ELECTRICAL M/C'S AND POWER UTILIZATION

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Electrical M/C's and Power Utilization

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MODULE 1. Electro motive force, reluctance, laws of magnetic circuits, determination of ampere-turns for series and parallel magnetic circuits

LESSON 1. Electro motive force, reluctance and laws of magnetic circuits

Production of Magnetic Field: When the current *i* amperes flows through a conductor, the magnetic flux is around as explained by right hand cork-screw rule. To determine the direction of flux created due flow of current in a conductor as wound round the path, consider the diagram in figure 1.1. The flux is along the central axis of the coil (solenoid coil) as shown.

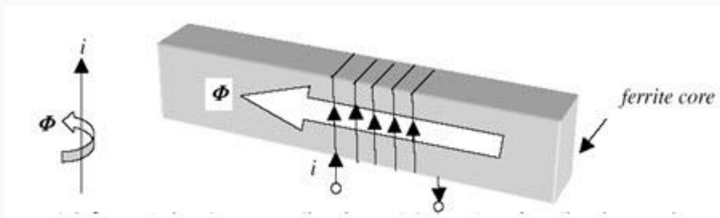


Fig. 1.1: Law relating the current direction and the resultant flux direction - cork-screw rule

Production of Magnetic Field

Consider the magnetic core shown in the diagram of Figure 1.2. It has a winding carrying a current of *i* amperes and *N* turns. It generates a magneto motive force (MMF) **F** of *N.i* ampere (A).

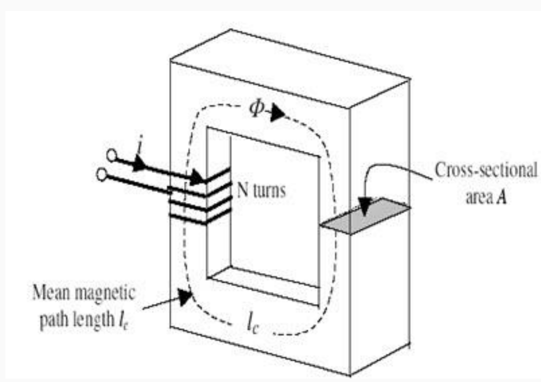


Fig. 1.2: A simple magnetic Core

Since the number of turns, N is dimensionless, the SI unit for MMF is just ampere, denoted by A. Magnetomotive Force (*mmf* drives the magnetic flux through the circuit and is analogous to emf in the electrical circuit. The magneto motive force of a circuit is measured by the work done in carrying a unit north pole through the entire circuit. The MMF creates a magnetic field in core having a **field intensity of *H* ampere-turns / meter** along the length of the magnetic path. Upon integrating the magnetic field intensity along the magnetic path, we get,

$\int H\, dl = N.i$ ampere (A) (1)

The above is the Ampere's law governing the production of a magnetic field by a current carrying coil. If the path of integration is the mean path length of the core *lc*, Ampere's law becomes;

$H.lc = N.i$ (2)

This MMF '**F**' drives through the magnetic core, a flux Φ Webers. The flux Φ can be related as:

$$\phi = \frac{N.i}{R} = \frac{F}{R}$$ (3)

The term R refers to reluctance of the magnetic core. The MMF has to drive the flux Φ against this reluctance R. The reluctance of the magnetic core may be given by the following expression:

$$R = \frac{l_c}{\mu.A_c}$$

(4)

Where *lc* refers to the mean length of the magnetic path in meters, *Ac* refers to the cross-sectional area of the flux path in meter2 and the term μ refers to the permeability of the magnetic material of the core.

The unit for R is 1/henry or 1/H. The unit for μ is H/m. The permeability of free space or air is μ **0** and is given by;

μ 0 = 4 Πx10-7 H/m (5)

Remember that each ferromagnetic material has its own relative permeability (μ**r**) and can be found from manufacturer where;

μ = μ **0** **x μr** **(6)**

Relative permeability is a convenient way to compare the magnetizability of different materials. For Example, the steels used in modern machines have relative permeability in the range 2000 to 6000. This means that, for a given amount of current, 2000 to 6000 times more flux is established in a piece of steel than in a corresponding area of air. Obviously, the metal in a transformer or a motor core may play an extremely important role in increasing and concentrating the magnetic flux in the device.

Since the permeability of iron is much higher than that of Air, a major portion of the flux in configuration, likes that of Figure 1.2, remains inside the core instead of traveling through the surrounding Air, which has lower permeability. The small portion of flux that does not travel through the iron core, but travels through Air path is called leakage flux. Treatment of leakage flux is very important in transformer and motors.

The flux density ***B*** may be defined as

$$B = \frac{\phi}{A_c} \qquad (7)$$

Flux Density (B) is the number of webers or of lines of induction per unit area, the area being taken at right angles to the direction of the flux.

Resolving Φ in the above equation using equation (3) and (4) we get,

$$\frac{\Phi}{A_c} = \frac{Ni/\Re}{A_c} = \frac{N.i}{A_c\left(l_c/A_c\mu\right)} = \mu.\frac{N.i}{l_c} = \mu.H \qquad (8)$$

The unit of the flux density is Weber/ mete**r2**, known as tesla (T). Thus, alternatively, the flux determined in (3) may be found as below

$$\Phi = \iint_S B.dS \qquad (9)$$

where *dS* is the differential unit of the cross-sectional area. If the flux density vector ***B*** is perpendicular to a plane of area *Ac*, and if the flux density ***B*** is constant throughout the area, then this equation reduces to

Φ = *B.Ac* (10)

Thus, the total flux in the core in Figure 1.2 due to the current *i* in the winding is

$$\Phi = B.A_c = \frac{\mu.N.i.A_c}{l_c} \qquad (11)$$

Based on the analogy between magnetic circuits and dc resistive circuits, Table below summarizes the corresponding quantities. Further, the laws of resistances in series and parallel also hold for reluctances. Analogy with dc Resistive Circuits.

DC Resistive Circuit	Magnetic Circuit
Current I (A)	Flux ϕ (Wb)
Voltage V (V)	Magnetomotive force (mmf) F
Resistance R = l/rA	Reluctance = l/µA (H^{-1})
Conductivity r (S/m)	Permeability µ (H/m)
I = V /R	ϕ = F / Reluctance
Current density J = I/A (A/m^2)	Flux density B = ϕ/A (Wb/m^2)

Magnetic Field Intensity, Relative Permeability and Reluctance

We define a new fundamental field quantity, the **magnetic field intensity H,** such that

$$H = \frac{B}{\mu_o} - M \ \ (A/m) \tag{12}$$

The Ampere law and other laws helps us to understand that magnetic flux behaves in the same manner as the current flowing in closed loops and many of magnetic circuits applications are of different shapes, sizes, and may come in unified or composite ferromagnetic materials.

Ampere's circuital law: the circulation of the magnetic field intensity around any closed path is equal to the free current flowing through the surface bounded by the path. Ampere's circuital law is most useful on determining the magnetic field caused by a current when cylindrical symmetry exists. When the magnetic properties of the medium are ***linear*** and ***isotropic,*** the magnetization is directly proportional to the magnetic field intensity:

$\mathbf{M} = \chi_m \mathbf{H}$,

where χ_m is called **magnetic susceptibility.**

LESSON 2. Determination of ampere-turns for series and parallel magnetic circuits

intro

Example 1

N turns of wire are wound around a toroidal core of a ferromagnetic material with permeability

μ. Determine **B**f, in the ferromagnetic core; **H**f in the core; and **H**g in the air gap.

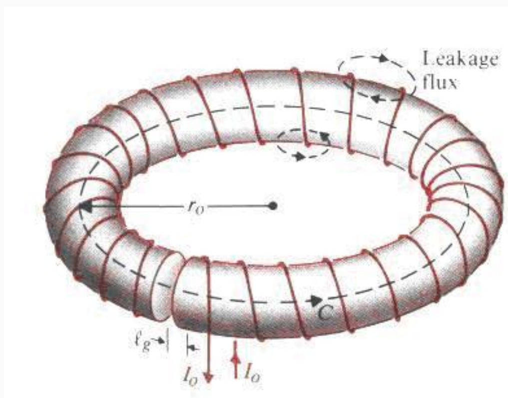


Fig. 1.3 Magnetic circuit

Applying Ampere's circuital law

$$\int_c H \cdot d\mathbf{l} = NI_o$$

If flux leakage is neglected, the same total flux will flow in both the ferromagnetic core and in the air gap. If the fringing effect of the flux in the air gap is also neglected, the magnetic flux density (B) in both the core and the air gap will also be the same.

However, because of the different permeabilities, the magnetic field intensities in both parts will be different. We have

$$B_f = B_g = \hat{\phi}\,\frac{Bf}{\mu}$$

where the f and g denote ferromagnetic and gap, respectively. In the ferromagnetic core,

$$H_f = \hat{\phi}\,\frac{B_f}{\mu}$$

and, in the air gap,

$$H_g = \hat{\phi}\,\frac{B_f}{\mu_o}$$

Ampere's law

$$\frac{B_f}{\mu}\left(2\pi r_o - l_g\right) + \frac{B_f}{\mu_o}\,l_g = NI_o \quad \text{and} \quad B_f = \hat{\phi}\,\frac{\mu_o \mu N I_o}{\mu_o\left(2\pi r_o - l_g\right) + \mu l_g}$$

We have

$$H_f = \hat{\phi}\,\frac{B_f}{\mu} = \hat{\phi}\,\frac{\mu_o N I_o}{\mu_o\left(2\pi r_o - l_g\right) + \mu l_g}$$

$$H_g = \hat{\phi} = \frac{\mu N I_o}{\mu_o\left(2\pi r_o - l_g\right) + \mu l_g}$$

If the radius of the cross section of the core is much smaller than the mean radius of the toroid, the magnetic flux density **B** in the core is approximately constant, and the magnetic flux in the circuit is

$$\Phi \cong BS$$

where S is the cross-sectional area of the core.

$$\phi = \frac{NI_o}{\left(2\pi r_o - l_g\right)/\mu S + l_g/\mu_o S}$$

$$\phi = \frac{V_m}{R_f + R_g}, \text{with } R_f = \frac{2\pi r_o - l_g}{\mu S} = \frac{l_f}{\mu S} \text{ and } R_g = \frac{l_g}{\mu_o S}$$

Both Rf and Rg are called **reluctance.**

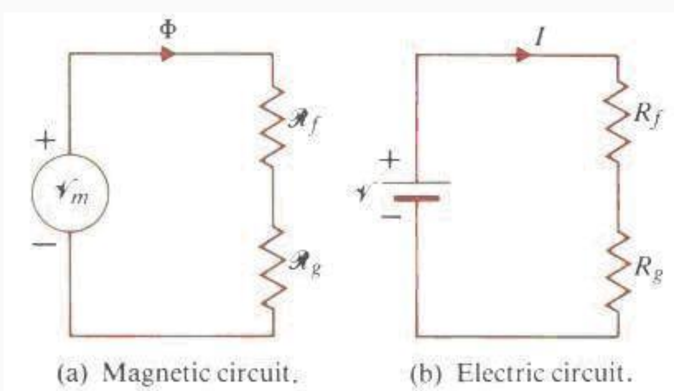


Fig. 1.4 Equivalent magnetic and electric circuit of the example1

Similar to Kirchhoff's voltage law, we may write, for any closed path in a magnetic circuit,

$$\sum_j N_j I_j = \sum_k R_k \phi_k$$

Around a closed path in a magnetic circuit the algebraic sum of ampere-turns is equal to the algebraic sum of the products of the reluctances and fluxes.

Kirchhoff's current law for a junction is consequence of $\nabla \cdot \mathbf{J} = 0$. Similarly $\nabla \cdot \mathbf{B} = 0$ leads to

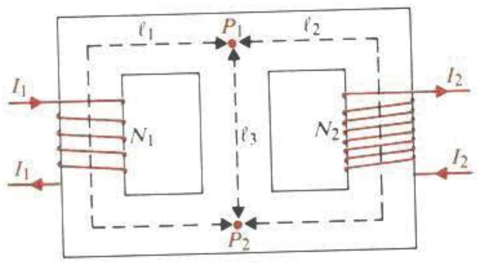
$\int_s \mathbf{B} \cdot d\,\mathbf{s} = 0$

Thus we have

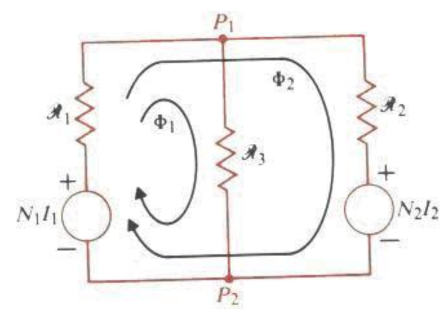
$\sum \Phi_j = 0$,

Which states that **the algebraic sum of all the magnetic fluxes flowing out of a junction in a magnetic circuit is zero.**

Example 2 Solve the magnetic circuit in **Fig. 1.5 (a)**



(a) Magnetic core with current-carrying windings.



(b) Magnetic circuit for loop analysis.

Solution

The reluctances are:

$$R_1 = \frac{l_1}{\mu S_c}, R_2 = \frac{l_2}{\mu S_c}, R_3 = \frac{l_3}{\mu S_c}$$

The two loop equations are

$$\text{loop1}: N_1I_1 = (R_1 + R_3)\phi_1 + R_1\phi_2;$$
$$\text{loop2}: N_1I_1 - N_2I_2 = R_1\phi_1 + (R_1 + R_2)\phi_2.$$

Solving these equations

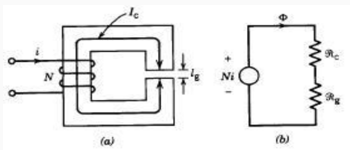
$$\phi_1 = \frac{R_2N_1I_1 + R_1N_2I_2}{R_1R_2 + R_1R_3 + R_2R_3}$$

This is analogous to the electric circuits methods such as voltages around the loop and adding parallel and series resistances, etc.

It is to be noticed with great interest that, most of the electromechanical energy devices cores have a small gap, as in transformers, motors and generators. One of the reasons for a narrow gap is that as the air gap of a coil is increased, the linear inductance region of the coil will also increase, and the coil will not saturate for the same excitation current; which would cause saturation if the air gap was not included.

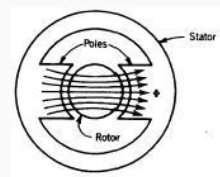
Magnetic circuit with air gap

Air gap requires much more mmf than the core



$$R_c = \frac{l_c}{\mu_c A_c} \qquad R_g = \frac{l_g}{\mu_g A_g} \qquad B_c = \frac{\Phi_c}{A_c}$$

$$Ni = H_c l_c + H_g l_g \qquad B_g = \frac{\Phi_g}{A_g}$$



Below is another example on magnetic parallel circuit with an air gap.

