the capacitor is

$${\nu _C}={1 \over C}\int {i\,\,dt}$$

$$={1 \over C}\int {{V \over R}{e^{ - tRC}}\,dt}$$

$$=-\left({{V \over {RC}} \times RC\,{e^{ - t/RC}}} \right) + \, = \, - V{e^{ - t/RC}} + c$$

At t=0, voltage across capacitor is zero

$c = V$

$$\nu_c = V(1 - e^{-t/RC})$$

The responses are shown in Fig.14.7.

Power in the resistor

$${P_R}=\nu _R^i=V{e^{ - t/RC}} \times {V \over R}{e^{ - t/RC}}={{V2} \over R}{e^{ - 2t/RC}}$$

Power in the capacitor

$${P_C}=\nu _c^i=V\left({1 - {e^{ - t/RC}}} \right){V \over R}{e^{ - t/RC}}$$

$$={{{V^2}} \over R}\left({{e^{ - 1/RC}} - {e^{ - 2t/RC}}} \right)$$

The responses are shown in Fig.14.8.

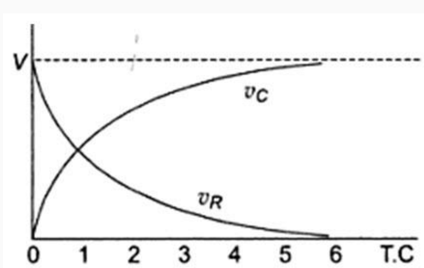


Fig.14.7

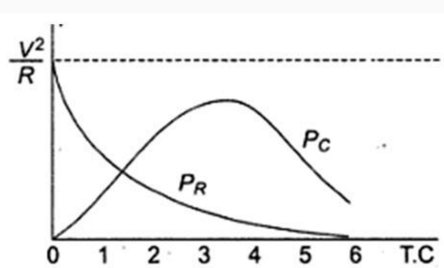


Fig.14.8

14.4. DC Response of an R-L-C Circuit

Consider a circuit consisting of resistance, inductance and capacitance as shown in Fig.14.9. The capacitor and inductor are initially uncharged, and are in series with a resistor. When switch S is closed at t=0, we can determine the complete solution for the current.

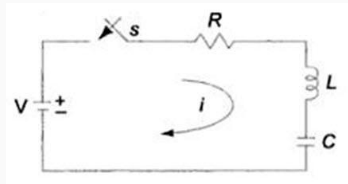


Fig14.9

Application of Kirchhoff's voltage law to the circuit results in the following differential equation.

$$V=Ri + L{{di} \over {dt}} + {1 \over C}\int {idt} \ldots\ldots\ldots\ldots\left({14.12} \right)$$

By differentiating the above equation, we have

$$0=R{{di} \over {dt}} + L{{{d^2}i} \over {d{t^2}}} + {1 \over c}i\ldots\ldots\ldots\ldots\left({14.13} \right)$$

or $${{{d^2}i} \over {d{t^2}}} + {R \over L}\,{{di} \over {dt}} + {1 \over {LC}}i=0\ldots\ldots\ldots\ldots\left({14.14} \right)$$

The above equation is a second order linear differential equation, with only complementary function. The particular solution for the above equation is zero. Characteristic equation for the above differential equation is

$$\left({{D^2} + {R \over L}D + {1 \over {LC}}} \right)=0\ldots\ldots\ldots\ldots\left({14.15} \right)$$

The roots of Eq.14.15 are

$${D^1},{D^2}=-{R \over {2L}} \pm \sqrt {{{\left({{R \over {2L}}} \right)}^2} - {1 \over {LC}}}$$

By assuming $${K_1}=-{R \over {2L}}and\,\,{K_2}=\sqrt {{{\left({{R \over {2L}}} \right)}^2} - {1 \over {LC}}}$$

$D_1 = K_1 + K_2$ and $D_2 = K_1 - K_2$

Here K_2 may be positive, negative or zero.

K_2 is positive, when ${\left({{R \over {2L}}} \right)^2}\, > 1/LC$

The roots are real and unequal, and give the over damped response as shown in Fig.14.10. Then Eq.14.14 becomes

$$\left[{D - \left({{K_1} + {K_2}} \right)} \right]\,\left[{D - \left({{K_1} - {K_2}} \right)} \right]\,\,i=0$$

The solution for the above equation is

$$i={c_1}{e^{\left({K1 + K2} \right)t}} + {c_2}{e^{\left({K1 - K2} \right)t}}$$

The current curve for the overdamped case is shown in Fig.14.10.

K_2 is negative, when $(R/2L)^2 < 1/LC$

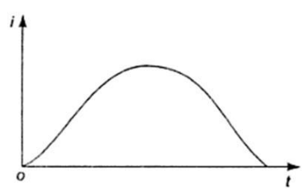
The roots are complex conjugate, and give the underdamped response as shown in Fig.14.11. Then Eq. 14.14 becomes

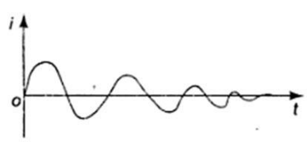
$$\left[{D - \left({{K_1} + j{K_2}} \right)} \right]\,\left[{D - \left({{K_1} - j{K_2}} \right)} \right]\,\,i=0$$

The solution for the above equation is

$$i={c_1}{e^{K1t}}\left[{{e_1}\,\,\cos \,{K_2}t + {c_2}\,\sin \,{K_2}t} \right]$$

The current curve for the underdamped case is shown in Fig.14.11.





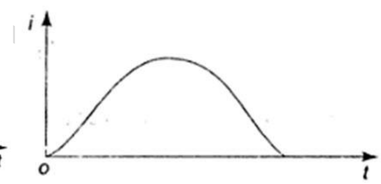


Fig. 14.10 **Fig. 14.11** **Fig14.12**

K_2 is zero, when $(R/2L)^2 = 1/LC$

The roots are equal, and give the critically damped response as shown in Fig.14.12.

Then Eq.14.14 becomes

$(D-K_1)\ (D-K_1)i = 0$

The solution for the above equation is

$$i={e^{{K_1}t}}\left({{c_1} + {c_2}t} \right)$$

The current curve for the critically damped case is shown in Fig.14.12.

LESSON 15. Sinusoidal Response of R-L & R-C Circuit

15.1. Sinusoidal Response of R-L Circuit

Consider a circuit consisting of resistance and inductance as shown in Fig.15.1. The switch, S, is closed at t=0. At t=0, a sinusoidal voltage V cos (wt+q) is applied to the series R-L circuit, where V is the amplitude of the wave and q is the phase angle. Application of Kirchhoff's voltage law to the circuit results in the following differential equation.

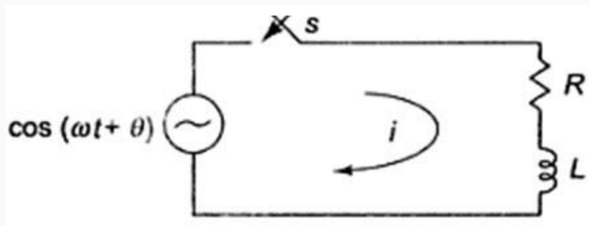


Fig.15.1

$$V\,\cos \,\left({\omega t + \theta } \right)=Ri + L{{di} \over {dt}}..\left({15.1} \right)$$

$${{di} \over {dt}} + {R \over L}i={V \over L}\,\,\cos \,\left({\omega t + \theta } \right)$$

The corresponding characteristic equation is

$$\left({D + {R \over L}} \right)i={{} \over L}\,\cos \,\left({\omega t + \theta } \right)..\left({15.2} \right)$$

For the above equation, the solution consists of two parts, viz. complementary function and particular integral.

The complementary function of the solution i is

$${i_c}=c{e^{ - t\left({R/L} \right)}}..\left({15.3} \right)$$

The particular solution can be obtained by using undetermined co-efficient.

By assuming $${i_p}=A\,\cos \,\left({\omega t + \theta } \right) + B\,\,\sin \,\left({\omega t + \theta } \right)..\left({15.4} \right)$$

$$i_p^'=A\omega \,\sin \,\left({\omega t + \theta } \right) + B\omega \,\cos \,\left({\omega t + \theta } \right)..\left({15.5} \right)$$

Substituting Eqs. 15.4 and 15.5 in Eq.15.2, we have

$$\left\{ { - A\omega \,\sin \,\left({\omega t + \theta } \right) + \,B\omega \,\cos \left({\omega t + } \right)} \right\} + {R \over L}\left\{ {A\,\cos \,\left({\omega t + \theta } \right) + B\,\,\sin \,\,\left({\omega t + \theta } \right)} \right\}={V \over L}\cos \,\left({\omega t + \theta } \right)$$

or
$$\left({ - A\omega + {{BR} \over L}} \right)\,\sin \,\left({\omega t+\theta}\right) + \left({B\omega + {{AR}\over L}}\right)\,\cos \,\,\left({\omega t + \theta }\right)={V \over L}\,\cos \,\left({\omega t + \theta }\right)$$

Comparing cosine terms and sine terms, we get

$$-A\omega+{{BR}\over L}=0$$

$$B\omega+{{AR} \over L}={V \over L}$$

From the above equations, we have

$$A=V{R \over {{R^2} + {{\left({\omega L} \right)}^2}}}$$

$$B=V{{\omega L} \over {{R^2} + {{\left({\omega L} \right)}^2}}}$$

Substituting the values of A and B in Eq.11.20, we get

$${i_p}=V{R \over {{R^2} + {{\left({\omega L} \right)}^2}}}\cos \,\left({\omega t + \theta } \right) + V{{\omega L} \over {{R^2} + {{\left({\omega L} \right)}^2}}}\,\sin \,\left({\omega t + \theta } \right)\ldots\ldots\ldots\left({15.6} \right)$$

Putting
$$M\,\cos\,\phi={{VR}\over{{R^2}+{{\left({\omega L}\right)}^2}}}$$

$$M\,\sin\,\phi=\,V{{\omega L}\over{{R^2}+{{\left({\omega L} \right)}^2}}}$$

to find M and ø, we divide one equation by the other

$${{M\,\sin \,\phi} \over {M\,\cos \,\phi}}=\,\tan \,\phi \,=\,{{\omega L} \over R}\,$$

Squaring both equation and adding, we get

$${M^2}\,{\cos ^2}\,\phi+{M^2}\,{\sin ^2}\,\phi={{{V^2}} \over {{R^2} + {{\left({\omega L} \right)}^2}}}$$

or
$$M={V \over {\sqrt {{R^2} + {{\left({\omega L} \right)}^2}} }}$$

The particular current becomes

$${i_p}={V\over{\sqrt{{R_2}+{{\left({\omega L}\right)}^2}}}}\,\cos\,\left({\omega t+\theta-{{\tan}^{-1}}{{\omega L}\over R}} \right)\ldots\ldots\ldots\left({15.7} \right)$$

The complete solution for the current
$$i={i_c} + {i_p}$$

$$i=c{e^{-t\left({R/L} \right)}} + {V \over{\sqrt{{R^2} + {{\left({\omega L} \right)}^2}}}}\,\cos \,\left({\omega t + \theta -{{\tan }^{-1}}{{\omega L} \over R}} \right)$$

Since the inductor does not allow sudden changes in currens, at t=0, i=o

$$c=-{V \over{\sqrt{{R^2} + {{\left({\omega L}\right)}^2}} }}\,\,\cos \,\,\left({\theta-{{\tan }^{-1}}{{\omega L}\over R}} \right)$$

The complete solution for the current is

$$i=\,{e^{-\left({R/L} \right)t}}\left[{{{-V} \over {\sqrt{{R^2} + {{\left({\omega L} \right)}^2}} }}\,\,\cos \,\,\left({\theta-{{\tan }^{-1}}{{\omega L} \over R}} \right)} \right]$$

$$+{V \over{\sqrt{{R^2}+{{\left({\omega L} \right)}^2}} }}\,\,\cos \,\,\left({\omega t +\theta-{{\tan }^{-1}}{{\omega L}\over R}}\right)$$

15.2. Sinusoidal Response of R-C Circuit

Consider a circuit consisting of resistance and capacitance in series as shown in Fig.15.2. The switch, S, is closed at t=0. At t=0, a sinusoidal voltage V cos (wt+q) is applied to the R-C circuit, where V is the amplitude of the wave and q is the phase angle. Applying Kirchhoff's voltage law to the circuit results in the following differential equation.

$$V\,\cos \,\left({\omega t + \theta } \right) = Ri + {1 \over C}\int {idt} \ldots\ldots\ldots\left({15.8} \right)$$

$$R{{di} \over {dt}} + {i \over C}=-V\,\,\omega \,\,\sin \,\left({\omega t + \theta } \right)$$

$$\left({D + {1 \over {RC}}i=-{{V\omega } \over R}\,\sin \,\left({\omega t + \theta } \right)} \right)\ldots\ldots\ldots\left({15.9} \right)$$

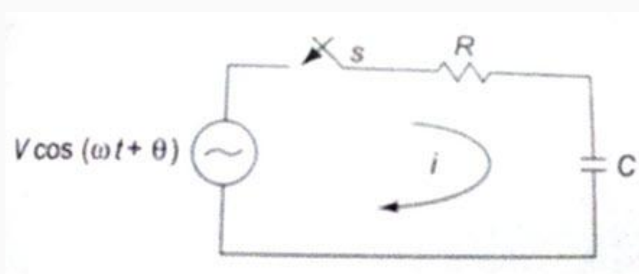


Fig. 15.2

The complementary function $i_c = ce^{-t/RC}$ $\ldots\ldots\ldots\left({15.10} \right)$

The particular solution can be obtained by using undermined coefficients.

$${i_p}=A\,\cos \,\left({\omega t + \theta } \right) + B\,\sin \,\left({\omega t + \theta } \right)\ldots\ldots\ldots\left({15.11} \right)$$

$$i_p^{'}=A\omega \,\sin \,\left({\omega t + \theta } \right) + B\omega \,\cos \,\left({\omega t + \theta } \right)\ldots\ldots\ldots\left({15.12} \right)$$

Substituting Eqs. 15.11 and 15.12 in Eq. 15.9, we get

$$\left\{ { - A\omega \,\sin \,\left({\omega t + \theta } \right) + B\omega \,\cos \,\left({\omega t + \theta } \right)} \right\} + {1 \over {RC}}\left\{ {A\,\cos \,\left({\omega t + \theta } \right) + B\,\sin \,\left({\omega t + \theta } \right)} \right\}$$

$$=-{{V\omega } \over {R\,}}\,\sin \,\left({\omega t + \theta } \right)$$

Comparing both sides, $$-A\omega+{B\over{RC}}=-{{V\omega}\over R}$$

$$B\omega+{A\over{RC}}=0$$

From which,

$$A = \frac{{VR}}{{{R^2} + {{\left({\frac{1}{{\omega c}}} \right)}^2}}}$$

and $$B = \frac{{ - V}}{{\omega C\left[{{R^2} + {{\left({\frac{1}{{\omega c}}} \right)}^2}} \right]}}$$

Substituting the values of A and B in Eq. 15.11, we have

$${i_p} = \frac{{VR}}{{{R^2}+{{\left({\frac{1}{{\omega c}}}\right)}^2}}}\,\cos\;\,\left({\omega t+\theta }\right)+\frac{{-V}}{{\omega C\left[{{R^2}+{{\left({\frac{1}{{\omega C}}}\right)}^2}}\right]}}\,\sin\,\left({\omega t+\theta } \right)$$

Putting $$M\,\cos\,\varphi=\frac{{VR}}{{{R^2}+{{\left({\frac{1}{{\omega C}}}\right)}^2}}}$$

and $$M\,\sin\,\varphi =\frac{V}{{\omega C\left[{{R^2}+{{\left({\frac{1}{{\omega C}}}\right)}^2}}\right]}}$$

To find M and ø, we divide one equation by the other,

$${{M\,\sin \,\phi } \over {M\,\cos \,\phi }}=\tan \,\phi={1 \over {\omega CR}}$$

Squaring both equations and adding, we get

$${M^2}\,{\cos ^2}\,\varphi +{M^2}\,{\sin ^2}\,\varphi =\frac{{{V^2}}}{{\left[{{R^2}+{{\left({\frac{1}{{\omega C}}}\right)}^2}}\right]}}$$

$$M =\frac{V}{{\sqrt{{R^2}+{{\left({\frac{1}{{\omega C}}}\right)}^2}}}}$$

The particular current becomes

$${i_p}=\frac{V}{{{R^2}+{{\left({\frac{1}{{\omega c}}}\right)}^2}}}\,\cos\;\left({\omega t+\theta+{{\tan}^{-1}}\frac{1}{{\omega CR}}}\right)..\left({15.13} \right)$$

The complete solution for the current $i=i_c+i_p$

$$i_p=c{e^{-\left({t/RC}\right)}}+\frac{V}{{{R^2}+{{\left({\frac{1}{{\omega c}}}\right)}^2}}}\,\cos\;\left({\omega t+\theta+{{\tan}^{-1}}\frac{1}{{\omega CR}}}\right)\ldots\ldots\ldots\ldots\ldots\ldots\left({15.14} \right)$$

Since the capacitor does not allow sudden changes in voltages at $t=0,\,i={V \over R}\cos \,\theta$

$$c=\frac{V}{R}\cos\,\theta-\frac{V}{{\sqrt{{R^2}+{{\left({\frac{1}{{\omega C}}}\right)}^2}}}}\cos\left({\theta+{{\tan }^{-1}}\frac{1}{{\omega CR}}}\right)$$

$$c=\frac{V}{R}\cos\,\theta-\frac{V}{{\sqrt{{R^2}+{{\left({\frac{1}{{\omega C}}}\right)}^2}}}}\cos\left({\theta+{{\tan}^{-1}}\frac{1}{{\omega CR}}}\right)$$

The complete solution for the current is

$$i = e-\left({t/RC}\right)\left[{\frac{V}{R}\cos \,\theta-\frac{V}{{\sqrt{{R^2}+{{\left({\frac{1}{{\omega C}}}\right)}^2}}}}\cos\,\left({\theta+{{\tan}^{-1}}\frac{1}{{\omega CR}}}\right)}\right]$$

$$+\frac{V}{{\sqrt{{R^2}+{{\left({\frac{1}{{\omega C}}}\right)}^2}} }}\cos\,\left({\omega t+\theta+{{\tan}^{-1}}\frac{1}{{\omega CR}}}\right)\ldots\ldots\ldots\ldots\ldots\ldots\left({15.15} \right)$$

LESSON 16. Sinusoidal Response of R-L -C Circuit

16.1. Sinusoidal Response of R-L-C Circuit

Consider a circuit consisting of resistance, inductance and capacitance in series as shown in Fig.16.1. Switch S is closed at t = 0. At t = 0, a sinusoidal voltage V cos (wt+q) is applied to the RLC series circuit, where V is the amplitude of the wave and q is the phase angle. Application of Kirchhoff's voltage law to the circuit results in the following differential equation.

$$V\;\cos \,\left({\omega t + \theta } \right)=Ri + L{{di} \over {dt}} + {1 \over C}\int {idt} \qquad \left({16.1} \right)$$

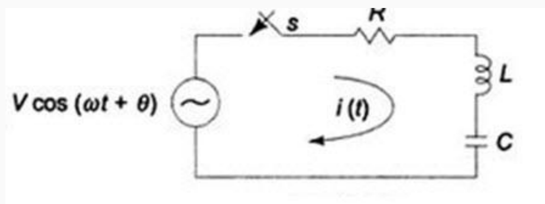


Fig16.1

Differentiating the above equation, we get

$$R{{di} \over {dt}} + L{{{d^2}i} \over {d{t^2}}} + i/C=-V\omega \,\sin \,\left({\omega t + \theta } \right)$$

$$\left({{D^2} + {R \over L}D + {1 \over {LC}}} \right)i=-{{V\omega } \over L}\,\sin \,\left({\omega t + \theta } \right) \qquad \left({16.2} \right)$$

The particular solution can be obtained by using undermined coefficients. By assuming

$${i_p}=A\,\cos \,\left({\omega t + \theta } \right) + B\,\sin \,\left({\omega t + \theta } \right) \qquad \left({16.3} \right)$$

$$i_p^'=A\omega \,\sin \,\left({\omega t + \theta } \right) + B\omega \,\cos \,\left({\omega t + \theta } \right) \qquad \left({16.4} \right)$$

$$i_p^{''}=A{\omega ^2}\,\cos \,\left({\omega t + \theta } \right) + B{\omega ^2}\,\sin \,\left({\omega t + \theta } \right) \qquad \left({16.5} \right)$$

Substituting i_p i'_p and i''_p in Eq.16.2, we have

$$\left\{ { - A\omega 2\,\cos \,\left({\omega t + \theta } \right) + B\,\sin \,\left({\omega t + \theta } \right)} \right\} + {R \over L}\left\{ { - A\omega \,\sin \,\left({\omega t + \theta } \right) + B\omega \,\cos \,\left({\omega t + \theta } \right)} \right\}$$

$$+ {1 \over {LC}}\left\{ {A\,\cos \,\left({\omega t + \theta } \right) + B\,\sin \,\left({\omega t + \theta } \right)} \right\}=-{{V\omega } \over L}\,\sin \left({\omega t + \theta } \right)..\left({16.6} \right)$$

Comparing both sides, we have

Sine coefficients

$$-B{\omega ^2} - A{{\omega R} \over L} + {B \over {LC}}=-{{V\omega } \over L}$$

$$A\left({{{\omega R} \over L}} \right) + B\left({{\omega ^2} - {1 \over {LC}}} \right)={{V\omega } \over L}..\left({16.7} \right)$$

Cosine coefficients

$$- A{\omega ^2} + B{{\omega R} \over L} + {A \over {LC}}=0$$

$$A\left({{\omega ^2} - {1 \over {LC}}} \right) - B\left({{{\omega R} \over L}} \right)=0..\left({16.8} \right)$$

Solving Eqs 16.7 and 16.8, we get

$$A =\frac{{V\times\frac{{{\omega^2}R}}{{{L^2}}}}}{{\left[{{{\left({\frac{{\omega R}}{L}}\right)}^2}-{{\left({{\omega ^2}=\frac{1}{{LC}}}\right)}^2}}\right]}}$$

$$B=\frac{{\left({{\omega^2}-\frac{1}{{LC}}}\right)V\omega}}{{L\left[{{{\left({\frac{{\omega R}}{L}}\right)}^2}-{{\left({{\omega^2}=\frac{1}{{LC}}}\right)}^2}}\right]}}$$

Substituting the values of A and B in Eq. 6.3, we get

$${i_p}=\frac{{V\frac{{{\omega ^2}R}}{{{L^2}}}}}{{\left[{{{\left({\frac{{\omega R}}{L}}\right)}^2}-{{\left({{\omega ^2}-\frac{1}{{LC}}}\right)}^2}}\right]}}\cos\,\left({\omega t+\theta}\right)$$

$$+\frac{{\left({{\omega ^2}-\frac{1}{{LC}}}\right)V\omega}}{{L\left[{{{\left({\frac{{\omega R}}{L}}\right)}^2}-{{\left({{\omega ^2}-\frac{1}{{LC}}}\right)}^2}}\right]}}\sin\,\left({\omega t +\theta}\right)................................\left({16.9} \right)$$

Putting $$M\,\cos\,\varphi=\frac{{V\frac{{{\omega^2}R}}{{{L^2}}}}}{{{{\left({\frac{{\omega R}}{L}}\right)}^2}-{{\left({{\omega ^2}-\frac{1}{{LC}}}\right)}^2}}}$$

and $$M\,\sin\,\varphi=\frac{{V\left({\omega 2-\frac{1}{{LC}}}\right)\omega}}{{L\left[{{{\left({\frac{{\omega R}}{L}}\right)}^2}-{{\left({{\omega ^2}-\frac{1}{{LC}}}\right)}^2}}\right]}}$$

To find M and ø we divide one equation by the other

or $${{M\,\sin \,\phi} \over {M\,\cos \,\phi }}=\tan \,\phi={{\left({\omega L - {1 \over {\omega C}}} \right)} \over R}$$

$$\phi=\tan^{-1}\left[{\left({\omega L - {1 \over {\omega C}}} \right)/R} \right]$$

Squaring both equations and adding, we get

$${M^2}\,{\cos ^2}\,\varphi \,+\,{M^2}\,{\sin ^2}\,\varphi \,=\frac{{{V^2}}}{{{R^2}+{{\left({\frac{1}{{\omega C}}-\omega L}\right)}^2}}}$$

$$M=\frac{V}{{\sqrt{{R^2}+{{\left({\frac{1}{{\omega C}}-\omega L}\right)}^2}}}}$$

The particular current becomes

$${i_p}=\frac{V}{{\sqrt {{R^2}+{{\left({\frac{1}{{\omega C}}-\omega L}\right)}^2}}}}\cos\left[{\omega t+\theta+{{\tan}^{-1}}\frac{{\left({\frac{1}{{\omega C}}-\omega L}\right)}}{R}}\right]\ldots\ldots\ldots\ldots\left({16.10} \right)$$

The complementary function is similar to that of DC series RLC circuit. To find out the complementary function, we have the characteristic equation

$$\left({{D^2} + {R \over L}D + {1 \over {LC}}} \right)=0\ldots\ldots\ldots\ldots\left({16.11} \right)$$

The roots of Eq.16.11, are

$${D_1},{D_2}={{ - R} \over {2L}} \pm \sqrt {{{\left({{R \over {2L}}} \right)}^2} - {1 \over {LC}}}$$

By assuming $${K_1}=-{R \over {2L}}and\,{K_2}=\sqrt {{{\left({{R \over {2L}}} \right)}^2} - {1 \over {LC}}}$$

$${D_1}={K_1} + {K_2}\,\,and\,\,{D_2}={K_1} - {K_2}$$

K_2 becomes positive, when $(R/2L)^2>1/LC$

The roots are real and unequal, which gives an overdamped response. Then Eq.16.11 becomes

$$\left[{D - \left({{K_1} + {K_2}} \right)} \right]\,\,\left[{D - {K_1} - {K_2}} \right]i=0$$

The complementary function for the above equation is

$${i_c}={c_1}{e^{\left({K1 + K2} \right)t}} + {c_2}{e^{\left({K1 - K2} \right)t}}$$

Therefore, the complete solution is

$$i={i_c} + {i_p}$$

$$ {i_c}={c_1}{e^{\left({K1+K2}\right)t}}+{c_2}{e^{\left({K1-K2}\right)t}}+\frac{V}{{\sqrt{{R^2}+{{\left({\frac{1}{{\omega C}}-\omega L}\right)}^2}}}}\cos\,\left[{\omega t +\theta+{{\tan}^{-1}}\left({\frac{1}{{\omega CR}}-\frac{{\omega L}}{R}} \right)} \right] $$

K_2 becomes negative, when $ {\left({{R \over {2L}}} \right)^2} < {1 \over {LC}} $

Then the roots are complex conjugate, which gives an under damped response, Equation 16.11 becomes.

$$ \left[{D - \left({{K_1} + j{K_2}} \right)} \right]\,\,\left[{D - {K_1} - j{K_2}} \right]i=0 $$

The solution for the above equation is

$$ {i_c}=e{K_2}^t\left[{{c_1}\,\cos \,{K_2}t + {c_2}\,\sin \,{K_2}t} \right] $$

Therefore, the complete solution is

$$ i={i_c} + {i_p} $$

$$ i=e{K_1}^t\left[{{c_1}\,\cos \,{K_2}t + {c_2}\,\sin \,{K_2}t} \right] $$

$$ +\frac{V}{{{{\sqrt{{R^2}+\left({\frac{1}{{\omega C}}-\omega L}\right)}}^2}}}\cos\,\left[{\omega t+\theta+{{\tan}^{-1}}\left({\frac{1}{{\omega CR}}-\frac{{\omega L}}{R}}\right)}\right] $$

K_2 becomes zero, when $ {\left({{R \over {2L}}} \right)^2}=1/LC $

Then the roots are equal which gives critically damped response. Then, Eq.16.11 becomes $(D-K_1)\,(D-K_1)\,I = 0$

The complementary function for the above equation is

$$ {i_c}={e^{K1t}}\left({{c_1} + {c_2}t} \right) $$

Therefore, the complete solution is $i=i_c + i_p$

$$ i = eK_1^t\left[{{c_1}+{c_2}t}\right]+\frac{V}{{\sqrt{{R^2}+}{{\left({\frac{1}{{\omega C}}-\omega L}\right)}^2}}}\cos\,\left[{\omega t+\theta+{{\tan }^{-1}}\left({\frac{1}{{\omega CR}}-\frac{{\omega L}}{R}}\right)}\right] $$

Module 8. Instantaneous and average power, power factor, reactive and apparent power

LESSON 17. Instantaneous and average power

In a purely resistive circuit, all the energy delivered by the source is dissipated in the form of heat by the resistance. In a purely reactive (indicative or capacitive) circuit, all the energy delivered by the source is stored by the inductor or capacitor in its magnetic or electric field during a portion of the voltage cycle, and then is returned to the source during another portion of the cycle, so that no net energy is transferred. When there is complex impedance in a circuit, part of the energy is alternately stored and returned by the reactive part, and part of it is dissipated by the resistance. The amount of energy dissipated is determined by the relative values of resistance and reactance.

Consider a circuit having complex impedance. Let v (t) = V_m cos wt be the voltage applied to the circuit and let i(t) = I_m cost (wt+θ) be the corresponding current flowing through the circuit. Then the power at any instant of time is

P(t) = v(t) i (t)

= V_m cost wt I_m cost (wt+θ) $$...\left({17.1} \right)$$

From Eq. 17.1, we get

$$P\left(t \right)=\,{{{V_m}{I_m}} \over 2}\,\left[{\cos \,(2\omega t + \theta) + \cos \theta } \right]..\left({17.2} \right)$$

Equation 17.2 represents instantaneous power. It consists of two parts. One is a fixed part, and the other is time-varying which has a frequency twice that of the voltage of current waveforms. The voltage, current and power waveforms are shown in figs. 17.1 and 17.2.

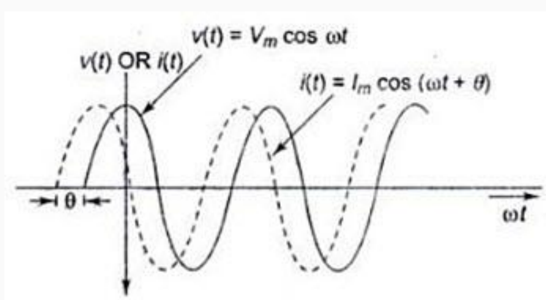


Fig. 17.1

Here, the negative portion (hatched) of the power cycle represents the power returned to the source. Figure 17.2 shows that the instantaneous power is negative whenever the voltage and current are of opposite sign. In fig. 17.2, the positive portion of the power is greater than the negative portion of the power, hence the average power is always positive, which is almost equal to the constant part of the instantaneous power (Eq. 17.2).

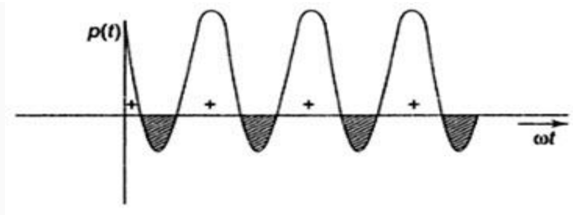


Fig. 17.2

The positive portion of the power cycle varies with the phase angle between the voltage and current waveforms. If the circuit is pure resistive, the phase angle between voltage and current is zero; then there is no negative cycle in the P (t) curve. Hence, all the power delivered by the source is completely dissipated in the resistance.

$P(t) = v(t)\, i(t)$

$= V_m I_m \cos^2 wt$

$$= \frac{V_m I_m}{2}\left(1 + \cos 2\omega t\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \left(17.3\right)$$

The waveform for Eq.17.3, is shown in fig. 17.3, where the power wave has a frequency twice that of the voltage or current. Here the average value of power is $V_m I_m/2$.