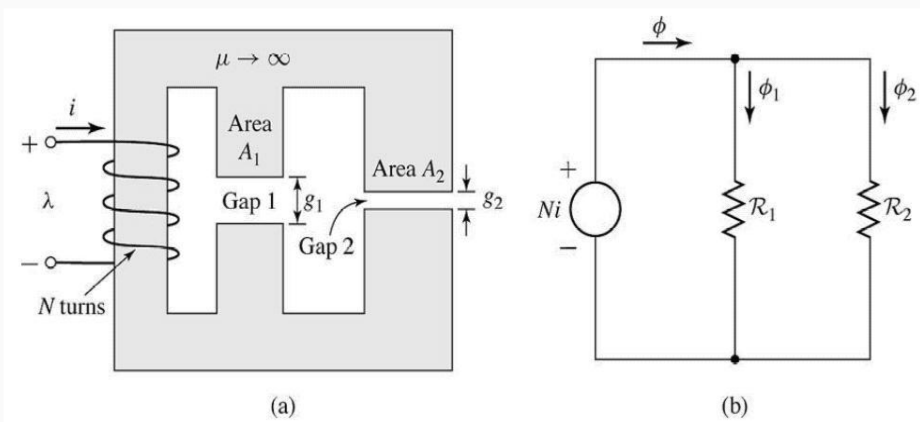


Fig. 1.6 Magnetic parallel circuit with an air gap

If we assume that there is no fringing then **Ac = Ag** (16)

But with an air-gap in a magnetic core, the flux fringes out into neighboring air paths as shown in Figure 1.6, these being of reluctance comparable to that of the gap. The result is non-uniform flux density in the air-gap (decreasing outward), enlargement of the effective air-gap area and a decrease in the average gap flux density. The fringing effect also disturbs the core flux pattern to some depth near the gap.

The effect of fringing increases with the air-gap length. Corrections for fringing in short gaps (as used in machines) are empirically made by adding one gap length to each of the two dimensions making up its area. For the core given in Figure 1.6, the air-gap reluctance, given by

$$\Re_g = \frac{l_g}{\mu_o A_g}$$

(17)

Should be calculated using an Ag which is greater than Ac.

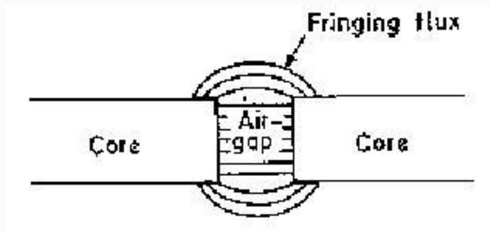


Fig. 1.7 Flux fringing at air gap.

It can be shown theoretically that the magnetic flux leaves and enters the surface of an infinitely permeable material, in a direction normal to the surface. This will be nearly so in ferromagnetic materials which have high permeability. In electrical machines a small amount of the tangential flux component present at iron surfaces will be neglected.

Adapted from EE360 course on Electric Energy Engineering, KFUPM Open Courseware

MODULE 2. Hysteresis and eddy current losses

LESSON 3. Hysteresis and eddy current losses

Before explaining hysteresis and eddy currents, let us look at some basic definitions and laws that govern electro magnetic induction

Lorentz Force

– The force on a moving charge in a magnetic field is proportional to the charge, velocity, and magnetic field strength.

Lenz's Law

–The induced currents in a conductor are in such a direction as to oppose the change in magnetic field that produces them

Ampere's Law

–The line integral of the magnetic field intensity around a closed path is equal to the algebraic sum of the currents flowing through the area enclosed by the path.

Faraday's law:

–Currents can be induced in wires by changing magnetic fields.

A time-varying magnetic field induces an electromotive force that produces a current in a closed circuit. This current flows in a direction such that it produces a magnetic field that tends to oppose the changing magnetic flux of the original time varying field.

Mathematically

$$emf(e) = \frac{d\lambda}{dt}$$

where λ = total flux linkages of closed path = $N\varphi$

$$e = \frac{dN\phi}{dt}$$

1. Time varying flux linking stationary path – transformer

2. Relative motion between steady flux and stationary path – synchronous generator under no-load conditions

3. A combination of previous two cases – induction

The above 3 points means that, voltage production in the above equations are dependent on flux, which is produced by AC flowing current. So if the closed path is a winding of transformer, then the flux will be produced and it can be received by another closed path windings and that what developed the transformer principle.

If coil connected to voltage source, V , current i will flow as shown in figure 2.1.

- Current will produce magnetic flux, Φ

- Total flux linkages of the coil containing N turns $\lambda = N\Phi$

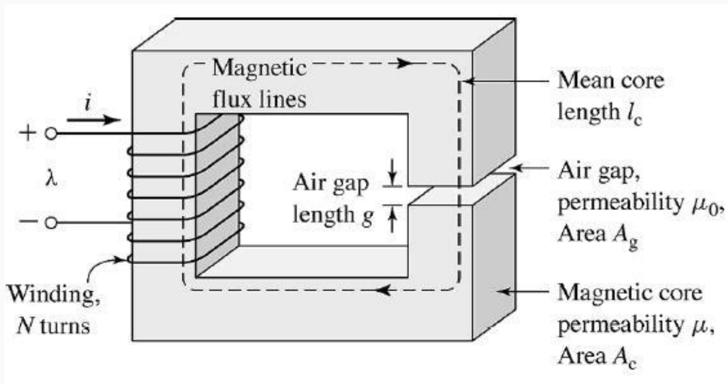


Fig. 2.1 Magnetic circuit

Magnetization curve or B-H curve

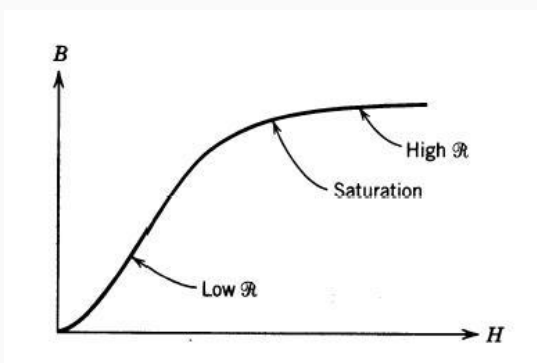


Fig. 2.2 Effect of reluctance on typical magnetization

- at low magnetic field intensity magnetic flux density increases almost (Fig. 2.2)linearly

- at higher values of magnetic field intensity the change of magnetic flux density is nonlinear - reaches saturation (Fig. 2.2)

Magnetic behavior of ferromagnetic materials

- The magnetic permeability (μ) is not constant as shown in figure 2.3:

B = Hm

Devices such as transformers have to operate in the linear (non-saturated) region.

B-H curve can be divided into regions; residual magnetism, linear portion, knee and saturation region.

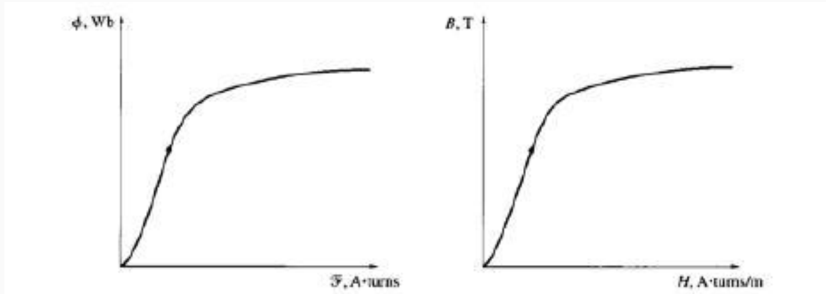


Fig. 2.3 B-H as it relates to Φ-I

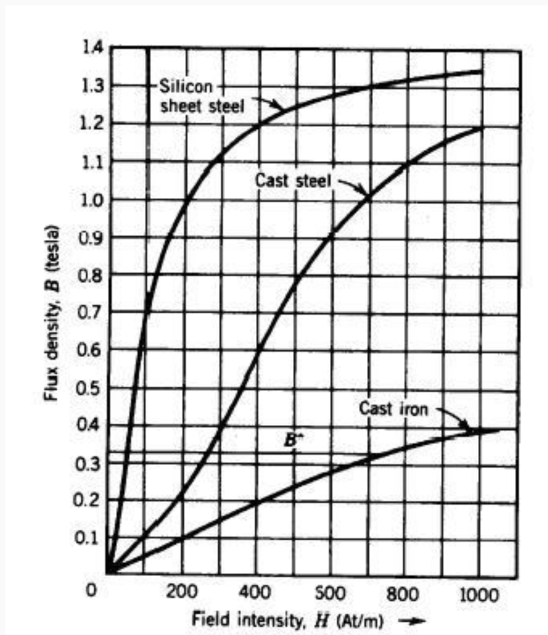


Fig. 2.4 Different B-H curves of some ferromagnetic materials

Each ferromagnetic material (Fig. 2.4) has its B-H- curve or magnetization curve.

LESSON 4. Hysteresis and eddy current losses

Hysteresis

As a varying voltage is applied across some windings, a time varying current will flow as starting from 0 to complete its sinusoidal cycle (Fig. 2.5). The magnetization that it will form is known as hysteresis loop (Fig. 2.6).

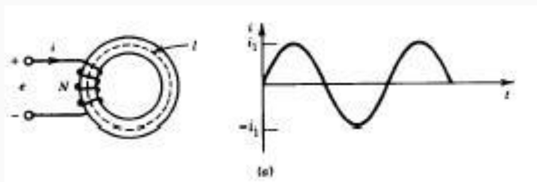


Fig. 2.5 Magnetic circuit applied with a varying voltage

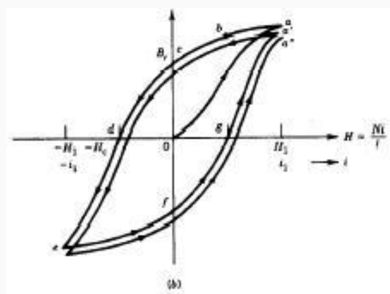


Fig. 2.6 Formation of hysteresis loop

- Initially unmagnetized – o
- *i* and *H* increased slowly – oa
- *i* and *H* removed : br residual flux density
- *H* reversed to –hc (coercivity): flux density vanishes
- During first period *B-H* curve will follow the bath oacdefga’ - the loop does not close

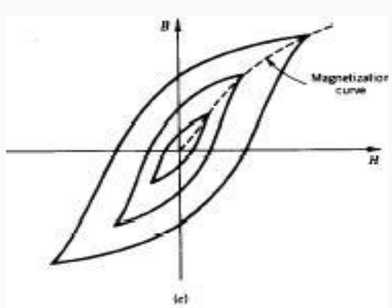


Fig. 2.7 Locus of the B-H curve

- The locus of the tip of the hysteresis loop is called the magnetization curve (Fig. 2.7)
- After few periods the loop almost closes the hysteresis loop (Fig. 2.8)
- B-H relation is nonlinear and multi- valued
- The *B* lags behind *H*

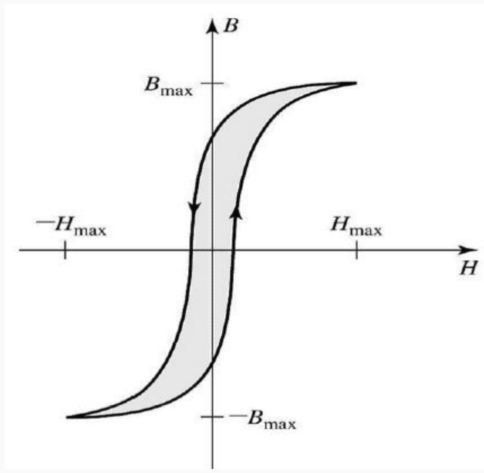


Fig. 2.8 Complete hysteresis loop

Hysteresis losses occur when the flux changes continuously both in value and direction. The magnetic material absorbs energy each cycle and dissipates it as heat. To reduce hysteresis losses, magnetic materials are selected that have a narrow hysteresis loop. The loss of power in the core due to the hysteresis effect is called hysteresis loss

As electric circuits have losses due to its resistance and flow of current, the magnetic circuits also have losses generated, by the varying action of flux and induced voltages, in cores due to magnetic field, namely hysteresis and Eddy current losses.

Eddy current loss

The other type of losses, that occurs in magnetic circuits, is the eddy current loss.

Time varying magnetic field induces eddy currents in conducting material Eddy currents occur when AC voltages are induced in a conductor by a changing magnetic field.

Eddy currents dissipate power as resistive losses in the conductor

To reduce eddy current losses, magnetic materials are laminated (for a given core size, eddy current losses decrease in proportion to the square of the number of laminations).

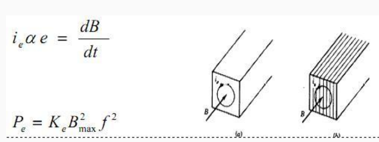


Fig. 2.9 Eddy current loss

A power loss proportional to *Ri*2 will be caused.

Eddy current loss can be reduced by:

– increasing the resistivity of the core material

– using laminated cores

Adapted from EE360 course on Electric Energy Engineering, KFUPM Open Courseware

MODULE 3. Transformer: principle of working, construction of single phase transformer

LESSON 5. Transformer: principle of working, construction of single phase transformer

Introduction

- For transmission and distribution networks to transfer large amounts of alternating current electricity over long distances with minimum losses and least cost, different voltage levels are required in the various parts of the networks.
- For example, the transfer of electricity efficiently over a long transmission line requires the use of high voltages. At the receiving end where the electricity is used, the high voltage has to be reduced to the levels required by the consumer.
- Transformers enable these changes in voltage to be carried out easily, cheaply and efficiently.
- A transformer used to increase the voltage is called a "step up" transformer, while that used to decrease the voltage is called a "step down" transformer.

What is a transformer?

– A device for increasing or decreasing an AC voltage

– Power yransformers, TV sets to provide high voltage to picture tubes, portable electronic device converters, transformers on the pole, etc. are few examples

A transformer consists of two coils of wires known as primary and secondary windings

– The two coils can be interwoven or linked by a laminated soft iron core to reduce eddy current losses

Basic components of transformers

A transformer (Fig. 3.1) consists of two coils electrically separate but linked by a common magnetic circuit of low reluctance formed by a laminated soft iron core. If one coil (the primary coil) is connected to an AC supply, an alternating magnetic flux is set up in the iron core.

This alternating magnetic flux passes through the secondary coil and induces and alternating voltage in the secondary coil.

The magnitude of the secondary voltage is directly proportional to the ratio of the number of turns in the secondary and primary windings and to the primary voltage.

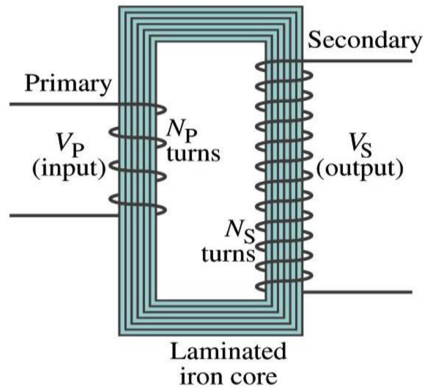


Fig. 3.1 Basic transformer

How do transformers work?

– A changing current through a coil of wire can create a changing magnetic field.

– Currents can be induced in other wires by these changing magnetic field.