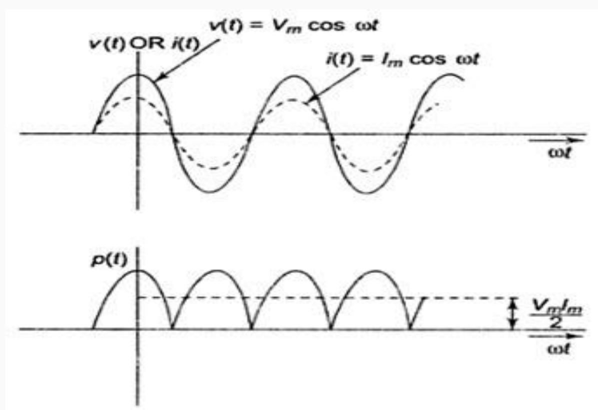


Fig. 17.3

When phase angle θ is increased, increased, the negative portion of the power cycle increases and lesser power is dissipated. When θ becomes $\frac{\pi}{2}$, the positive and negative portions of the power cycle are equal. At this instant, the power dissipated in the circuit is zero, i.e. the power delivered to the load is returned to the source.

17.2 Average power

To find the average value of any power function, we have to take a particular time interval from t1 to t2; by integrating the function from t_1 to t_2 and dividing the result by the time interval t_2-t_1, we get the average power.

Average power $$P = \frac{1}{t_2 - t_1}\int\limits_{t_1}^{t_2} P(t)\,dt \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \left(17.4\right)$$

In general, the average value over one cycle is

$$P_{av}={1 \over T}\int\limits_{0_1}^T {P\left(t \right)dt} \ldots\ldots \left({17.5} \right)$$

By integrating the instantaneous power P(t) in Eq. 17.5 over one cycle, we get average power

$$P_{av}={1\over T}\,\,\int\limits_0^T{}\left\{{{{V_m}{I_m}}\over 2}\left[{\cos\left({2\omega t+\theta}\right)+\cos \theta } \right]\,dt} \right\}$$

$$={1 \over T}\,\,\int\limits_0^T {{{{V_m}{I_m}} \over 2}\left[{\cos \left({2\omega t + \theta } \right)} \right]\,\,\,dt\,\, + \,{1 \over T}\,\int\limits_0^T {{{{V_m}{I_m}} \over 2}\cos \theta \,dt} } \ldots\ldots \left({17.6} \right)$$

In Eq.17.6, the first term becomes zero, and the second term remains. The average power is therefore

$$P_{av}={{{V_m}{I_m}} \over 2}\cos \,\,\,\,\theta \,W \ldots\ldots \left({17.7} \right)$$

We can write Eq. 17.7 as

$$P_{av}=\left({{{{V_m}} \over {\sqrt 2 }}} \right)\,\,\left({{{{I_m}} \over {\sqrt 2 }}} \right)\cos \,\,\,\theta \ldots\ldots \left({17.8} \right)$$

In Fq. 6.8, $V{}_m/\sqrt 2 \,\,and\,\,{I_m}/\sqrt 2 \,\,$ are the effective values of both voltage and current.

$$P_{av}\,=\,{V_{eff}}\,{I_{eff}}\,\cos \,\theta$$

To get average power, we have to take the product of the effective values of both voltage and current multiplied by cosine of the phase angle between voltage and the current.

If we consider a purely resistive circuit, the phase angle between voltage and current is zero. Hence, the average power is

$$P_{av}={1 \over 2}{V_m}{I_m}={1 \over 2}I_m^2R$$

If we consider a purely reactive circuit (i.e. purely capacitive or purely inductive), the phase angle between voltage and current is 90^0. Hence, the average power is zero or P_{av} = o.

If the circuit contains complex impedance, the average power is the power dissipated in the resistive part only.

LESSON 18. Power factorand apparent power

18.1 Apparent Power and Power Factor

The power factor is useful in determining useful power (true power) transferred to a load. The highest power factor is 1, which indicates that the current to a load is in phase with the voltage across it (i.e.in the case of resistive load). When the power factor is 0, the current to a load is 90^0 out of phase with the voltage (i.e. in case of reactive load).

Consider the following equation

$${P_{av}}={{{V_m}{I_m}} \over 2}\cos \theta \,W...\left({18.1} \right)$$

In terms of effective values

$${P_{av}}={{{V_m}} \over {\sqrt 2 }}\,{{{I_m}} \over {\sqrt 2 }}\cos \theta \,$$

$$=\,{V_{eff}}\,{I_{eff}}\,\cos \,\theta \,W...\left({18.2} \right)$$

The average power is expressed in watts. It means the useful power transferred from the course to the load, which is also called true power. If we consider a dc source applied to the network, true power is given by the product of the voltage and the current. In case of sinusoidal voltage applied to the circuit, the product of voltage and current is not the true power or average power. This product is called apparent power. The apparent power is expressed in volt amperes, or simply VA.

Apparent power = $V_{eff}I_{eff}$

In Eq. 18.2, the average power depends on the value of cosθ; this is called the power factor of the circuit.

$$Power\,\,factor\,\left({pf} \right)=\cos \,\theta \,={{{P_{av}}} \over {{V_{eff}}\,\,{I_{eff}}}}$$

Therefore, power factor is defined as the ratio of average power to the apparent power, whereas apparent power is the product of the effective values of the current and the voltage. Power factor is also defined as the factor with which the volt amperes are to be multiplied to get true power in the circuit.

In the case of sinusoidal sources, the power factor is the cosine of the phase angle between voltage and current

$$pf\,\, = \,\,\cos \,\,\theta$$

As the phase angle between voltage and total current increase, the power factor decreases. The smaller the power factor, the smaller the power dissipation. The power factor varies from 0 to 1. For purely resistive circuits, the phase angle between voltage and current is zero, and hence the power factor is unity. For purely reactive circuits, the phase angle between voltage and current is 90^0, and hence the power factor is zero. In an RC circuit, the power factor is referred to as leading power factor because the current leads the voltage. In an RL circuit, the power factor is referred to as lagging power factor because the current lags behind the voltage.

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LESSON 19. Reactive power and power triangle

19.1. Reactive Power

We know that the average power dissipated is

$$P_{av}=V_{eff}\left[I_{eff}\,\,\cos\,\theta \right]\ldots\ldots\left(19.1 \right)$$

From the impedance triangle shown in fig. 19.1

$$\cos\;\theta\,=\,{R \over \left[Z \right]}\ldots\ldots\left(19.2 \right)$$

$$and\,\,V_{eff}\,\,=\,\,I_{eff}\,\,Z\ldots\ldots\left(19.3 \right)$$

If we substitute Eqs (19.2) and (19.3) in Eq. (19.1), we get

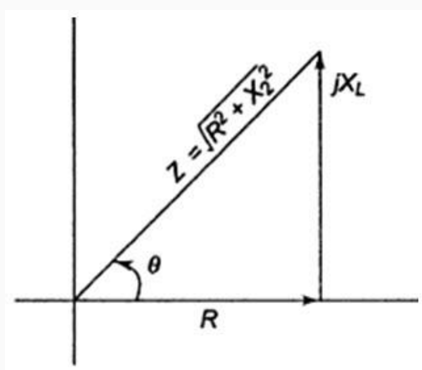


Fig.19.1

$$P_{av}=I_{eff}\,\,Z\,\left[I_{eff}\,{R \over Z} \right]$$

$$=\,\,\,I_{eff}^2\,\,R\,\,watts\ldots\ldots\left(19.4 \right)$$

This gives the average power dissipated in a resistive circuit.If we consider a circuit consisting of a pure inductor, the power in the inductor.

$$P_r=i\,v_L\ldots\ldots\left(19.5 \right)$$

$$=iL\,{di \over dt}$$

Consider $i=I_m\,\sin\,(\omega t\, + \,\theta)$

Then $$P_r=I_m^2\,\sin \,\left({\omega t + \theta } \right)L\omega \,\,\cos \,\,\left({\omega t + \theta } \right)$$

$$={{I_m^2} \over 2}\,\left({\omega L} \right)\sin \,\,2\,\,\left({\omega t + \theta } \right)$$

$$P_r=I_{eff}^2\left({\omega L} \right)\,\sin \,2\,\left({\omega t + \theta } \right) \ldots\ldots\ldots \left({19.6} \right)$$

From the above equation, we can say that the average power delivered to the circuit is zero. This is called reactive power. It is expressed in volt-amperes reactive (VAR).

$$P_r=I_{eff}^2\,{X_L}\;\,VAR \ldots\ldots\ldots \left({19.7} \right)$$

From Fig.19.1, we have

$$X_L=Z\sin \,\theta \ldots\ldots\ldots \left({19.8} \right)$$

Substituting Eq. 19.8 in Eq. 19.7, we get

$$P_r=I_{eff}^2\,Z\,\sin \,\theta \;$$

$$=\left({{I_{eff}}Z} \right){I_{eff}}\sin \,\theta$$

$$=\,{V_{eff}}\,{I_{eff}}\,\,\sin \,\theta \,\,VAR$$

19.2. The Power Triangle

A generalized impedance phase diagram is shown in Fig.19.2. A phasor relation for power can also be represented by a similar diagram because of the fact that true power P_{av} and reactive power Pr differ from R and X by a factor I^2_{eff} as shown in Fig. 19.2.

At any resultant in time, P_a is the total power that appears to be transferred between the course and reactive circuit. Part of the apparent power is true power and part of it is reactive power.

$$P_a=I_{eff}^2\,Z$$

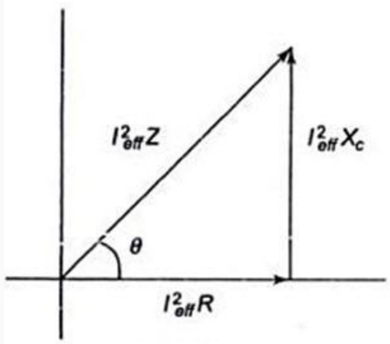
The power triangle is shown in Fig19.2

From Fig. 19.3, we can write

$$P_{true}={P_a}\cos \,\theta$$

or average power $P_{av} = P_a \cos \theta$

and reactive power $P_r = P_a \sin \theta$



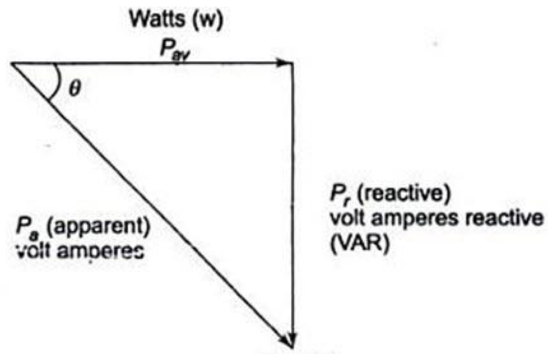


Fig.19.2 Fig. 19.3

Module 9. Concept and analysis of balanced polyphase circuits

LESSON 20. Polyphase System

20.1. Polyphase System

In an ac system it is possible to connect two or more number of individual circuits to a common polyphase source. Though it is possible to have any number of sources in a polyphase system, the increase in the available power is not significant beyond the three-phase system. The power generated by the same machine increases 41.4 per cent from single phase to two-phase, and the increase in the power is 50 per cent from single phase to three-phase. Beyond three-phase, the maximum possible increase is only seven per cent, but the complications are many. So, an increase beyond three-phase does not justify the extra complications. In view of not justify the extra complications. In view of this, it is only in exceptional cases where more than three phases are used. Circuits supplied by six, twelve and more phases are used in high power radio transmitter stations. Two-phase systems are used to supply two-phase servo motors in feedback control systems.

In general, a three-phase system of voltages (currents) is merely a combination of three single phase systems of voltages (currents) of which the three voltages (currents) differ in phase by 120 electrical degrees from each other in a particular sequence. One such three-phase system of sinusoidal voltages is shown in Fig.20.1.

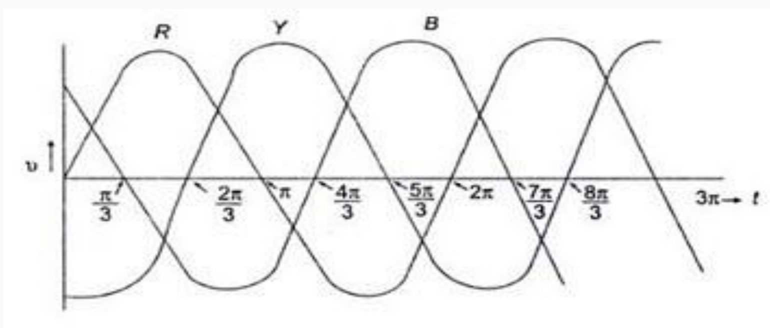


Fig. 20.1

20.2. Advantages of a Three-Phase System

It is observed that the polyphase, especially three-phase, system has many advantages over the single phase system, both from the utility point of view as well as from the consumer point of view. Some of the advantages are as under.

1. The power in a single phase circuit is pulsating. When the power factor of the circuit is unity, the power becomes zero 100 times in a second in a 50Hz supply. Therefore, single phase motors have a pulsating torque. Although the power supplied by each phase is pulsating, the total three-phase power supplied to a balanced three-phase

circuit is constant at every instant of time. Because of this, three-phase motors have an absolutely uniform torque.

2. To transmit a given amount of power over a given length, a three-phase transmission circuit requires less conductor material than a single-phase circuit.

3. In a given frame size, a three-phase motor or a three-phase generator produces more output than its single phase counterpart.

4. Three-phase motors are more easily started than single phase motors. Single phase motors are not self-starting, whereas three-phase motors are.

In general, we can conclude that the operating characteristics of a three-phase apparatus are superior than those of a similar single phase apparatus. All three-phase machines are superior in performance. Their control equipment's are smaller, cheaper, lighter in weight and more efficiency. Therefore, the study of three phase circuits is of great importance.

20.3. Generation of Three-Phase Voltages

Three-phase voltages can be generated in a stationary armature with a rotating field structure, or in a rotating armature with a stationary field as shown in Fig.20.2 (a) and (b).

Single phase voltages and currents are generated by single phase generators as shown in Fig. 20.3 (a). The armature (here a stationary armature) of such a generator has only one winding, or one set of coils. In a two-phase generator the armature has two distinct winding, or two sets of coils that are displaced 90^0 (electrical degrees) apart, so that the generated voltages in the two phases have 90^0 phase displacement as shown in Fig. 20.3 (b). Similarly, three-phase voltages are generated in three separate but identical sets of winding or coils that are displaced by 120 electrical degrees in the armature, so that the voltages generated in them are 120^0 apart in time phase. This arrangement is shown in Fig. 20.3 (c). Here RR' constitutes one coil (R-phase); YY' another coil (Y-phase), and BB' constitutes the third phase (B-phase). The field magnets are assumed in clockwise rotation.

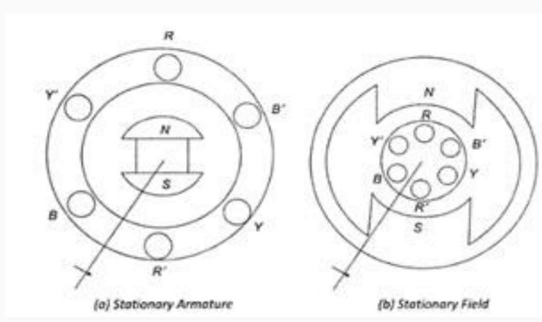


Fig. 20.2

The voltage generated by a three-phase alternator is shown in Fig.20.3 (d). The three voltages are of the same magnitude and frequency, but are displaced from one another by 120^0. Assuming the voltages to be sinusoidal, we can write the equations for the instantaneous values of the voltages of the three phases. Counting the time from the instant when the voltage in phase R is zero. The equations are

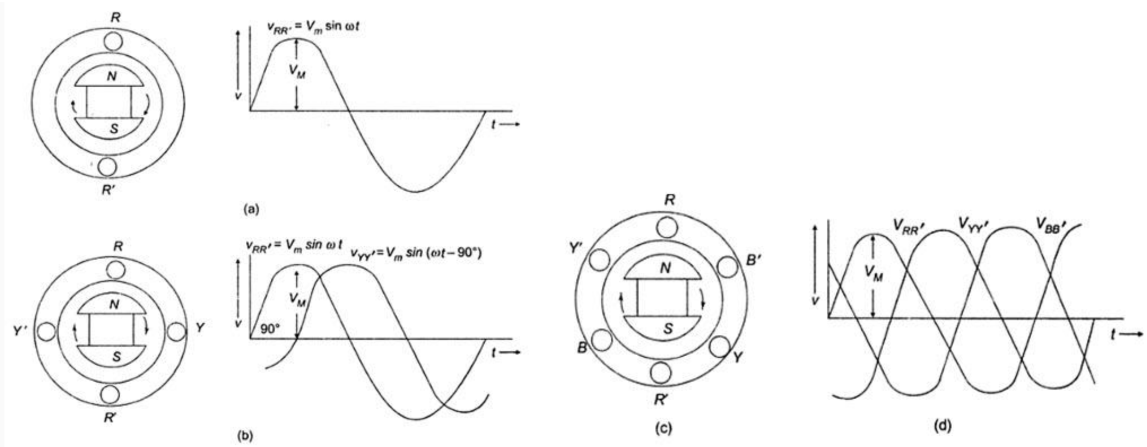


Fig.20.3

$V_{RR'} = V_m \sin wt$

$V_{yy'} = V_m \sin (wt - 120^0)$

$V_{BB'} = V_m \sin (wt - 244^0)$

At any given instant, the algebraic sum of the three voltages must be zero.

20.4. Phase Sequence

Here the sequence of voltages in the three phases are in the order $V_{RR'}$ - $V_{yy'}$ - $V_{BB'}$ and they undergo changes one after the other in the above mentioned order. This is called the phase sequence. It can be observed that this sequence depends on the rotation of the field. If the field system is rotated in the anticlockwise direction, then the sequence of the voltages in the three-phases are in the order $V_{RR'}$ - $V_{yy'}$ - $V_{BB'}$; briefly we say that the sequence is RBY. Now the equations can be written as

$V_{RR'} = V_m \sin wt$

$V_{BB'} = V_m \sin (wt - 244^0)$

$V_{yy'} = V_m \sin (wt - 120^0)$

20.5 Inter connection of three –phase sources and loads

20.5.1 Inter connection of three –phase sources

In a three-phase alternator, there are three independent phase windings or coils. Each phase or coil has two terminals, viz. start and finish. The end connections of the three sets of the coils may be brought out of the machine, to form three separate single phase sources to feed three individual circuits as shown in Fig. 20.4 (a) and (b).

The coils are inter-connected to form a wye (Y) or delta (D) connected three-phase system to achieve economy and to reduce the number of conductors, and thereby, the complexity in the circuit. The three-phase sources so obtained serve all the functions of the three separate single phase sources.

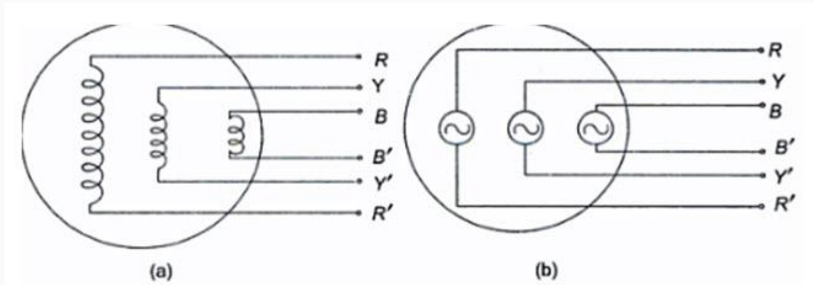
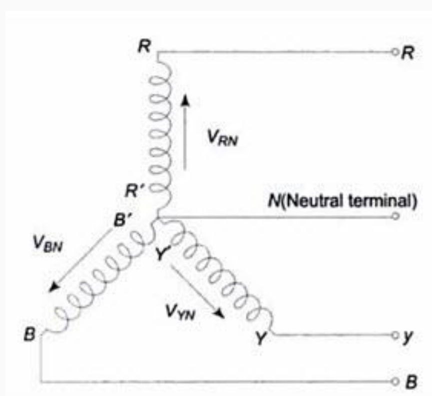


Fig.20.4

Wye or Star-Connection

In this connection, similar ends (start or finish) of the three phases are joined together within the alternator a shown in Fig.20.5. The common terminal so formed is referred to as the neutral point (N), or neutral terminal. Three lines are run from the other free ends (R, Y, B) to feed power to the three-phase load.

Figure 20.5 represents a three-phase, four-wire, star-connected system. The terminals R,Y and B are called the line terminals of the source. The voltage between any line and the neutral point is called the phase voltage ($V_{RN'}$ V_{YN} and V_{BN}), while the voltage between any two lines is called the line voltage ($V_{RY'}$ V_{YB} and V_{BR}). The currents flowing through the phases are called the phase currents, while those flowing in the lines are called the line currents. If the neutral wire is not available for external connection, the system is called a three-phase, three-wire, star-connected system. The system so formed will supply equal line voltages displaced 120^0 from one another and acting simultaneously in the circuit like three independent single phase sources in the same frame of a three-phase alternator.



20.5

Delta or Mesh-Connection

In this method of connection the dissimilar ends of the windings are joined together, i.e. *R′* is connected to *Y, Y′* to *B* and *B′* to as shown in Fig. 20.6.

The three line conductors are taken from the three junctions of the mesh or delta connection to feed the three-phase load. This constitutes a three-phase, three-wire, delta-connected system. Here there is no common terminal; only three line voltages $V_{RY'}$ V_{YB} and V_{BR} are available.

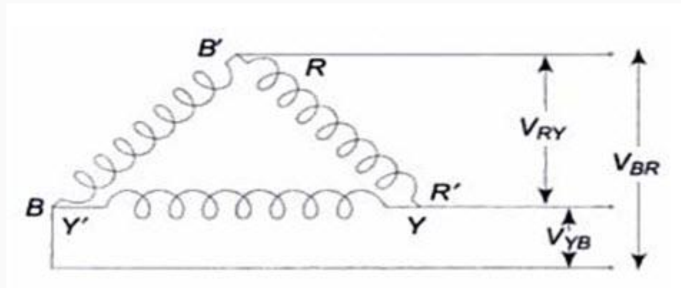
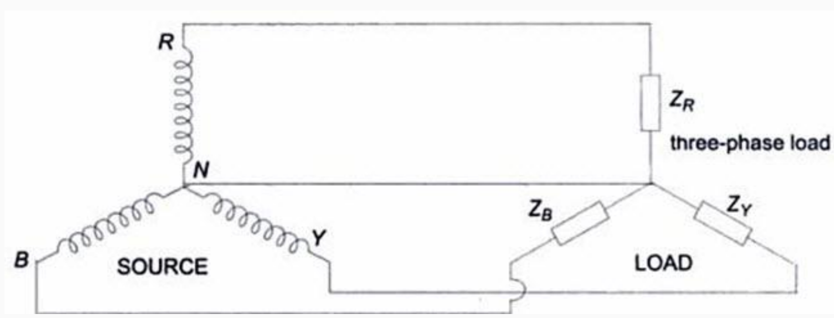


Fig.20.6

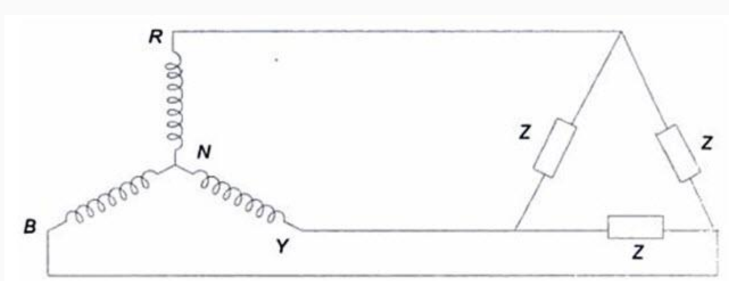
These line voltages are also referred to as phase voltages in the delta connected system. when the sources are connected in delta, loads can be connected only across the three line terminals, R, Y and B. In general, a three-phase source, star or delta, can eb either balanced or unbalanced. A balanced three-phase source is one in which the three individual sources have equal magnitude, with 120^0 phase difference as shown in Fig. 20.3 (d).

Interconnection of Loads

The primary question in a star or delta-connected three-phase supply is how to apply the load to the three-phase supply. An impedance, or load, connected across any two terminals of an active network (source) will draw power from the source, and is called a single phase load. Like alternator phase windings, load can also be connected across any two terminals, or between neutral and neutral terminal (if the source is star-connected). Usually the three-phase load impedances are connected in star or delta formation, and then connected to the three-phase source as shown in Fig.20.7 (a) and 20.7 (b).



(A)



(B)

20.7 (b)

Figure 20.7 (a) represents the typical inter-connections of loads and sources in a three-phase star system, and is of a three-phase four wire system. A three line and neune -phase star connected load is connected to a three-phase star-connected source, terminal to terminal, and both the neutrals are joined with a fourth wire. Figure 20.7 (b) is a three-phase, three-wire system. a three-phase, delta connected load is connected to a three-phase star-connected source, terminal to terminal, as shown in Fig.20.7 (b). When either source or load, or both are connected in delta, only three wires will suffice to connect the load to source.

Just as in the case of a three-phase source, a three-phase load can be either balanced or unbalance. A balanced three-phase load is one in which all the branches have identical impedances, i.e. each impedance has the same magnitude and phase angle. The resistive and reactive components of each phase are equal. Any load which does not satisfy the above requirements is said to be an unbalanced load.

20.6. Star to Delta and Delta to Star Transformation

While dealing with currents and voltages in loads, it is often necessary to convert a star load to delta load, and vice-versa. The delta (Δ) connection of resistances can be replaced by an equivalent start (Y) connection and vice-versa. Similar methods can be applied in the case of networks containing general impedances in complex from. So also with ac, where the same formulae hold good, except that resistances are replaced by the impedances. These formulae can be applied even if the loads are unbalanced. Thus, considering Fig. 20.8 (a), star load can be replaced by an equivalent delta-load with branch impedances as shown.

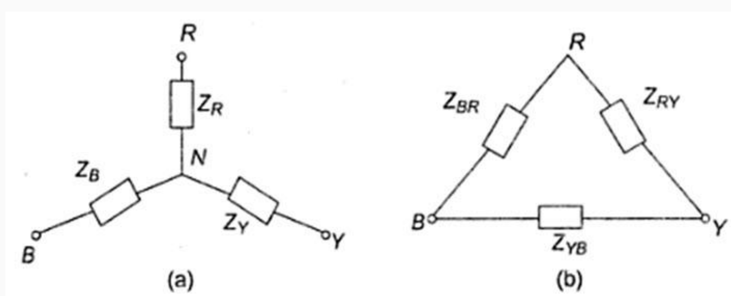


Fig. 20.8

Delta impedances, in terms of star impedances, are

$$Z_{RY}=\frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B}$$

$$Z_{YB}=\frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R}$$

and $$Z_{BR}=\frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y}$$

The converted network is shown in Fig.20.8 (b). Similarly, we can replace the delta load of Fig.20.9 (b) by an equivalent star load with branch impedances as

$$Z_R={{Z_{RY}}{Z_{BR}}} \over {{Z_{RY}} + {Z_{YB}} + {Z_{BR}}}$$

$$Z_Y={{Z_{RY}}{Z_{YR}}} \over {{Z_{RY}} + {Z_{YB}} + {Z_{BR}}}$$

and $$Z_B={{Z_{BR}}{Z_{YB}}} \over {{Z_{RY}} + {Z_{YB}} + {Z_{BR}}}$$

It should be noted that all impedances are to be expressed in their complex form.

Balanced Star-Delta and Delta-Star Conversion

If the three-phase load is balanced, then the conversion formulae in Section 20.6 get simplified. Consider a balanced star-connected load having an impedance Z_1 in each phase as shown in Fig.20.9 (a).

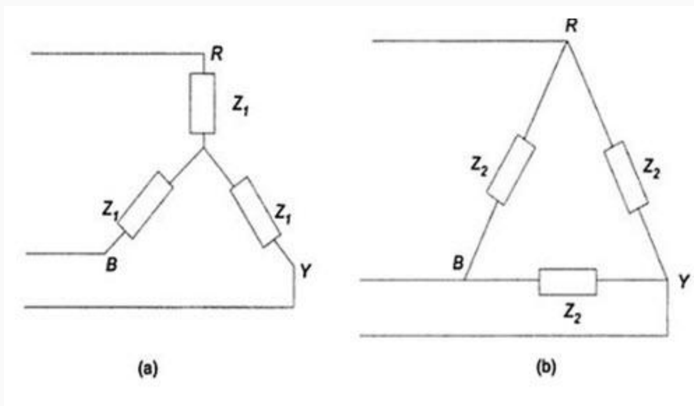


Fig.20.9

Let the equivalent delta-connected load have in impedance of Z_2 in each phase as shown in Fig.20.9 (b). Applying the conversion formulae from Section 20.6 for delta impedances in terms of star impedances, we have

$$Z_2 = 3Z_1$$

Similarly, we can express star impedances in terms of delta $Z_1 = Z_2/3$.

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LESSON 21. Voltage, Current and Power in a Star and Delta Connected System

21.1 Star-Connected System

Figure 21.1 shows a balanced three-phase, Y-connected system. The voltage induced in each winding is called the phase voltage (V_{Ph}). Likewise $V_{RN'}$ V_{YN} and V_{BN} represent the rms values of the induced voltages in each phase. The voltage available between any pair of terminals is called the line voltage (V_1). Likewise $V_{RY'}$ V_{YB} and V_{BR} are known as line voltages. The double subscript notation is purposefully used to represent voltages and currents in polyphase circuits. Thus, V_{RY} indicates a voltage V between points R and Y, with R being positive with respect to point Y during its positive half cycle.

Similarly, V_{YB} means that Y is positive with respect to point B during its positive half cycle, it also means that V_{RY} = -$V_{YR'}$

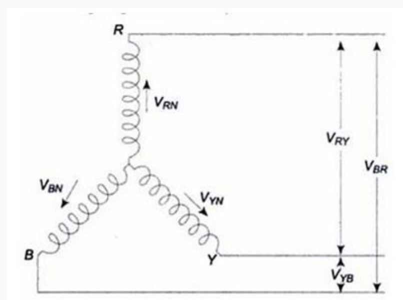

Fig .21.1

Voltage Relation

The phasors corresponding to the phase voltages constituting a three-phase system can be represented by a phasor diagram as shown in Fig.21.2.

From Fig.21.2, considering the lines R, Y and B, the line voltage V_{RY} is equal to the phasor sum of V_{RN} and V_{NY} which is also equal to the phasor difference of V_{RN} and V_{YN} (V_{NT}=-V_{YN}). Hence, V_{RY} is found by compounding V_{RN} and V_{YN} reserved.

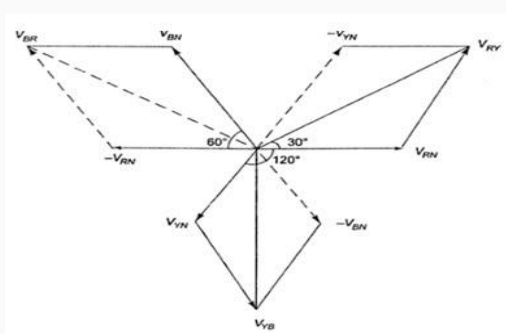

Fig.21.2

To subtract V_{YN} from $V_{RN'}$ we reverse the phasor V_{YN} and find its phasor sum with V_{RN} as shown in Fig.21.2. The two phasors, V_{RN} and $-V_{YN'}$ are equal in length and are 600 apart.

$$\left| {{V_{RN}}} \right|=-\left| {{V_Y}_N} \right|={V_{ph}}$$

$${V_{RY}}=2{V_{ph}}\cos \,60/2=\sqrt 3 \,\,{V_{ph}}$$

Similarly, the line voltage V_{YB} is equal to the phasor difference of V_{YN} and $V_{BN'}$ and is equal to $\sqrt 3 \,{V_{ph}}.$ The line voltage V_{BR} is equal to the phasor difference of V_{BN} and $V_{RN'}$ and is equal to $\sqrt 3 \,{V_{ph}}.$ Hence, in a balanced star-connected system

(i) Line voltage = $\sqrt 3 \,{V_{ph}}.$

(ii) All line voltages are equal in magnitude and are displaced by 120^0, and

(iii) All line voltages are 30^0 ahead of their respective phase voltages (from Fig.21.2).

Current Relations

Figure 21.3 (a) shows a balanced three-phase, wye-connected system indicating phase currents and line currents. The arrows placed alongside the currents $I_{R'}$ I_Y *and* I_B flowing in the three phases indicate the directions of currents when they are assumed to be positive and not the directions at that particular instant. The phasor diagram for phase currents with respect to their phase voltages is shown in Fig.21.3 (b). All the phase currents are displaced by 120^0 with respect to each other, 'ø' is the phase angle between phase voltage and phase current (lagging load is assumed). For a balanced load, all the phase currents are equal in magnitude. It can be observed from Fig.21.3 (a) that each line conductor is connected in series with its individual phase winding. Therefore, the current in a line conductor is the same as that in the phase to which the line conductor is connected.

$$I_L = I_{ph} = I_R = I_Y = I_B$$

It can be observed from Fig. 21.3 (b) that the angle between the line (phase) current and the corresponding line voltage is $(30 + ø)^0$ for a lagging load. Consequent, if the load is leading, then the angle between the line (phase) current and corresponding line voltage will be $(30 - ø)^0$.

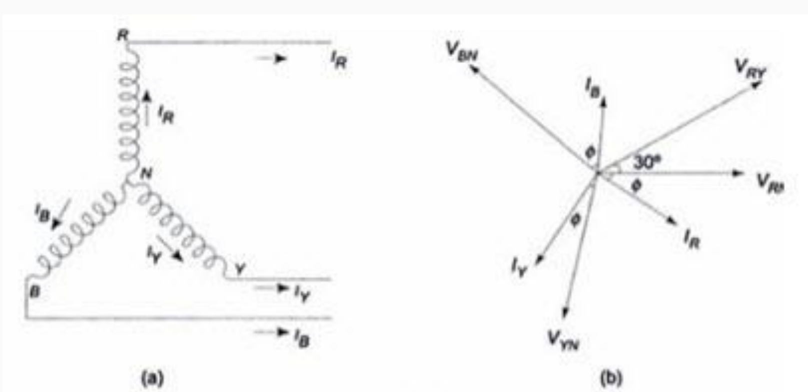

Fig.21.3

Power in the Star-Connected Network

The total active power or true power in the three-phase load is the sum of the powers in the three phases. For a balanced load, the power in each load is the same; hence total power = 3 x power in each phase

or $P = 3 ´ V_{ph} ´ I_{ph} \cos ø$

It is the usual practice to express the three-phase power in terms of line quantities as follows.

$${V_L}=\sqrt 3 \,\,{V_{ph'}}\,{I_L}={1_{ph}}$$

$$P=\sqrt 3 \,{V_L}{I_L}\,\cos \,\phi W$$

or $\sqrt 3 {V_L}{I_L}\,\cos \,\phi$ is the active power in the circuit.

Total reactive power is given by

$$Q=\sqrt 3 {V_L}{I_L}\,\sin \,\phi \,VAR$$

Total apparent power or volt-amperes

$$=\sqrt 3 {V_L}{I_L}\,VA$$

n-phase Star System

It is to be noted that star and mesh are general terms applicable to any number of phases; but wye and delta are special cases of star and mesh when the system is a three-phase system. Consider an n-phase balanced star system with two adjacent phases as shown in Fig.21.4 (a). Its vector diagram is shown in Fig.21.4 (b).

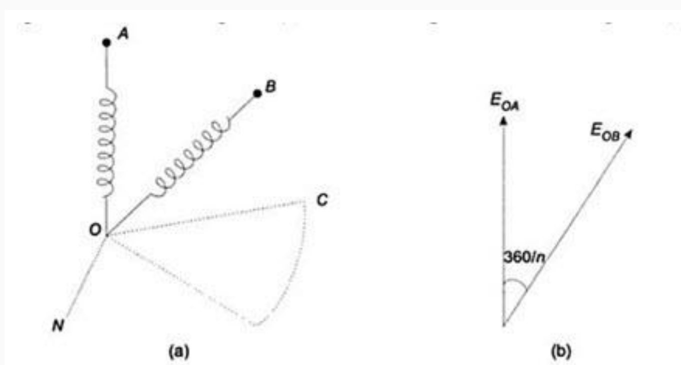

Fig. 21.4

The angle of phase differences between adjacent phase voltages is 360^0 /n. Let E_{ph} be the voltage of each phase. The line voltage, i.e. the voltage between A and B is equal to E_{AB} = E_L – E_{AO} + E_{OB}. The vector addition is shown in Fig.21.4 (c). It is evident that the line current and phase current are same.

$E_{AB} = E_{AO} + E_{OB}$

Consider the parallelogram *OABC*.

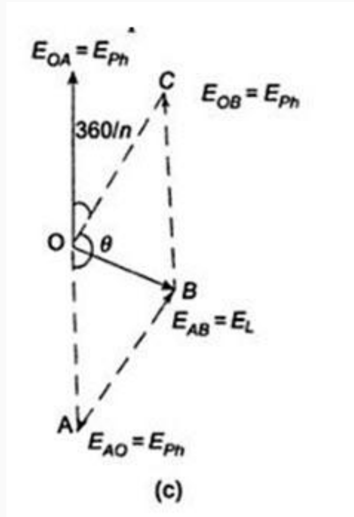

Fig.21.4 (c)

$$OB=\sqrt {O{C^2} + O{A^2} + 2 \times OA \times OC \times COS\theta }$$

$$=\,\sqrt {E_{ph}^2 + E_{ph}^2 + 2E_{ph}^2\,\cos \left({{{180}^0} - {{{360}^0}} \over n}} \right)}$$

$$=\,\sqrt {2E_{ph}^2\,\,2E_{ph}^2\,\cos {{{360}^0}} \over n}}$$

$$=\sqrt 2 \,{E_{ph}}\sqrt {\left[{1 - \cos 2\left({{{{180}^0}} \over n}} \right)} \right]}$$

$$=\sqrt 2 \,{E_{ph}}\sqrt {2{{\sin }^2}\left({{{{180}^0}} \over n}} \right)}$$

$${E_L}=2{E_{ph}}\sin \,{{180} \over n}$$

The above equation is a general equation for line voltage, for example, for a three-phase system, $n = 3$; $E_L = 2\ E_{ph} \sin 60^0$ = $\sqrt 3 {E_{ph}}.$

21.2. Voltage, Current and Power in a Delta Connected System

Delta-Connected System

Figure 21.5 shows a balanced three-phase, three-wire, delta-connected system. This arrangement is referred to as mesh connection because it forms a closed circuit. It is also known as delta connection because the three branches in the circuit can also be arranged in the shape of delta (Δ).

From the manner of interconnection of the three phases in the circuit, it may appear that the three phase are short-circuited among themselves. However, this is not the case. Since the system is balanced, the sum of the three voltages round the closed mesh is zero; consequently, no current can flow around the mesh when the terminals are open.

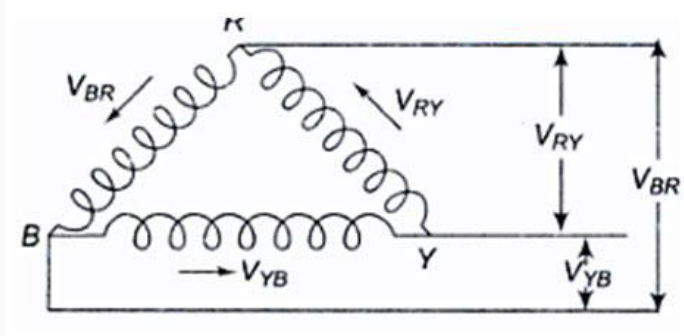

Fig.21.5

The arrows placed alongside the voltages, $V_{RY'}$ V_{YB} and $V_{BR'}$ of the three phases indicate that the terminals R, Y and B are positive with respect to Y, B and R, respectively, during their respective positive half cycles.

Voltage Relation

From Fig.21.6, we notice that only one phase is connected between any two lines. Hence, the voltage between any two lines (V_1) is equal to the phase voltage (V_{ph}).

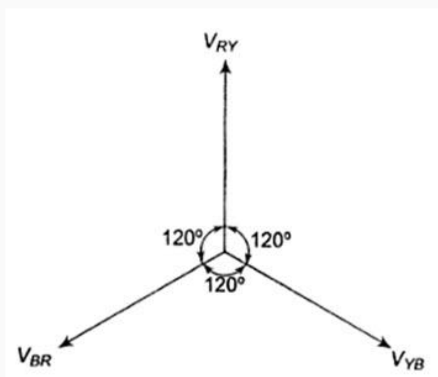

Fig.21.6

Since the system is balanced, all the phase voltages are equal, but displaced by 120^0 from one another as shown in the phasor diagram in Fig.21.6. The phase sequence RYB is assumed.

$$\,\left| {{V_{RY}}} \right|=\left| {{V_{YB}}} \right|=\left| {{V_{BR}}} \right|={V_L}={V_{ph}}$$

Current Relation

In Fig.21.7 we notice that, since the system is balanced, the three phase currents (I_{ph}), i.e. $I_{R'}$ $I_{Y'}$ I_B are equal in magnitude but displaced by 120^0 from one another as shown in Fig.21.7(b). $I_{1'}$ I_2 and I_3 are the line currents (I_L) i.e. I_1 is the line current in line 1 connected to the common point of R. Similarly, I_2 and I_3 are the line currents in lines 2 and 3, connected to common points Y and B, respectively. Though here all the line currents are directed outwards, at no instant will all the three line currents flow in the same direction, either outwards or inwards. Because the three line currents are displaced 120^0 from one another, when one is positive, the other two might both be negative, or one positive and one negative.

Also it is to be noted that arrows placed alongside phase currents in Fig.21.7 (a), indicate the direction of currents when they are assumed to be positive and not their actual direction at a particular instant. We can easily determine the line currents in Fig. 21.7 (a), I_1,I_2 and I_2 by applying KCL at the three terminals R, Y and B, respectively. Thus, the current in line 1, $I_1 = I_R - I_B$; i.e. the current in any line is equal to the phasor difference of the currents in the two phases attached to that line. Similarly, the current in line 2, $I_2 = I_Y - I_{R'}$ and the current in line 3, $I_3 = I_B - I_Y$.

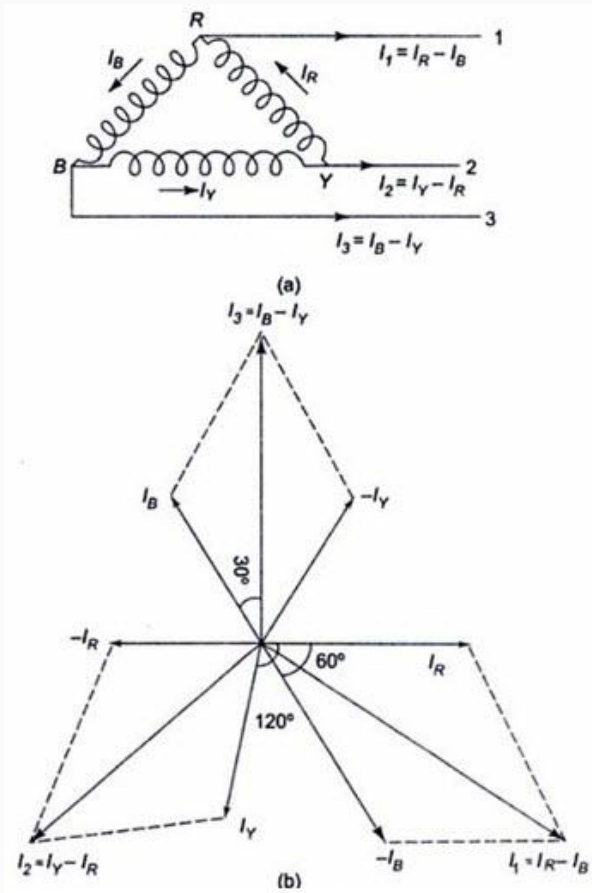

Fig. 21.7

The phasor addition of these currents is shown in Fig.21.7 (b). from the figure,

$I_1 = I_R - I_B$

$$I_1=\sqrt{I_R^2 + I_B^2 + 2 I_R I_B \, \cos \, 60^0}$$

$$I_1=\sqrt{3}\, I_{ph'} \quad \text{since} \; I_R=I_B=I_{ph}$$

Similarly, the remaining two line currents, I2 and I3′ are also equal to $\sqrt 3$ times the phase currents; i.e. $=\sqrt 3 \, I_{ph}$

As can be seen from Fig.21.7 (b), all the line currents are equal in magnitude but displaced by 120^0 from one another; and the line currents are 30^0 behind the respective phase currents.

Power in the Delta-Connected System

Obviously the total power in the delta circuit is the sum of the powers in the three phases. Since the load is balanced, the power consumed in each phase is the same. Total power is equal to three times the power in each phase.

Power per phase = $V_{ph}\, I_{ph}$ cos ø

Where ø is the phase angle between phase voltage and phase current.

Total power P = 3 ´ $V_{ph}\, I_{ph}$ *cos* ø

In terms of line quantities

$$P=\sqrt 3 {V_L}{I_L}\,\cos \,\phi \,\,W$$

Since $${V_{ph}}={V_L}\,\,and\,\,{I_{ph}}={{{I_L}} \over {\sqrt 3 }}$$

For a balanced system, whether star or delta, the expression for the total power is the same.

n-Phase Mesh System

Figure 21.8 (a) shows part of an n-phase balanced mesh system. Its vector diagram is shown in Fig.21.8 (b).

Let the current in line *BB′* be I_L. This is same in all the remaining lines of the n-phase system. $I_{AB'}\, I_{BC}$ are the phase currents in *AB* and *BC* phases respectively. The vector addition of the line current is shown in Fig.21.8(c). It is evident from the Fig.21.8 (b) that the line and phase voltages are equal.

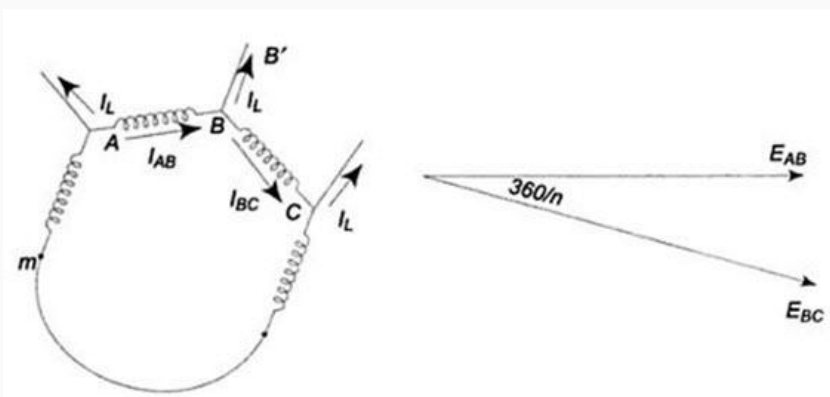

Fig.21.8 (a) **Fig.21.8 (b)**

$I_{BB} = I_L = I_{AB} + I_{CB}$

$= I_{AB} - I_{BC}$

Consider the parallelogram *OABC.*

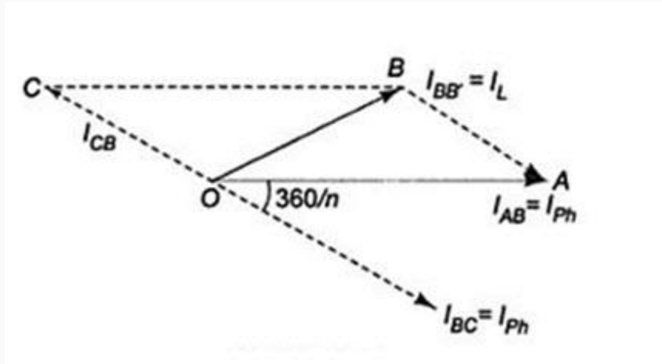

Fig.21.8 (c)

$$OB=\sqrt {O{A^2} + O{C^2} + 2 \times OA \times OC \times \cos \left({180 - {{360} \over n}} \right)}$$

$$=\sqrt {I_{ph}^2\,\,I_{ph}^2\,\,2I_{ph}^2\,\,\cos \,\,{{360} \over n}}$$

$$=\sqrt 2 \,\,{I_{ph}}\sqrt {1\,\,\,\cos 2\left({{{180} \over n}} \right)}$$

$$=\sqrt 2 \,\,{I_{ph}}\sqrt {2\,\,\,{{\cos }^2}\left({{{180} \over n}} \right)}$$

$${I_L}\,=2{I_{ph\,}}\,\sin \,{{180} \over n}$$

The above equation is a general equation for the line current in a balanced n-phase mesh system.

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LESSON 22. Three-Phase Balanced Circuits

22.1. Three-Phase Balanced Circuits

The analysis of three-phase balanced system is presented in this section. It is no way different from the analysis of AC systems in general. The relation between voltages, currents and power in delta-connected and star-connected systems has already been discussed in the previous sections. We can make use of those relations and expressions while solving the circuits.

Balanced Three-Phase System-Delta Load

Figure 22.1 (a) shows a three-phase, three-wire, balanced system supplying power to a balanced three-phase delta load. The phase sequence is RYB. We are required to find out the currents in all branches and lines.

Let us assume the line voltage $V_{RY}=V \angle 0^0$ as the reference phasor. Then the three source voltages are given by

$V_{RY}=V\angle 0^0$ V

$V_{YB}=V\angle =120^0$ V

$V_{BR}=V\angle 240^0$ V

These voltages are represented by phasors in Fig.22.1 (b). Since the load is delta-connected, the line voltage of the source is equal to the phase voltage of the load. The current in phase RY, I_R will lag (lead) behind (ahead of) the phase voltage V_{RY} by an angle f as dictated by the nature of the load impedance. The angle of lag of I_r with respect to $V_{YB'}$ as well as the angle of lag of I_B with respect to V_{BR} will be f as the load is balanced. All these quantities are represented in Fig. 22.1 (b).

If the load impedance is $Z \angle ø$, the current flowing in the three load impedances are then

$$I_R=\frac{V_{RY} - 0}{Z - \varphi}=\frac{V}{Z}\angle-\varphi$$

$$I_Y=\frac{V_{YB} - 120}{Z - \phi}=\frac{V}{Z}\angle-120^0-\phi$$

$$I_B=\frac{V_{BR} - 120}{Z - \phi}=\frac{V}{Z}\angle-120^0-\phi$$

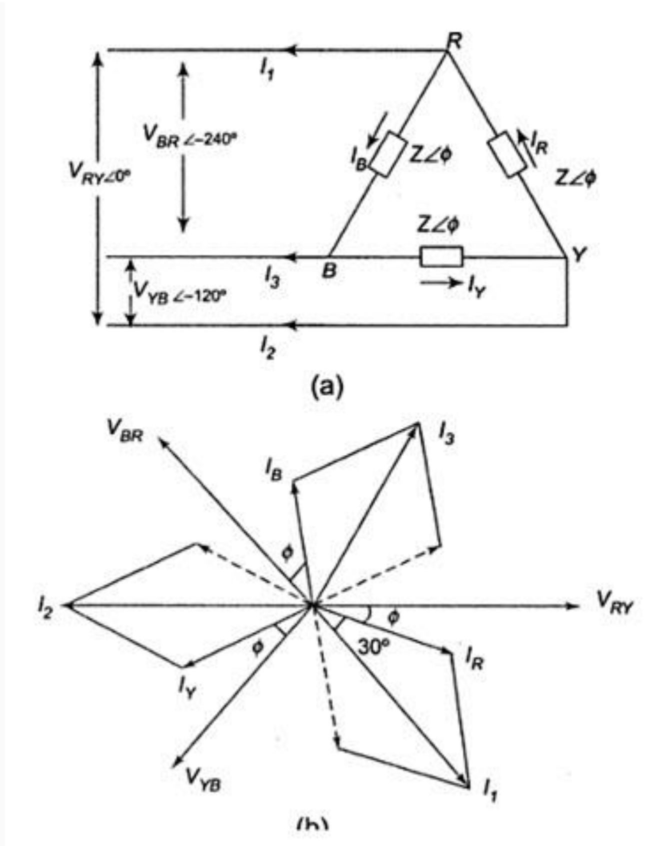

Fig.22.1

The line currents are $\sqrt 3$ times the phase currents, and are 30^0 behind their respective phase currents.

Current in line 1 is given by

$$I_1=\sqrt 3 \left| {V \over Z} \right|\,\angle \left(-\phi-30^0 \right),\,\,\text{or}\,\,I_R - I_B\,\,\left(\text{phasor}\,\,\text{difference} \right)$$

Similarly, the current in line 2

$$I_2=\sqrt 3 \left| {V \over Z} \right|\,\angle \left(-120-\varphi-30^0 \right),\,\,\,$$

or $$I_Y - I_R\,\,\left(\text{phasor}\,\,\text{difference}\right)\,\,\, = \,\,\sqrt 3\left| {V \over Z} \right|\,\,\angle \left(-\phi-150^0\right),\,\,\text{and}$$

$$I_3=\sqrt 3 \left| {V \over Z} \right|\,\angle \left(-120-\phi-30^0 \right),\,\,\text{or}\,\,I_B - I_Y\,\,\left(\text{phasor}\,\,\text{difference} \right)\,\,\,$$

$$=\sqrt 3 \left| {V \over Z} \right|\, - \left(120-\phi\right)$$

To draw all these quantities vectorially, $V_{RY}=V\angle 0^0$ is taken as the reference vector.

Balanced Three Phase System-Star Connected Load

Figure 22.2(a) shows a three-phase, three wire system supplying power to a balanced three phase star connected load. The phase sequence RYB is assumed.

In star connection, whatever current is flowing in the phase is also flowing in the line. The three line (phase) currents are $I_{R'}$ I_Y and I_B.

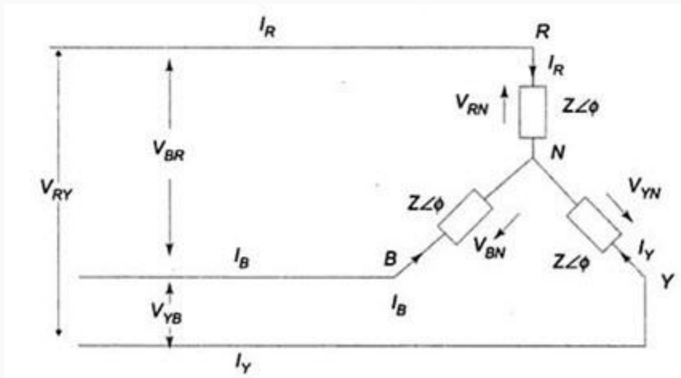

Fig.22.2(a)

$V_{RN'}$ V_{YN} and V_{BN} represent three phase voltage of the network, i.e. the voltage between any line and neutral. Let us assume the voltage $V_{RN} = V\angle 0^0$ as the reference phasor. Consequently, the phase voltage.

$$V_{RN}\,=\,V\,\angle {0^0}$$

$$V_{YN}\,=\,V\,\angle-{120^0}$$

$$V_{BN}\,=\,V\,\angle-{240^0}$$

Hence $$I_R={{V_{RN}} \over {Z\angle \phi }}={{V - 0} \over {Z - \phi }}=\left| {{V \over Z}} \right|\angle-\phi$$

$$I_Y={{V_{YN}}\over {Z - \phi }}={{V - 120} \over {Z - \phi }}=\left| {{V \over Z}} \right|\angle-{120^0}-\phi$$

$$I_B={{V_{BN}} \over {Z - \phi }}={{V - 240} \over {Z - \phi }}=\left| {{V \over Z}} \right|\angle-{240^0}-\phi$$

As seen from the above expressions, the currents, $I_{R'}$ I_Y and $I_{B'}$ are equal in magnitude and have a 120^0 phase difference. The disposition of these vectors is shown in Fig.22.2(b). Sometimes, a 4th wire, called neutral wire is run from the neutral point, if the source is also star-connected. This given three-phase, four-wire star-connected system. however, if the three line currents are balanced, the current in the fourth wire is zero; removing this connecting wire between the source neutral and load neutral is, therefore, not going to make any change in the condition of the system. the availability of the neutral wire makes it possible to use all the three phase voltages, as well as the three line voltages. Usually, the neutral is grounded for safety and for the design of insulation.

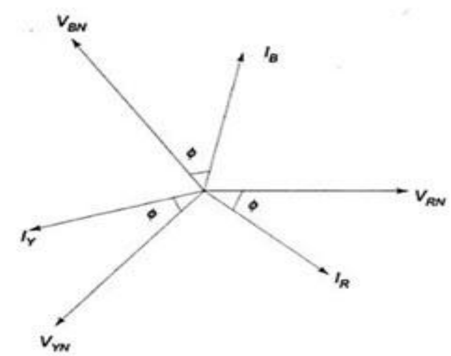

Fig. 22.2 (b)

It makes no different to the current flowing in the load phases, as well as to the line currents, whether the sources have been connected in star or in delta, provided the voltage across each phase of the delta connected source is \[\sqrt 3\] times the voltage across each phase of the star-connected source.

22.10. Three-Phase-Unbalanced Circuits

Types of Unbalanced Loads

An unbalance exists in a circuit when the impedances in one or more phases differ from the impedances of the other phases. In such a case, line or phase currents are different and are displaced from one another by unequal angles. So far, we have considered balanced loads connected to balanced systems. It is enough to solve problems, considering one phase only on balanced loads, the conditions on other two phases being similar. Problems on unbalanced three-phase loads are difficult to handle because conditions in the three phases are different. However, the source voltages are assumed to be balanced. If the system is a three-wire system, the currents flowing towards the load in the three liens must add to zero at any given instant. If the system is a four-wire system, the sum of the three outgoing line currents is equal to the return current in the neutral wire. We will now consider different methods to handle unbalanced star-connected and delta-connected loads. In practice, we may come across the following unbalanced loads:

(i) Unbalanced delta-connected load

(ii) Unbalanced three-wire star-connected load, and

(iii) Unbalanced four-wire star-connected load.

Unbalanced Delta-connected Load

Figure 22.3 shows an unbalanced delta-load connected to a balanced three-phase supply.

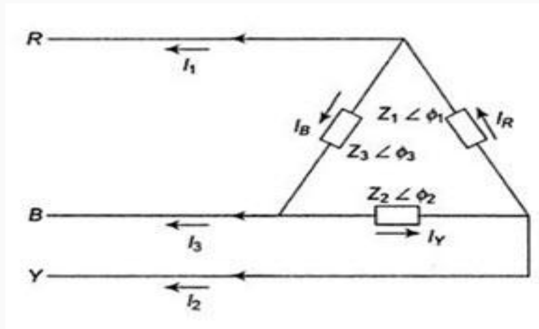

Fig. 22.3

The unbalanced delta-connected load supplied from a balanced three-phase supply does not present any new problems because the voltage across the load phase is fixed. It is independent of the nature of the load and is equal to the line voltage of the supply. The current in each load phase is equal to the line voltage divided by the impedance of that phase. The line current will be the phasor difference of the corresponding phase currents, taking V_{RY} as the reference phasor.

Assuming RYB phase sequence, we have

$$V_{RY}=V\,\angle 0^0\,\,V,\,\,V_{YB}=V\,\,\angle -120_0\,V,\,\,V_{BR}\,=\,\,V\,\,\angle\,-240^0\,\,V$$

Phase currents are

$$I_R={V_{RY} \over Z_1\angle \phi}={V\angle 0^0 \over Z_1\angle \phi_1}=\left| {V \over Z_1} \right|\,\angle \phi_1\,A$$

$$I_Y={V_{YB} \over Z_2\angle \phi_2}={V\angle-120^0 \over Z_2\angle \phi_2}=\left| {V \over Z_2} \right|\,\angle -120^0 - \phi_2\,A$$

$$I_B={V_{BR} \over Z_3\angle \phi_3}={V\angle-120^0 \over Z_3\angle \phi_3}=\left| {V \over Z_3} \right|\,\angle -120^0 - \phi_3\,A$$

The three line currents are

$I_1 = I_R - I_B$ phasor difference

$I_2 = I_Y - I_R$ phasor difference

$I_3 = I_B - I_Y$ phasor difference

Unbalanced Four Wire-Connected Load

Figure 22.4 shows an unbalanced star load connected to a balanced 3-phase, 4-wire supply.

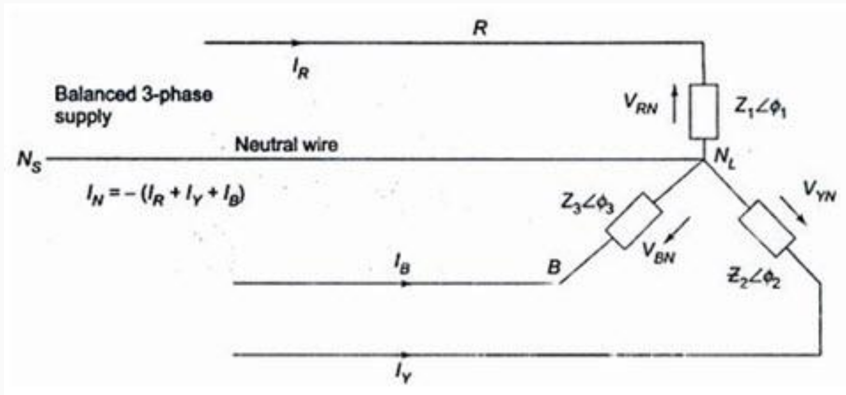

Fig. 22.4

The star point, NL′ of the load is connected to the star point, NS of the supply. It is the simplest case of an unbalanced load because of the presence of the neutral wire; the star points of the supply NS (generator) and the load NL are at the same potential. It means that the voltage across each load impedance is equal to the phase voltage of the supply (generator), i.e. the voltage across the three load impedances are equalized even though load impedances are unequal. However, the current in each phase (or line) will be different. Obviously, the vector sum of the currents in the three lines is not zero, but is equal to neutral current. Phase currents can be calculated in similar way as that followed in an unbalanced delta-connected load.

Taking the phase voltage $VRN=V\angle {0^{0}}V$ as reference, and assuming RYB phase sequences; we have the three phase voltages as follows.

$$VRN=V\angle {0^{0}}V,{V_{YN}}=V\angle-{120^{0}}V,{V_{BN}}=V\angle-{240^{0}}$$

The phase currents are

$${I_R}={{{V_{RN}}} \over {{Z_1}}}={{V\,\angle {0^0}} \over {{Z_1}\angle {\phi _1}}}A=\left| {{V \over {{Z_1}}}} \right|\angle-{\phi _1}A$$

$${I_Y}={{{V_{YN}}} \over {{Z_2}}}={{V\,\angle {{120}^0}} \over {{Z_1}\angle{\phi _2}}}A=\left|{{V\over{{Z_2}}}}\right|\angle-{120^0}-{\phi _2}A$$

$${I_B}={{{V_{BN}}} \over {{Z_3}}}={{V\,\angle {{120}^0}} \over {{Z_3}\angle{\phi _3}}}A=\left|{{V\over {{Z_3}}}}\right|\angle-{240^0}-{\phi _3}A$$

Incidentally, IR′ IY and IB are also the line currents; the current in the neutral wire is the vector sum of the three line currents.