– Therefore, the primary coil current must have AC.

– The iron core of the transformer is not required but it does increases the efficiency a great deal.

Theory of the transformer

The operation of a transformer is based on two principles:

- A voltage is induced in a conductor when the conductor passes through a magnetic field. The same effect is produced if the conductor is stationary but the magnetic field in which it is located varies; and
- A current passing through a conductor will develop a magnetic field around the conductor.

Note: In this discussion on transformers, the term magnetic "flux" will usually be used instead of magnetic "field".

- A magnetic field is the space or region surrounding a magnet or a current carrying conductor, in which magnetic effects can be detected.
- The strength of the magnetic field is generally expressed in terms of magnetic flux density (magnetic flux per square meter). Magnetic flux refers to the magnetic lines of force.

Transformer equation

- The transformer equation does not work for Direct current since there is no change of magnetic flux

- If NS>NP, the output voltage is greater than the input so it is called a step- up transformer while NS<NP is called step-down transformer
- Now, it looks like energy conservation is violated since we can get more emf from smaller ones, right?

– Wrong! Wrong! Wrong! Energy is always conserved!

– A well designed transformer can be more than 99% efficient

– The power output is the same as the input.

$$V_p I_p = V_s I_s$$

Transformer Equation

$$\frac{V_p}{I_p} = \frac{I_s}{I_p} = \frac{N_p}{N_s}$$

- When an AC voltage is applied to the primary, the changing B it produces will induce voltage of the same frequency in the secondary wire
- So how would we make the voltage different?

– By varying the number of loops in each coil

– From Faraday’s law, the induced emf in the secondary is

$$V_s = N_s \frac{d\theta_B}{dt}$$

The input primary voltage is

$$V_p = N_p \frac{d\theta_B}{dt}$$

Since $\frac{d\phi_t}{dt}$ is the same $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ = Transformer Ratio, this is the most important relationship by which most of the transformer variables are governed and it is denoted (a)

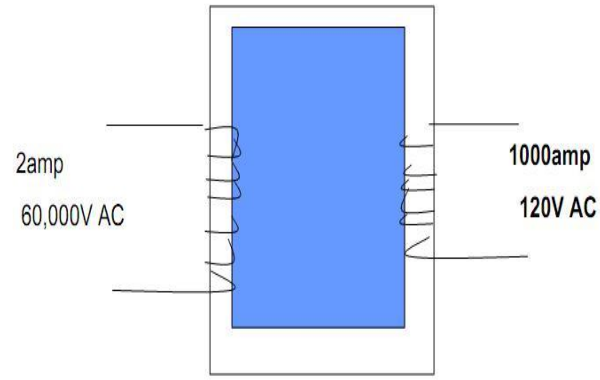


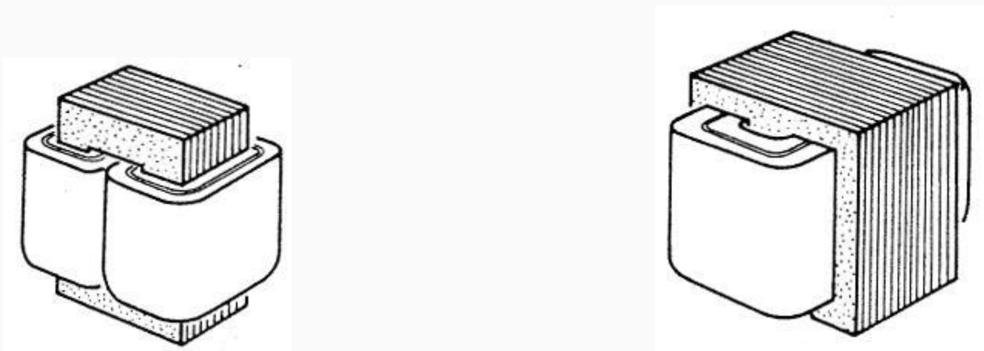
Fig. 3.2 Induction and the transformer

Remember that the relative number of turns dictates the output current and voltage as seen above and by transformer equations.

LESSON 6. Construction of single phase transformer

Transformers types

Transformers are manufactured in many types the most widely used in power systems are classified with their core types as seen below. Core type (Fig. 3.3 A) where each of the windings are wound on one leg of the core, while the shell core type (Fig. 3.3 B), in which both windings are wound on the same leg. Each type has its own advantages and disadvantages. Core type is very reliable and easy to maintain, but take larger space, however, shell type is smaller but not reliable.

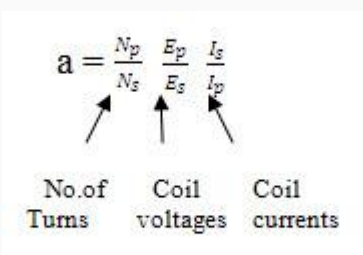


A) Core type   B) Shell type

Figure 3.3 Single phase transformer construction

Ideal Transformer:

This is the most known relationships that relates the induction between the two coils (primary/Secondary) of an ideal transformer to turns ratio and their currents



Ideal Unloaded Transformers

- Winding resistances are zero, no leakage inductance and iron loss
- Magnetization current generates a flux that induces voltage in both windings

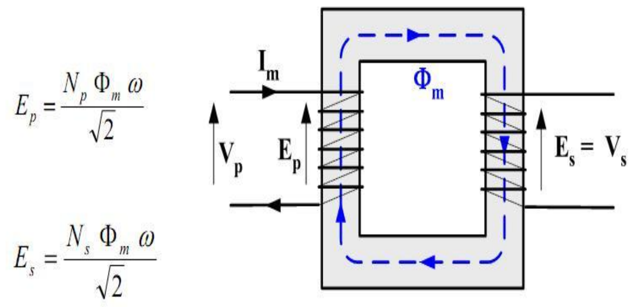


Fig. 3.4 Current, voltages and flux in an unloaded ideal transformer

Ideal loaded transformer

- Loaded transformer , means current will be also feeding the load and that secondary side voltage, current (load Side) will be calculated using the previous relations

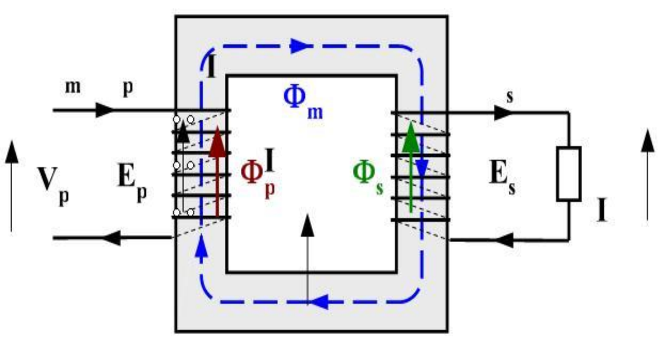


Fig. 3.5 Currents and fluxes in a loaded ideal transformer

Turn ratio:

$$a = T = \frac{N_p}{N_s} = \frac{E_p}{E_s}$$

Input and outpour are equal

$$E_p I_p^* = S_p = S_s = E_s I_s^*$$

Leading to

$$a = T = \frac{E_p}{E_s} = \frac{I_s}{I_p}$$

Ideal transformers equivalent circuit

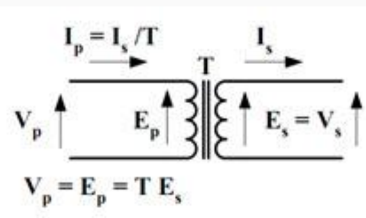


Fig. 3.6 Equivalent circuit of an ideal transformer

Transferring impedances through a transformer

$$\mathbf{Z}_P = \frac{\mathbf{V}_P}{\mathbf{I}_P} = \frac{T\,\mathbf{V}_S}{\left(\frac{\mathbf{I}_S}{T}\right)} = T^2\,\frac{\mathbf{V}_S}{\mathbf{I}_S} \qquad \mathbf{Z}_P = T^2\,\mathbf{Z}_{load}$$

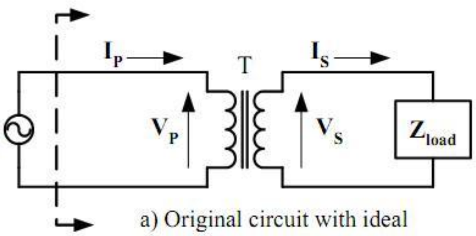


Fig. 3.7 Thevenin equivalent of transformer circuit

Transferring impedances through a transformer

To transfer impedances of either side then the square of the turns ratio is used.

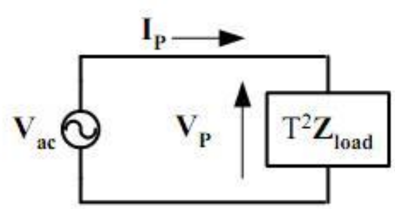


Fig. 3.8 Equivalent circuit when secondary impedance is transferred to primary side and ideal transformer eliminated

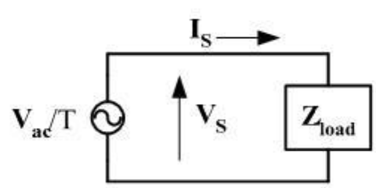


Fig. 3.9 Equivalent circuit when primary source is transferred to secondary side and ideal transformer eliminated

Example: A 200 kV A, 6600 V/400 V, 50 Hz single-phase transformer has 80 turns on the secondary. Calculate:

(i) the appropriate values of the primary and secondary currents;

(ii) the approximate number of primary turns

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = a$$

$$N_1 = \frac{V_1 N_2}{V_2} = (a)\,N_2$$

6600/400 (80)= 1220 turns

Transformer rating = Primary Voltage x Primary current

= Secondary Voltage x Secondary Current

For Primary 200 KVA = (6600) (I1)

I_1 = 200KVA/6600v = 30.3 A

For Secondary 200 KVA= 400 I2

I_2 = 200 KVA/400v = 500 A

A single-phase transformer has 1000 turns on the primary and 200 turns on the secondary. The no-load current is 3 A at a power factor of 0.2 lagging.

Calculate the primary current and power factor when the secondary current is 280 A at a power factor of 0.8 lagging. Assume the voltage drop in the windings to be negligible.

Answer: 58.3 A, 0.78 pf lagging.

Adapted from EE360 course on Electric Energy Engineering, KFUPM Open Courseware

MODULE 4. EMF equation, phase diagram on load, leakage reactance, transformer on load, equivalent circuit, voltage regulation

LESSON 7. Transformer EMF equation and phase diagram on load

No load phasor diagram

A transformer is said to be under no load condition, when no load is connected across the secondary is kept opened and no current is carried by the secondary windings. The phasor diagram under no load condition can be drawn starting with φ as the reference phasor as shown in figure 4.1.

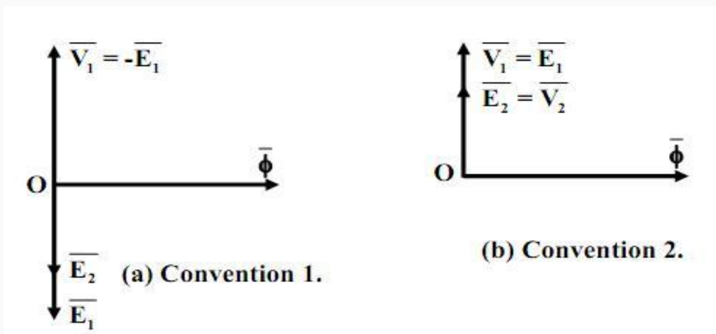


Fig. 4.1 No load phasor diagram following two conventions.

In convention 1, phasors E1 and E2 are drawn 180° out of phase with respect to V1 in order to convey that the respective power flow directions of these two are opposite. The second convention results from the fact that the quantities v1(t), e1(t) and e2(t) vary in unison then why not show them as co-phasal and keep remember the power flow business in one's mind. Also remember vanishingly small magnetizing current is drawn from the supply creating the flux and in time phase with the flux.

Transformer under loaded condition

In this lesson, we shall study the behavior of the transformer when loaded. A transformer gets loaded when we try to draw power from the secondary. In practice loading can be imposed on a transformer by connecting impedance across its secondary coil. It will be explained how the primary reacts when the secondary is loaded. It will be shown that any attempt to draw current/power from the secondary, is immediately responded by the primary winding by drawing extra current/power from the source. We shall also see that mmf balance will be maintained whenever both the windings carry currents. Together with the mmf balance equation and voltage ratio equation, invariance of Volt-Ampere (VA or KVA) irrespective of the sides will be established.

We have seen that the secondary winding becomes a seat of emf and ready to deliver power to a load if connected across it when primary is energized. Under no load condition, power drawn is zero as current drawn is zero for ideal transformer. However when loaded, the secondary will deliver power to the load and same amount of power must be sucked in by the primary from the source in order to maintain power balance. We expect the primary current to flow now. Here we shall examine in somewhat detail the mechanism of drawing extra current by the primary when the secondary is loaded. For a fruitful discussion on it let us quickly review the dot convention in mutually coupled coils.

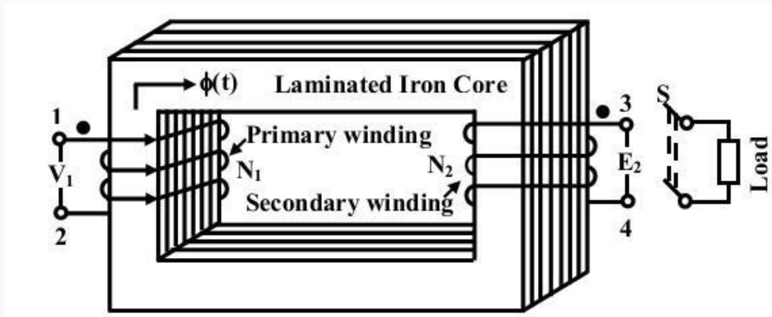


Fig. 4.2 Typical transformer

Dot convention

The primary of the transformer is energized from a.c source and potential of terminal 1 with respect to terminal 2 is v12 = Vmaxsinωt. Naturally polarity of 1 is sometimes +ve and some other time it is –ve. The dot convention helps us to determine the polarity of the induced voltage in the secondary coil marked with terminals 3 and 4. Suppose at some time t, we find that terminal 1 is +ve and it is increasing with respect to terminal 2. At that time what should be the status of the induced voltage polarity in the secondary – whether terminal 3 is +ve or –ve? If possible let us assume terminal 3 is –ve and terminal 4 is positive. If that be current the secondary will try to deliver current to a load such that current comes out from terminal 4 and enters terminal 3. Secondary winding therefore, produces flux in the core in the same direction as that of the flux produced by the primary. So core flux gets strengthened in inducing more voltage. This is contrary to the dictate of Lenz's law which says that the polarity of the induced voltage in a coil should be such that it will try to oppose the cause for which it is due. Hence terminal 3 can not be –ve.

If terminal 3 is +ve, then we find that secondary will drive current through the load leaving from terminal 3 and entering through terminal 4. Therefore flux produced by the secondary clearly opposes the primary flux fulfilling the condition set by Lenz's law. Thus when terminal 1 is +ve, terminal 3 of the secondary too has to be positive. In mutually coupled coils dots are put at the appropriate terminals of the primary and secondary merely to indicative the status of polarities of the voltages. Dot terminals will have at any point of time identical polarities. In the transformer of figure 4.2, it is appropriate to put dot markings on terminal 1 of primary and terminal 3 of secondary. It is to be noted that if the sense of the windings are known (as in figure 4.2), then one can ascertain with confidence where to place the dot markings without doing any testing whatsoever. In practice however, only a pair of primary terminals and a pair of secondary terminals are available to the user and the sense of the winding can not be ascertained at all. In such cases the dots can be found out by doing some simple tests such as polarity test or d.c kick test.

If the transformer is loaded by closing the switch S, current will be delivered to the load from terminal 3 and back to 4. Since the secondary winding carries current it produces flux in the anti clock wise direction in the core and tries to reduce the original flux. However, KVL in the primary demands that core flux should remain constant no matter whether the transformer is loaded or not. Such a requirement can only be met if the primary draws a definite amount of extra current in order to nullify the effect of the mmf produced by the secondary. Let it be clearly understood that net mmf acting in the core is given by: mmf due to vanishingly small magnetizing current + mmf due to secondary current + mmf due to additional primary current. But the last two terms must add to zero in order to keep the flux constant and net mmf eventually be once again be due to vanishingly small magnetizing current. If I2 is the magnitude of the secondary currents I2 and I2′ is the additional current drawn by the primary and then following relation must hold good.

$$N_1 I'_2 = N_2 I_2$$

$$\text{or } I'_2 = \frac{N_2}{N_1} I_2$$

$$= \frac{I_2}{a}$$

$$\text{where, } a = \frac{N_1}{N_2} = \text{turns ratio}$$

To draw the phasor diagram under load condition, let us assume the power factor angle of the load to be θ2, lagging. Therefore the load current phasor I2 , can be drawn lagging the secondary terminal voltage E2 by θ2 as shown in the figure 4.3 as per the convention 2.

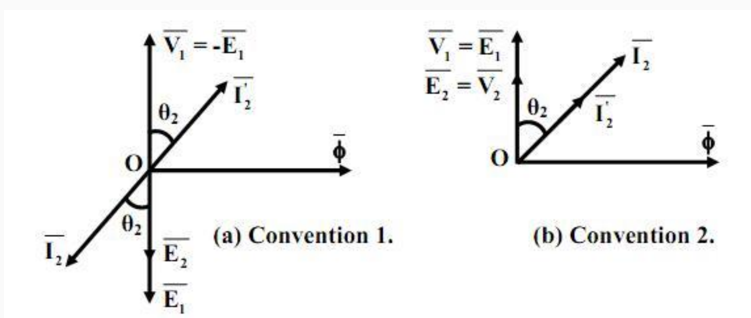


Fig. 4.3. Phasor diagram under load

At this stage, let it be suggested to follow one convention only and we select convention 2 for that purpose. Now,

Volt-Ampere delivered to the load = V2I2 = E2I2

$$= aE_1 \frac{I_1}{a}$$

$$= E_1 I_1 = V_1 I_1 = \text{Volt - Ampere drawn from the supply}$$

Thus we note that for an ideal transformer the output VA is same as the input VA and also the power is drawn at the same power factor as that of the load.

LESSON 8. Transformers on load

Practical transformer

- Winding resistance
- Flux leakage
- Finite permeability
- Core losses

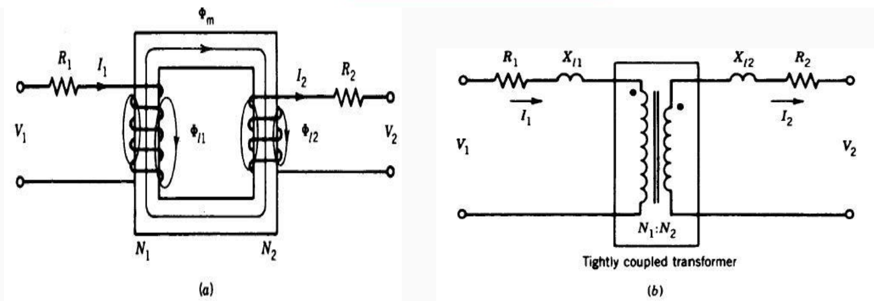


Fig. 4.4 Model of practical transformer

Transformer model

- physical reasoning
- mathematic model of coupled circuits
- Winding resistance in series with leakage inductance
- Magnetizing inductance in parallel with core resistance

Referred equivalent circuits

- Practical transformer is equivalent to lumped parameters circuit and ideal transformer

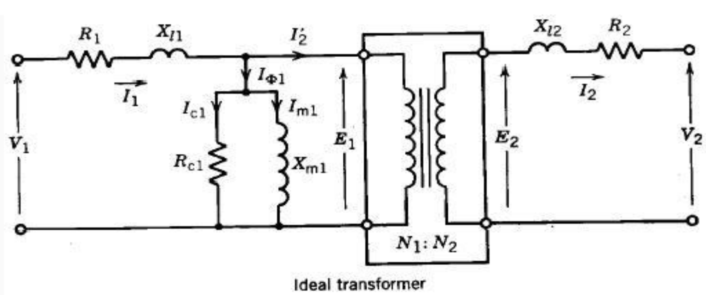


Fig. 4.5 Equivalent circuit of ideal transformer

The ideal transformer can be shifted to either side as in Figure 4.6 below and the circuit parameters reduce to the appropriate values

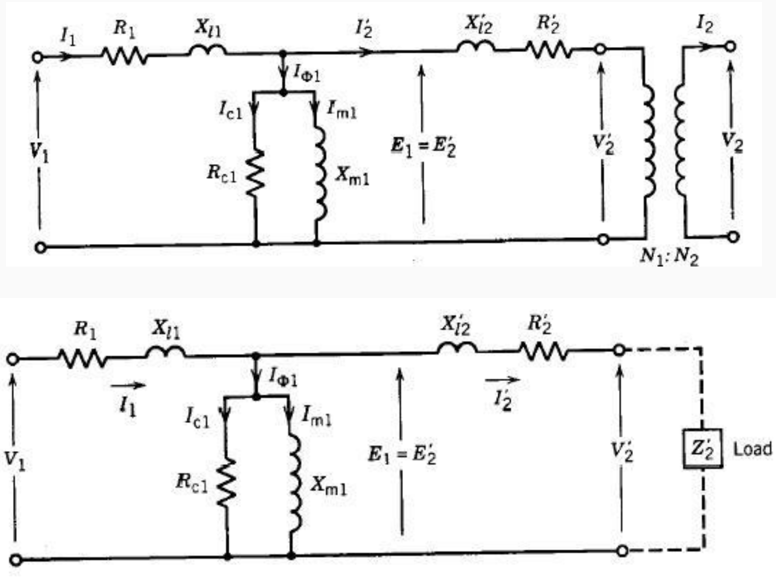


Fig. 4.6 Reduction of parameter in equivalent circuit

$E_1=E'_2= aE_2$

$V'_2=aV_2$

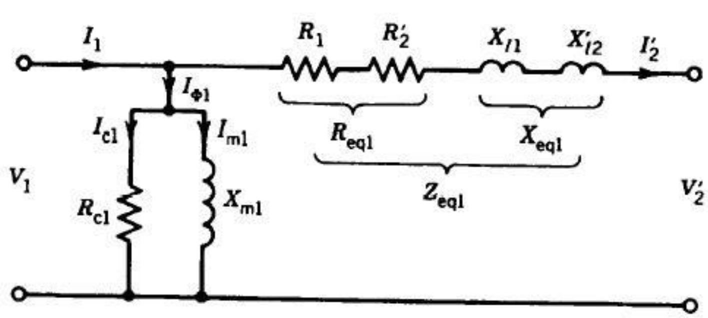
$I'_2=I_2/a$

$X'_{12}=a2x_{12}$

$R'_2=a2R_2$

Approximate equivalent circuits

- I1R1 and I1X*l*1 are small Therefore, |E1| = |V1|
- Shunt branch can be moved to supply terminal
- IΦ small (5% of rated current) Shunt branch removed



(a) $V'_2 = aV_2$, $I'_2 = I_2/a$

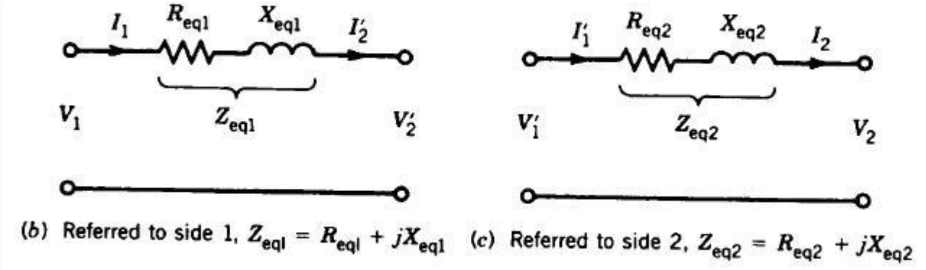


Fig. 4.7. Approximate equivalent parameters

Determination of equivalent circuit parameters

- No-load test (rated voltage on one side whereas the other side is open)

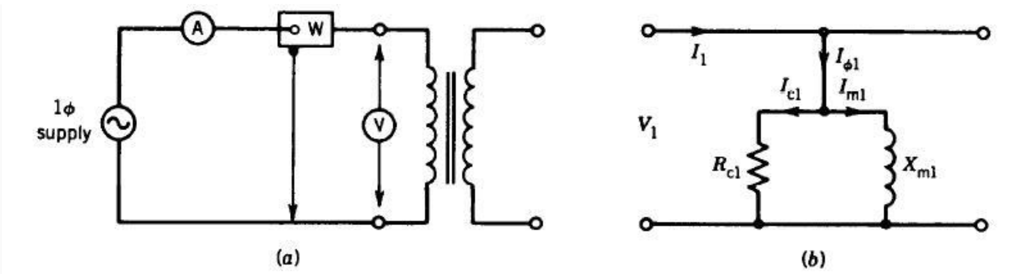


Fig. 4.8. Equivalent circuit parameters under no load test

LESSON 9. Equivalent circuit and voltage regulation of transformers

intro

Short-circuit test (rated current on one side whereas the other side is short-circuited)

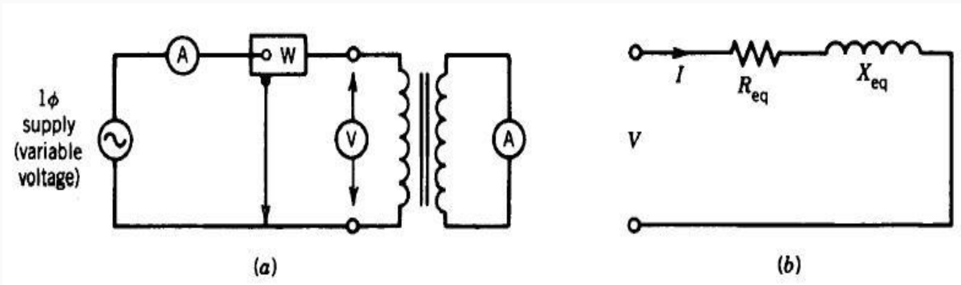


Fig. 4.9 Equivalent circuit parameters under short circuit test

Example

- An Engineer needs to know the parameters of a 46KVA Transformer which has a 2300V/230V winding. His results are:
- Open circuit test: 230V 11.2A 1150W
- Short circuit test: 160V 28.0A 1150W
- We must first determine which side low or high the test was performed on.

For the open circuit test we compare the tested voltage to the rated voltage of the transformer.

- In this example we see that the open circuit test voltage is the same as the Low side rated operating voltage, thus we know the test was performed on the low side and the high side was left open.
- Next we need to determine which side the short circuit test was performed on, so we compare the current this time.

$$I_L = \frac{S}{V_L} = \frac{46000W}{230V} = 200A$$

$$I_H = \frac{S}{V_H} = \frac{46000W}{2300V} = 20A$$

- Open circuit test: 230V 11.2A 1150W -H.V. Left open, and tested on low side
- Short circuit test: 160V 28.0A 1150W -L.V. Shorted, and tested on high side
- So we know that the Open Circuit parameters are referred to primary.

- So we will use the referred to primary parameters for the open circuit test.

What we know:

$$V = 230,\ I_0 = 11.2A,\ S = 1150W$$

$$R_C = \frac{V^2}{S} = \frac{230^2}{1150} = 46\,\Omega$$

$$\theta = \cos^{-1}\left[\frac{S}{VI}\right] = \cos^{-1}\left[\frac{1150}{230 * 11.2}\right] = 63.5^0$$

$$I_\phi = I_0 \sin\theta = 11.6 \sin 63.5^0 = 10A$$

$$X_\phi = \frac{V}{I_\phi} = \frac{230}{10} = 23\,\Omega$$

- This means our equivalent calculations are referred to the H.V. side. So in effect what we are calculating are our $X2$ and $R2$ values. If we want our $X1$ and $R1$ values, we must divide by our transformation ratio.

$$R_2 = \frac{S}{I_{s.c.}} = \frac{1150}{20} = 2.875\,\Omega$$

$$X_2 = \sqrt{\left(\frac{V_{s.c.}}{I_{s.c.}}\right)^2 - R_2} = \sqrt{\left(\frac{160}{20}\right)^2 - 2.875\Omega} = 7.818j\,\Omega$$

- To complete our primary referred circuit we must find our $R1$ and $X1$ values. And given the fact that the transformer is stepping up voltage from 230 to 2300, we can see that it is a 1:10 ratio, or $N1 = 1$, $N2 = 10$.

$$R_1 = \left(\frac{N_1}{N_2}\right)^2 R_2 = \left(\frac{1}{10}\right)^2 2.875\Omega = 0.02875\Omega$$

$$X_1 = \left(\frac{N_1}{N_2}\right)^2 jX_2 = \left(\frac{1}{10}\right)^2 7.818j\Omega = 0.07818j\Omega$$

$$\overline{E}_1 = I_1(R_1 + X_1) = I_2 \frac{N_2}{N_1}(R_1 + X_1) = 200A(0.02875 + j0.07818) = 200A\angle 0^0 R_{eq}\angle\gamma^0$$

- Now here we must convert our resistance into a singular vector for multiplication.
- We will momentarily ignore our j operator to get the magnitude of the vector

$$R_{eq} = \sqrt{0.02875^2 + 0.07818^2} = 0.0833\Omega$$

Now we can find E'_1: $\angle\gamma^0 = \tan^{-1}\frac{X_1}{R_1} = \tan^{-1}\frac{0.07818}{0.02875} = \angle 69.8^0$

$$\overline{E}_1 = 200\angle 0^0 0.0833\angle 69.8^0 = 200 * .0833\angle(0 + 69.8)^0 = 16.66\angle 69.8^0\,V$$

$$\overline{E}_2 = \frac{N_2}{N_1}E_1 = 166.6\angle 69.8^0$$

So you can see from the determination that 6.6 volts are due to internal losses in the transformer itself. Knowing this magnitude, an engineer can design a simulation with a source to the exact specifications and losses.