Unbalanced Three Wire Star-Connected Load

In a three-phase, four-wire system if the connection between supply neutral and load neutral is broken, it would result in an unbalanced three-wire star-load. This type of load is rarely found in practice, because all the three wire star loads are balanced. Such as system is shown in Fig.22.5. Note that the supply star point (N_S) is isolated from the load star point (N_L). The potential of the load star pint is different from that of the supply star point. The result is that the load phase voltages are not equal to the supply phase voltage; and they are not only unequal in magnitude, but also subtend angles other than 120^0 with one another. The magnitude of each phase voltage depends upon the individual phase loads. The potential of the load neutral point changes according to changes in the impedances of the phases that is why sometimes the load neutral is also called a floating neutral point. All-star-connected, unbalanced loads supplied from polyphase systems without a neutral wire have floating neutral point. The phasor sum of the three unbalanced line currents is zero. The phase voltage of the load is not \[1\sqrt 3\] of the line voltage. The unbalanced three-wire star load is difficult to deal with. It is because load phase voltages cannot be determined directly from the given supply line voltages. There are many methods to solve such unbalanced Y-connected loads. Two frequently used methods are presented here. They are

(i) Star-delta conversion method, and

(ii) The application of Millman's theorem

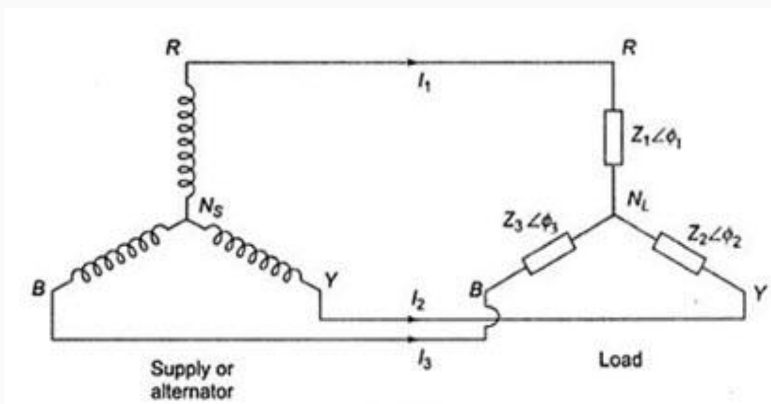

Fig. 22.5

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Module 10. Laplace transform method of finding step response of DC circuits

LESSON 23. Laplace Transform method of finding step response of DC circuits

23.1. Definition of the Laplace Transform

The Laplace transform is a powerful analytical technique that is widely used to study the behavior of linear, lumped parameter circuits. Laplace transforms are useful in engineering, particularly when the driving function has discontinuities and appears for a short period only.

In circuit analysis, the input an output functions do not exist forever in time. For causal functions, the function can be defined as f(t) u(t). The integral for the Laplace transform is taken with the lower limit at t = 0 in order to include the effect of any discontinuity at t = 0.

Consider a function f(t) which is to be continuous and defined for values of $t \geq 0$. The Laplace transform is then

$$L\left[{f\left(t \right)} \right]=F\left(s \right)=\int\limits_{-\infty }^\infty{{e^{-st}}f(t)dt=\int\limits_0^\infty{f(t){e^{-st}}dt}} \ldots\ldots\ldots\ldots\left({23.1} \right)$$

is a continuous function for $t \geq 0$multiplied by e^{-st} which is integrated with respect to t between the limits 0 and ∞. The resultant function of the variables is called Laplace transform of . Laplace transform is a function of independent variables corresponding to the complex variable in the exponent of e^{-st}. The complex variable S is, in general, of the form $S = \sigma + j\omega$ and σ and ω being the real and imaginary parts respectively. For a function to have a Laplace transform, it must satisfy the condition $\int\limits_0^\infty{f(t){e^{-st}}dt<\infty.}$ Laplace transform changes the time domain function $f(t)$ to the frequency domain function F(s). Similarly, inverse Laplace transformation converts frequency domain function F(s) to the time domain function $f(t)$ as follows.

$${L^{ - 1}}\left[{F\left(s \right)} \right]=f\left(t \right)={1 \over {2\pi j}}\int\limits_{-j}^{+j} {F(s)\,{e^{st}}\,ds} \ldots\ldots\ldots\ldots\left({23.2} \right)$$

Here, the inverse transform involves a complex integration. can be represented as a weighted integral of complex exponentials. We will denote the transform relationship between $f(t)$ and F(s) are

$$f(t)\buildrel L \over \longleftrightarrow F(s)$$

In Eq. 23.1, if the lower limit is 0 then the transform is referred to as one sided, or unilateral, Laplace transform. In the two-sided, or bilateral, Laplace transform, the lower limit is -∞.

In the following discussion, we divide the Laplace transforms into two types: functional transforms and operational transforms. A functional transform is the Laplace transform of a specific function, such as sinwt, t, e^{-at}, and so on. An operational transform defines a general mathematical property of the Laplace transform, such as binding the transform of the derivative of $f(t)$. Before considering functional and operational transforms, we used to introduce the step and impulse functions.

23.2. Step Function

In switching operations abrupt changes may occur in current and voltages. On some functions discontinuity may appear at the origin. We accommodate these discontinuities mathematically by introducing the step and impulse functions.

Figure 23.1 shows the step function. It is zero for $t<0$. It is denoted by $k\,u(t)$.

Mathematically it is defined as

$k\,u(t) = 0,\ t<0$

$k\,u(t) = k,\ t>0 \quad ....\left({23.3} \right)$

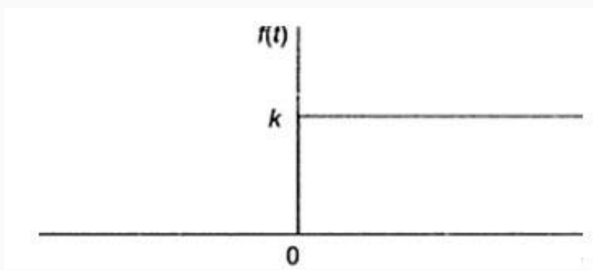


Fig.23.1

If k is 1, the function defined by Eq. (23.3) is the unit step. The step function is not defined at t=0. In situations where we need to define the transition between 0 and 0^+, we assume that it is linear and that

$$K\,u(0) = 0.5\,K \quad ....\left({23.4} \right)$$

Figure 23.2 shows the linear transition from 0 to 0^+.

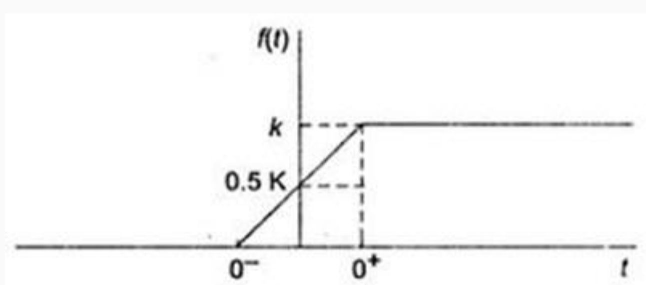


Fig. 23.2

A discontinuity may occur at some time other than t = 0, for example, in sequential switching. The step function occurring at t = a when a > 0 is shown in Fig.23.3. A step occurs at t = a is expressed as ku(t-a). thus

k u(t-a) = 0, t<a

k u(t-a) = k, t>a \[...\left({23.5} \right)\]

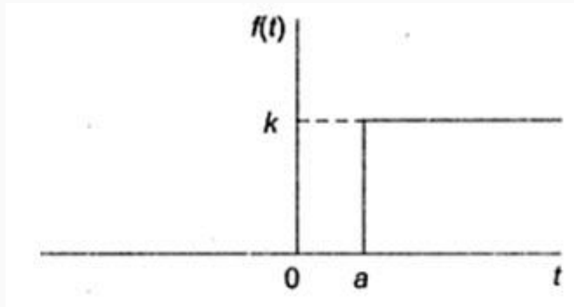


Fig.23.3

If a>0, the step occurs to the right of the origin, and if a<0, the step occurs to the left of the origin. Step function is 0 when the argument t-a is negative, and it is k when the argument is positive.

A step function equal to k for t<0 is written as ku(a-t). Thus

k u(a-t) = k, t<a

k u(a-t) = 0, t>0 \[...\left({23.6} \right)\]

The discontinuity is to the left of the origin when a<0. A step function k u(a-t) for a>0 is shown in Fig.23.4.

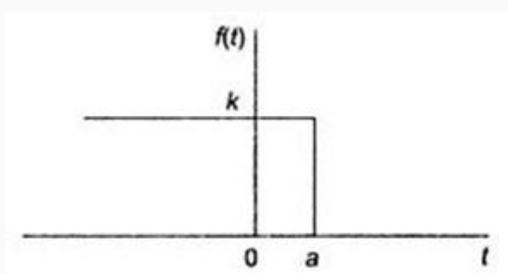


Fig.23.4

Step function is useful to define a finite-width pulse, by adding two step functions. For example, the function k[u(t – 1) – u(t – 3)] has the value k or 1<t<3 and the value 0 everywhere else, so it is a finite-width pulse of height k initiated at t = 1 and terminated at t = 3. Here, u(t – 1) is a function “turning on” the constant value k at t = 1, and the step function –u(t = 3) as “turning off” the constant value k at t = 3. We use step functions to turn on and turn off linear functions.

23.3 Impulse Function

An impulse is a signal of infinite amplitude and zero duration. In general, an impulse signal doesn't exist in nature, but some circuit signals come very close to approximating this definition. Due to switching operations impulsive voltages and currents occur in circuit analysis. The impulse function enables us to define the derivative at a discontinuity, and thus to define the Laplace transform of that derivative.

To define derivative of a function at a discontinuity, consider that the function varies linearly across the discontinuity as shown in Fig.23.5.

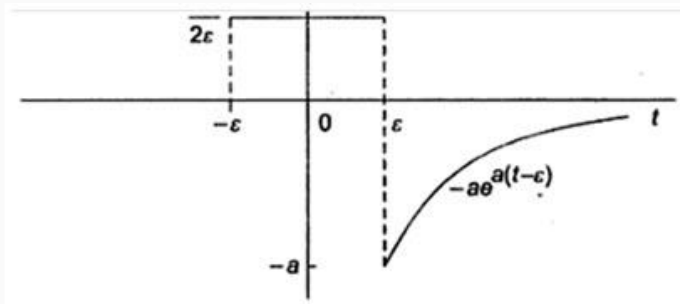


Fig.23.5

In the Fig.23.5 shown as ε→0, an abrupt discontinuity occurs at the origin. When we differentiate the function, the derivative between -εand +ε is constant at a value of ${1 \over {2\varepsilon }}$. For t>ε, the derivative is $-ae^{-a(t-\varepsilon)}$. The derivative of the function shown in Fig. 23.5 and shown in Fig.23.6.

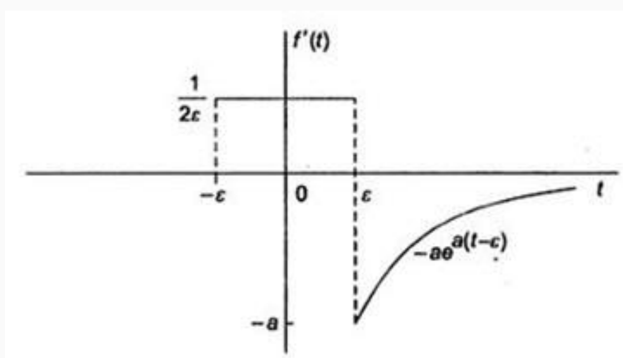


Fig.23.6

As ε approaches zero, the value of $f'\left(t \right)$ between ±ε approaches infinity. At the same time, the duration of this large value is approaching zero. Furthermore, the area under $f'\left(t \right)$ between ±ε remainsfs constant as ε→0. In this example, the area is unity. As ε approaches zero, we say that the function between ±ε approaches a unit impulse function; denoted δ(t). Thus the derivative of $f'\left(t \right)$ at the origin approaches a unit impulse function as ε approaches zero, or

$$f\left(0\right)\to\delta\left(t \right)\,\,as\,\,\varepsilon\to 0$$

If the area under the impulse function curve is other than unity, the impulse function is denoted by K δ(t), where K is the area. K is often referred to as the strength of the impulse function.

Mathematically, the impulse function is defined

$$\int\limits_{-\infty }^\infty{K\,\delta \left(t \right)\,dt = k} \quad \ldots\ldots\ldots\ldots\ldots\ldots \left({23.7} \right)$$

$$\delta(t) = 0,\ t \neq 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \left({23.8} \right)$$

Equation (23.7) states that the area under the impulse function is constant. This area represents the strength of the impulse. Equation (23.8) states that the impulse is zero everywhere except at t = 0. An impulse that occurs at t = a is denoted K δ(t – a). The graphical symbol is shown in Fig.23.7. The impulse K δ(t-a) is also shown in Fig.23.7.

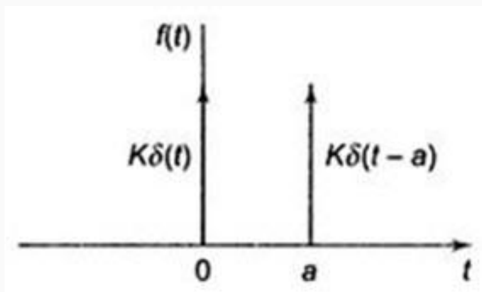


Fig.23.7

An important property of the impulse function is the shifting property, which is expressed as

$$\int\limits_{-\infty }^\infty{f\left(t \right)\,\delta \left({t - a} \right)\,dt=f\left(a \right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \left({23.9} \right)$$

Equation 23.19 shows that the impulse function shifts out everything except the value of $f\left(t \right)$ at t = a. The value of δ(t-a) is zero everywhere except at t=a, and hence the integral can be written

$$I=\int\limits_{-\infty}^\infty{f\left(t\right)\,\delta\left({t-a}\right)\,dt=\int\limits_{a-\in}^{a+\in}{f\left(t\right)\,\delta\left({t-a}\right)dt}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \left({23.10} \right)$$

But because $f\left(t \right)$ is continuous at a, it takes on the value $f\left(a \right)$ as t → a, so

$$I=\int\limits_{a-\in }^{a+ \in }{f\left(a\right)\,\delta \left({t - a} \right)\,dt = f\left(a \right)\int\limits_{a - \in }^{a + \in }{\,\delta \left({t - a} \right)dt=f\left(a \right)} } \quad \ldots\ldots\ldots\ldots\ldots\ldots \left({23.11} \right)$$

We use the shifting property of the impulse function to find its Laplace transform.

$$L\left[{\delta \left(t \right)} \right]=\int\limits_{0-}^\infty{\delta \left(t \right){e^{-st}}\,\,dt=\int\limits_{0-}^\infty{\delta \left(t \right)\,\,dt = 1} } \ldots\ldots\ldots\ldots \left({23.12} \right)$$

which is important Laplace transform pair that we make good use of the circuit analysis.

We can also define the derivatives of the impulse function and the Laplace transform of these derivatives.

The function illustrated in Fig.23.8 (a) generates an impulse function as $\varepsilon \to 0$. Figure 23.8 (b) shows the derivative of the impulse generating function, which is defined as the derivative of the impulse [$\delta'(t)$] as $\varepsilon \to 0$. The derivative of the impulse function sometimes is referred to as a moment function, or unit doublet.

To find the Laplace transform of $\delta'(t)$, we simply apply the defining integral to the function shown in Fig.23.6 (b) and, after integrating, let $\varepsilon \to 0$. Then,

$$L\left\{ {\delta '\left(t \right)} \right\}=\lim \left[{\int\limits_{-\varepsilon }^0 {{1 \over 2}{e^{-st\,}}\,dt}+\int\limits_{{0^ +}}^\varepsilon{\left({{{-1} \over {{\varepsilon ^2}}}} \right){e^{ - st}}\,\,dt} } \right]$$

$$=\,\,\lim \,\,{{{e^{s\varepsilon }} + {e^{ - s\varepsilon }} - 2} \over {s{\varepsilon ^2}}}$$

$$=\,\,\lim \,\,{{s{e^{s\varepsilon }} - {e^{ - s\varepsilon }}} \over {2\varepsilon s}}$$

$$= \,\,\lim \,\,{{{s^2}{e^{s\varepsilon }} + {s^2}{e^{ - s\varepsilon }}} \over {2s}}$$

$$=s \ldots\ldots\ldots\ldots \left({23.13} \right)$$

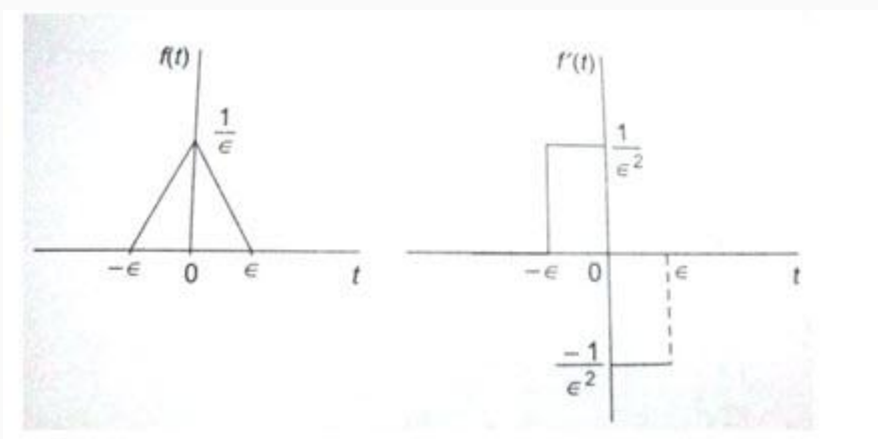


Fig.23.8(a) **Fig.23.8(b)**

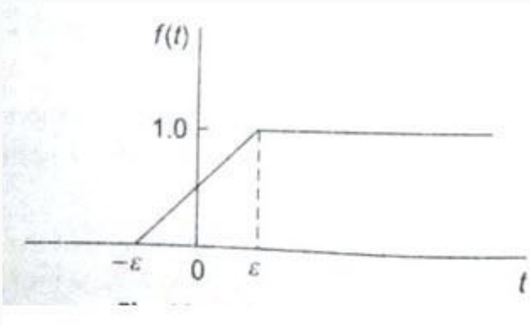
For the nth derivative of the impulse, we find that its Laplace transform simply is s^n; that

is , $$L\left\{ {\delta '\left(t \right)} \right\} = {s^n} \ldots\ldots\ldots\ldots \left({23.14} \right)$$

An impulse function can be thought of as a derivative of a step function, that is

$$\delta \left(t \right)={{du\left(t \right)} \over {dt}}\text{...}\left({23.15} \right)$$

Figure 23.9 (a) approaches a unit step function as ε→ 0. The function shown in Fig. 23.9 (b), the derivative of the function in 23.9 (b), approaches a unit impulse as ε→ 0.



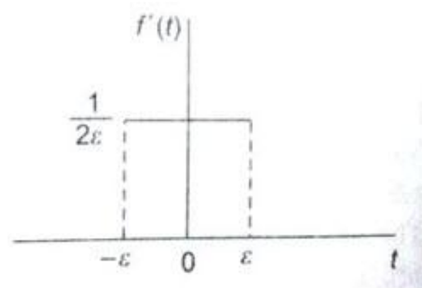


Fig.23.9 (a) **Fig.23.9 (b)**

The impulse function is an extremely useful concept in circuit analysis where discontinuities occur at the origin.

LESSON 24. Functional and operational Transforms

24.1. Functional Transforms

A functional transform is simply the Laplace transform of a special function of t. Because we are limiting our introduction to the unilateral, or one-sided, Laplace transform, we define all functions to be zero for t < 0.

(i) The unit step function \[f\left(t \right) = u\left(t \right)...\left({24.1} \right)\]

Where u(t) = 1 for t > 0

= 0 for t < 0

\[L\left[{\delta \left(t \right)}\right]=\int\limits_0^\infty{f\left(t \right){e^{-st}}\,\,dt}\]

\[\,\int\limits_0^\infty{1e-st\,\,dt={{e-st}\over {-s}}} l\nolimits_0^\infty={1\over s}\]

\[L\left[{u\left(t \right)} \right]={1 \over s}...\left({24.2} \right)\]

(ii) Exponential function \[f\left(t \right)={e^{ - at}}\]

\[L\left({{e^{-at}}}\right)=\int_0^\infty{e - at.{e^{-st}}\,dt}\]

\[=\int_0^\infty{{e^{-\left({s+a}\right)t}}={{-1}\over{s+a}}\left[{{e^{-\left({s + a}\right)t}}}\right]_0^\infty={1\over {s + a}}}\]

\[L\left[{{e^{-at}}}\right]={1\over{s + a}}...\left({24.3} \right)\]

(iii) The cosine function : cos wt \[...\left({24.4} \right)\]

\[L\left({\cos\,\omega t} \right)=\int\limits_0^\infty{\cos \,\omega t\,\,{e^{-st}}dt}\]

\[=\int\limits_0^\infty{{e^{-st}}\left[{{{{e^{j\omega t}}+{e^{-j\omega t}}}\over 2}}\right]}\,dt\]

\[={1 \over 2}\left[{\int\limits_0^\infty{{e^{-\left({s-j\omega }\right)t}}dt+\int\limits_0^\infty{{e^{-\left({s + j\omega}\right)t}}dt}}} \right]\]

\[={1 \over 2}\left[{{{{e^{-\left({s-j\omega t}\right)t}}}\over{s-j\omega}}}\right]_0^\infty\,\, + \,{1\over 2}\left[{{{{e^{-\left({s+j\omega t} \right)t}}} \over {s + j\omega }}} \right]_0^\infty\]

$$={1 \over 2}\left[{{1 \over {s - j\omega }}+{1\over {s + j\omega }}}\right]={s \over {{s^2} + {\omega ^2}}}$$

$$L\left({\cos \,\,\omega t} \right)={s \over {{s^2} + {\omega ^2}}}...\left({24.5} \right)$$

(iv) The sine function : sin wt $...\left({24.6} \right)$

$$L\left({\sin \,\omega t} \right)=\int\limits_0^\infty{\sin \,\omega t\,\,{e^{ - st}}dt}$$

$$=\int\limits_0^\infty{{e^{-st}}\left[{{{{e^{j\omega t}}-{e^{-j\omega t}}}\over {2j}}} \right]} \,dt$$

$$={1 \over {2j}}\left[{\int\limits_0^\infty{{e^{-\left({s-j\omega }\right)t}}dt-\int\limits_0^\infty{{e^{-\left({s+j\omega}\right)t}}dt}}}\right]$$

$$={1 \over {2j}}\left\{ {\left[{{{{e^{ - \left({s - j\omega t} \right)t}}} \over {s - j\omega }}} \right]_0^\infty \,\, + \left[{{{{e^{ - \left({s + j\omega t} \right)t}}} \over {s + j\omega }}} \right]_0^\infty } \right\}$$

$$={1\over {2j}}\left[{{1\over {s - j\omega }} + {1 \over {s + j\omega }}} \right]={\omega\over {{s^2} + {\omega ^2}}}$$

$$L\left({\sin \,\,\omega t} \right)={\omega\over {{s^2} + {\omega^2}}}...\left({24.7} \right)$$

(v) The function t”, where n is a positive integer

$$L\left({t} \right)=\int\limits_0^\infty{t.{e^{ - st}}dt} ....\left({24.8} \right)$$

$$=\left[{{{t{e^{-st}}} \over {-s}}}\right]_0^\infty-\int\limits_0^\infty{{{{e^{-st}}} \over {-s}}n{t^{n - 1}}dt}$$

$$={n \over s}\int\limits_0^\infty{{e^{-st}}\,t{n^{-1}}\,dt}$$

$$={n \over s}L\left({{t^{n - 1}}} \right)...\left({24.9} \right)$$

Similarly $$L\left({{t^{n - 1}}} \right)={{n - 1} \over s}L\left({{t^{n - 2}}} \right)$$

By taking Laplace transformations of t^{n-2}, t^{n-3} … and substituting in the above equation, we get

$$L\left({t} \right)={n \over s}{{n - 1} \over s}{{n - 2} \over s}......{2 \over s}{1 \over s}L\left({{t^{n - m}}} \right)$$

$$= \frac{\angle n}{s^n} L\left(t^0\right) = \frac{\angle n}{s^n} \times \frac{1}{s} = \frac{\angle n}{s^{n+1}}$$

$$L\left(t \right)={1 \over {s^2}} \quad \ldots\ldots \left({24.10} \right)$$

(vi) The hyperbolic sine and cosine function

$$L\left({\cosh \,\,at} \right)\,=\,\int\limits_0^\infty{\cosh \,at\,{e^{ - st}}dt} \quad \ldots\ldots \left({24.11} \right)$$

$$=\int\limits_0^\infty{\left[{{{e^{at}}+{e^{-at}}}\over 2}\right]}{e^{-st}}\,\,dt$$

$$={1 \over 2}\int\limits_0^\infty{{e^{-\left({s - a} \right)t}}\,dt} + {1 \over 2}\int\limits_0^\infty{{e^{-\left({s + a} \right)t\,}}dt}$$

$$={1 \over 2}{1 \over {s - a}} + {1 \over 2}{1 \over {s + a}}={s \over {{s^2} - {a^2}}} \quad \ldots\ldots \left({24.12} \right)$$

Similarly,

$$L\left({\sinh \,at} \right)=\int\limits_0^\infty{\sinh \,\left({at} \right){e^{ - st\,}}\,dt} \quad \ldots\ldots \left({24.13} \right)$$

$$=\int\limits_0^\infty{\left[{{{e^{at}} - {e^{ - at}}} \over 2} \right]} {e^{ - st}}\,\,dt$$

$$={1 \over 2}\int\limits_0^\infty{{e^{ - \left({s - a} \right)t}}\,dt} + {1 \over 2}\int\limits_0^\infty{{e^{ - \left({s + a} \right)t\,}}dt}$$

$$={1 \over 2}{1 \over {s - a}} + {1 \over 2}{1 \over {s + a}}={s \over {{s^2} - {a^2}}} \quad \ldots\ldots \left({24.14} \right)$$

Table 24.1 List of Laplace Transform Pairs

Type	$f\left(t \right)$	F(s)
Impulse	δ(t	1
Step	U(t)	${1 \over s}$
ramp	t	${1 \over {s^2}}$
exponential	e^{-at}	${1 \over {s + a}}$
sine	sin ωt	${\omega \over {{s^2} + {\omega^2}}}$
cosine	cos ωt	${s \over {{s^2} + {\omega^2}}}$
Hyperbolic sine	sinh at	${a \over {{s^2} - {a^2}}}$
Hyperbolic cosine	cosh at	${s \over {{s^2} - {a^2}}}$

damped ramp	te^{-at}	$${1 \over {\left({s + a} \right)}}$$
Damped sine	e^{-at} sin ωt	$${\omega\over{{{\left({s + a} \right)}^2} + {\omega^2}}}$$
Damped cosine	e^{-at} cos ωt	$${{s + a} \over {{{\left({s + a} \right)}^2} + {\omega ^2}}}$$

24.2. Operational Transforms

Operational transforms indicate how mathematical operations performed on either and F(s) are converted into the opposite domain. The operations of primary interest are

(1) Multiplication by a constant

(2) Addition (subtraction)

(3) Differentiation

(4) Integration

(5) Translations in the time domain

(6) Translation in the frequency domain and

(7) Scale charging

Multiplication by Constant

From the defining integral, if

$$L\left[{f\left(t \right)} \right]=F\left(s \right)$$

then $$L\left\{ {K\,f\left(t \right)} \right\}=K\,F\left(s \right)\ldots\ldots\ldots\left({24.15} \right)$$

Consider a function $f\left(t \right)$ multiplied by a constant K.

The Laplace transform of this function is given by

$$L\left[{K\,f\left(t \right)} \right]=\int\limits_0^\infty{Kf\left({t\,{e^{ - st}}\,dt} \right)} \ldots\ldots\ldots\left({24.16} \right)$$

$$=K\int\limits_0^\infty{f\left(t \right)\,{e^{ - st}}\,dt=LF\left(s \right)} \ldots\ldots\ldots\left({24.17} \right)$$

This property is called linearly property.

Addition (Subtraction)

Addition (subtraction) in the time domain translates into addition (subtraction) in the frequency domain.

This if

$${f_1}\left(t \right)\buildrel L \over \longleftrightarrow {F_1}\left(s \right)\,and$$

$${f_2}\left(t \right)\buildrel L \over \longleftrightarrow {F_2}\left(s \right),\,then$$

$$L\left[{{f_1}\left(t \right) \pm {f_2}\left(t \right)} \right]={F_1}\left(s \right) \pm {F_2}\left(s \right)..\left({24.18} \right)$$

Consider two functions ${f_1}\left(t \right)$ and ${f_2}\left(t \right)$. The Laplace transform of the sum or difference of these two functions is given by

$$L\left\{ {{f_1}\left(t \right) \pm {f_2}\left(t \right)} \right\}=\int\limits_0^\infty{\left\{ {{f_1}\left(t \right) \pm {f_2}\left(t \right)} \right\}{e^{ - st}}dt}$$

$$=\int\limits_0^\infty{{f_1}\left(t \right){e^{ - st}}dt \pm \int\limits_0^\infty{{f_2}\left(t \right){e^{ - st}}dt} }$$

$$={F_1}\left(s \right) \pm {F_2}\left(s \right)$$

$$L\left\{ {{f_1}\left(t \right) \pm {f_2}\left(t \right)} \right\}={F_1}\left(s \right) \pm {F_2}\left(s \right)..\left({24.19} \right)$$

The Laplace transform of the sum of the two or more functions is equal to the sum of transforms of the individual function. This is called superposition property, if we can use of linearity and superposition properties jointly, we have

$$L\left[{{K_1}{f_1}(t) + {K_2}{f_2}(1)} \right]={K_1}L\left[{{f_1}(t} \right] + {K_2}L\left[{{F_2}(t} \right]$$

$$={K_1}{F_1}(s) + {K_2}{f_2}(s)..\left({24.20} \right)$$

Integration

If a function $f\left(t \right)$ is continuous, then the Laplace transform of its integral $\int {f\left(t \right)} \,\,dt$ is given by

$$L\left[{\int\limits_0^t {f\left(t \right)} \,dt} \right]={1 \over s}F\left(s \right)..\left({24.21} \right)$$

By definition

$$L\left[{\int\limits_0^t {f\left(t \right)} \,dt}\right]=\int\limits_0^\infty{}\left[{\int\limits_0^t {f\left(t \right)} \,dt} \right]{e^{ - st}}\,dt\ldots\ldots\ldots\ldots\ldots\ldots\left({24.22} \right)$$

Integrating by parts, we get

$$= \left[{{{{e^{ - st}}} \over {-s}}\int\limits_0^t {f\left(t \right)dt} } \right]_0^\infty+{1 \over s}\int\limits_0^\infty{{e^{ - st}}f} \left(t \right)dt\ldots\ldots\ldots\ldots\ldots\ldots\left({24.23} \right)$$

Since, the first term is zero, we have

$$L\left[{\int\limits_0^t {f\left(t \right)dt} } \right]={1 \over s}L\left[{f\left(t \right)} \right]={{F\left(s \right)} \over s}\ldots\ldots\ldots\ldots\ldots\ldots\left({24.24} \right)$$

Differentiation of Transforms

If the Laplace transform of the function exists, then the derivative of the corresponding transform with respect to s in the frequency domain is equal to its multiplication by t in the time domain.

i.e. $$L\left[{tf\left(t \right)} \right]={{ - d} \over {ds}}F\left(s \right)\ldots\ldots\ldots\ldots\ldots\ldots\left({24.25} \right)$$

By definition

$${d \over {ds}}F\left(s \right)={d \over {ds}}\int\limits_0^\infty{f\left(t \right){e^{ - st}}dt} \ldots\ldots\ldots\ldots\ldots\ldots\left({24.26} \right)$$

Since s and t are independent variables, and the limits 0, ∞ are constants not depending on s, we can differentiate partially with respect to s within the integration and then integrate the function obtained with respect to t.

$${d \over {ds}}F\left(s \right)= {d \over {ds}}\int\limits_0^\infty{f\left(t \right){e^{ - st}}dt}$$

$$=\int\limits_0^\infty{f\left(t \right)\left[{ - t{e^{ - st}}} \right]} \,dt$$

$$=-\int\limits_0^\infty{\left\{ {tf\left(t \right)} \right\}} {e^{ - st}}dt=-L\left[{tf\left(t \right)} \right]$$

Hence $$L\left[{tf\left(t \right)} \right]={{ - d} \over {ds}}F\left(s \right)\ldots\ldots\ldots\ldots\ldots\ldots\left({24.27} \right)$$

Integration of Transforms

If the Laplace transform of the function $f\left(t \right)$ exists, then the integral of corresponding transform with respect to s in the complex frequency domain is equal to its division by t in the time domain.

\[i.e.\,\,\,\,\,\,L\left[{{{f\left(t \right)} \over t}} \right]=\int\limits_s^\infty{F\left(s \right)ds} ....\left({24.28} \right)\]

i.e. \[f\left(t \right) \leftrightarrow F\left(s \right)\]

\[F\left(s \right)=L\left[{f\left({}\right)}\right]=\int\limits_0^\infty{f\left(t \right){e^{-st}}\,dt} ....\left({24.29} \right)\]

Integrating both sides from s to ∞

\[\int\limits_s^\infty{F\left(s\right)\,ds=\int\limits_s^\infty{}\left[{\int\limits_0^\infty {f\left(t\right){e^{-st}}dt}}\right]} ds..\left({24.30} \right)\]

By changing the order of integration, we get

\[=\int\limits_0^\infty{f\left(t \right)} \left[{\int\limits_s^\infty{{e^{ - st}}ds} } \right]dt..\left({24.31} \right)\]

\[=\int\limits_0^\infty{f\left(t \right)} \left({{{{e^{ - st}}} \over t}} \right)dt\]

\[=\int\limits_{0t}^\infty{}\left[{{{f\left(t \right)} \over t}} \right]\,\,{e^{ - st}}\,dt = L\left[{{{f\left(t \right)} \over t}} \right]..\left({24.32} \right)\]

\[\int\limits_0^\infty{F\left(s \right)} ds = L\left[{{{f\left(t \right)} \over t}} \right]..\left({24.33} \right)\]

Translation in the time Domain

If the function \[f\left(t \right)\] has the transform F(s), then the Laplace transform of \[f\left({t - a} \right)\,\,u\left({t - a} \right)\] is e^{-as} F(s). By definition

\[L\left[{f\left({t - a} \right)\,u\left({t - a} \right)} \right]=\int\limits_0^\infty{\left[{f\left({t - a} \right)\,u\left({t - a} \right)} \right]\,{e^{ - st}}\,\,dt} ....\left({24.34} \right)\]

Since \[f\left({t - a} \right)\,u\left({t - a}\right)=0 &for\,\,t < a\]

\[=f\left({t - a} \right)\,\,\,\,\,\,for\,\,t > a\]

\[L\left[{f\left({t - a} \right)\,u\left({t - a} \right)} \right]=\int\limits_0^\infty{f\left({t - a} \right)\,{e^{ - st}}\,\,dt} ....\left({24.35} \right)\]

\[Put\,\,t - a\,=\,\tau\,then\,\tau+a=t\]

\[dt=d\tau\]

Therefore, the above becomes

$$L\left[{f\left({t - a}\right)\,u\left({t - a} \right)}\right]\int\limits_0^\infty{f\left(t \right){e^{-s\left({\tau+a} \right)}}} d\tau \ldots\ldots\ldots\ldots \left({24.36} \right)$$

$$={e^{-as}}\int\limits_0^\infty{f\left(\tau\right){e^{-ST}}d\tau=} {e^{-as}}F\left(s \right)\ldots\ldots\ldots\ldots \left({24.37} \right)$$

$$L\left[{f\left({t - a} \right)\,\,u\left({t - a} \right)} \right]={e^{ - as}}F(s)\ldots\ldots\ldots\ldots \left({24.38} \right)$$

Translation in the time domain corresponds to multiplication by an exponential in the frequency domain.

Translation in the Frequency Domain

If the function $f\left(t \right)$ has the transform F(s), then Laplace transform of ${e^{ - at}}f(t)is\,\,F(s + a)$

By definition, $$F\left(s \right)=\int\limits_0^\infty{f\left(t \right)} {e^{ - st}}dt\ldots\ldots\ldots\ldots \left({24.39} \right)$$

and therefore, $$F\left({s + a} \right)=\int\limits_0^\infty{f\left(t \right)} {e^{ - (s + a)}}dt\ldots\ldots\ldots\ldots \left({24.40} \right)$$

$$=\int\limits_0^\infty{{e^{ - at}}f\left(t \right)} {e^{ - st}}dt = L\left[{{e^{ - at}}f\left(t \right)} \right]\ldots\ldots\ldots\ldots \left({24.41} \right)$$

$$F(s + a)=L\left[{{e^{ - at}}f(t)} \right]\ldots\ldots\ldots\ldots \left({24.42} \right)$$

Similarly, we have

$$L\left[{{e^{at}}f(t)} \right]=F(s - a)\ldots\ldots\ldots\ldots \left({24.43} \right)$$

Translation in the frequency domain corresponds to multiplication by an exponential in the time domain.

Scale Changing

The scale change property gives the relationship between and F(s) when the time variable is multiplied by a positive constant.

$$L\left\{ {f\left({at} \right)} \right\}={1 \over a}F\left({{s \over a}} \right),a > 0\ldots\ldots\ldots\ldots \left({24.44} \right)$$

By definition

$$L\left[{f\left({at} \right)} \right]=\int\limits_0^\infty{f\left({at} \right)} {e^{ - st}}dt\ldots\ldots\ldots\ldots \left({24.45} \right)$$

Put $at=\tau$

$$dt={1 \over a}d\tau$$

$$L\left[{f\left({at} \right)} \right]=\int\limits_0^\infty{f\left(\tau\right)} {e^{{s \over a}\tau}}.{1 \over a}d\tau$$

$$={1 \over a}\int\limits_0^\infty{f\left(\tau\right){e^{{s \over a}\tau}}} d\tau$$

$$={1 \over a}F\left({{s \over a}} \right)..\left({24.46} \right)$$

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LESSON 25. Laplace Transform of Periodic Functions and inverse transforms

25.1. Laplace Transform of Periodic Functions

Periodic functions appear in many practical problems. Let function $f(t)$ be a periodic function which satisfies the condition $f(t)$ = $f(t + T)$ for all t>0 where T is period of the function.

$$L[f(t)] = \int_0^T f(t)e^{-st}\,dt + \int_T^{2T} f(t)e^{-st}\,dt + \int_{nT}^{(n+1)T} f(1)e^{-st}\,dt + \text{...}(25.1)$$

$$= \int_0^T f(t)e_{-st}\,dt + \int_0^T f(t)e - ste^{-sT}\,dt + \ldots + \int_0^T f(t)e^{-st}e^{-nsT}\,dt + \ldots$$

$$= \left(1 + e^{-sT} + e - 2sT + \ldots + e^{-nsT} + \ldots\right)\int_0^T f(t)e^{-st}\,dt \text{..}(25.2)$$

$$= \frac{1}{1 - e^{-sT}}\int_0^T f(t)e^{-st}\,dt \text{..}(25.3)$$

25.2 Inverse Transforms

So far, we have discussed Laplace transform of functions $f(t)$. If the function is a rational function of s, which can be expressed in the form of a ratio of two polynomial in s such that no non-integral powers of s appear in the polynomials. In fact, for liner, lumped parameter circuits whose component values are constant, the s-domain expressions for the unknown voltages and currents are always rational functions of s. If we can inverse transform rational functions of s, we can solve for the time domain expressions for the voltages and currents.

In general, we need to find the inverse transform of a function that has the form.

$$F\left(s \right) = {{N\left(s \right)} \over {D\left(s \right)}}={{{a_n}{s^n} + {a_{n - 1}}{s^{n - 1}} + ... + {a_1}s + {a_0}} \over {{b_m}{s^m} + {b_{m - 1}}{s^{m - 1}} + ... + {b_1}s + {b_0}}}\text{..}\left({25.4} \right)$$

The coefficients a and b are real constants, and the exponents m and n are positive integers. The ratio ${{N\left(2 \right)} \over {D\left(2 \right)}}$ is called a paper rational function if m>n, and an improper rational function if m ≤ n. Only a proper rational function can be expanded as a sum of partial fractions.

Partial Fraction Expansion: Proper Rational Functions

A proper rational function is expanded into a sum of partial fractions by writing a term or a series of terms for each root of D(s). Thus D(s) must be in factored form before we can make a particle fraction expansion. The roots of D(s) are either (1) real and distinct (2) complex and distinct (3) real and repeated or (H) complex and repeated.

When the roots are real and distinct

In this case $$F\left(s \right)={{N\left(s \right)} \over {D\left(s \right)}}...\left({25.5} \right)$$

Where D(s) = (s-a) (s-b) (s-c) $$...\left({25.6} \right)$$

Expanding F(s) into partial fractions, we get

$$F(s)={A \over {s - a}} + {B \over {s - b}} + {C \over {s - c}}...\left({25.7} \right)$$

To obtain the constant A, multiplying Eq. (25.7) with (s-a) and putting s = a, we get

$$F(s)\,(s - a)\,{\,_{s = a}}=A$$

Similarly, we can get the other constants,

$$B=(s - b)\,\,F{(s)_{s = b}}$$

$$C=(s - c)\,\,F{(s)_{s = c}}$$

When roots are real and repeated

In this case $$F\left({s={{N\left(s \right)} \over {D\left(s \right)}}} \right)$$

where $$D\left(s \right)={\left({s - a} \right)^n}\,\,{D_1}\left(s \right)$$

The partial fraction expansion of F(s) is

$$F\left(s \right)={{{A_0}} \over {{{\left({s - a} \right)}^n}}} + {{{A_1}} \over {{{\left({s - a} \right)}^{n - 1}}}} + ... + {{{A_{n - 1}}} \over {s - a}} + {{{N_1}\left(s \right)} \over {{D_1}\left(s \right)}}...\left({25.8} \right)$$

Where $${{{N_1}\left(s \right)} \over {{D_1}\left(s \right)}}$$ represents the remainder terms of expansion.

To obtain the constant $A_{0'}$ $A_{1'}$ … $A_{n-1'}$ let us multiply both sides of Eq.(25.8) by $(s-a)^n$.

Thus

$(s – a)^n F(s) = F_1(s) = A_0 + A_1 (s – a) + A_2 (s – a)^2 + …$

$+A_{n-1} (s – a)^{n-1} + R(s) (s – a)^n$ $$....................\left({25.9} \right)$$

Where R(s) indicates the remainder terms

Putting s = a, we get

$A_0 = (s – a)^n F(s)/_{s=a}$

Differentiating Eq. (13.117) with respect to s, and putting s = a, we get

$${A_1}={d \over {ds}}{F_1}\left(s \right){|_{s = a}}$$

Similarly, $${A_2}={1 \over {2!}}{{{d^2}} \over {d{s^2}}}{F_1}\left(s \right){|_{s = a}}$$

In general, $${A_n}={\left. {{1 \over {n!}}{{{d^n}{F_1}\left(s \right)} \over {d{s^n}}}} \right|_{s = a}}\text{..}\left({25.10} \right)$$

When roots are distinct complex roots of D(s)

Consider a function

$$F\left(s\right)={{N\left(s\right)}\over{D\left(s\right)\left({s-\alpha+j\beta}\right)\left({s-\alpha-j\beta}\right)}}\text{..}\left({25.11} \right)$$

The partial fraction expansion of F(s) is

$$F\left(s\right)={A\over{s-\alpha-j\beta}}+{B\over{s-\alpha+j\beta}}+{{{N_1}\left(s \right)} \over {{D_1}\left(s \right)}}\text{..}\left({25.12} \right)$$

where $${{{N_1}\left(s \right)} \over {{D_1}\left(s \right)}}$$ is the remainder term.

Multiplying Eq. (25.12) by (s-a-jβ) and putting

S = a + jβ.

we get $$A={{N\left({\alpha+j\beta }\right)}\over{{D_1}\left({\alpha+j\beta}\right)\left({+2j\beta}\right)}}\text{..}\left({25.13} \right)$$

Similarly $$B={{N\left({\alpha+j\beta } \right)} \over {\left({-2j\beta}\right){D_1}\left({\alpha-\beta } \right)}}\text{..}\left({25.14} \right)$$

In general, B = A* where A* is complex conjugate of A.

If we denote the inverse transform of the complex conjugate terms as *f* (t)

$$f\left(t \right)={L^{ - 1}}\left[{{A \over {s - \alpha- j\beta }} + {B \over {s - \alpha+j\beta }}} \right]\text{..}\left({25.15} \right)$$

$$={L^{ - 1}}\left[{{A \over {s - \alpha-j\beta }} + {B \over {s - \alpha+j\beta }}} \right]$$

where A and A* are conjugate terms.

If we denote A = C+jD, then

B = C – jD = A*

$$f(t) = e^{at}\left(Ae^{j\beta t} + A^* e^{-j\beta t}\right)$$

When roots are repeated and complex of D(s)

The complex roots always appear in conjugate pairs and that the coefficients associated with a conjugate pair are also conjugate, so that only half the Ks need to be evolved.

Consider the function $$F\left(s \right)={{768} \over {{\left({{s^2} + 6s + 2s} \right)}^2}} \ldots\ldots\ldots \left({25.17} \right)$$

By factoring the denominator polynomial, we have

$$F\left(s \right)={{768} \over {{\left({s + 3 - j4} \right)}^2}{{\left({s + 3 + j4} \right)}^2}}$$

$$={{{K_1}} \over {{\left({s + 3 - j4} \right)}^2}} + {{{K_2}} \over {s + 3 - j4}}$$

$$+ {{K_1^*} \over {{\left({s + 3 + j4} \right)}^2}} + {{K_2^*} \over {s + 3 + j4}} \ldots\ldots\ldots \left({25.18} \right)$$

Now we need to evaluate only K_1 and K_2 because K^*_1 and K^*_2 are conjugate values. The value of $K_{1'}$ is

$$K1={\left.{{768}\over{{\left({s+3+j4}\right)}^2}}\right|_{s=-3+j4}}={{768}\over{{\left({j8}\right)}^2}}=-12 \ldots\ldots\ldots \left({25.19} \right)$$

The value of K_2 is

$$K2={d \over {ds}}{\left[{{768} \over {{\left({s + 3 + j4} \right)}^2}} \right]_{s=-3+j4}}$$

$$=-{\left. {{2\left({768}\right)}\over{{\left({s+3+j4}\right)}^3}}\right|_{s=-3 + j4}}=-{{2\left({768} \right)}\over {{\left({j8} \right)}^3}}$$

$$=-j3=3\angle -90^0 \ldots\ldots\ldots \left({25.20} \right)$$

Partial Fraction Expansion: Improper Rational Function

An improper rational function can always be expanded into a polynomial plus a proper rational function. The polynomial is then inverse –transformed into impulse functions and derivatives of impulse functions.

$$F\left(s \right)={{{S^4} + 13_s^3 + 66_s^2 + {{200}_s} + 300} \over {{s^2} + 9s + 20}} \ldots\ldots\ldots \left({25.21} \right)$$

Dividing the denominator into the numerator until the remainder is proper rational function gives

$$F\left(s \right)={s^2} + 4s + 10 + {{30s + 100} \over {{s^2} + 9s + 20}}..\left({25.22} \right)$$

Now we expand the proper rational function into a sum of partial fractions

$${{30s + 100} \over {{s^2} + 9s + 20}}={{30s + 100} \over {\left({s + 4} \right)\,\,\left({ + 5} \right)}} = {{ - 20} \over {s + 4}} + {{50} \over {s + 5}}..\left({25.23} \right)$$

Substituting Eq. (25.23) into Eq. (25.22) yields

$$F\left(s \right)={s^2} + 4s + 10 - {{20} \over {s + 4}} + {{50} \over {s + 5}}..\left({25.24} \right)$$

By taking inverse transform, we get

$$f\left(t \right)={{{d^2}\delta \left(t \right)} \over {d{t^2}}} + 4{{d\delta \left(t \right)} \over {dt}} + 10\,\,\delta \left(t \right) - \left({20{e^{ - 4t}} - 50{e^{ - 5t}}} \right)u\left(t \right)..\left({25.25} \right)$$

25.3. Initial and Final Value Theorems

The initial and final value theorems are useful because they enable us to determine from $F\left(s \right)$ the behavior of $f\left(t \right)$ at 0 and ∞. Hence we can check the initial and final value of to sec if they conform with known circuit behavior, before actually finding the inverse transform of $F\left(s \right)$.

The initial –value theorem states that

$$\lim \,f\left(t \right)=\lim SF(s)..\left({25.26} \right)$$

and the final-value theorem states that

$$\lim \,f\left(t \right)=\lim SF(s)..\left({25.27} \right)$$

The initial-value theorem is based on the assumption that contains no impulse functions.

To prove, initial value theorem, we start with the operational transform of the fits derivative.

$$L\left[{{{df} \over {dt}}} \right]=SF\left(s \right) - f\left(0 \right)$$

$$=\int\limits_0^\infty{{{df} \over {dt}}{e^{ - st}}dt} ....\left({25.28} \right)$$

Now we take the limit as → ∞

$${\lim }\limits_{s \to \infty} \left[{SF\left({s - f\left(0 \right)} \right)} \right]={\lim }\limits_{s \to \infty } \int\limits_0^\infty{{{df} \over {dt}}} {e^{ - st}}\,\,dt..\left({25.29} \right)$$

The right hand side of the above equation becomes zero as s®¥

$${\lim }\limits_{s \to \infty} \left[{SF\left(s \right) - f\left(0 \right)} \right]= 0$$

$${\lim }\limits_{s \to \infty} SF\left(s \right) = f\left(0 \right)={\lim }\limits_{s \to \infty} f\left(t \right)..\left({25.30} \right)$$

The proof of the final value theorem also starts with Eq. (25.31). Here we take the limit as $s\to 0$.

$${\lim }\limits_{s \to \infty} \left[{SF\left(s\right)-f\left(0 \right)} \right]={\lim }\limits_{s \to 0} s\left({\int\limits_0^\infty{{{df} \over {dt}}{e^{ - st}}\,dt} } \right)..\left({25.31} \right)$$

$${\lim }\limits_{s \to 0} \left[{SF\left(s \right) - f\left(0 \right)} \right] = \left[{f\left(t \right)} \right]_0^\infty ....\left({25.32} \right)$$

$${\lim }\limits_{s \to 0} \left[{SF\left(s \right) - f\left(0 \right)} \right]={\lim }\limits_{t \to \infty} \,f\left(t \right) - f\left(0 \right)$$

Since $f\left(0 \right)$ is not a function of s, it gets cancelled from both sides.

$${\lim }\limits_{t \to \infty } f\left(t \right)={\lim }\limits_{s \to 0} \,FS\left(0 \right)..\left({25.33} \right)$$

The final-value theorem is useful only if $f\left(\infty\right)$ exists.

Application of initial and final value theorems

Consider the transform pair given by

$${{100\left({s + 3} \right)} \over {\left({s + 6} \right)\left({{s^2} + 6s + 25} \right)}} \leftrightarrow \left[{212{e^{ - 6t}} + 20{e^{ - 3t}}\cos \left({4t - {{53.13}^0}} \right)} \right]\,u\left(t \right)......................................\left({25.34} \right)$$

The initial value theorem gives

$${\lim}\limits_{s\to\infty}\left[{SF\left(s\right)={\lim}\limits_{s\to\infty}}\right]\frac{{100{s^2}\left({1+\frac{3}{s}}\right)}}{{{s^3}\left[{1+\frac{6}{s}}\right]\left[{1+\frac{6}{s}+\frac{{25}}{{{s^2}}}}\right]}}=0.............\left({25.35}\right)$$

$${\lim}\limits_{t\to 0}f\left(t\right)=\left[{-12+20\cos\left({-{{53.13}^0}} \right)} \right]\left(1 \right)=-12 + 12 = 0..\left({25.36} \right)$$

The final value theorem gives

$${\lim }\limits_{s \to 0} \,SF\left(s \right)={\lim }\limits_{s \to 0} {{100s\left({s + 3} \right)} \over {\left({s + 6} \right)\left({{s^2} + 6s + 25} \right)}} = 0..\left({25.37} \right)$$

$$\lim\limits_{t \to \infty} \,f\left(t \right)=\lim\limits_{t \to \infty} \left[{ - 12{e^{ - 6t}} + 20{e^{ - 3t}}\cos \left({4t - {{53.13}^0}} \right)} \right]u\left(t \right) = 0 \qquad \left({25.38} \right)$$

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Module 11. Series and parallel resonance

LESSON 26. Series Resonance

26.1. Series Resonance

In many electrical circuits, resonance is a very important phenomenon. The study of resonance is very useful, particularly in the area of communications. For example, the ability of a radio receiver to select a certain frequency, transmitted by a station and to eliminate frequencies from other stations is based on the principle of resonance. In a series RLC circuit, the current lags behind, or leads the applied voltage depending upon the values of X_L and X_C X_L causes the total current to lag behind the applied voltage, while X_C causes the total current to lead the applied voltage. When $X_L > X_C$, the circuit is predominantly inductive, and when X= > X_L, the circuit is predominantly capacitive. However, if one of the parameters of the series RLC circuit is varied in such a way that the current in the circuit is in phase with the applied voltage, then the circuit is said to be in resonance.

Consider the series RLC circuit shown in Fig. 26.1.

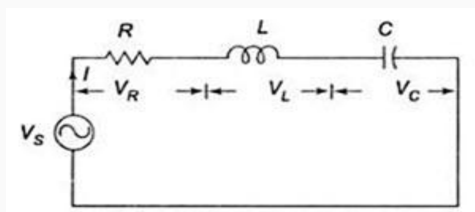

Fig. 26.1

The total impedance for the series RLC circuit is

$$Z=R + j\,\left({X_L^{} - {X_C}} \right)\,=\,R + j\,\left({\omega L - {1 \over {\omega C}}} \right)$$

It is clear from the circuit that the current $I= V_S/Z$.

The circuit is said to be in resonance if the current is in phase with the applied voltage. In a series RLC circuit, series resonance occurs when $X_L=X_C$. The frequency at which the resonance occurs is called the resonant frequency. Since $X_L=X_C$, the impedance in a series RLC circuit is purely resistive. At the resonant frequency, f_r, the voltages across capacitance and inductance are equal in magnitude. Since they are 180^0 out of phase with each other, they cancel each other and, hence zero voltage appears across the LC combination.

At resonance

$${X_L}={X_C}\,\,i.e.\,\,\omega L\,=\,{1 \over {\omega C}}$$

Solving for resonant frequency, we get

$$2\pi {f_r}L\,=\,{1 \over {2\pi {f_r}C}}$$

$$f_r^2={1 \over {4{\pi ^2}LC}}$$

$${f_r}=\,{1 \over {2\pi \sqrt {LC} }}$$

In a series RLC circuit, resonance may be produced by varying the frequency, keeping L and C constant; otherwise, resonance may be produced by varying either L or C for a fixed frequency.

26.2. Impedance and Phase Angle of a series Resonant Circuit

The impendence of a series RLC circuit is

$$\left| Z \right|\,=\,\sqrt {{R^2} + {{\left({\omega L - {1 \over {\omega C}}} \right)}^2}}$$

The variation of X_C and X_L with frequency is shown in Fig. 26.2.

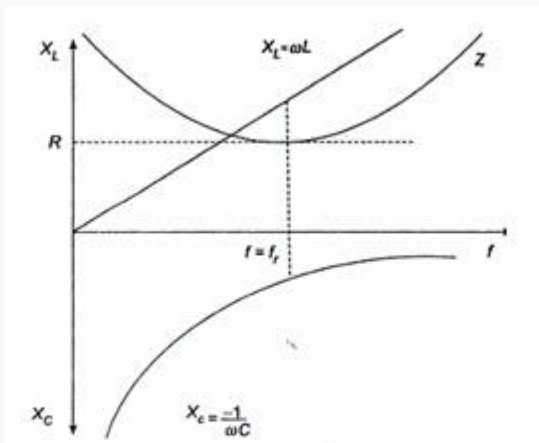

Fig.26.2

At zero frequency, both X_C and Z are infinitely large, and X_L is zero because at zero frequency the capacitor acts as an open circuit and the inductor acts as a short circuit. As the frequency increases, X_C decreases and X_L increases. Since X_C is larger than X_L, at frequencies below the resonant frequency f_r, Z decreases along with X_C. At resonant frequency $f_r, X_C = X_L,\, Z = R.$ At frequencies above the resonant frequency f_r, X_L is larger than X_C, causing Z to increase. The phase angle as a function of frequency is shown in Fig.26.3.

At a frequency below the resonant frequency, current leads the source voltage because the capacitive reactance is greater than the inductive reactance. The phase angle decreases as the frequency approaches the resonant value, and is 0^0 at resonance. At frequencies above resonance, the current lags behind the source voltage, because the inductive reactance is greter than capacitive reactance. As the frequency goes higher, the phase angle approaches 90^0.

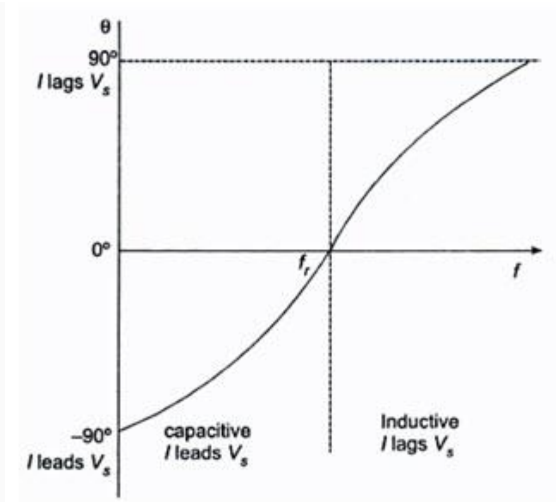

Fig.26.3

26.3. Voltages and Currents in a series resonant circuit

The variation of impedance and current with frequency is shown in Fig.26.4.

At resonant frequency, the capacitive reactance is equal to inductive reactance, and hence the impedance is minimum. Because of minimum impedance, maximum current flows through the circuit. The current variation with frequency is plotted.

The voltage drop across resistance, inductance and capacitance also varies with frequency. At $f=0$, the capacitor acts as an open circuit and blocks current. The complete source voltage appears across the capacitor. As the frequency increases, X_C decreases and S_L increases, causing total reactance X_C-X_L to decrease. As a result, the impedance decreases and the current increases. As the current increases, V_R also increases, and both V_C and V_L increase.

When the frequency reaches is resonant value ${f_r}$, the impedance is equal to R, and hence, the current reaches its maximum value, and V_R is at maximum value.

As the frequency is increased above resonance, X_L-X_C to increase. As a result continues to decrease, causing the total reactance. As a result there is an increase in impedance and a decrease in current. As the current decreases, V_R also decreases, and both V_C and V_L decrease. As the frequency becomes very high, the current approaches zero, both V_R and V_C approach zero, and V_L approaches V_S.

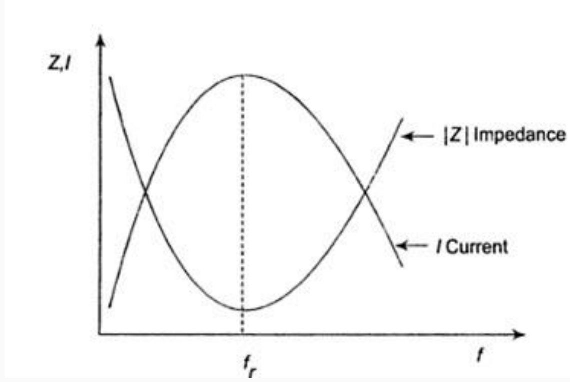

Fig. 26.4

The response of different voltages with frequency is shown in Fig. 26.5

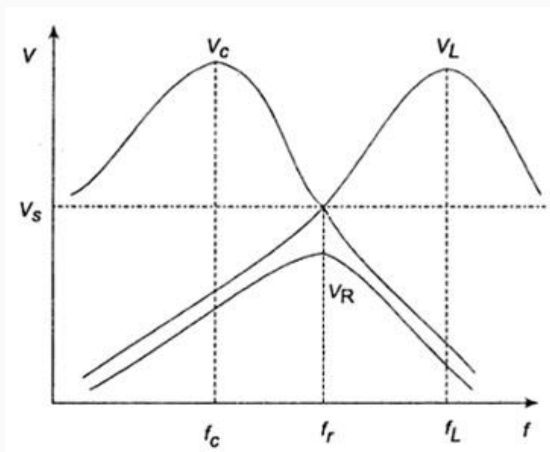

Fig. 26.5

The drop across the resistance reaches its maximum when $f={f_r}$. The maximum voltage across the capacitor occurs at $f={f_e}$. Similarly, the maximum voltage across the inductor occurs at $f={f_L}$.

$${V_L}=I{X_L}$$

Where $I={V \over Z}$

$$= \,{{\omega LV} \over {\sqrt {{R^2} + {{\left({\omega L - {I \over {\omega C}}} \right)}^2}} }}$$

To obtain the condition for maximum voltage across the inductor, we have to take the derivative of the above equation with respect to frequency, and make it equal to zero.

$${{d{V_L}} \over {d\omega }}=0$$

If we solve for ω, we obtain the value of ω when V_L is maximum

$${{d{V_L}} \over {d\omega }}={d \over {d\omega }}\left\{ {\omega LV{{\left[{{R^2} + {{\left({\omega L - {1 \over {\omega C}}} \right)}^2}} \right]}^{ - 1/2}}} \right\}$$

$$LV{\left({{R^2} + {\omega ^2}{L^2} - {{2L} \over C} + {1 \over {{\omega ^2}{C^2}}}} \right)^{ - 1/2}}$$

$$\frac{-\frac{\omega LV}{2}\left(R^2+\omega^2L^2-\frac{2L}{C}+\frac{1}{\omega^2C^2}\right)\left(2\omega L^2-\frac{2}{\omega^2C^2}\right)}{R^2+\omega^2L^2-\frac{2L}{C}+\frac{1}{\omega^2C^2}}=0$$

From this

$$R^2-\frac{2L}{C}+2/\omega^2C^2=0$$

$$\therefore \quad \omega L=\sqrt{\frac{2}{2LC-R^2C^2}}=\frac{2}{\sqrt{LC}}\sqrt{\frac{2}{2-\frac{R^2C}{L}}}$$

$$f_L=\frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{1-\frac{R^2C}{2L}}}$$

Similarly, the voltage across the capacitor is

$$V_C=IX_C=\frac{1}{\omega C}$$

$$\therefore V_C=\frac{V}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}\times\frac{1}{\omega C}$$

To get maximum value $${{d{V_C}} \over {d\omega }}=0$$

If we solve for ω, we obtain the value of ω when V_C is maximum.

$${{d{V_C}} \over {d\omega }}=\omega C{1 \over 2}{\left[{{R^2} + {{\left({\omega L - {1 \over {\omega C}}} \right)}^2}} \right]^{ - 1/2}}\left[{2\left({\omega L - {1 \over {\omega C}}} \right)\,\left({L + {1 \over {{\omega ^2}C}}} \right)} \right]$$

$$+ \sqrt {{R^2} + {{\left({\omega L - {1 \over {\omega C}}} \right)}^2}\,\,C = 0}$$

From this

$$\omega _C^2\,=\,{1 \over {LC}}\, - \,{{{R^2}} \over {2L}}$$

$${\omega _C}\,=\sqrt {{1 \over {LC}}\, - \,{{{R^2}} \over {2L}}} \,$$

$${f_C}\,=\,{1 \over {2\pi }}\sqrt {{1 \over {LC}}\, - \,{{{R^2}} \over {2L}}} \,$$

The maximum voltage across the capacitor occurs below the resonant frequency; and the maximum voltage across the inductor occurs above the resonant frequency.

26.4. Bandwidth of an RLC Circuit

The bandwidth of any system is the range of frequencies for which the current or output voltage is equal to 70.7% of its value at the resonant frequency, and it is denoted by BW, Figure 26.6 shows the response of the series RLC circuit.