Voltage regulation

No load V2=V1/a

Loaded V2=V1/a ± ΔV2

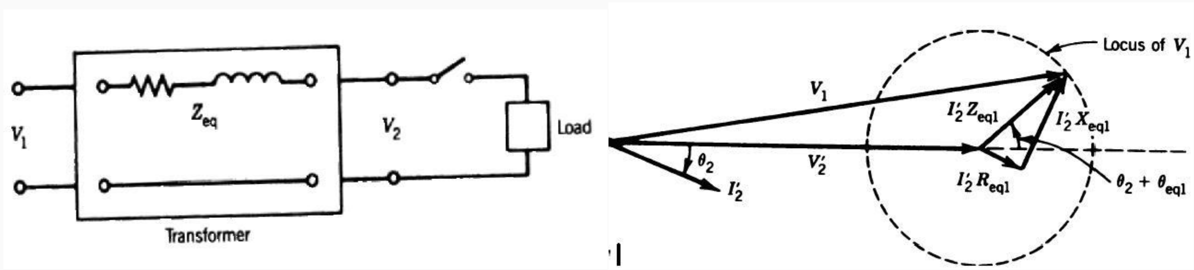


Fig. 4.10 Voltage regulation

Voltage regulation =

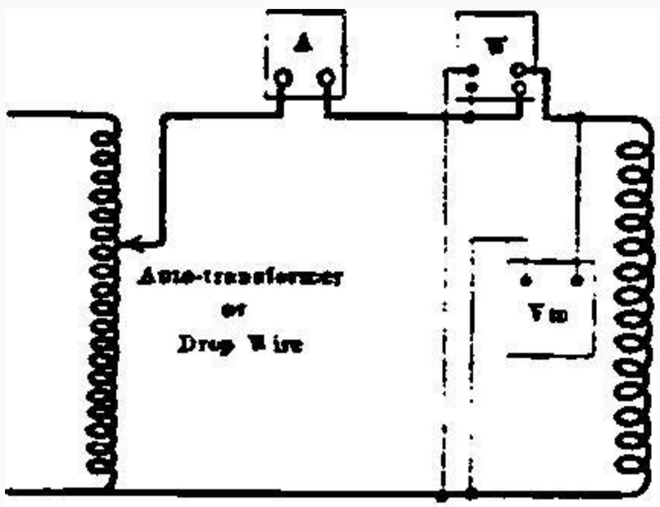
$$\frac{Vnl - \mathrm{Vfl}}{\mathrm{Vfl}} x100\ \%$$

MODULE 5. Power and energy efficiency, open circuit and short circuit tests, principles

LESSON 10. Power and energy efficiency, open circuit and short circuit tests, principles

Open-circuit Test

Figure 5.1 shows a transformer having the low side connected to an alternating source of supply and the high side open-circuited. Either an auto-transformer or a drop wire is shown as a means of varying the voltage supplied to the low side of the transformer. A voltmeter, an ammeter and a wattmeter are connected in the primary circuit. The voltmeter reads the voltage across the primary terminals, the ammeter reads the no-load current, and the wattmeter reads the power taken by the transformer under these conditions.



5.1 Connections for open-circuit test

This power goes to supply the primary I^2R loss and the core loss of the transformer. As the exciting current is very small, the primary I^2R loss due to it may be neglected. Therefore, the wattmeter reads the transformer core loss. If the primary voltage be varied and the core loss be determined for different values of voltage, a curve is obtained showing the relation of core loss to voltage. At no load the flux is practically proportional to the terminal voltage, as the primary impedance drop due to the no-load current is negligible. The eddy-current loss varies as the square of the voltage and the hysteresis loss as the 1.6 power of the voltage. The core loss will increase, therefore, nearly as the square of the voltage, as shown below (a).

Transformers are usually so designed that the most economical use of materials is obtained. Therefore, the core is operated at as high a flux density as the allowable core loss will permit. Figure 5.2 (a) shows that a slight increase of voltage, above rated voltage, produces a very large percentage increase in core loss. As transformers are rated by their maximum safe operating temperatures, this increased core loss may cause overheating of the transformer. Therefore, the effect of operating transformers at over-voltage is to produce a large increase in temperature.

If the magnetizing current be plotted as abscissa, and the voltage as ordinates, a saturation curve similar to that of figure 5.2 (b) is obtained. The point marked "rated voltage" is the point on the saturation curve at which transformers are generally operated, and is well beyond the knee of the curve. Outside the question of increased core loss, the usual transformer cannot be operated at a voltage very much in excess of its rated voltage, for the exciting current increases very rapidly with small increase in voltage, as indicated in figure 5.2 (b). The flux density in the core is determined primarily by the permissible core loss. Open-hearth annealed sheet steel, such as is used in dynamos, can be used for transformer cores. For a given flux density and frequency, however, silicon steel has much less core loss pen* unit volume than open-hearth steel, the effect of the silicon being to increase the electrical resistance, and hence reduce the eddy-current loss. Because of its small core loss, silicon steel may be operated safely at very high flux densities. The greater cost of silicon steel is more than offset by the saving in iron and in copper, and in the general reduction of the transformer dimensions.

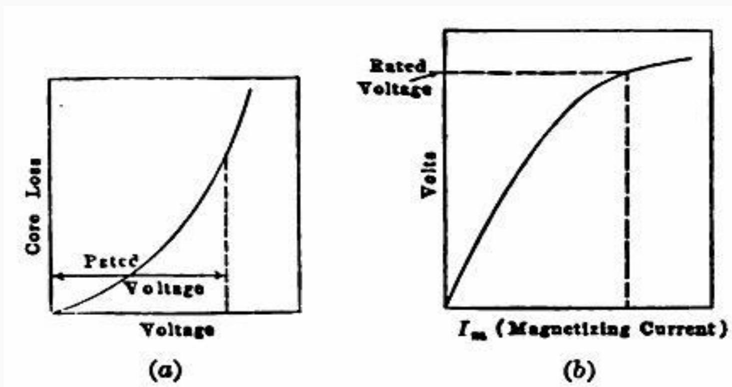


Fig. 5.2. Characteristics; (a) Relation of core loss to voltage in a transformer (b) Relation of magnetising current to voltage in a transformer.

To obtain the true value of the exciting current; the current measured by the ammeter, should be resolved into two components, one of *which* lies along the voltage $-E_1$ or V and is shown as I_c in the figure 5.3 ($-E_1$ and V are practically equal at no load). This current $I_c = I_O \cos q$ is the energy component of the current and supplies the core losses. The quadrature component $I_m = I_O \sin q$ is the true magnetizing current.

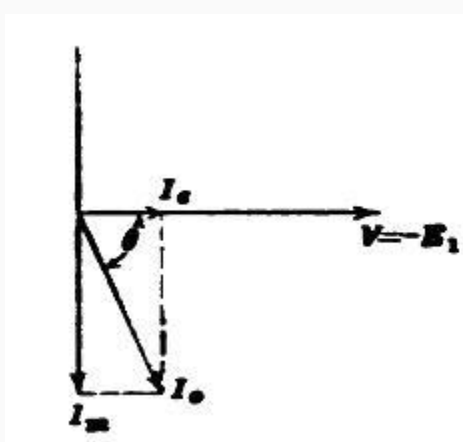


Fig. 5.3 Phasor diagram

Short circuit test

Figure 5.4 shows the transformer short-circuited on the secondary side.

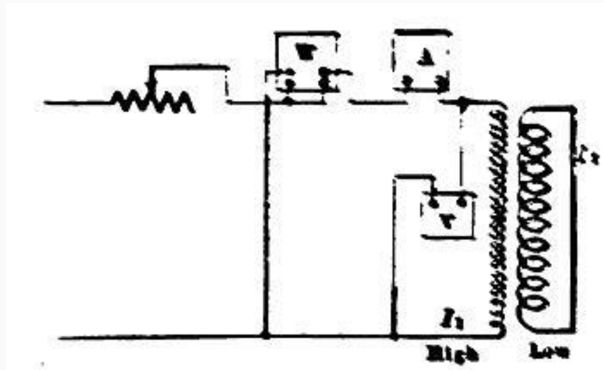


Fig. 5.4 Short circuit test

In a transformer, the impedance drop seldom exceeds 5 per cent of the rated voltage. If the 2,200 volt side of a transformer in figure 5.4 be used as the primary, the voltage necessary to send rated current through the windings on short-circuit is about 5 per cent of 2,200, or 110 volts, which is a standard voltage for instrument coils. If the secondary of the transformer were rated at 220 volts, the voltage at short-circuit would be only 11 volts and the current would also be high. At this low voltage, high precision could not be obtained with ordinary instruments. When a primary current I_1 flows, the secondary current I_2 is equal to $I_1 (N_1/N_2)$. Therefore, no need of using an ammeter for measuring I_2. The power delivered to the transformer, goes to supply three losses; the primary copper loss, $I_1^2R_1$, the secondary copper loss, $I_2^2R_2$, and the core loss at short-circuit. The core loss is negligible, as 5 per cent, primary voltage means only about 2.5 per cent of the rated value of flux, since half the impressed voltage on short-circuit is consumed in the primary impedance drop. The core loss at 2 or 3 per cent, of the rated flux is so small as to be negligible, for the core loss varies nearly as the square of the flux. Therefore, the power at short circuit

$$P = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} = I_2^2 R_{02}$$

where R_{01} and R_{02} are the transformer equivalent resistances referred to the primary and secondary, respectively. The value of equivalent resistance as found in this manner may be checked with the value determined by measuring the resistance of each winding with direct current. The ratio of effective to ohmic resistance is only a few per cent, greater than unity in most transformers. Figure 5.3 shows the equivalent circuit vector diagram for the short-circuit test. This diagram is merely same as that of open circuit test, except that V2 now equals zero and all quantities are now referred to the primary side. It will be recognized that the entire voltage V1 is consumed in the impedance drops of the two windings. From this it is obvious that if Z_{01} be the equivalent impedance of the transformer, referred to the primary side,

$$Z_{01} = \frac{V_1}{I_1}$$

$$Z_{02} = Z_{01}\left(\frac{N_2}{N_1}\right)^2$$

Knowing the equivalent impedance and equivalent resistance, the equivalent reactance is readily found as $X_0 = \sqrt{Z_0^2 - R_0^2}$ for either primary or secondary side. In making the short-circuit and the open-circuit tests, the question of instrument losses should be investigated and correction made if this be found necessary. As the losses in a transformer are very small, the power taken by the instruments may be a considerable percentage of the power being measured.

Regulation and Efficiency

The data obtained from the short-circuit and open-circuit tests are sufficient to compute the regulation and the efficiency of the transformer at any load. As the equivalent resistance and reactance referred to either side are known, it is merely necessary to determine the regulation. The procedure will be demonstrated by an example which follows. It has been pointed out that with constant voltage, the mutual flux of the transformer is practically constant from no load to full load. It usually does not vary more than from 1 to 3 per cent. Therefore, the core loss is practically constant at all loads and may be determined by the open-circuit test. For most purposes, it is necessary merely to measure the loss at the rated voltage of the transformer. The only other losses are the primary and secondary copper losses. These can be calculated readily, knowing the resistances of primary and secondary, or they may be computed from the equivalent resistance determined at short-circuit. The efficiency of the transformer may then be computed, since the losses are known. That is, the efficiency

$$Eff. = \frac{V_2 I_2 (P.F.)}{V_2 I_2 (P.F.) + Core\ Loss + I_1^2 R_1 + I_2^2 R_2}$$

$$= \frac{V_2 I_2 (P.F.)}{V_2 I_2 (P.F.) + Core\ Loss + I_2^2 R_{02}}$$

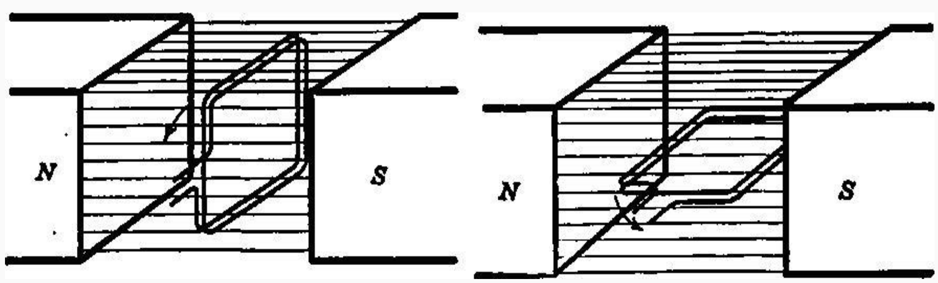
MODULE 6. Operation and performance of DC machine (generator and motor)

LESSON 11. Operation and performance of DC machine (generator and motor)

Generator

Definition.—A generator is a machine which converts mechanical energy into electrical energy. This is accomplished by means of an armature carrying conductors upon its surface, acting in conjunction with a magnetic field. Electrical power is generated by the relative motion of the armature conductors and the magnetic field. In the direct-current generator the field is usually stationary and the armature rotates. In most types of alternating-current generators the armature is stationary and the field rotates. Either the armature or the field is driven by mechanical power applied to its shaft.

Generated Electromotive Force- If the flux linking a coil is varied in any way, an



(a) Maximum lines passing through coil (b) No lines passing through coll

Fig. 6.1 Simple coil rotating in a magnetic field.

electromotive force is induced in the turns of the coil. The action of the generator is based on this principle. The flux linking the armature coils is varied by the relative motion of the armature and field.

A coil revolves in a uniform magnetic field produced by a north and a south pole. In Fig. 6.1 (a) the coil is perpendicular to the magnetic field and in this position the maximum possible flux links the coil. If the coil be rotated counter-clockwise a quarter of a revolution, it will lie in the position shown in Fig. 6.1 (b). As the plane of the coil is parallel to the flux no lines link the coil in this position. Therefore, in a quarter revolution the flux which links the coil has been decreased by t lines. The average voltage induced in the coil during this period is, therefore,

$$e = Nj(10^{-8}) / t$$

where, N is the number of turns in the coil and t the time required for a quarter revolution. But $t = 1/4R$ where R = the revolutions per second. Therefore, the average voltage during a quarter revolution is $e = 4NRj(10^{-8})$ volts

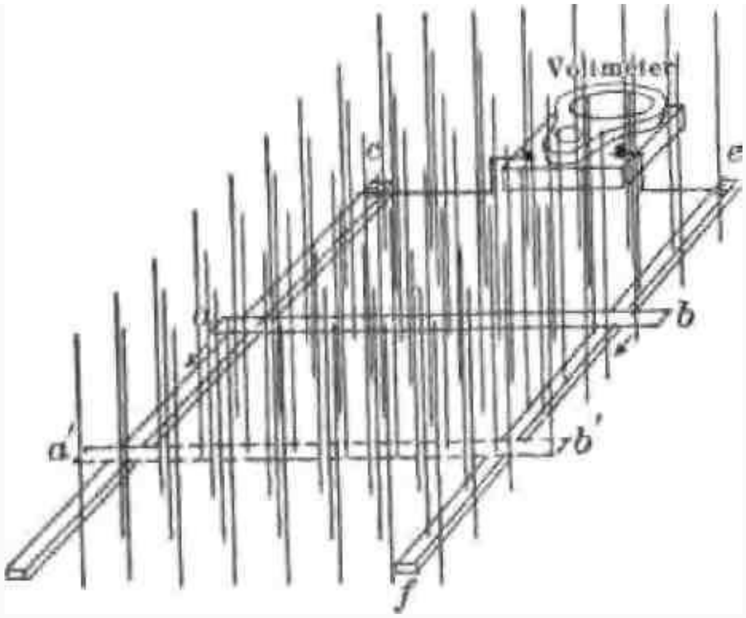


Fig. 6.2. Conductor cutting a uniform magnetic field

The generation of electromotive force in a moving coil of this type, which is similar to those used in dynamos, may also be analyzed by considering the total electromotive force as being due to the sum of the electromotive forces generated in each side of the coil. The electromotive force of one turn is the sum of the electromotive forces in each conductor forming the sides of the turn, since these conductors are connected in series by the end connections of the turn. The individual electromotive forces are then considered as being generated in the conductor rather than induced in the coil.