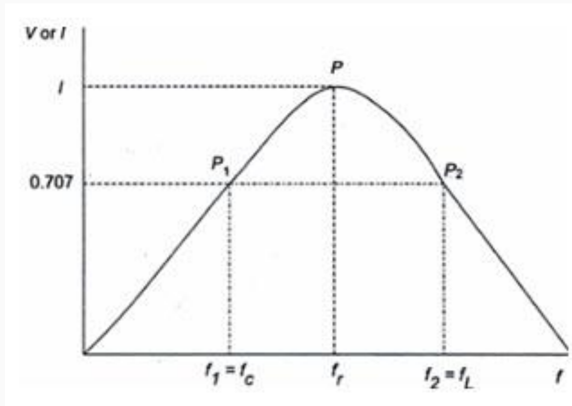

Fig. 26.6

Here the frequency f_1 is the frequency at which the current is 0.707 times the current at resonant value, and it is called the lower cut-off frequency. The frequency f_2 is the frequency at which the current is 0.707 times the current at resonant value (i.e. maximum value), and is called the upper cut-off frequency. The band width, or BW, defined as the frequency difference between $f_2 \, and \, f_1$.

$$BW = f_2 - f_1$$

The unit of BW is hertz (Hz).

If the current at P_1 is $0.707I_{max}$, the impedance of the circuit at this point is $\sqrt{2}R,$ and hence

$$\frac{1}{\omega_1 C} - \omega_1 L = R \quad \ldots\ldots (26.1)$$

Similarly, $$\omega_2 L - \frac{1}{\omega_2 C} = R \quad \ldots\ldots (26.2)$$

If we equate both the above equations, we get

$$\frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$$

$$L(\omega_1 + \omega_2) = \frac{1}{C}\left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right) \quad \ldots\ldots (26.3)$$

From Eq. 26.3, we get

$$\omega_1 \omega_2 = \frac{1}{LC}$$

we have $$\omega _r^2={1 \over {LC}}$$

$$\omega _r^2={\omega _1}{\omega _2}\ldots\ldots\left({26.4} \right)$$

If we add Eqs 8.1 and 8.2, we get

$${1 \over {{\omega _1}C}} - {\omega _1}L + {\omega _2}L - {1 \over {{\omega _2}C}}=2R$$

$$\left({{\omega _2} - {\omega _1}} \right)L + {1 \over C}\left({{{{\omega _2} - {\omega _1}} \over {{\omega _1}{\omega _2}}}} \right) = 2R\ldots\ldots\left({26.5} \right)$$

Since $$C={1 \over {\omega _r^2L}}$$

and $${\omega _1}{\omega _2}=\omega _r^2$$

$$\left({{\omega _2} - {\omega _1}} \right)L + {{\omega _r^2\left({{\omega _2} - {\omega _1}} \right)} \over {\omega _r^2}} = 2R\ldots\ldots\left({26.6} \right)$$

From Eq. 26.6, we have

$${\omega _2} - {\omega _1} = {R \over L}\ldots\ldots\left({26.7} \right)$$

$${f_2} - {f_1}={R \over {2\pi L}}\ldots\ldots\left({26.8} \right)$$

or $$BW\,=\,{R \over {2\pi L}}$$

from Eq. 26.8, we have

$${f_2} - {f_1}={R \over {2\pi L}}$$

$${f_r} - {f_1}={R \over {4\pi L}}$$

$$\,{f_2} - {f_1}={R \over {4\pi L}}$$

The lower frequency limit $${f_1}={f_r} - {R \over {4\pi L}}\ldots\ldots\left({26.9} \right)$$

The upper frequency limit $${f_2}={f_r} - {R \over {4\pi L}}\ldots\ldots\left({26.10} \right)$$

If we divide the equation on both sides by ${f_r}$, we get

$${{{f_2} - {f_1}} \over {{f_r}}}={R \over {2\pi {f_r}L}}\ldots\ldots\left({26.11} \right)$$

Here an important property of a coil is defined. It is the ratio of the reactance of the coil to its resistance. This ratio is defined as the Q of the coil. Q is known as a figure of merit, it is also called quality factor and is an indication of the quality of a coil.

$$Q{{{X_1}} \over R}={{2\pi {f_r}L} \over R}\ldots\ldots\left({26.12} \right)$$

If we substitute Eq.26.11 in Eq.26.12, we get

$${{{f_2} - {f_1}} \over {{f_r}}}={1 \over Q}\ldots\ldots\left({26.13} \right)$$

The upper and lower cut-off frequencies are sometimes called the half-power frequencies. At these frequencies the power from the source is half of the power delivered at the resonant frequency.

At resonant frequency, the power is

$${P_{\max }}=I_{\max }^2R$$

At frequency , the power is

Similarly, at frequency ${f_2}$, the power is ${P_1}={\left({{{I_{\max }}} \over {\sqrt 2 }} \right)^2}\,\,R = {{I_{\max }^2R} \over 2}$

$${P_2}={\left({{{I_{\max }}} \over {\sqrt 2 }} \right)^2}\,R$$

$$={{I_{\max }^2R} \over 2}$$

The response curve in fig.26.6 is also called the selectively curve of the circuit. Selectivity indicates how well a resonant circuit responds to a certain frequency and eliminates all other frequencies. The narrower the bandwidth, the greater the selectivity.

26.5. The Quality Factor (Q) and its Effect on Bandwidth

The quality factor, Q, is the ratio of the reactive power in the inductor or capacitor to the true power in the resistance in series with the coil or capacitor.

The quality factor

$$Q=2\pi\times {{\max imum\,energy\,\,stored} \over {energy\,\,dissipated\,per\,cycle}}$$

In an inductor, the max energy stored is given by ${{L{I^2}} \over 2}$

Energy dissipated per cycle = ${\left({{I \over {\sqrt 2 }}} \right)^2}\,\,R \times T\, = \,{{{I^2}RT} \over 2}$

Quality factor of the coil

$$Q = 2\pi \times \frac{2}{\frac{I^2 R}{2} \times \frac{1}{f}}$$

$$={{2\pi fL} \over R}={{\omega L} \over 2}$$

Similarly, in a capacitor, the max energy stored is given by $${{C{V^2}} \over 2}$$

The energy dissipated per cycle $$={\left({I/\sqrt 2 } \right)^2}R \times T$$

The quality factor of the capacitance circuit

$$Q=\frac{2\pi \frac{1}{2} C\left(\frac{1}{\omega C}\right)^2}{\frac{I^2}{2} R \times \frac{1}{f}}=\frac{1}{\omega CR}$$

In series circuits, the quality factor $$Q={{\omega L} \over R}={1 \over {\omega CR}}$$

We have already discussed the relation between bandwidth and quality factor, which is $$Q={{{f_r}} \over {BW}}$$.

A higher value of circuit Q results in a smaller bandwidth. A lower value of Q causes a larger bandwidth.

26.6. Magnification in Resonance

If we assume that the voltage applied to the series RLC circuit is V, and the current at resonance is I, then the voltage across L is $V_L=IX_L=(V/R)\ \omega,L$

Similarly, the voltage across C

$${V_C} = I{X_C}={V \over {R{\omega _r}C}}$$

Since $Q = 1/\omega_r CR = \omega_r L/R$

Where w_r is the frequency at resonance.

Therefore $V_L = VQ$

$V_C = VQ$

The ratio of voltage across either L or C to the voltage applied at resonance can be defined as magnification.

Magnification = $Q = V_L / V$ or V_C / V

LESSON 27. Parallel Resonance

27.1. Parallel Resonance

Basically, parallel resonance occurs when $X_C=X_L$. The frequency at which resonance occurs is called the resonant frequency. When $X_C=X_L$, the two branch currents are equal in magnitude and 180^0 out of phase with each other. Therefore, the two currents cancel each other out, and the total current is zero. Consider the circuit shown in Fig.26.1. The condition for resonance occurs when $X_L=X_C$.

In fig. 26.1, the total admittance

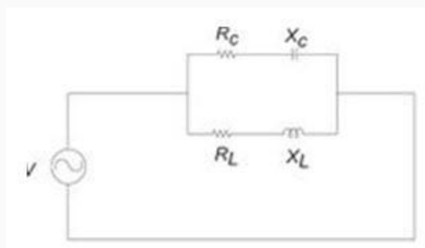

Fig. 27.1

$$Y={1 \over {{R_L} + j\omega L}} + {1 \over {{R_C} - \left({j/\omega C} \right)}}$$

$$= \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C + (j/\omega C)}{R_C^2 + \frac{1}{\omega^2 C^2}}$$

$$= \frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega^2 C^2}} + j\left\{ \left[\frac{1/\omega C}{R_C^2 + \frac{1}{\omega^2 C^2}} \right] - \left[\frac{\omega L}{R_L^2 + \omega^2 L^2} \right] \right\} \ldots\ldots\ldots\ldots\ldots\ldots(27.1)$$

At resonance the susceptance part becomes zero

$$\therefore \frac{\omega_r L}{R_L^2 + \omega_r^2 L^2} = \frac{\frac{1}{\omega_r C}}{R_C^2 + \frac{1}{\omega_r^2 C^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(27.2)$$

$${\omega _r}L\left[{R_C^2 + {1 \over {\omega _r^2{C^2}}}} \right]={1 \over {{\omega _r}C}}\left[{R_L^2 + \omega _r^2{L^2}} \right]$$

$$\omega _r^2\left[{R_C^2 + {1 \over {\omega _r^2{C^2}}}} \right] = {1 \over {LC}}\left[{R_L^2 + \omega _r^2{L^2}} \right]$$

$$\omega _r^2R_C^2 - {{\omega _r^2} \over C}={1 \over {LC}}R_L^2 - {1 \over {{C^2}}}$$

$$\omega _r^2\left[{R_C^2 - {L \over C}} \right]={1 \over {LC}}\left[{R_L^2 - {L \over C}} \right]$$

$${\omega _r}={1 \over {\sqrt {LC} }}\sqrt {{{R_L^2 - \left({L/C} \right)} \over {R_C^2 - \left({L/C} \right)}}} \qquad \left({27.3} \right)$$

The condition for resonant frequency is given by EQ.8.16. As a special case, if $R_L = R_C$, then Eq.27.3 becomes

$${\omega _r}={1 \over {\sqrt {LC} }}$$

Therefore $${f_r}={1 \over {2\pi \sqrt {LC} }}$$

27.2. Resonant Frequency for a Tank Circuit

The parallel resonant circuit is generally called a tank circuit because of the fact that the circuit stores energy in the magnetic field of the coil and in the electric field of the capacitor. The stored energy is transferred back and forth between the capacitor and coil and vice-versa. The tank circuit is shown in Fig.27.2. The circuit is said to be in resonant condition when the susceptance part of admittance is zero.

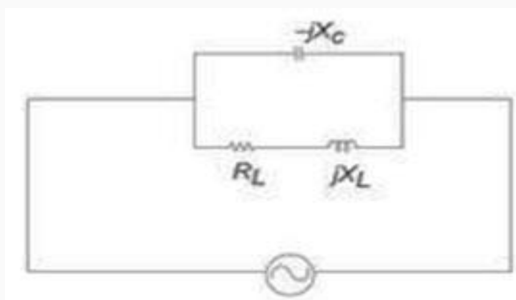

Fig.27.2

The total admittance is

$$Y={1 \over {{R_L} + j{X_L}}} + {1 \over { - j{X_C}}} \qquad \left({27.4} \right)$$

Simplifying Eq. 27.4, we have

$$Y={{{R_L} - j{X_L}} \over {R_L^2 + X_L^2}} + {j \over {{X_C}}}$$

$$={{{R_L}} \over {R_L^2 + X_L^2}} + j\left[{{1 \over {{X_C}}} - {{{X_L}} \over {R_L^2 + X_L^2}}} \right]$$

To satisfy the condition for resonance, the susceptance part is zero.

$${1 \over {{X_C}}}={{{X_L}} \over {R_L^2 + X_L^2}} \qquad \left({27.5} \right)$$

$$\omega C={{\omega L} \over {R_L^2 + {\omega ^2}{L^2}}} \qquad \left({27.6} \right)$$

From Eq.27.6, we get

$$R_L^2 + {\omega ^2}{L^2}={L \over C}$$

$${\omega ^2}{L^2}={L \over C} - R_L^2$$

$${\omega ^2}={1 \over {LC}} - {{R_L^2} \over {{L^2}}}$$

$$\,\omega=\sqrt {{1 \over {LC}} - {{R_L^2} \over {{L^2}}}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \left({27.7} \right)$$

The resonant frequency for the tank circuit is

$${f_r}={1 \over {2\pi }}\sqrt {{1 \over {LC}} - {{R_L^2} \over {{L^2}}}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \left({27.8} \right)$$

27.3. Variation of Impedance with Frequency

The impedance of a parallel resonant circuit is maximum at the resonant frequency and decreases at lower and higher frequencies as shown in Fig. 27.3.

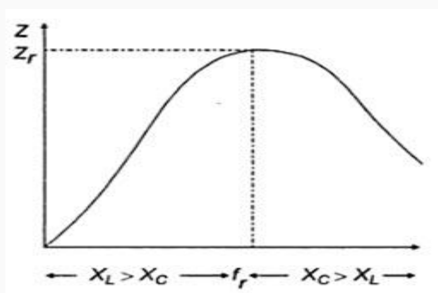

Fig.27.3

At very low frequencies, X_L is very small and X_C is very large, so the total impedance is essentially inductive. As the frequency increases, the impedance also increases, and the inductive reactance dominates until the resonant frequency is reached. At this point X_L=X_C and the impedance is at its maximum. As the frequency goes above resonance, capacitive reactance dominates and the impedance decreases.

27.4. Q Factor of Parallel Resonance

Consider the parallel RLC circuit shown in Fig. 27.4.

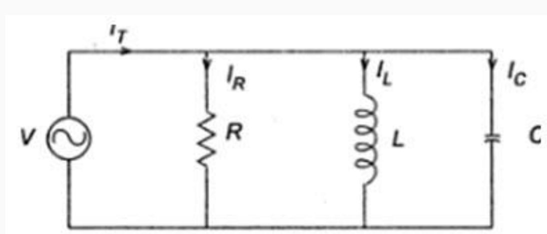

Fig.27.4

In the circuit shown, the condition for resonance occurs when the susceptance part is zero.

Admittance Y = G + jB $$\ldots\ldots\left({27.9} \right)$$

$$={1 \over R} + j\omega C + {1 \over {j\omega L}}$$

$$={1 \over R} + j\left({\omega C + {1 \over {\omega L}}} \right)\ldots\ldots\left({27.10} \right)$$

The frequency at which resonance occurs is

$${\omega _r}C - {1 \over {{\omega _r}L}}=0\ldots\ldots\left({27.11} \right)$$

The voltage and current variation with frequency is shown in Fig. 27.5. At resonant frequency, the current is minimum.

The bandwidth, $BW=f_2-f_1$

For parallel circuit, to obtain the lower power half frequency,

$${\omega _1}C - {1 \over {{\omega _1}L}}={1 \over R}\ldots\ldots\left({27.12} \right)$$

From Eq.27.12, we get

$$\omega _1^2 + {{{\omega _1}} \over {RC}} - {1 \over {LC}}=0\ldots\ldots\left({27.13} \right)$$

If we simplify Eq.27.13, we get

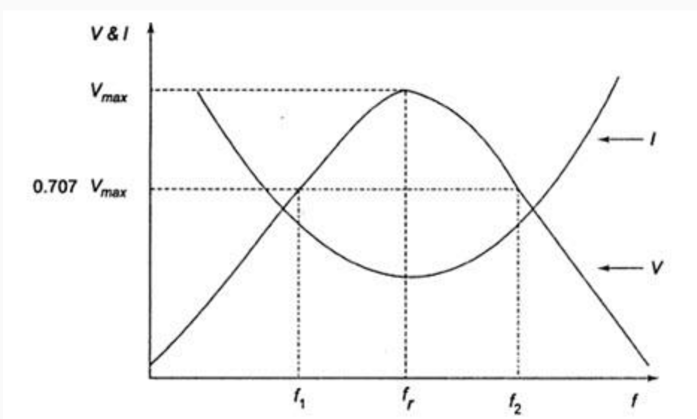

Fig.27.5

$${\omega _1}={{ - 1} \over {2RC}} + \sqrt {{{\left({{1 \over {2RC}}} \right)}^2} + {1 \over {LC}}} \ldots\ldots\left({27.14} \right)$$

Similarly, to obtain the upper half power frequency

$${\omega _2}C - {1 \over {{\omega _2}L}}={1 \over R}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\left({27.15} \right)$$

From Eq.27.15, we have

$${\omega _2}={1 \over {2RC}} + \sqrt {{{\left({{1 \over {2RC}}} \right)}^2} + {1 \over {LC}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\left({27.16} \right)$$

Bandwidth $BW={\omega _2} - {\omega _1}={1 \over {RC}}$

The quality factor is defined as $${Q_r}={{{\omega _r}} \over {{\omega _2}{\omega _1}}}$$

$${Q_r}={{{\omega _r}} \over {1/RC}}={\omega _r}RC$$

In other words,

$$Q=2\pi\times {{\max imum\,\,energy\,\,stored} \over {Energy\,\,dissipated\,\,/\,cycle}}$$

In the case of an inductor,

The maximum energy stored $={1 \over 2}L{I^2}$

Energy dissipated per cycle $={\left({{I \over {\sqrt 2 }}} \right)^2} \times R \times T$

The quality factor

$$Q = 2\pi \times \frac{1/2\left(LI^2\right)}{\frac{I^2}{2}R \times \frac{1}{f}}$$

$$\therefore\ \ Q = 2\pi \times \frac{\frac{1}{2}L\left(\frac{V}{\omega L}\right)^2 R}{\frac{V^2}{2} \times \frac{1}{f}}$$

$$= \frac{2\pi f LR}{\omega^2 L^2} = \frac{R}{\omega L}$$

For a capacitor, maximum energy stored = ½ (CV^2)

Energy dissipate per cycle $=P \times T={{{V^2}} \over {2 \times R}} \times {1 \over f}$

The quality factor

$$Q = 2\pi \times \frac{1/2\left(CV^2\right)}{\frac{V^2}{2R} \times \frac{1}{f}}$$

$$=2\pi fCR=\omega CR$$

Module 12. Classification of filters

LESSON 28. Classification of filters

28.1. Classification of filters

Wave filters were first invented by G.ACampbell and O.I. Lobel of the Bell Telephone Laboratories. A filter is a reactive network that freely passes the desired bands of frequencies while almost totally suppressing all other bands. A filter is constructed from purely reactive elements, for otherwise the attenuation would never become zero in the pass band of the filter network. Filters differ from simple resonant circuits in providing a substantially constant transmission over the band which they accept; this band may lie between any limits depending on the design. Ideally, filters should produce no attenuation in the desired band, called the transmission band or pass band, an should provide total or infinite attenuation at all other frequencies, called attenuation band or stop band. The frequency which separates the transmission band and the attenuation band is defined as the cut-off frequency of the wave filters, and is designated by fc.

Filter networks are widely used in communication systems to separate various voice channels in carrier frequency telephone circuits. Filters also find applications in instrumentation, telemetering equipment, etc. where it is necessary to transmit or attenuate a limited range of frequencies.

A filter may, principle, have any number of pass bands separated by attenuation bands. However, they are classified into four common types, viz. low pass, high pass, band pass and band elimination.

Decibel and Neper

The attenuation of a wave filter can be expressed in decibels or nepers. Neper is defined as the natural logarithm of the ratio of input voltage (or current) to the output voltage (or current), provided that the network is properly terminated in its characteristic impedance Z_0.

From Fig. 28.1 (a) the number of nepers, $$N={\log _e}\left[{{{V_1}} \over {{V_2}}} \right]\,or\,\,{\log _e}\left[{{{I_1}} \over {{I_2}}} \right]$$

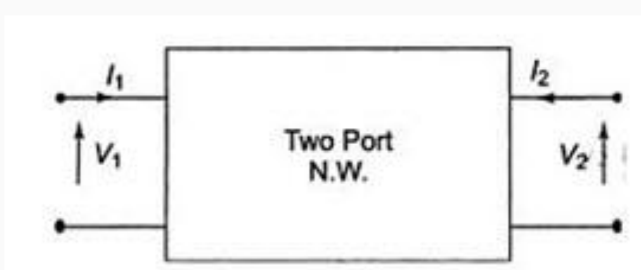

Fig. 28.1 (a)

A neper can also be expressed in terms of input power, P_1 and the output power P_2 as $N = \frac{1}{2} \log_e P_1/P_2$.

A decibel is defined as en times the common logarithms of the ratio of the input power to the output power.

Decibel $$D=10\,{\log _{10}}{{P_1} \over {P_2}}$$

The decibel can be expressed in terms of the ratio of input voltage (or current) and the output voltage (or current).

$$D=10\,{\log _{10}}\left[{{{P_1} \over {P_2}}} \right]\,\,=20\,\,{\log _{10}}\left[{{{I_1} \over {I_2}}} \right]$$

One decibel is equal to 0.115 N.

Low Pass Filter

By definition, a low pass (LP) filter is one which passes without attenuation all frequencies up to the cut-off frequency f_c, and attenuates all other frequencies greater than f_c. The attenuation characteristic of an ideal LP filter is shown in Fig.28.1 (b). This transmits currents of all frequencies from zero up to the cut-off frequency. The band is called pass band or transmission band. Thus, the pass band for the LP filter is the frequency range 0 to f_c. The frequency range over which transmission does not take place is called the stop band or attenuation band. The stop band for a LP filter is the frequency range above f_c.

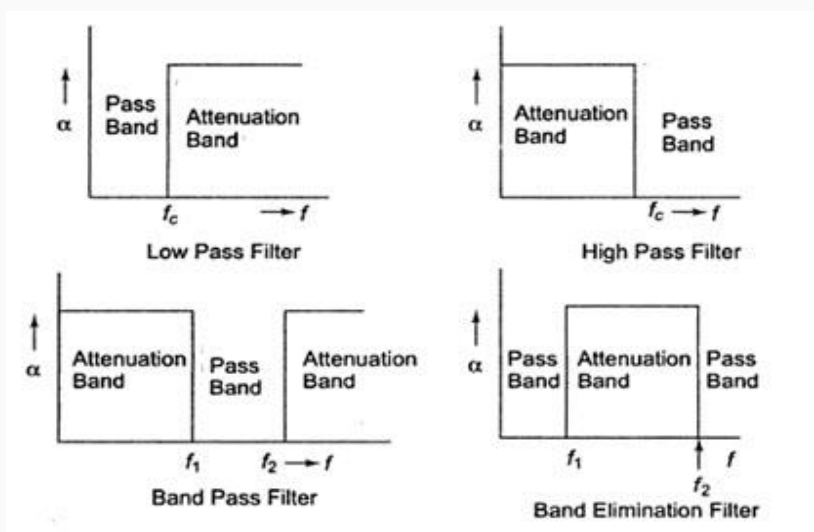

Fig.28.1 (b)

High Pass Filter

A high pass (HP) filter attenuates all frequencies below a designated cut-off frequency, f_c, and passes all frequencies above f_c. Thus the pass band of this filter is the frequency range above f_c, and the stop band is the frequency range below f_c. The attenuation characteristic of a HP filter is shown in Fig.28.1 (b).

Band Pass Filter

A band pass filter passes frequencies between two designated cut-of frequencies and attenuates all other frequencies. It is abbreviated as BP filter. As shown in Fig.28.1 (b), a BP filter has two cut-off frequencies and will have the pass band $f_2 - f_1$; f_1 is called the lower cut-off frequency, while f_2 is called the upper cut-off frequency.

Band Elimination Filter

A band elimination filter passes all frequencies lying outside a certain range, while it attenuates all frequencies between the two designated frequencies. It is also referred as band stop filter. The characteristic of an ideal band elimination filter is shown in fig.28.1 (b).

All frequencies between f_1 and f_2 will be attenuated while frequencies below f_1 and above f_2 will be passed.

28.2. Filter Networks

Ideally a filter should have zero attenuation in the pass band. This condition can only be satisfied if the elements of the filter are dissipation less, which cannot be realized in practice. Filters are designed with an assumption that the elements of the filters are purely reactive. Filters are made of symmetrical T, or p sections. T and p sections can be considered as combinations of unsymmetrical L sections as shown in Fig.28.2.

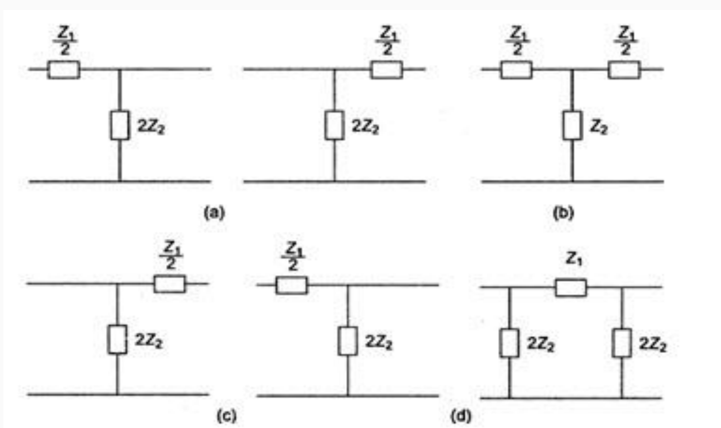

Fig.28.2

The ladder structure is one of the commonest forms of filter network. A cascade connection of several T and \[\pi\] sections constitutes a ladder network. A common form of the ladder network is shown in Fig.28.3.

Figure 28.3 (a) represents a T section ladder network, whereas Fig.28.3 (b) represents the \[\pi\] section ladder network. It can be observed that both networks are identical except at the ends.

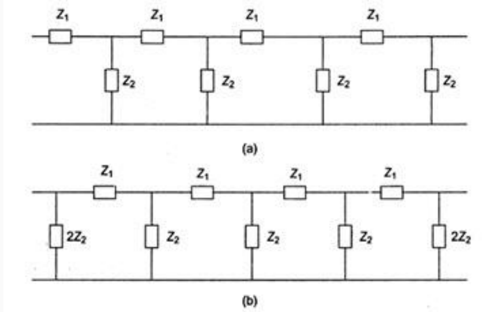

Fig. 28.3

28.3. Equations of Filter Networks

The study of the behavior of any filter requires the calculation of its propagation constant \[\gamma\] , attenuation \[\alpha\] , phase shift β and its characteristic impedance Z_0.

T-Network

Consider a symmetrical T-network as shown in Fig.28.4

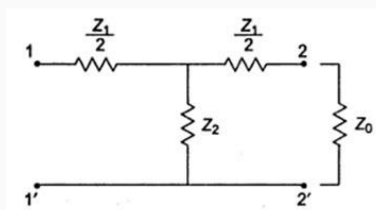

Fig.28.4

If the image impedances at port 1-1′ and port 2-2′ are equal to each other, the image impedance is then called the characteristic, or the iterative impedance, Z_0. Thus, if the network in Fig.28.4 is terminated in Z_0, its input impedance will also be Z_0. The value of input impedance fro the T-network when it is terminated in Z_0 is given by

$$Z_{in} = \frac{Z_1}{2} + \frac{Z_2\left(\frac{Z_1}{2} + Z_0\right)}{\frac{Z_1}{2} + Z_2 + Z_0}$$

also $$Z_{in} = Z_0$$

$$Z_0 = \frac{Z_1}{2} + \frac{2Z_2\left(\frac{Z_1}{2} + Z_0\right)}{Z_1 + 2Z_2 + 2Z_0}$$

$$Z_0 = \frac{Z_1}{2} + \frac{(Z_1Z_2 + 2Z_2Z_0)}{Z_1 + 2Z_2 + 2Z_0}$$

$$Z_0 = \frac{Z_1^2 + 2Z_1Z_2 + 2Z_1Z_0 + 2Z_1Z_2 + 4Z_0Z_2}{2(Z_1 + 2Z_2 + 2Z_0)}$$

$$4Z_0^2 = Z_1^2 + 4Z_1Z_2$$

$$4Z_0^2 = \frac{Z_1^2}{4} + Z_1Z_2$$

The characteristic impedance of a symmetrical T-section is

$${Z_{0T}}=\sqrt {{{Z_1^2} \over 4} + Z_1^{}{Z_2}} \text{ ....} \left({28.1} \right)$$

Z_{0T} can also be expressed in terms of open circuit impedance Z_{OC} and short circuit impedance Z_{SC} of the T-netowrk. From Fig.17.4, the open circuit impedance $Z_{OC}^{}={{{Z_1}} \over 2} + {Z_2}\,\,\text{and}$

$$Z_{sc} = \frac{Z_1}{2} + \frac{\frac{Z_1}{2} + Z_2}{\frac{Z_1}{2} + Z_2}$$

$$Z_{sc} = \frac{Z_1^2 + 4Z_1Z_2}{2Z_1 + 4Z_2}$$

$$Z_{OC} \times Z_{SC} = Z_1Z_2 + \frac{Z_1^2}{4}$$

$$= Z_{OT}^2 \ \ or \ \ Z_{OT} = \sqrt{Z_{OC}Z_{SC}} \text{ ....} (28.2)$$

Propagation Constant of T-Network

By definition the propagation constant g of the network in Fig.28.5 is given by

$\gamma = \log_e I_1/I_2$

Writing the mesh equation for the 2nd mesh, we get

$$I_1Z_2 = I_2\left(\frac{Z_1}{2} + Z_2 + Z_0\right)$$

$$\frac{I_1}{I_2} = \frac{\frac{Z_1}{2} + Z_2 + Z_0}{Z_2}$$

$$\frac{I_1}{I_2} = \frac{\frac{Z_1}{2} + Z_2 + Z_0}{Z_2} = e^{\gamma}$$

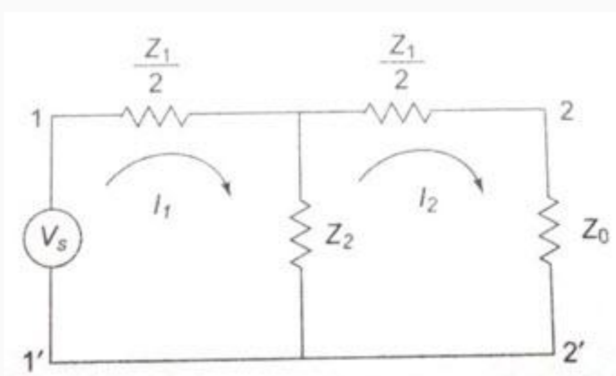

Fig.28.5

$$ {{{Z_1}} \over 2} + {Z_2} + {Z_0}={Z_2}{e^\gamma } $$

$$ {Z_0}={Z_2}\left({{e^\gamma } - 1} \right) - {{{Z_1}} \over 2} \ldots \left({28.3} \right) $$

The characteristic impedance of a T-network is given by

$$ {Z_{OT}}=\sqrt {{{Z_1^2} \over 4} + {Z_1}{Z_2}} \ldots \left({28.4} \right) $$

Squaring Eqs. 28.3 and 28.4 and subtracting Eq.28.4 from Eq.28.3, we get

$$ Z_2^2{\left({{e^\gamma } - 1} \right)^2} + {{Z_1^2} \over 4} - {Z_1}{Z_2}\left({{e^\gamma } - 1} \right) - {{Z_1^2} \over 4} - {Z_1}{Z_2}=0 $$

$$ Z_2^2{\left({{e^\gamma } - 1} \right)^2} - {Z_1}{Z_2}\left({1 + {e^\gamma } - 1} \right)=0 $$

$$ Z_2^2{\left({{e^\gamma } - 1} \right)^2} - {Z_1}{Z_2}{e^\gamma }=0 $$

$$ {Z_2}{\left({{e^\gamma } - 1} \right)^2} - {Z_1}{e^\gamma }=0 $$

$$ {\left({{e^\gamma } - 1} \right)^2}={{{Z_1}{e^\gamma }} \over {{Z_2}}} $$

$$ {e^\gamma } + 1 - 2{e^\gamma }={{{Z_1}} \over {{Z_2}{e^{ - \gamma }}}} $$

Rearranging the above equation, we have

$$ {e^{ - \gamma }}\left({{e^{2\gamma }} + 1 - 2{e^\gamma }} \right) = {{{Z_1}} \over {{Z_2}}} $$

$$ \left({{e^\gamma } + {e^{ - \gamma }} - 2} \right)={{{Z_1}} \over {{Z_2}}} $$

Dividing both sides by 2, we have

$$ {{{e^\gamma } + {e^{ - \gamma }}} \over 2}=1 + {{{Z_1}} \over {2{Z_2}}} $$

$$ \text{Cosh}\ \gamma =1 + {{{Z_1}} \over {2{Z_2}}} \ldots \left({28.5} \right) $$