Consider the conductor ab, free to slide along the two metal rails cd Fig. 6.2. The rails are connected at one end 'ce' by a voltmeter. A magnetic field having a density of B lines per sq. cm. passes perpendicularly through the plane of the rails and conductor. Let the conductor ab move at a uniform velocity to the position a'b'. While this movement is taking place, the voltmeter will indicate a certain voltage.

The electromotive force in volts generated by a single conductor which cuts a magnetic field is

$$e = Blv\,(10^{-8}) \qquad (93)$$

where B, I and v are mutually perpendicular. B is the flux density of the field in gausses, I the length of conductor in centimeters, and v the velocity of the conductor in.centimeters per second.

That the electromotive force induced by a change of the flux linked with a coil is the same as that obtained by considering the emf generated by the cutting of magnetic lines by the conductor which make up the coil may be illustrated by a concrete example. Let the flux have a density of 100 lines per cm^2. The distance ab is 30 cm. and aa' is 20 cm. The conductor ab moves at a uniform velocity to position a'b' in 0.1 second. What is the electromotive force across 'ce'?

The change of flux linking the coil is:

j = 30 X 20 X 100 = 60,000 lines.

This change occurs in 0.1 second and v = 20/0.1= 200 cm s^{-1}.

e = 100 X 30 X 200 X 10~* = 0.006 volt.

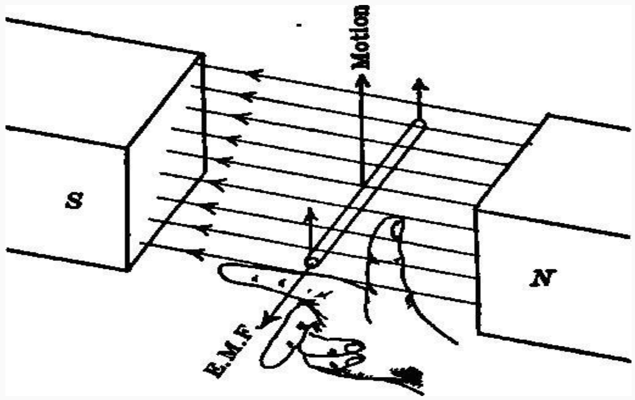


Fig. 6.3 Fleming's right-hand rule: Fore finger along lines of force; Thumb in direction of motion; Middle finger gives direction of induced emf.

Direction of Induced Electromotive Force: Fleming's Right-hand Rule definite relation exists among the direction of the flux, the direction of motion of the conductor and the direction of the electromotive force in the conductor just as a definite relation exists between the direction of current and of the flux which it produces.

If the fore-finger points along the lines of flux and the thumb in the direction of motion of the conductor, the middle finger will point in the direction of the induced electromotive force.

Voltage generated by the revolution of a coil. A coil of a single turn is shown in Fig.6.4 below. The coil rotates in a counter-clockwise direction at a uniform speed in a uniform magnetic field. As the coil assumes successive positions, the electromotive force induced in it changes. When it is in position (1) the electromotive force generated is zero, for in this position neither conductor is cutting magnetic lines, but rather is moving parallel to these lines. When the coil reaches position (2), (shown dotted) its conductors are cutting across the lines obliquely and the electromotive force has a value indicated

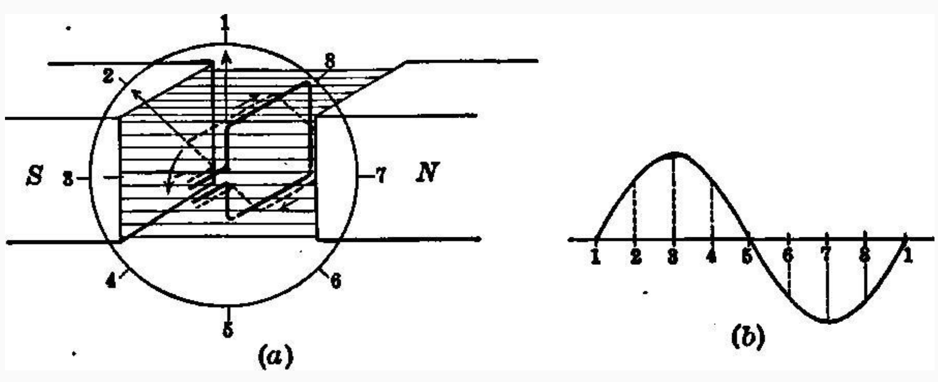
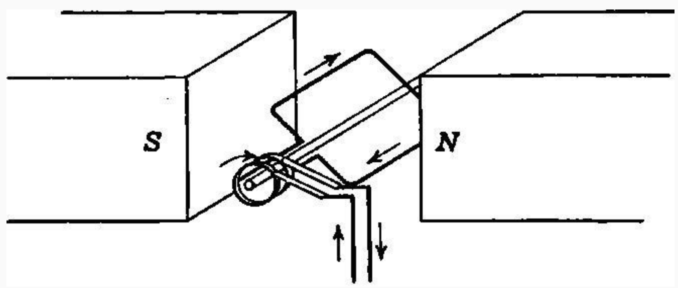


Fig. 6.4. Emf induced in a coil rotating at constant speed in a uniform magnetic field.

at (2) shown in (b). When the coil reaches position (3) the conductors are cutting the lines perpendicularly and are therefore cutting at the maximum possible rate. Hence the electromotive force is a maximum when the coil is in this position. At position (4) the electromotive force is less, due to a lesser rate of cutting. At position (5) no lines are being cut and as in (1) there is no electromotive force. In position (6) the direction of the electromotive force in the conductors will have reversed as each conductor is under a pole of opposite sign to that for positions (1) to (5). The electromotive force increases to a negative maximum at (7) and then decreases until the coil again reaches position (1). After this the coil merely repeats the cycle.

This induced electromotive force is alternating and an emf. varying in the manner shown is called a sine wave of electromotive force. This alternating electromotive force may be impressed on an external circuit by means of two slip-rings as shown below. Each ring is continuous and insulated from the other ring and from the shaft. A spring loaded carbon brush rests on each ring and conducts the current from the coil to the external circuit. If a direct current is desired, that is, one whose direction is always the same, such rings cannot be used. A direct current must always flow into the external circuit in the same direction.



Current taken from rotating coil by means of slip-rings.

As the coil current, must necessarily be alternating, since the emf. which produces it is alternating, this current must be rectified before it is allowed to enter the external circuit. This rectification can be accomplished by using a split ring such as shown in Fig. 6.5 below. This is split by saw cuts at two points diametrically opposite each other. The two ends of the coil are connected one to each of the sections or segments so produced.

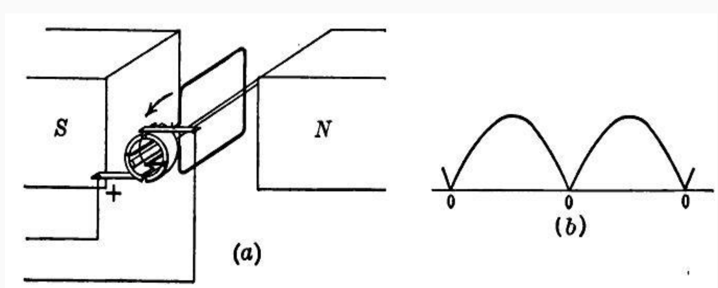


Fig. 6.5. Rectifying effect of a split ring or commutater.

A careful consideration will show that, as the direction of the current in the coil reverses, its connections to the external circuit are simultaneously reversed. Therefore, the direction of flow of the current in the external circuit is not changed. The brushes pass over the cuts in the ring when the coil is perpendicular to the magnetic field or when it is in the so-called neutral plane and is generating no voltage. These neutral points are marked 0-0-0 in figure 6.5 (b). Here it will be seen that the negative half of the wave has been reversed and so made positive.

A voltage with a zero value twice in each cycle, as shown above, could not be used commercially for direct current service. Also a single-coil machine would have a small output for its size and weight. The electromotive force wave may be improved upon by the use of two coils and four commutator segments as shown in the figure 6.6. This gives an open circuit type of winding, since it is impossible to start at any one commutator segment and return to this segment again by following through the entire winding. In this particular arrangement the full electromotive force generated in each coil is not utilized, as one coil passes out of contact with the brushes at points a, a, a in the next figure 6.6 (b), and the voltage shown by the dotted lines is not utilized.

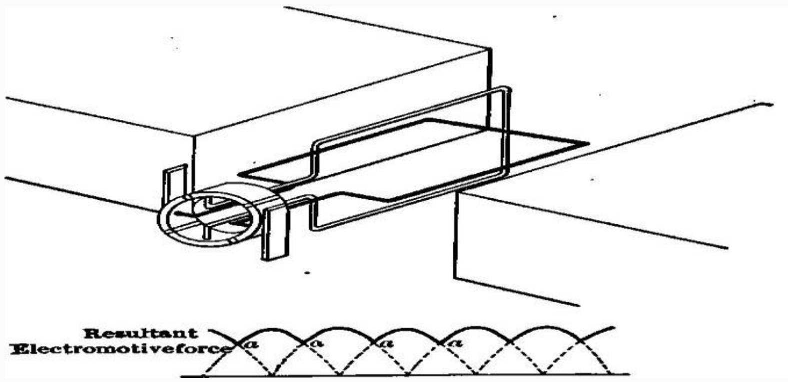


Fig. 6.6 Effect of two coils and four commutator segments upon the electro motive force wave.

MODULE 7. EMF and torque equations, armature reaction, commutation, excitation of DC generator and their characteristics

LESSON 12. EMF and torque equations of DC machines

Electromotive Force in an Armature.

The path of the magnetic flux from the poles of a generator into the armature, and a curve showing the flux distribution are shown in figure 7.1. The ordinate at each point is proportional to the flux density in the air-gap at that point. The maximum flux density is given by the ordinate B_{max}. The positive ordinates of the distribution curve are north pole flux entering the armature and the negative ordinates are flux leaving the armature and entering a south pole.

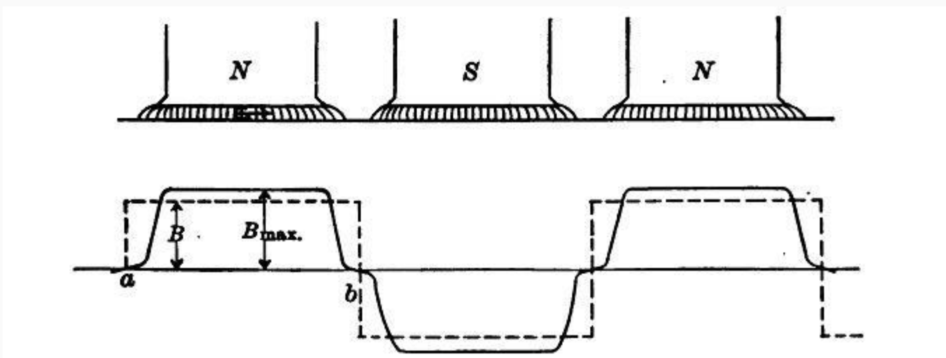


Fig. 7. 1 Flux distribution at no load of a D. C. generator.

The total flux leaving a north pole is given by the area under one of the positive parts of the distribution curve. Similarly, the total flux leaving the armature is the area of one of the negative parts of the distribution curve. Each positive part and each negative part of the curve may be replaced by a rectangle having the same area. The height of this rectangle will be B maxwells / cm^2, which is equal to the average value of the flux density under an entire pole pitch.

Now to determine the average electromotive force induced in a single conductor as it passes through the flux of successive poles. Let the total flux leaving a north pole or entering a south pole be φ maxwells. Let A be the pole area in cm^2, l the active length of the conductor in cm., s the speed of the armature in revolutions per second, and P the number of poles. When the conductor passes through the distance ab, or one pole pitch, the average induced voltage, by equation studied earlier is

$e = Blv\ 10^{-8}$

where, B is the average flux density, I the active length of the conductor in cm., and v the velocity of the conductor in cm. per second.

Then v = ab/t, where 't' is the time required for the conductor to traverse ab.

Therfore, e = Bl (ab/t) 10^{-8} = (φ/t) 10^{-8}

since Bl (ab) gives the total flux between the points a and b as cut by the conductor and is therefore equal to φ.

The time t = 1/sP

Therefore, the average voltage per conductor is e = (φsP) 10^{-8}

If there are Z such conductors and p paths through the armature, there must be Z / p such conductors in series. Hence the total voltage generated between brushes is

E = (φsPZ) / ($p10^8$)

Example.—A 900 r.p.m., 6-pole generator has a simplex lap winding. There are 300 conductors on the armature. The poles are 10 cm^2 and the average flux density is 50,000 lines per cm^2. What is the voltage induced between brushes?

φ = 10 × 10 × 50,000 = 5,000,000 lines

s = 900/60 = 15 r.p.s ; P = 6; p = 6

E = 5,000,000 ×15 × 6 × 300 / 6 × 10^8 = **225 volts**.

The Saturation Curve or No volt characteristic curve:

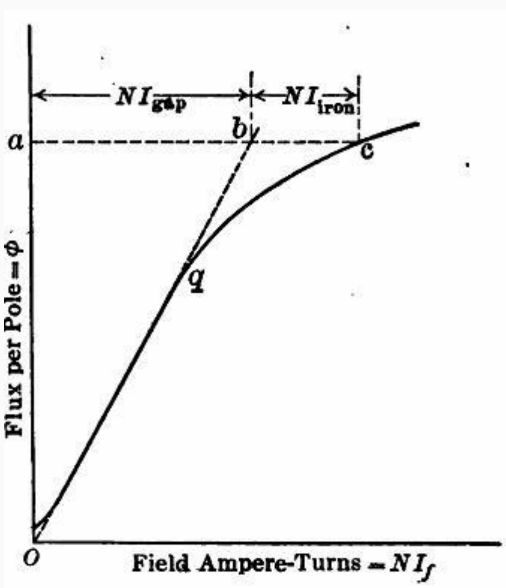


Fig. 7. 2 Saturation curve

The last equation may be written as

$$E = \left(\frac{PZ}{60\,p10^8}\right)\phi S$$

where S = r.p.m.

The quantity within the brackets is constant for a given machine and may be denoted by K. Therefore E = KφS. i.e. the induced emf. in a machine is directly proportional to the flux and to the speed. If the speed is kept constant, the induced voltage is directly proportional to the flux, φ.

The flux is produced by the field ampere-turns, and as the turns on the field remain constant, the flux is a function of the field current. It is not directly proportional to the field current because of the varying permeability of the magnetic circuit.

Figure 7.2 shows the relation existing between the field ampere-turns and the flux per pole. The flux does not start at zero ordinarily but at some value slightly greater, owing to the residual magnetism in the machine. At first the line is practically straight, as most of the reluctance of the magnetic circuit is in the air-gap. At the point 'q', the iron begins to be saturated arid the curve falls away from the straight line.

The number of field ampere-turns for the air-gap and for the iron can be approximately determined for any point on the curve.

Let it be required to determine the ampere-turns for the gap and for the iron at the point c. From the origin draw ob tangent to the saturation curve and also draw the horizontal line ac. The line ob is the magnetization curve of the air gap, if the reluctance of the iron at low saturation be neglected. Therefore, the ampere-turns required by the gap are equal to ab and those required by the iron are equal to bc.

The induced voltage is proportional to the flux, if the speed is maintained constant. Two curves as shown in Fig. 7. 3 below can so be plotted for 1,200 r.p.m. and the other for 900 r.p.m. The curves are similar, any ordinate of the lower curve being 900/1,200 of the value of the corresponding ordinate of the upper curve. Thus, at ordinate 'ac'

ab/ac = 900/1200 Also at a'b'/a'c' = 900/1200

If the saturation curve of a generator for one speed is available, saturation curves for other speeds may be readily found by the method.

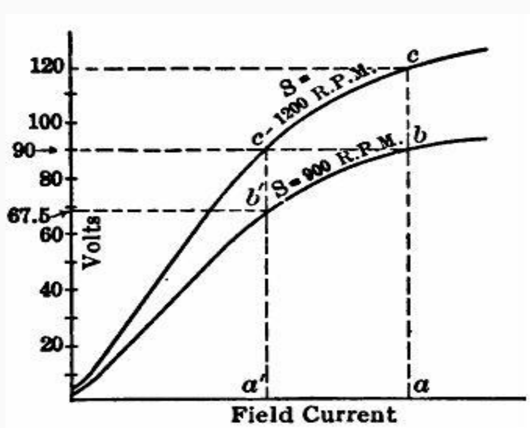
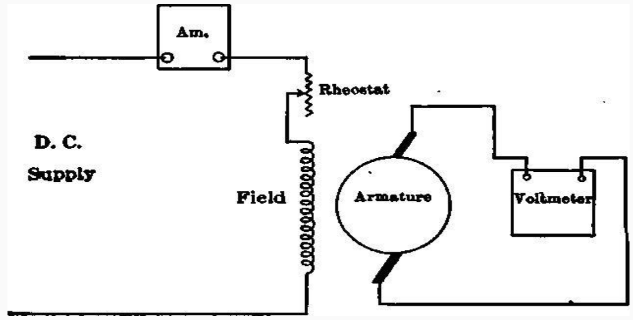


Fig. 7. 3 Saturation curves different speeds.

Determination of saturation curve.



To determine, the saturation curve experimentally, connect the field, in series with an ammeter, across a direct source of power. A voltmeter is connected across the armature terminals under no load. The readings of voltage for each field current is plotted as the saturation curve. As the voltage drop within the armature due to no load current is negligible, the terminal volts and the induced volts under these conditions are identical.

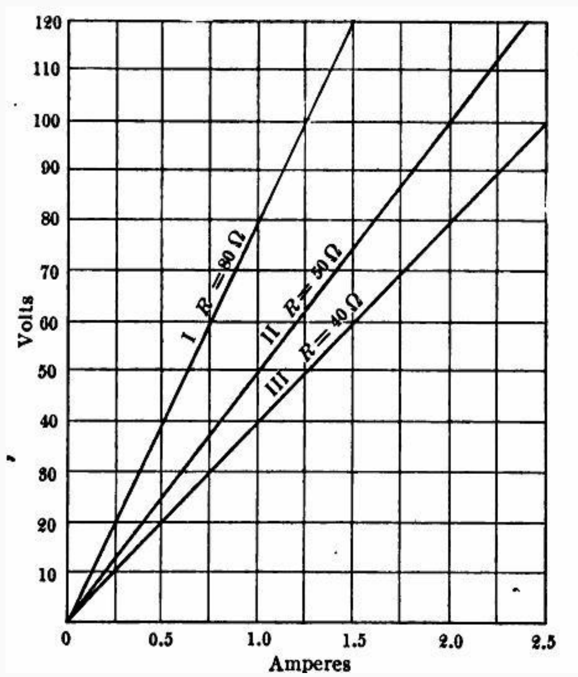


Fig. 7. 4 Field resistance lines.

Field Resistance Line: If the current in a resistance be plotted against volts, a straight line passing through the origin results. For example, if the resistance of a field circuit be 50 ohms, the current will be 2 amperes when the voltage is 100 volts; 1.5 amperes when the voltage is 75 volts, and 1 ampere when the voltage is 50 volts. This relation is shown in the Fig.7.4 below for some values of resistances. It may be noted that the higher the resistance the greater the slope of the resistance line. The slope of the line is equal to the field resistance in ohms.

LESSON 13. DC generators and their characteristics - Armature reaction and commutation

Types of generators

There are three general types of generator in common use, the shunt, the compound and the series. In the shunt type (Fig. 7. 5**)** the field circuit is connected across the armature terminals, usually in series with a rheostat. The shunt field, therefore, must have a comparatively high resistance that it does not take too great a proportion of the generator current. The compound generator is similar to the shunt, but has an additional field winding connected in series with the armature or load. The series generator is excited entirely by a winding of comparatively fewer turns connected in series with the armature and load.

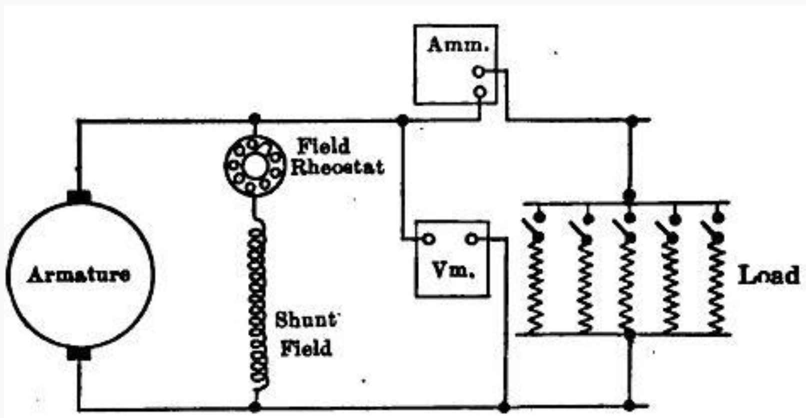


Fig. 7. 5 Shunt generator connections.

Shunt Generator.

Fig. 7. 6 below shows the saturation curve of a shunt generator and its shunt field resistance line drawn on the same plot. This field has a resistance of 24 ohms, so that at 120 volts it takes 5 amp and so on. At the instant of starting a generator the induced voltage is zero. As the generator is brought up to speed there will be small voltage ‘oa’, here about 4 volts, induced in the armature due to the residual magnetism of the machine. This exists across the shunt field coil, because it is connected across the armature terminals. The resulting field current due to this voltage is obtained by drawing a horizontal line from a until it meets the field resistance line at b. The current in this particular case is ob' or about 0.2 ampere. By looking at the saturation curve, it is seen that for this field current, the induced voltage, b'c, is about 8 volts. The 8 volts produces about 0.33 ampere in the field, as may be seen by projecting across to the field resistance line at d. This field current od’ produces a voltage d'e, which in turn produces a higher value of field current. Thus each value of field current produces a voltage in excess of its previous value and this increased voltage in turn increases the field current, which is cumulative. The machine will continue to build up until point ‘f’ is reached, where the field resistance line crosses the saturation curve. The machine will not build up beyond this point for the following reasons:

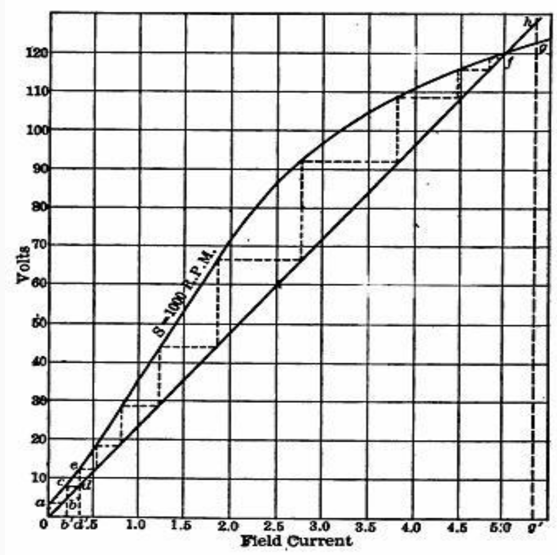
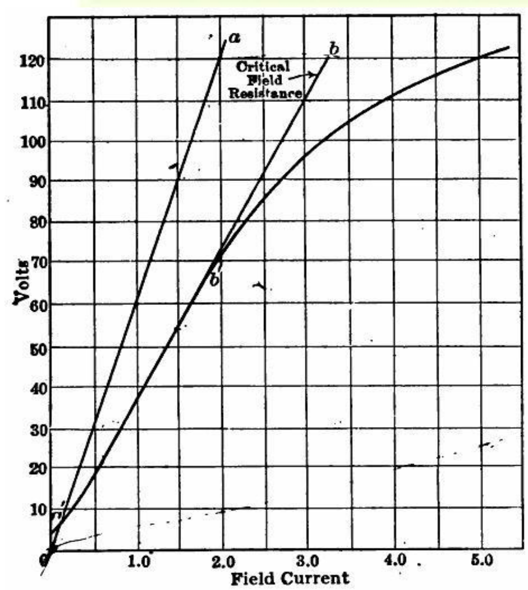


Fig. 7.6 Method of shunt generator building up.

Critical field resistance

If the resistance of the field be increased to say 60 ohms, the field resistance line will be represented by oa in Fig. 7.7. This line crosses the saturation curve at point a', corresponding to about 6 volts. Therefore, with this value of field resistance, the generator will not build up beyond a'. If the field resistance be slowly decreased until the field resistance line reaches ob, the generator will start building up rapidly. It will of course stop building up voltage at the point b'. The value of the field resistance corresponding to ob is called the critical field resistance. In this particular case the resistance is 120/3.25 or 36.1 ohms.

Armature Reaction



Critical field resistance

The next Fig. 7.8 shows the flux passing from the field poles through an armature when there is no current in the armature conductors. This flux is produced entirely by the ampere-turns of the field. The neutral plane, which is a plane perpendicular to the flux, coincides with the geometrical neutral axis of the poles. At the right is shown a vector F which represents the mmf. producing this flux, in magnitude and direction. At right angles to this vector F is the neutral plane.

In figure 7.8 (b), there is no current in the field coils, but the armature conductors are shown as carrying current. This current is in the same direction in the armature conductors as it would be were the generator under load. The current obviously flows in the same direction in all the conductors that lie under one pole. The current is shown as flowing into the paper on the left-hand side of the armature. (This current direction as obtained by Fleming's right-hand rule). These conductors combine their mmf.'s to send a flux downward through the armature, as shown in the diagram 7.8 (b), this direction determined by the corkscrew rule. The conductors on the right-hand side of the armature are shown as carrying current coming out of the paper. They also combine their mmf.'s to send a flux downward through the armature. The conductors on both sides of the armature combine their mmfs in such a manner as to send flux down through the armature. The direction of this flux is perpendicular to the polar axis. To the right of the figure the armature mmf. is represented in direction and magnitude by the vector F_A. Figure (c) shows the result obtained when the field current and the armature current are acting simultaneously, which occurs when the generator is under load.

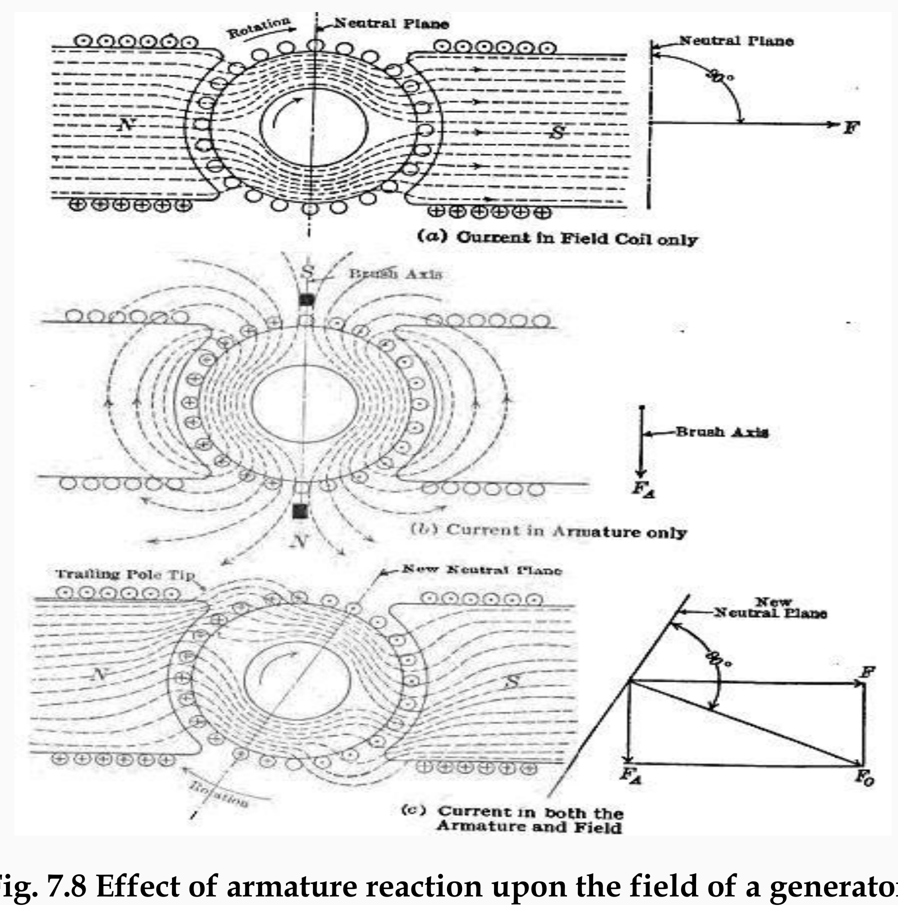


Fig. 7.8 Effect of armature reaction upon the field of a generator.