Still another expression may be obtained for the complex propagation constant in terms of the hyperbolic tangent rather than hyperbolic cosine.

$$ \sinh \,\gamma=\sqrt {\cos \,{h^2}\gamma-1} $$

$$ =\sqrt {{{\left({1 + {{{Z_1}} \over {2{Z_2}}}} \right)}^2}-1}=\sqrt {{{{Z_1}} \over {{Z_2}}} + {{\left({{{{Z_1}} \over {2{Z_2}}}} \right)}^2}} $$

$$ \text{Sinh}\,\,\gamma=\,{1 \over {{Z_2}}}\sqrt {{Z_1}{Z_2} + {{Z_1^2} \over 4}}={{{Z_{0T}}} \over {{Z_2}}} \ldots \left({28.6} \right) $$

Dividing by Eq.28.6 by Eq.28.5, we get

$$\tanh \gamma = \frac{Z_{0T}}{Z_2 + \frac{Z_1}{2}}$$

But $${Z_2} + {{{Z_1}} \over {{Z_2}}}={Z_{0C}}$$

Also from Eq. 28.2, $$\tanh \,\,\gamma=\sqrt {{{{Z_{sc}}} \over {{Z_{0c}}}}}$$

$$\tanh \,\,\gamma=\sqrt {{{{Z_{sc}}} \over {{Z_{oc}}}}}$$

Also $$\sinh \,{\gamma\over 2}=\sqrt {{1 \over 2}\left({\cosh \,\,\gamma-1} \right)}$$

Where $$\cosh \,\,\gamma=1 + \left({{Z_1}/2{Z_2}} \right)$$

$$=\sqrt {{{{Z_1}} \over {4{Z_2}}}} \text{ ....} \left({28.7} \right)$$

π -Network

Consider asymmetrical p-section shown in Fig.28.6. When the network is terminated in Z_0 at port 2-2′, it input impedance is given by

$$Z_{in} = \frac{2Z_2\left[Z_1 + \frac{2Z_2\, Z_0}{2Z_2 + Z_0}\right]}{Z_1 + \frac{2Z_2 + Z_0}{2Z_2 + Z_0} + 2Z_2}$$

By definition of characteristic impedance, $Z_{in} = Z_0$

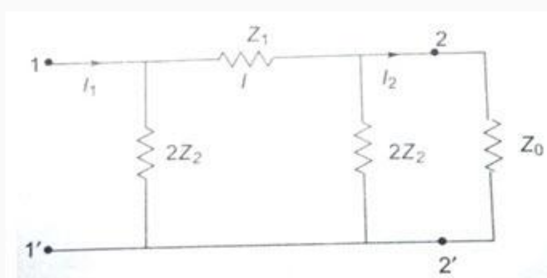

Fig.28.6

$$Z_0 = \frac{2Z_2\left[Z_1 + \frac{2Z_2\, Z_0}{2Z_2 + Z_0}\right]}{Z_1 + \frac{2Z_2 + Z_0}{2Z_2 + Z_0} + 2Z_2}$$

$$Z_0Z_1 + \frac{2Z_2Z_0^2}{2Z_2 + Z_0} + 2Z_0Z_2 = \frac{2Z2(2Z_1Z_2 + Z_0Z_1 + 2Z_0Z_2)}{(2Z_2 + Z_0)}$$

$$2Z_0Z_1Z_2 + Z_1Z_0^2 + 2Z_0^2Z_2 + 4Z_2^2Z_0 + 2Z_2Z_0^2$$
$$= 4Z_1Z_2^2 + 2Z_0Z_1Z_2 + 4Z_0Z_2^2$$

$$Z_1 Z_0^2 + 4Z_2 Z_0^2 = 4Z_1 Z_2^2$$

$$Z_0^2(Z_1 + 4Z_2) = 4Z_1 Z_2^2$$

$$Z_0^2 = \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2}$$

Rearranging the above equation leads to

$$ {Z_0}=\sqrt {{{{Z_1}{Z_2}} \over {1 + {Z_1}/4{Z_2}}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \left({28.8} \right) $$

which is the characteristic impedance of a symmetrical pnetwork,

$$ {Z_{0\pi }}=\sqrt {{{{Z_1}{Z_2}} \over {{Z_1}{Z_2} + Z_1^2/4}}} $$

From Eq. 28.1 $$ {Z_{OT}}=\sqrt {{{Z_1^2} \over 4} + {Z_1}{Z_2}} $$

$$ {Z_{0\pi }}={{{Z_1}{Z_2}} \over {{Z_{OT}}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \left({28.9} \right) $$

Z_{0p} can be expressed in terms of the open circuit impedance Z_{0c} and short circuit impedance Z_{SC} of the p network shown in Fig.28.6 exclusive of the lead Z_0.

$$ by\,\,\,\,{Z_{0C}}={{2{Z_2}\left({{Z_1} + 2{Z_2}} \right)} \over {{Z_1} + 4{Z_2}}} $$

Similarly, the input impedance at port 1-1′ when port 2-2′ is short circuited is given by

$$ {Z_{sc}}={{2{Z_1}{Z_2}} \over {2{Z_2} + {Z_1}}} $$

Hence $$ {Z_{oc}} \times {Z_{sc}}={{4{Z_1}Z_2^2} \over {{Z_1} + 4{Z_2}}} = {{{Z_1}{Z_2}} \over {1 + {Z_1}/4{Z_2}}} $$

Thus from Eq. 28.8

$$ {Z_{0\pi }}=\sqrt {{Z_{oc}}{Z_{sc}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \left({28.10} \right) $$

Propagation Constant of p-Network

The propagation constant of a symmetrical p-section is the same as that for a symmetrical T-section.

i.e. $$ \cosh \,\,\gamma=1 + {{{Z_1}} \over {2{Z_2}}} $$

LESSON 29. Classification of pass Band and Stop Band

29.1. Classification of pass Band and Stop Band

It is possible to verify the characteristics of filters from the propagation constant of the network. The propagation constant γ, being a function of frequency, the pass band, stop band and the cut-off point, i.e. the point of separation between the two bands, can be identified. For symmetrical T or p-section, the expression for propagation constant γ in terms of the hyperbolic functions is given by Eqs.28.5 and 17.7 in section 28.3. From Eq. 28.7, $\sinh {\gamma\over 2}=\sqrt {{{Z_1} \over {4Z_2}}}$

If Z_1 and Z_2 are both pure imaginary values, their ratio, and hence $Z_1/4Z_2$, will be a pure real number. Since Z_1 and Z_2 may be anywhere in the range from $-j\propto$ t_0 + $j\propto$, $Z_1/4Z_2$ may also have any real value between the infinite limits. Then $\sinh {\gamma\over 2}=\sqrt {Z_1} /\sqrt {4Z_2}$ will also have infinite limits, but may be either real or imaginary depending upon whether $Z_1/4Z_2$ is positive or negative.

We known that the propagation constant is a complex function $\gamma = \alpha + j\beta$, the real part of the complex propagation constant α , is a measure of the change in magnitude of the current or voltage in the network, known as the attenuation constant. β is a measure of the difference in phase between the input and output currents or voltages, known as phase shift constant. Therefore α and β take on different values depending upon the range of $Z_1/4Z_2$, From Eq. 28.7, we have

$$\sinh {\gamma \over 2}=\sinh \left({\alpha\over 2} + {j\beta \over 2} \right)=\sinh {\alpha\over 2}\cos {\beta\over 2} + j\cosh {\alpha\over 2}\sin {\beta\over 2}$$

$$=\sqrt {{Z_1} \over {4Z_2}} \qquad \ldots\ldots\ldots\ldots \left(29.1 \right)$$

Case A

If Z_1 and Z_2 are the same type of reactances, then $\left[{{Z_1} \over {4Z_2}} \right]$ is real and equal to say a+x,

The imaginary part of the Eq. 29.1 must be zero.

$$\cosh {\alpha\over 2}\sin {\beta\over 2}=0 \qquad \ldots\ldots\ldots\ldots \left(29.2 \right)$$

$$\sinh {\alpha\over 2}\cos {\beta\over 2}=x \qquad \ldots\ldots\ldots\ldots \left(29.3 \right)$$

α and β must satisfy both the above equations.

Equation 29.2 can be satisfied if $\beta/2 = 0$ or $n\pi$, where n = 0, 1,2,…, then cos $\beta/2 = 1$ and sinh

$$\alpha /2=x=\sqrt {{{Z_1}} \over {4{Z_2}}}$$

That x should be always positive implies that

$$\sqrt {{{Z_1}} \over {4{Z_2}}}>0\,and\,\alpha=2\,{\sinh ^{-1}}\sqrt {{{Z_1}} \over {4{Z_2}}} \quad\ldots\ldots\ldots\ldots\left({29.4} \right)$$

Since $\alpha \neq 0$, it indicates that the attenuation exists.

Case B

Consider the case of Z_1 and Z_2 being appositive type of reactances, i.e. $Z_1/4Z_2$ is negatie, making $\sqrt {{Z_1}/4{Z_2}}$ imaginary and equal to say jx

The real part of the Eq.29.1 must be zero.

$$\sinh \,{\alpha\over 2}\cos {\beta\over 2}=0\quad\ldots\ldots\ldots\ldots\left({29.5} \right)$$

$$\cosh \,{\alpha\over 2}\sin {\beta\over 2}=x\quad\ldots\ldots\ldots\ldots\left({29.6} \right)$$

(i) When $\alpha = 0$; from Eq. 29.5, sinh $\alpha/2 = 0$. And from Eq.29.6 sin $\beta /2=x=\sqrt {{Z_1}/4{Z_2}}$. But the sine can have a maximum value of 1. Therefore, the above solution is valid only for negative $Z_1/4Z_2$, and having maximum value of unity. It indicates the conduction of pass band with zero attenuation and follows the condition as

$$- 1 \le {{{Z_1}} \over {4{Z_2}}} \le 0$$

$$\beta=2{\sin ^{-1}}\sqrt {{{Z_1}} \over {4{Z_2}}} \quad\ldots\ldots\ldots\ldots\left({29.7} \right)$$

(ii) When $\beta = \pi$, from Eq.29.5, cos $\beta/2$ = . And from Eq.29.6, sin$\beta/2 = \pm1$;

cosh a2 = $x=\sqrt {{Z_1}/4{Z_2}}$.

Since cosh $\alpha/2 \geq 1$, this solution is valid for negative $Z_1/4Z_2$, and having magnitude greater than, or equal to unity. It indicates the condition of stop band since $\alpha = 0$.

$$-\alpha\le {{{Z_1}} \over {4{Z_2}}} \le-1$$

$$\alpha=2{\cos ^{-1}}\sqrt {{{Z_1}} \over {4{Z_2}}} \quad\ldots\ldots\ldots\ldots\left({29.8} \right)$$

It can be observed that there are three limits for case A and B. Knowing the values of Z1 and Z2, it is possible to determine the case to be applied to the filter. Z1 and Z2 are made of different types of reactances, or combinations of reactances, so that, as the frequency changes, a filter may pass from one case to another. Case A and (ii) in case B are attenuation bands, whereas (i) in case B is the transmission band.

The frequency which separates the attenuation band from pass band or vice versa is called cut-off frequency. The cut-off frequency is denoted by f_c, and is also termed as nominal frequency. Since Z_0 is real in the pass band and imaginary in an attenuation band, f_c is the frequency at which Z_0 cahnges from being real to being imaginary. These frequencies occur at

$${{Z_1} \over {4{Z_2}}}=0\,\,or\,\,{Z_1}=0 \qquad (29.8a)$$

$${{Z_1} \over {4{Z_2}}}=1\,\,or\,\,{Z_1} + 4{Z_2}=0 \qquad (29.8b)$$

The above conditions can be represented graphically, as in Fig. 29.1

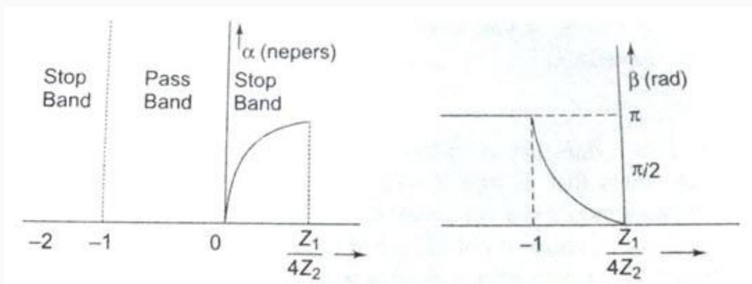

Fig.29.1

29.2. Characteristic Impedance in the Pass and Stop Bands

Referring to the characteristic impedance of a symmetrical T-network, from Eq.28.1 we have

$${Z_{0T}}=\sqrt {{{Z_1^2} \over 4} + {Z_1}{Z_2}}=\sqrt {{Z_1}{Z_2}\left({1 + {{{Z_1}} \over {4{Z_2}}}} \right)}$$

If Z_1 and Z_2 are purely reactive, let $Z_1 = jx_1$ and $Z_2 = jx_2$, then

$${Z_{0T}}=\sqrt {{x_1}{x_2}\left({1 + {{{Z_1}} \over {4{Z_2}}}} \right)} \qquad (29.9)$$

A pass band exists when x_1 and x_2 are of opposite reactance and

$$-1 < {{{x_1}} \over {4{x_2}}} < 0$$

Substituting these conditions in Eq.29.9, we find that Z_{0T} is positive and real. Now consider the stop band. A stop band exists when x_1 and x_2 are of the same type of reactances; then $x_1/4x_2 > 0$. Substituting these conditions in Eq.29.9, we find that Z_{OT} is purely imaginary in this attenuation region. Another stop band exists when x_1 and x_2 are of the same type of reactances, but with $x_1/4x_2 < -1$. Then from Eq.29.9, Z_{0T} is again purely imaginary in the attenuation region.

Thus, in a pass band if a network is terminated in a pure resistance R_0 ($Z_{0T} = R_0$), the input impedance is R_0 and the network transmits the power received from the source to the R_0 without any attenuation. In a stop band Z_{0T} is reactive. Therefore, if the network is terminated in a pure reactance (Z_0 = pure reactance), the input impedance is reactive, and cannot receiver or transmit power. However, the network transmits voltage and current with 90^0 phase difference and with attenuation. It has already been shown that the characteristic impedance of a symmetrical p-section can be expressed in terms of T. Thus, from Eq.29.9,

$$Z_{0p} = Z_1Z_2/Z_{0T}.$$

Since Z_1 and Z_2 are purely reactive, $Z_{0\backslash[\backslash pi\backslash]}$ is real if Z_{0T} is real, and Z_{0x} is imaginary if Z_{0T} is imaginary. Thus the conditions developed for T = sections are valid for \[\pi\] sections.

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Module 13. Constant-k, m-derived, terminating half network and composite filters

LESSON 30. Constant-k filters

30.1. Constant – K Low Pass Filter

A network, either *T* or π, is said to be of the constant-*k* type if Z_1 and Z_2 of the network satisfy the relation

$$Z_1 Z_2 = k^2 \quad \ldots\ldots\ldots\ldots \left({30.1} \right)$$

where Z_1 and Z_2 are impedance in the T and π sections as shown in Fig.17.8. Equation 17.20 states that Z_1 and Z_2 are inverse if their product is a constant, independent of frequency. *k* is a real constant, that is the resistance. *k* is often termed as design impedance or nominal impedance of the constant *k*-filter.

The constant *k, T* or π type filter is also known as the prototype because other more complex networks can be derived from it. A prototype T and π-sections are shown in

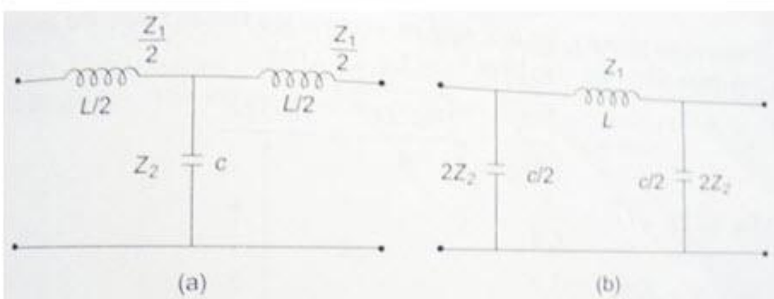

Fig. 30.1

Fig.30.1 (a) and (b), where Z_1 = jω_L and Z_2 = 1/jω_C. Hence Z_1Z_2= ${L \over C}={k^2}$ which is independent of frequency.

$${Z_1}{Z_2}={k^2}={L \over C}\,\,\,or\,\,k=\sqrt {{L \over C}} \quad \ldots\ldots\ldots\ldots \left({30.2} \right)$$

Since the product Z_1 and Z_2 is constant, the filter is a constant-k type. From Eq. 29.8 (a) the cut-off frequencies are $Z_1/4Z_2 = 0$.

i.e. $${{ - {\omega ^2}LC} \over 4}=0$$

i.e. $$f=0\,\,and\,\,{{{Z_1}} \over {4{Z_2}}}=-1$$

$${{ - {\omega ^2}LC} \over 4}=-1$$

or $${f_c}={1 \over {\pi \sqrt {LC} }} \quad \ldots\ldots\ldots\ldots \left({30.3} \right)$$

The pass band can be determined graphically. The reactances of Z_1 and $4Z_2$ will vary with frequency as drawn in Fig.30.2. The cut-off frequency at the intersection of the curves Z_1 and $4Z_2$ is indicated as f_c. On the X-axis as $Z_1 = -4Z_2$ at cut-off frequency, the pass band lies between the frequencies at which $Z_1 = 0$, and $Z_1=-4Z_2$.

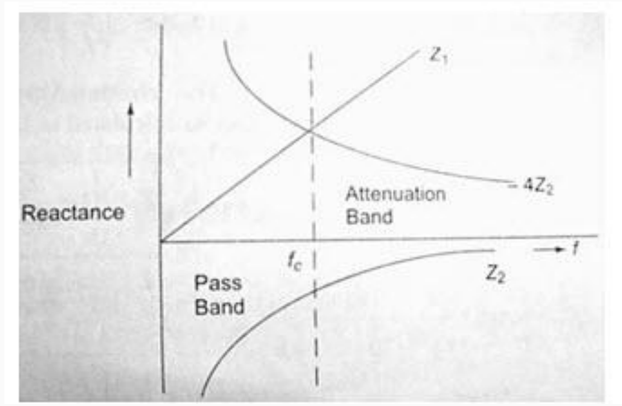

Fig.30.2

All the frequencies above f_c lie in a stop or attenuation band. Thus, the network is called a low-pass filter. We also have from Eq.28.7 (previous chapter) that

$$\sinh {\gamma\over 2}=\sqrt {{{{Z_1}} \over {4{Z_2}}}}=\sqrt {{{ - {\omega ^2}LC} \over 4}}={{J\omega \sqrt {LC} } \over 2}$$

From Eq. 30.3 $\sqrt {LC}={1 \over {{f_c}\pi }}$

$$\sinh {\gamma\over 2}={{j2\pi f} \over {2\pi {f_c}}}=j{f \over {{f_c}}}$$

We also know that in the pass band

$$-1<{{{Z_1}} \over {4{Z_2}}}< 0$$

$$-1<{{ - {\omega ^2}LC} \over 4}< 0$$

$$-1<-\left({{f \over {{f_c}}}} \right)< 0$$

or $${f \over {{f_c}}} < 1$$

and $$\beta=2{\sin ^{ - 1}}\left({{f \over {{f_c}}}} \right);\,\,\alpha=0$$

In the attenuation band,

$${{{Z_1}} \over {4{Z_2}}}<-1,\,\,i.e.{f \over {{f_c}}} < 1$$

$$\alpha=2{\cosh ^{ - 1}}\left[{{{{Z_1}} \over {4{Z_2}}}} \right]=2{\cosh ^{ - 1}}\left[{{f \over {{f_c}}}} \right];\,\beta=\pi$$

The plots of α and β for pass and stop bands are shown in Fig.30.3.

Thus, from Fig.30.3, a = 0, b = 2 $\sinh^{-1}$ $\left({{f \over {{f_c}}}} \right)for\,\,f < {f_c}$

$$\alpha=2{\cosh ^{ - 1}}\left({{f \over {{f_c}}}} \right);\,\beta=\pi \,\,for\,\,f > fc\,\,f < {f_c}$$

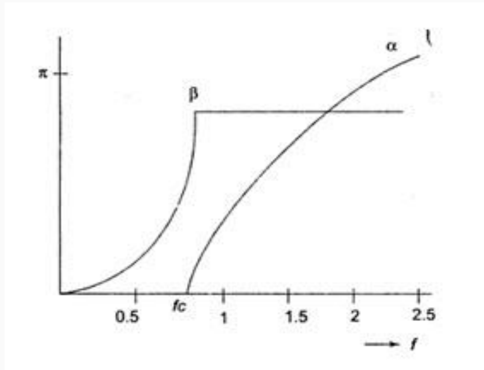

Fig.30.3

The characteristic impedance can be calculated as follows.

$${Z_{0T}}=\sqrt {{Z_1}{Z_2}\left({1 + {{{Z_1}} \over {4{Z_2}}}} \right)}$$

$$=\sqrt {{L \over C}\left({1 + {{{\omega ^2}LC} \over 4}} \right)}$$

$${Z_{0T}}=k{\sqrt {1 - \left({{f \over {{f_c}}}} \right)} ^2}..\left({30.4} \right)$$

From Eq.30.4, Z_{0T} is real when $f < f_c$ i.e. in the pass band at $f = f_c$, $Z_{0T} = 0$; and for $f > f_c$, Z_{0T} is imaginary in the attenuation band, rising to infinite reactance at infinite frequency. The variation of Z_{0T} with frequency is shown in Fig.30.4.

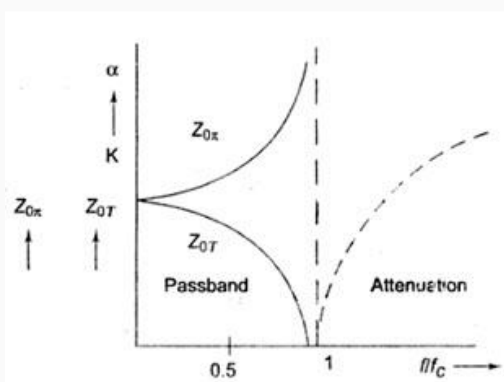

Fig.30.4

Similarly, the characteristic impedance of a π-network is given by

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{k}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(30.5)$$

The variation of $Z_{0\pi}$ with frequency is shown in fig. 30.4 for $f < f_c$, $Z_{0\pi}$ is real; at $f = f_c$, $Z_{0\pi}$ is infinite, and for $f > f_c$, Z_{0T} is imaginary. A low pass filter can be designed from the specifications of cut-off frequency and load resistance.

All cut-off frequency, $Z_1 = -4Z_2$

$$j{\omega _c}L={{ - 4} \over {j{\omega _c}C}}$$

$${\pi ^2}f_c^2LC=1$$

Also we know that $k=\sqrt {L/C}$ is called the design impedance or the load resistance

$${k_2}={L \over C}$$

$${\pi ^2}f_c^2{k^2}{C^2}=1$$

$$C={1 \over {\pi \,{f_c}k}}\,\text{gives}\,\,\text{the}\,\,\text{value}\,\,\text{of}\,\,\text{the}\,\,\text{shunt}\,\text{capacitance}$$

$$\text{and}\,L={k^2}C={k \over {\pi {f_c}}}\,\text{gives}\,\text{the}\,\text{value}\,\text{of}\,\text{the}\,\text{series}\,\text{inductance.}$$

30.2. Constant K-High Pass Filter

Constant K-high pass filter can be obtained by changing the positions of series and shunt arms of the networks shown in Fig.30.1. The prototype high pass filters are shown in Fig.30.5, where $Z_1 = -j/\omega_C$ and $Z_2 = j\omega L$.

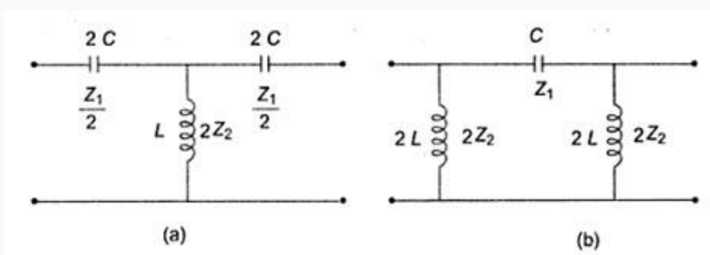

Fig.30.5

Again, it can be observed that the product of Z_1 and Z_2 is independent of frequency, and the filter design obtained will be of the constant k type. Thus, Z_1Z_2 are given by

$${Z_1}{Z_2}={{ - j} \over {\omega C}}j\omega L={L \over C}={k^2}$$

The cut-off frequencies are given by $Z_1 = 0$ and $Z_1 = -4Z_2$.

$${Z_1}\,=\,0\,\text{indicates}\,{j \over {\omega C}}=0,\,\,\text{or}\,\,\omega\to \alpha$$

From $Z_1 = -4Z_2$

$${{ - j} \over {\omega C}}=-4j\omega L$$

$${\omega ^2}LC={1 \over 4}$$

or $${f_c}={1 \over {4\pi \sqrt {LC} }}\qquad\ldots\ldots\left({30.6} \right)$$

The reactance of Z_1 and Z_2 are sketched as functions of frequency as shown in Fig.30.6.

As seen from Fig.30.6, the filter transmits all frequencies between $f = f_c$ and $f = \propto$. The point f_c from the graph is a point at which $Z_1 = -4Z_2$.

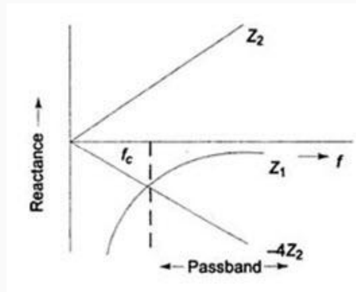

Fig.30.6

$$\sinh {\gamma\over 2}=\sqrt {{{{Z_1}} \over {4{Z_2}}}}=\sqrt {{{ - 1} \over {4{\omega ^2}LC}}}$$

$$From\,Eq.17.25,{f_c}={1 \over {4\pi \sqrt {LC} }}$$

$$\sqrt {LC}={1 \over {4\pi {f_c}}}$$

$$\sinh {\gamma\over 2}=\sqrt {{{ - {{\left({4\pi } \right)}^2}{{\left({{f_c}} \right)}^2}} \over {4{\omega ^2}}}}=j{{{f_c}} \over f}$$

In the pass band, $- 1 < {{{Z_1}} \over {4{Z_2}}} < 0,\alpha=0\,\,or\,the\,region\,which\,{{{f_c}} \over f} < 1\,is\,a\,pass\,band$

$$\beta=2{\sin ^{ - 1}}\left({{{{f_c}} \over f}} \right)$$

In the attenuation band ${{{Z_1}} \over {4{Z_2}}}<-1,i.e.{{{f_c}} \over f} > 1$

$$\alpha=2{\cosh ^{ - 1}}\left({{{{Z_1}} \over {4{Z_2}}}} \right)$$

$$=2{\cos ^{ - 1}}\left({{{{f_c}} \over f}} \right);\,\,\beta=-\pi$$

The plots of a and b for pass and stop bands of a high pass filter network are shown in Fig.30.7

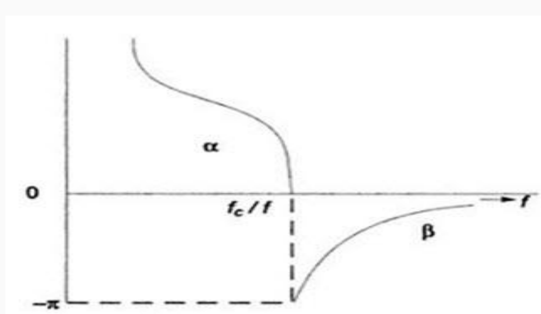

Fig.30.7

A high pass filter may be designed similar to the low pass filter by choosing a resistive load r equal to the constant k, such that $$R=k=\sqrt {L/C}$$

$${f_c}={1 \over {4\pi \sqrt {L/C} }}$$

$${f_c}={k \over {4\pi L}}={1 \over {4\pi Ck}}$$

Since $$\sqrt C={L \over k},$$

$$L={k \over {4\pi {f_c}}}and\,C={1 \over {4\pi {f_c}k}}$$

The characteristic impedance can be calculated using the relation

$${Z_{0T}}=\sqrt {{Z_1}{Z_2}\left({1 + {{{Z_1}} \over {4{Z_2}}}} \right)}=\sqrt {{L \over C}\left({1 - {1 \over {4{\omega ^2}LC}}} \right)}$$

$${Z_{0T}}=k\sqrt {1 - {{\left({{{{f_c}} \over f}} \right)}^2}}$$

Similarly, the characteristic impedance of a p-network is given by

$${Z_{0\pi }}={{{Z_1}{Z_2}} \over {{Z_{0T}}}}={{{k_2}} \over {{Z_{0T}}}}$$

$$= \frac{k}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \quad \text{...} (30.7)$$

The plot of characteristic impedance with respect to frequency is shown in Fig.30.8

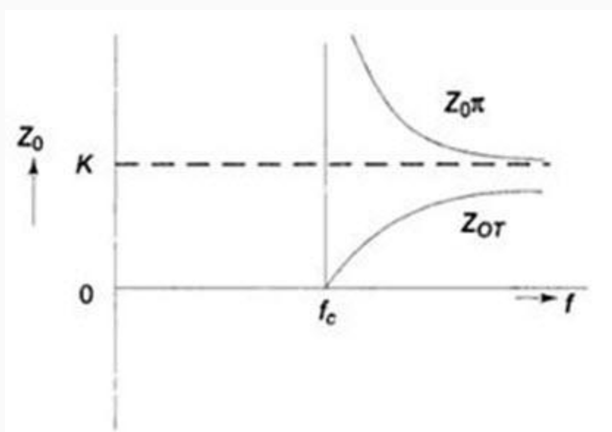

Fig.30.8

LESSON 31. m-derived filters

31.1 m-Derived T-Section

It is clear from previous chapter Figs 30.3 & 30.7 that the attenuation is not sharp in the stop band for k-type filters. The characteristic impedance, Z_0 is a function of frequency and varies widely in the transmission band. Attenuation can be increased in the stop band by using ladder section, i.e. by connecting two or more identical sections. In order to join the filter sections, it would be necessary that their characteristic impedance be equal to each other at all frequencies. If their characteristic impedances match at all frequencies, they would also have the same pass band. However, cascading is not a proper solution from a practical point of view. This is because practical elements have a certain resistance, which gives rise to attenuation in the pass band also. Therefore, any attempt to increase attenuation in stop band by cascading also results in an increase of 'a' in the pass band. If the constant k section is regarded as the prototype, it is possible to design a filter to have rapid attenuation in the stop band, and the same characteristic impedance as the prototype at all frequencies. Such a filter is called m-derived filter. Suppose a prototype T-network shown in Fig.31.1 (a) has the series arm modified as shown in Fig.31.1 (b), where m is a constant. Equating the characteristic impedance of the networks in Fig.31.1, we have

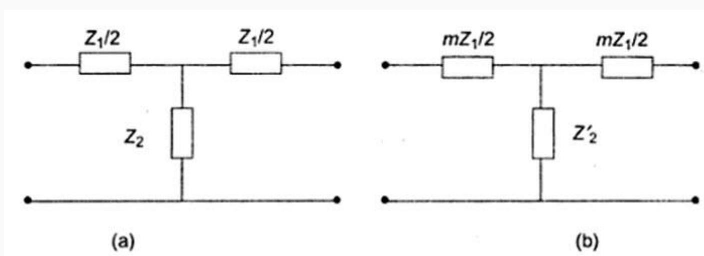

Fig.31.1

$Z_{0T} = Z_{0T'}$

where $Z_{0T'}$ is the characteristic impedance of the modified (m-derived) T-network

$$\sqrt {{{Z_1^2} \over 4} + {Z_1}{Z_2}}=\sqrt {{{{m^2}Z_1^2} \over 4} + m{Z_1}Z_2^1}$$

$${{Z_1^2} \over 4} + {Z_1}{Z_2}={{{m^2}Z_1^2} \over 4} + m{Z_1}Z_2^1$$

$$m{Z_1}Z_2^1={{Z_1^2} \over 4}\left({1 - {m^2}} \right) + {Z_1}{Z_2}$$

$$Z_2^1={{{Z_1}} \over {4m}}\left({1 - {m^2}} \right) + {{{Z_2}} \over m}\ldots\ldots\left({31.1} \right)$$

It appears that the shunt arm Z_2^1 consider of two impedance in series as shown in Fig.31.2

From Eq.31.1, ${{1 - {m^2}} \over {4m}}$ should be positive to realize the impedance Z_2^1 physically, i.e. 0<m<1. Thus m-derived section can be obtained from the prototype by modifying its series and shunt arms. The same technique can be applied to π section network. Suppose a prototype p-network shown in Fig.31.3 (a) has the shunt arm modified as shown in Fig.31.3 (b).

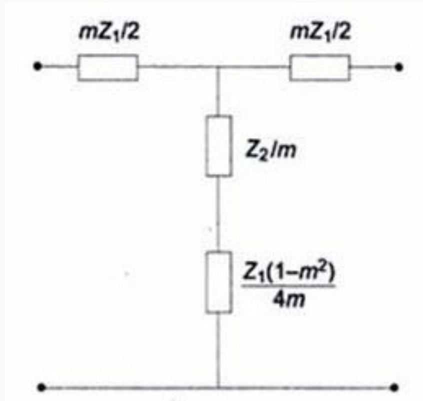

Fig.31.2

The characteristic impedances of the prototype and its modified sections have to be equal for matching.

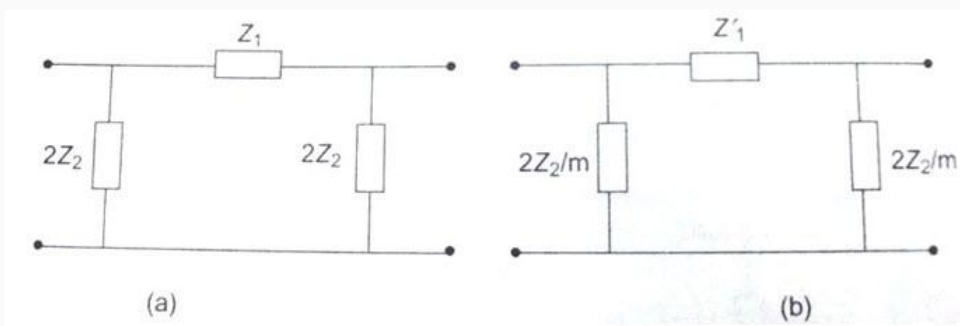

Fig.31.3

$$Z0\pi=Z_{0\pi }^1$$

where $Z_{0\pi }^1$ is the characteristic impedance of the modified (m-derived) π-network.

$$\therefore \quad \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}} = \sqrt{\frac{Z_1^{'}\frac{Z_2}{m}}{1+\frac{Z_1^{'}}{4.Z_2/m}}}$$

Squaring and cross multiplying the above equation results as under.

$$\left({4{Z_1}{Z_2} + mZ_1^{'}{Z_1}} \right)={{4Z_1^{'}{Z_2} + {Z_1}Z_1^{'}} \over m}$$

$$Z_1'\left(\frac{Z_1}{m}+\frac{4Z_2}{m}-mZ_1\right)=4Z_1Z_2$$

Or $$Z_1' = \frac{Z_1Z_2}{\frac{Z_1}{4m}+\frac{Z_2}{m}-\frac{mZ_1}{4}}$$

$$= \frac{Z_1Z_2}{\frac{Z_2}{m}+\frac{Z_1}{4m}\left(1-m^2\right)}$$

$$Z_1' = \frac{Z_1Z_2\frac{4m^2}{\left(1-m^2\right)}}{\frac{Z_2 4m_2}{m\left(1-m_2\right)}+Z_1 m} = \frac{mZ_1\frac{Z_2 4m}{\left(1-m^2\right)}}{mZ_1+\frac{Z_2 4m}{\left(1-m^2\right)}} \quad \text{.......................................}(31.2)$$

It appears that the series arm of the m-derived π section is a parallel combination of mZ_1 and $4mZ_2/1-m^2$. The derived m section is shown in Fig.31.4

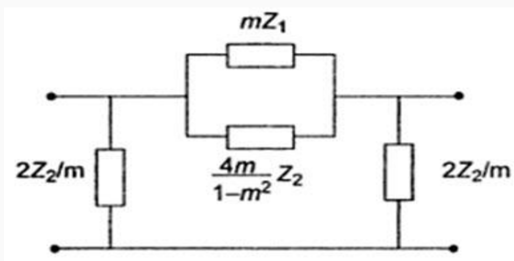

Fig.31.4

m-Derived Low Pass Filter

In Fig.31.5, both m-derived low pass T and π filter sections are shown. For the T-section shown Fig.31.5 (a), the shunt arm is to be chosen so that it is resonant at some frequency f_x above cut-off frequency f_c its impedance will be minimum or zero. Therefore, the output is zero and will correspond to infinite attenuation at this particular frequency. Thus, at f_x.

$$\frac{1}{m\omega_r C}=\frac{1-m^2}{4m}\omega_r L,$$ where ω, is the resonant frequency

$$\frac{1-m^2}{4m}C$$

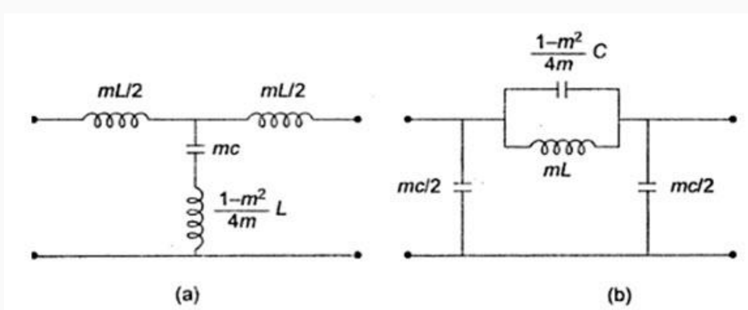

Fig.31.5

$$\omega_r^2={4 \over {\left({1 - {m^2}} \right)LC}}$$

$$f_r={1 \over {\pi \sqrt {LC\left({1 - {m^2}} \right)} }}=f_x$$

Since the cut-off frequency for the low pass filter is $f_c={1 \over {\pi \sqrt {LC} }}$

$$f_x={{f_c} \over {\sqrt {1 - {m^2}} }}\ldots\ldots\ldots\ldots\ldots\ldots\left({31.3} \right)$$

or $$m=\sqrt {1 - {{\left({{{f_c}} \over {{f_\alpha }}} \right)}^2}} \ldots\ldots\ldots\ldots\ldots\ldots\left({31.4} \right)$$

If a sharp cut-off is desired, f_x should be near to f_c, From Eq.31.3, it is clear that for the smaller the value of m, $f_{\propto}$ comes close to f_c, . Equation 31.4 shows that if f_c and $f_{\propto}$ are specified, the necessary value of m may then be calculated. Similarly, for m –derived π section, the inductance and capacitance in the series arm constitute a resonant circuit. Thus, at f_x a frequency corresponds to infinite attenuation, i.e. at f_x.

$$m\omega_r L = \frac{1}{\left(\frac{1-m^2}{4M}\right)\omega_r C}$$

$$\omega_r^2 = \frac{4}{LC(1-m^2)}$$

$$f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}}$$

Since, $f_c={1 \over {\pi \sqrt {LC} }}$

$$f_r={{f_c} \over {\sqrt {1 - {m^2}} }}=f_\alpha \ldots\ldots\ldots\ldots\ldots\ldots\left({31.5} \right)$$

Thus for both m-derived low pass networks for a positive value of m (0< m <1), $f_x > f_c$. Equations 31.4 or 31.5 can be used to choose the value of m, knowing f_c and f_r. After the value of m is evaluated, the elements of the T or p-network can be found from Fig.31.5. The variation of attenuation for a low pass m-derived section can be verified from α = 2 $\cosh^{-1} \sqrt {{Z_1}/4{Z_2}} \,for\,{f_c} < f < {f_\alpha }.$ For $Z_1 = j\omega L$ and $Z_1 = -J/\omega C$ for the prototype.

$$\therefore \qquad \alpha = 2\cosh^{-1}\frac{m\frac{f}{f_c}}{\sqrt{1-\left(\frac{f}{f_\alpha}\right)^2}}$$

And $$\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_1}\right|} = 2\sin^{-1}\frac{m\frac{f}{f_c}}{\sqrt{1-\left(\frac{f}{f_c}\right)^2(1=m)^2}}$$

Figure 31.6 shows the variation of α , β and Z_0 with respect to frequency for an m-derived low pas filter.

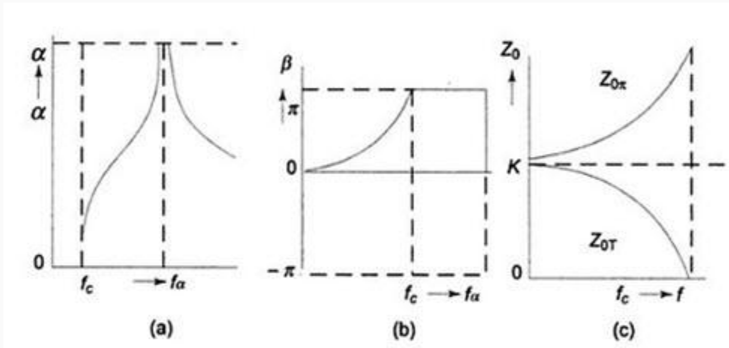

Fig.31.6

m-derived High Pass Filter

In Fig.31.7 both m-derived high pass T and π-sections are shown.

If the shunt arm in T-section is series resonant, it offers minimum or zero impedance. Therefore, the output is zero and, thus, at resonance frequency, or the frequency corresponds to infinite attenuation.

$$\omega_r \frac{L}{m} = \frac{1}{\omega_r \frac{4m}{1-m^2} C}$$

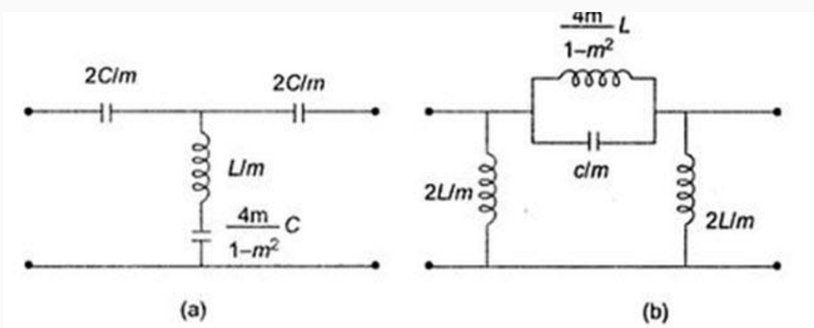

Fig.31.7

$$\omega_r^2 = \omega_\alpha^2 = \frac{1}{\frac{L}{m}\frac{4m}{1-m^2}C} = \frac{1-m^2}{4LC}$$

$$\omega_\alpha = {\sqrt{1 - m^2} \over 2\sqrt{LC}} \text{ or } \, f_\alpha = {\sqrt{1 - m^2} \over 4\pi\sqrt{LC}}$$

From Eq.30.6, the cut-off frequency f_c of a high pass prototype filter is given by

$$f_c = {1 \over 4\pi\sqrt{LC}}$$

$$f_\alpha = f_c\sqrt{1 - m^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \left(31.6\right)$$

$$m = \sqrt{1 - \left(\frac{f_\alpha}{f_c}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \left(31.7\right)$$

Similarly, for the m-derived π-section, the resonant circuit is constituted by the series arm inductance nd capacitance. Thus, at fx.

$$\frac{4m}{1-m^2}\omega_r L = \frac{1}{\frac{\omega_r}{m}C}$$

$$\omega_r^2 = \omega_\alpha^2 = \frac{1-m^2}{4LC}$$

$$\omega_\alpha = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \; or \; f_\alpha = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

Thus, the frequency corresponding to infinite attenuation is the same for both sections.

Equation 31.7 may be used to determine m for a given fx and fc. The elements of the m-derived high pass T or π-sections can be found from Fig.31.7. The variation of α, β and Z_0 with frequency is shown in Fig.31.8

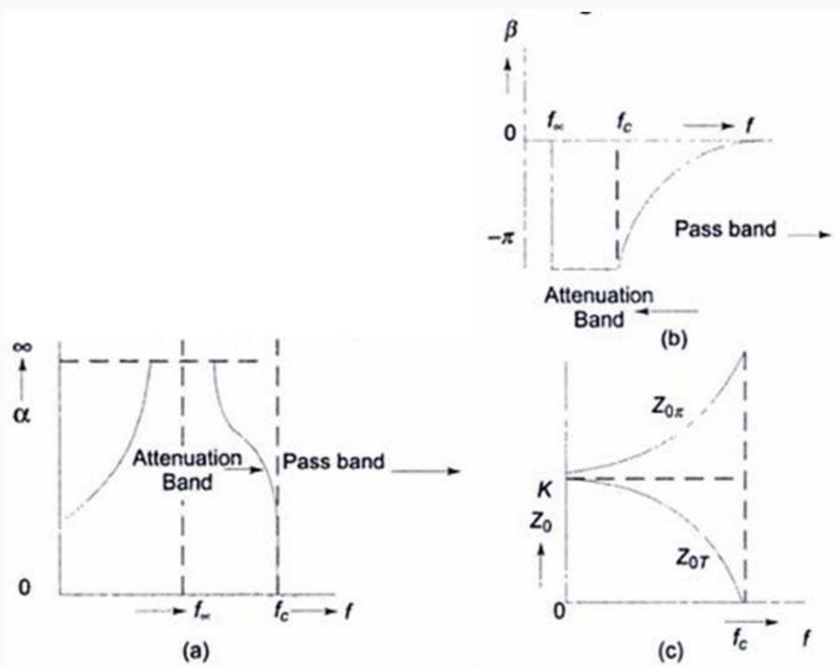

Fig. 31.8

LESSON 32. Terminating half network and composite filters

32.1. Band Pass Filter

A band pass filter is one which attenuates all frequencies below a lower cut-off frequency f_1 and above an upper cut-off frequency f_2. Frequencies lying between f_1 and f_2 comprise the pass band, and are transmitted with zero attenuation. A band pass filter may be obtained by using a low pass filter followed by a high pass filter in which the cut-off frequency of the LP filter is above the cut-off frequency of the HP filter, the overlap thus allowing only a band of frequencies to pass. This is not economical in practice; it is more economical to combine the low and high pass functions into a single filter section.

Consider the circuit in Fig.32.1, each arm has a resonant circuit with same resonant frequency, i.e. the resonant frequency of the series arm and the resonant frequency of the shunt arm are made equal to obtain the band pass characteristic.

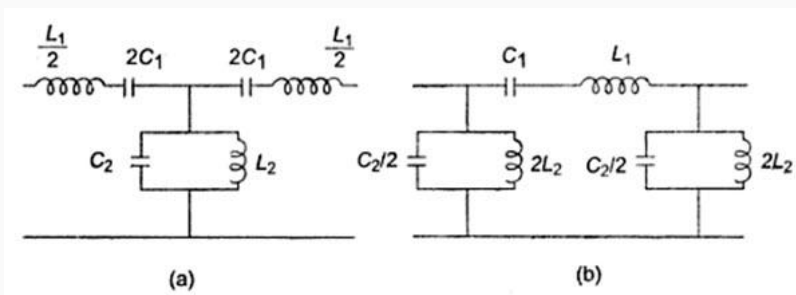

Fig.32.1

For this condition of equal resonant frequencies.

$$\frac{\omega_0 L_1}{2}=\frac{1}{2\omega_0 C_1}\,\,\text{for}\,\,\text{the}\,\,\text{sereis}\,\,\text{arm}$$

From which, $$\omega_0^2 L_1 C_1=1 \ldots\ldots\ldots\ldots \left(32.1\right)$$

and $$\frac{1}{\omega_0 C_2}=\omega_0 L_2\,\text{for}\,\text{the}\,\text{shunt}\,\text{arm}$$

from which, $$\omega_0^2 L_2 C_2=1 \ldots\ldots\ldots\ldots \left(32.2\right)$$

$$\omega_0^2 L_1 C_1=1=\omega_0^2 L_2 C_2$$

$$L_1 C_1=L_2 C_2 \ldots\ldots\ldots\ldots \left(32.3\right)$$

The impedance of the series arm, Z_1 is given by

$$Z_1=\left(j\omega L_1-\frac{j}{\omega C_1}\right)=j\left(\frac{\omega^2 L_1 C_1-1}{\omega C_1}\right)$$

The impedance of the shunt arm, Z_2 is given by

$$Z_2 = \frac{j\omega L_2 \dfrac{1}{j\omega C_2}}{j\omega L_2 + \dfrac{1}{j\omega C_2}} = \frac{j\omega L_2}{\omega C_1}$$

$${Z_1}{Z_2}=j\left({{{\omega ^2}{L_1}{C_1} - 1} \over {\omega {C_1}}} \right)\left({{j\omega {L_2}} \over {1 - {\omega ^2}{L_2}{C_2}}} \right)$$

$$={{ - {L_2}} \over {{C_1}}}\left({{{\omega ^2}{L_1}{C_1} - 1} \over {1 - {\omega ^2}{L_2}{C_2}}} \right)$$

From Eq.32.3, $L_1C_1 = L_2C_2$

$${Z_1}{Z_2}={{{L_2}} \over {{C_1}}}={{{L_1}} \over {{C_2}}}={k^2}$$

Where k is constant. Thus, the filter is a constant k-type. Therefore, for a constant k-type in the pass band.

$$- 1 < {{{Z_1}} \over {4{Z_2}}} < 0,\,\,and\,\,at\,cut - off\,frequency$$

$Z_1 = 4Z_2$

$$Z_1^2=-4{Z_1}{Z_2}=-4{k^2}$$

$${Z_1}=\pm j2k$$

i.e. the value of Z_1 at lower cut-off frequency is equal to the negative of the value of Z_1 at the upper cut-off frequency.

$$\left({{1 \over {j{\omega _1}{C_1}}} + j{\omega _1}{L_1}} \right)=-\left({{1 \over {j{\omega _2}{C_1}}} + j{\omega _2}{L_1}} \right)$$

or $$\left({{\omega _1}{L_1}{1 \over {{\omega _1}{C_1}}}} \right)=\left({{1 \over {{\omega _2}{C_1}}} - {\omega _2}{L_1}} \right)$$

$$\left({1 - \omega _1^2{L_1}{C_1}} \right)={{{\omega _1}} \over {{\omega _2}}}\left({\omega _2^2{L_1}{C_1} - 1} \right)\ldots\ldots\ldots\ldots\ldots\left({32.4} \right)$$

From Eq.32.1, $${L_1}{C_1}={1 \over {\omega _0^2}}$$

Hence Eq.32.4 may be written as

$$\left({1 - {{\omega _1^2} \over {\omega _0^2}}} \right)={{{\omega _1}} \over {{\omega _2}}}\left({{{\omega _2^2} \over {\omega _0^2}} - 1} \right)$$

$$\left({\omega _0^2 - \omega _1^2} \right){\omega _2}={\omega _1}\left({\omega _2^2 - \omega _0^2} \right)$$

$$\omega _0^2{\omega _2} - \omega _1^2{\omega _2}={\omega _1}\omega _2^2 - {\omega _1}\omega _0^2$$

$$\omega _0^2\left({\omega _1^{} + {\omega _2}} \right)={\omega _1}{\omega _2}\left({{\omega _2} + {\omega _1}} \right)$$

$$\omega _0^2={\omega _1}{\omega _2}$$

$${f_0}=\sqrt {{f_1}{f_2}} \text{...}\left({32.5} \right)$$

$${Z_1}=-2jk$$

Thus, the resonant frequency is the geometric mean of the cut-off frequencies. The variation of the reactances with respect to frequency is shown in Fig.32.2

Design. If the filter is terminated in a load resistance R=K, then at the lower cut-off frequency.

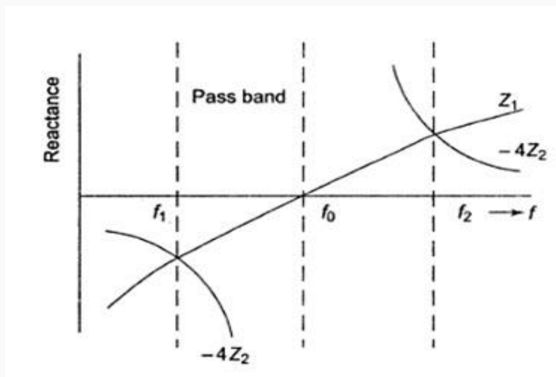

Fig.32.2

$$\left({{1 \over {j{\omega _1}{C_1}}} + j{\omega _1}{L_1}} \right)=-2jk$$

$${1 \over {{\omega _1}{C_1}}}-{\omega _1}{L_1}=2k$$

$$1 - \omega _1^2{C_1}{L_1} - 2k{\omega _1}{C_1}$$

Since $${L_1}{C_1}={1 \over {\omega _0^2}}$$

$$1 - {{\omega _1^2} \over {\omega _0^2}}=2k{\omega _1}{C_1}$$

or $$1 - {\left({{{{f_1}} \over {{f_0}}}} \right)^2}=4\pi k{f_1}{C_1}$$

$$1 - {{f_1^2} \over {{f_1}{f_2}}}=4\pi k{f_1}{C_1} \quad \left({{f_0}=\sqrt {{f_1}{f_2}} } \right)$$

$${f_2} - {f_1}=4\pi k{f_1}{f_2}{C_1}$$

$${C_1}={{{f_2} - {f_1}} \over {4\pi k{f_1}{f_2}}}\text{..}\left({32.6} \right)$$

Since $${L_1}{C_1}={1 \over {\omega _0^2}}$$

$$L_1 = \frac{1}{\omega_0^2 C_1} = \frac{4\pi k f_1 f_2}{\omega_0^2 \left(f_2 - f_1 \right)}$$

$$L_1 = \frac{k}{\pi \left(f_2 - f_1 \right)} \quad \ldots\ldots \left(32.7 \right)$$

To evaluate the values for the shunt arm, consider the equation

$$Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = k^2$$

$$L_2 = C_1 k_2 = \frac{\left(f_2 - f_1 \right) k}{4\pi f_1 f_2} \quad \ldots\ldots \left(32.8 \right)$$

and $$C_2 = \frac{L_1}{k^2} = \frac{1}{\pi \left(f_2 - f_1 \right) k} \quad \ldots\ldots \left(32.9 \right)$$

Equations 32.6 through 32.9 are the design equations of a prototype band pass filter. The variation of a, b with respect to frequency is shown in Fig.32.3.

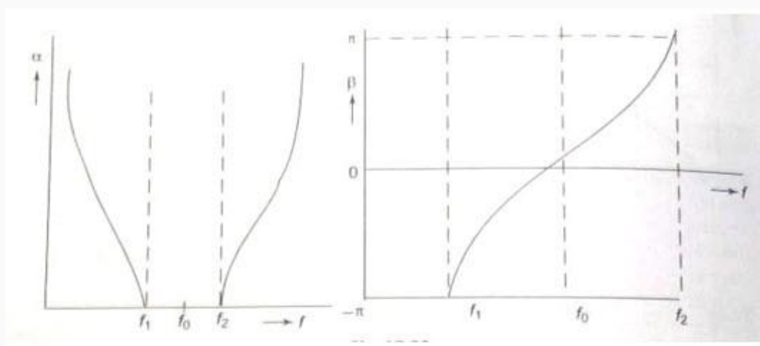

Fig.32.3

32.2. Band Elimination Filter

A band elimination filter is one which passes without attenuation all frequencies less than the lower cut-of frequency f_z, and greater than the upper cut-off frequency f_2. Frequencies lying between f_1 and f_2 are attenuated. It is also known as band stop filter. Therefore, a band stop filter can be realized b connecting a low pass filter in parallel with a high pass section, in which the cut-off frequency of low pass filter is below that of a high pass filter. The configurations of T and π constant k band to sections are shown in Fig.32.4. The band elimination filter is designed in the same manner as is the band pass filter.

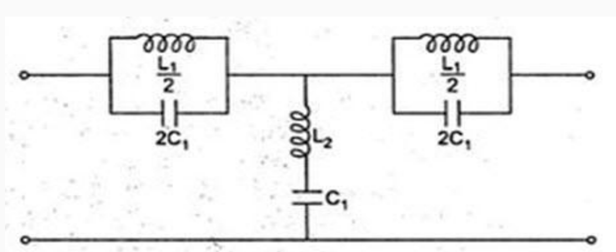

Fig.32.4

As for the band pass filter, the series and shunt arms are chosen to resonate at the same frequency ω_0. Therefore, from Fig.32.1 (a), for the condition of equal resonant frequencies.

$$\frac{\omega_0 L_1}{2} = \frac{1}{2\omega_0 C_1}\,\,\text{for}\,\,\text{the}\,\,\text{series}\,\,\text{arm}$$

or $$\omega_0^2 = \frac{1}{L_1 C_1} \qquad \ldots\ldots\ldots\ldots \left(32.10\right)$$

$$\omega_0 L_2 = \frac{1}{\omega_0 C_2}\,\,\text{for}\,\,\text{the}\,\,\text{shunt}\,\,\text{arm}$$

$$\omega_0^2 = \frac{1}{L_2 C_2} \qquad \ldots\ldots\ldots\ldots \left(32.11\right)$$

$$\frac{1}{L_1 C_1} = \frac{1}{L_2 C_2} = k$$

Thus $L_1C_1 = L_2C_2$

It can be also verified that

$$Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1} = k^2 \qquad \ldots\ldots\ldots\ldots \left(32.13\right)$$

and $$f_0 = \sqrt{f_1 f_2} \qquad \ldots\ldots\ldots\ldots \left(32.14\right)$$

At cut-off frequencies, $Z_1 = -4Z_2$

Multiplying both sides with Z_2, we get

$$Z_1 Z_2 = -4Z_2^2 = k^2$$

$$Z_2 = \pm j\frac{k}{2} \qquad \ldots\ldots\ldots\ldots \left(32.15\right)$$

If the load is terminated in a load resistance, R = k, then at lower cut-off frequency

$$Z_2 = j\left(\frac{1}{\omega_1 C_2} - \omega_1 L_2\right) = j\frac{k}{2}$$

$$\frac{1}{\omega_1 C_2} - \omega_1 L_2 = \frac{k}{2}$$

$$1 - \omega_1^2 C_2 L_2 = \omega_1 C_2 \frac{k}{2}$$

From Eq. 32.11, $L_2 C_2 = \frac{1}{\omega_0^2}$

$$1 - \frac{\omega_1^2}{\omega_0^2} = \frac{k}{2}\omega_1 C_2$$

$$1 - \left(\frac{f_1}{f_0}\right)^2 = k\pi f_1 C_2$$

$$C_2 = \frac{1}{k\pi f_1}\left[1 - \left(\frac{f_1}{f_0}\right)^2\right]$$

since $f_0=\sqrt{f_1 f_2}$

$$C_2=\frac{1}{k\pi}\left[\frac{1}{f_1}-\frac{1}{f_2}\right]$$

$$C_2=\frac{1}{k\pi}\left[\frac{f_2-f_1}{f_1 f_2}\right]\ldots\ldots\ldots\ldots\left(32.16\right)$$

From Eq.32.11, $\omega_0^2=\frac{1}{L_2 C_2}$

$$L_2=\frac{1}{\omega_0^2 C_2}=\frac{\pi k f_1 f_2}{\omega_0^2\left(f_2-f_1\right)}$$

Since $f_0=\sqrt{f_1 f_2}$

$$L_2=\frac{k}{4\pi\left(f_2-f_1\right)}\ldots\ldots\ldots\ldots\left(32.17\right)$$

Also from Eq.32.13,

$$k^2=\frac{L_1}{C_2}=\frac{L_2}{C_1}$$

$$L_1=k^2 C_2=\frac{k}{\pi}\left(\frac{f_2-f_1}{f_1 f_2}\right)\ldots\ldots\ldots\ldots\left(32.18\right)$$

and $$C_1=\frac{L_2}{k^2}\ldots\ldots\ldots\ldots\left(32.19\right)$$

$$=\frac{1}{4\pi k\left(f_2-f_1\right)}$$

The variation of the reactances with respect to frequency is shown in Fig.32.5

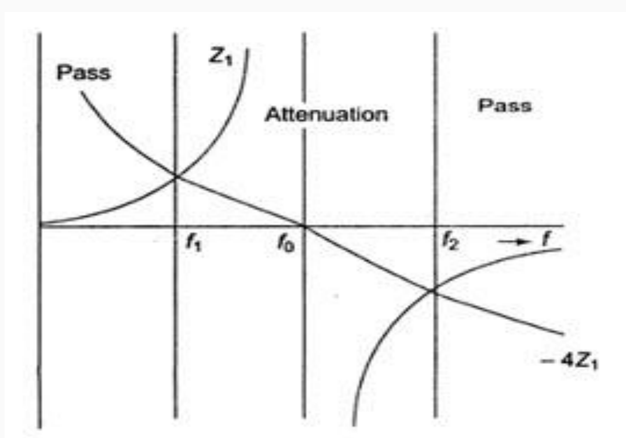

Fig.32.5

Equation 32.16 through Eq.32.19 are the design equations of a prototype band elimination filter. The variation of \[\alpha\], β with respect to frequency is shown in Fig.32.6

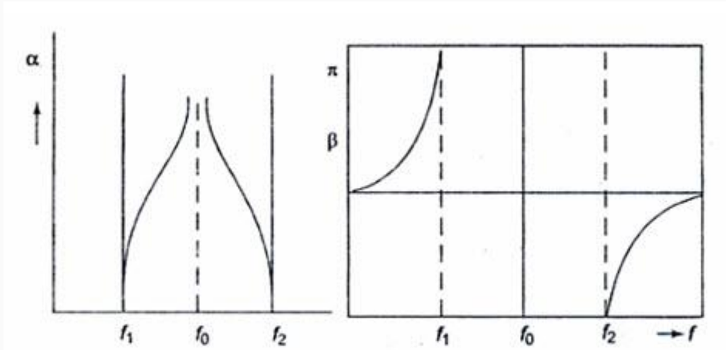

Fig.32.6

32.3 Composite filter

A composite filter is an electronic filter consisting of multiple filter sections of two or more different types.

The method of filter design determines the properties of filter sections by calculating the properties they have in an infinite chain of such sections. In this, the analysis parallels transmission line theory on which it is based. Filters designed by this method are called parameter filters, or just filters. An important parameter of filters is their impedance, the impedance of an infinite chain of identical sections.

The basic sections are arranged into a ladder network of several sections, the number of sections required is mostly determined by the amount of stop band rejection required. In its simplest form, the filter can consist entirely of identical sections. However, it is more usual to use a composite filter of two or three different types of section to improve different parameters best addressed by a particular type. The most frequent parameters considered are stop band rejection, steepness of the filter skirt (transition band) and impedance matching to the filter terminations. Filters are linear filters and are invariably also passive in implementation.

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