The armature emf crowds the symmetrical field flux shown in 7.8 (a) into the upper pole tip in the north pole and into the lower pole tip in the south pole. As the generator armature is shown rotating in a clockwise direction, the flux is crowded into the trailing pole tip and is weakened in the two leading pole tips.

To the right of figure 7.8 (c), the armature reaction is as vectors. The field vector F and the armature vector F_A add to form the resultant field vector F_0. The direction of F_0 is downward and to the right, which corresponds to the general direction of the resultant flux. As the magnetic neutral plane is perpendicular to the resultant field and hence has turned angularly. But we know that the brushes should be set so that they short-circuit the coil undergoing commutation as it is passing through the neutral plane.

As the load current varies, depending on its magnitude will be shifting the magnetic neutral plane accordingly. So the brushes if placed at the original geometric neutral plane would commute the armature when there is a current in the conductors resulting in sparking and subsequent pitting of commutator. If the brushes are advanced to correspond to the advance of the neutral plane, all the conductors to the left of the two brushes must still carry current into the paper, and those to the right must carry current out of the paper. The direction of the armature field moves with the brushes. Its axis always lies along the brush axis. Therefore F_A, instead of pointing vertically downward, now points downward and to the left, as is shown by the vectors. F_A may be resolved into two components, F_D parallel to the polar axis and F_c perpendicular to this axis. F_D acts in direct opposition to F_A, the main field and reduces the total flux and so is called the demagnetizing component of armature reaction (Fig. 7.9). F_c acts at right angles to F and produces distortion and is called the cross-magnetizing component of armature reaction.

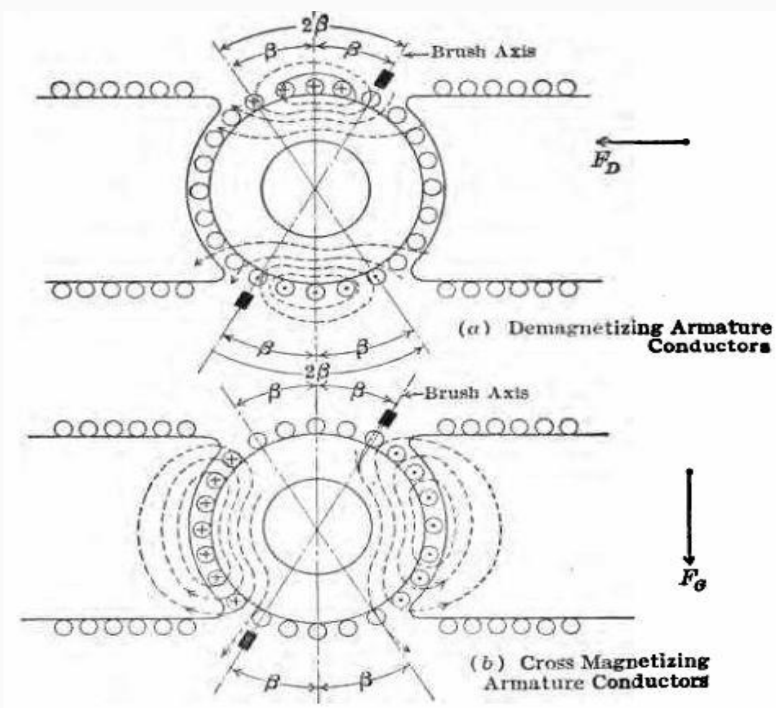


Fig. 7.9 Demagnetizing and cross-magnetizing components of armature reaction

LESSON 14. Shunt generator

The shunt generator

i) Characteristics: If a shunt generator, after building up to voltage, be loaded, the terminal voltage will drop (Fig. 7.10). This drop in voltage will increase with increase of load. Such a drop in terminal voltage is undesirable, especially when it occurs in generators which supply power to lamps. It is very important to know the voltage at the terminals of a generator for each value of current that it delivers, because the ability to maintain its voltage under load conditions determines in a large measure the suitability of a generator for certain specified service.

To test a generator, rated load should first be applied and the field current adjusted until rated voltage is obtained.

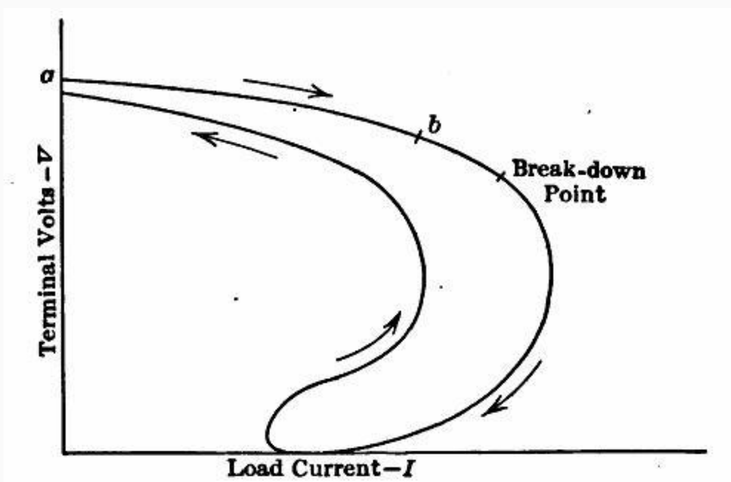


Fig. 7.10 Load Current—/Shunt generator characteristic.

The load should then be cut off and the no-load volts read on the voltmeter. The load should then be gradually applied, reading the volts and the current for each load. The speed of the generator should be maintained constant throughout. If the readings be plotted as shown, the shunt characteristic results. If in a small generator, the load be carried far enough, a rapid decrease of voltage will occur. This is called the break-down point of the generator. Further application of load results in a very rapid decrease of voltage and beyond a certain point any attempt at increase of load results in a decrease of current rather than an increase. The load may even be carried to short-circuit conditions and yet the current will actually decrease as short-circuit is approached. This is due to the fact that the field is short-circuited and any low current flowing at short-circuit is due to the residual magnetism of the machine only.

If the external resistance be now increased, the voltage will rise slowly and will ultimately reach a value that at which it started. The fact that the voltage follows a different curve when the short-circuit is removed is primarily due to hysteresis. When the load is being applied, the voltage is dropping and the iron is on the part of the cycle represented by c. When the voltage starts to increase, it returns along the path a,

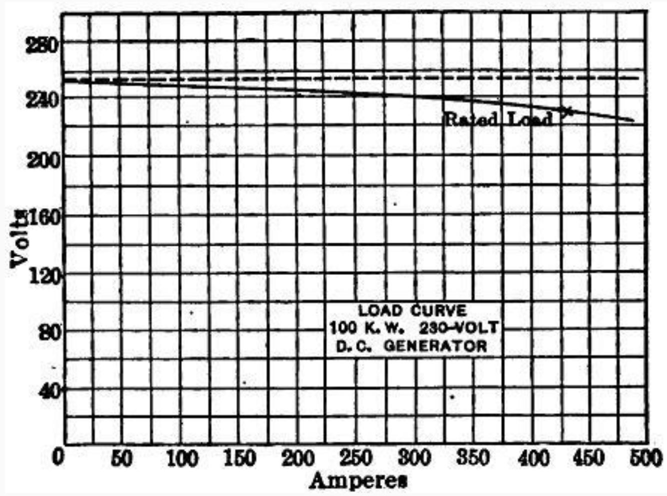


Fig. 7.11 Typical shunt characteristic.

There is less flux for a given field current and consequently less voltage is induced in the machine upon the return curve. This, together with a lesser field current resulting from the lower voltage, accounts for the return curve lying below the other.

In practice, machines are operated only on the portion ab of the characteristic. Figure 7.11 shows the typical curve for a 100-KW., 230-volt generator. The rated current is 100,000/230 = 435 amperes. The generator field rheostat is set so that the generator terminal voltage is 230 volts when it is delivering this load of 435 amperes.

There are three reasons for the drop in voltage (Fig. 7.12) under load of a shunt generator:

(1) The terminal voltage is less than the induced voltage by the resistance drop in the armature. That is, the terminal voltage

$V = E - I_a R_c$

where E is the induced volts, J« the armature current and R_a the armature resistance.

(2) Armature reaction weakens the field and so reduces the induced voltage.

(3) The drop in terminal voltage due to (1) and (2) results in a decreased field current. This in turn results in a lesser induced voltage.

Example.— The voltage induced within the armature of a shunt generator is 600 volts. The armature resistance is 0.1 ohm. What is the terminal voltage when the machine delivers 200 amp?

Applying equation, V - 600 - (200 X 0.1) « 600 - 20 - 680 volts

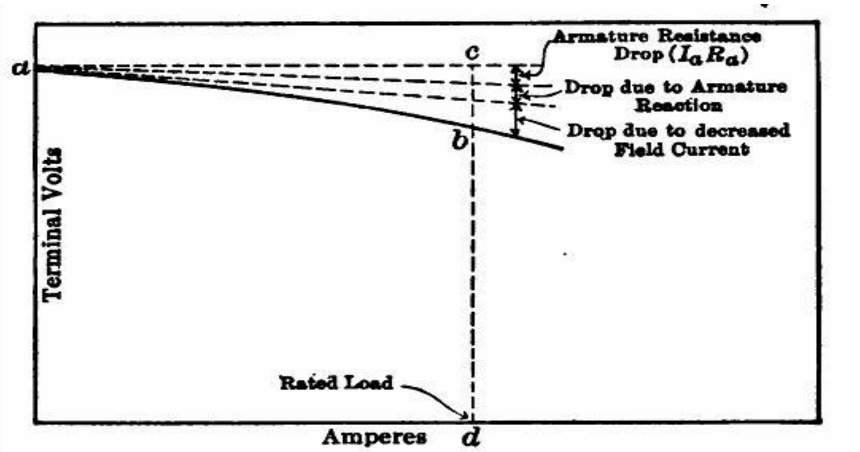


Fig. 7.12 Effect of each of these three factors.

Generator Regulation

The ability of a generator to maintain its voltage under load is a measure of its suitability for constant potential service. The regulation shows quantitatively the amount the voltage varies from rated load to no load.

The definition of regulation is the rise in voltage between rated load and no load. This is usually expressed as a percentage. Regulation may be more specifically defined as follows:

$$\text{Regulation} = 100\frac{\text{no load - rated load}}{\text{rated load}}\text{volts(per cent)}$$

Total Characteristic: The shunt characteristic is the relation existing between load current and terminal volts (Fig. 7.13). The total characteristic is the relation between armature current and induced volts. The armature current differs from the load current by the amount of current flowing in the field.

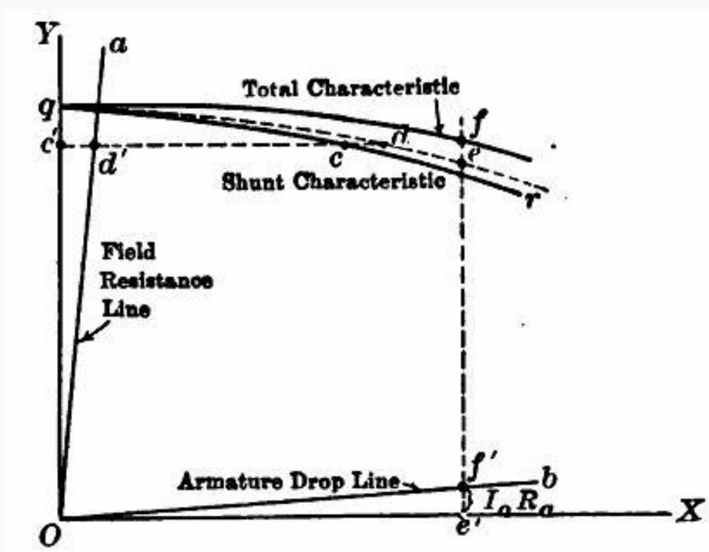


Fig. 7.13 Total characteristic of shunt generator

The armature current,

$I_a = I + I_f$

when I is the load current and I_f the shunt field current. The induced volts

$E = V + I_a R_a$

Where, V is the terminal voltage and R_a the armature resistance, including brush and brush contact resistance. The total characteristic is the curve showing the relation of I_a and E. It may be found graphically from the shunt characteristic as follows:

Example.—A 20-KW, 220-volt, shunt generator has an armature resistance of 0.07 ohm and a shunt afield resistance of 100 ohms. What power is developed in the armature when it delivers its rated output?

Rated current

$$I = \frac{20{,}000}{220} = 90.9 \, amp.$$

Field current

$$I_1 = \frac{220}{100} = 2.2 \, amp.$$

Armature current

$$I_a = 90.9 + 2.2 = 93.1 \text{ amp.}$$

Induced volts

$$E = 220 + (93.1 \times 0.07) = 226.5 \text{ volts.}$$

Power developed in armature

$$P = 226.5 \times 93.1 = 21.1 \text{kw.} \quad \text{Ans.}$$

LESSON 15. Compound and Series generators

The Compound Generator:

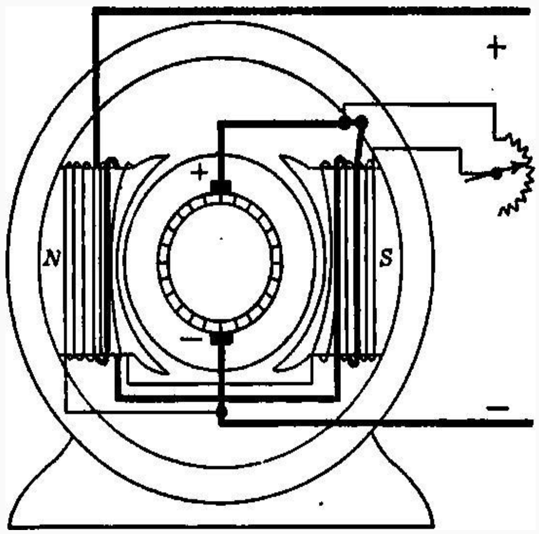


Fig. 7.15 Compound generator connections.

The drop in voltage with load, which is characteristic of the shunt generator, makes this type of generator undesirable where constant voltage is essential. This applies particularly to lighting circuits, where a very slight change of voltage makes a material change in the power of lamps. A generator may be made to produce a substantially constant voltage, or even a rise in voltage as the load increases, by placing on the field core a few turns which are connected in series with the load.

These turns are connected' so as to aid the shunt turns when the generator ers current. As the load increases,the current through the series turns also increases and, therefore, the flux through the armature increases. The effect of this increased flux is to increase the induced voltage. By proper adjustment of the series ampere-turns, this increase in armature voltage is made to balance the drop in voltage due to armature reaction and that due to the resistance drop in the armature. If the terminal voltage is maintained substantially constant, the field current will not drop as the load increases. Therefore, the three causes of voltage drop, namely, armature reaction, I_aR drop, and drop in field current, are neutralized more or less completely by the effect of the series ampere-turns.

The shunt field may be connected directly across the armature terminals, Fig.7.15 (a) above, in which case the machine is called short shunt. If the shunt field be connected across the machine terminals outside the series field, as in fig 7.15 (b), the machine is long shunt. The operating characteristic is about the same in either case (Fig. 7.16).

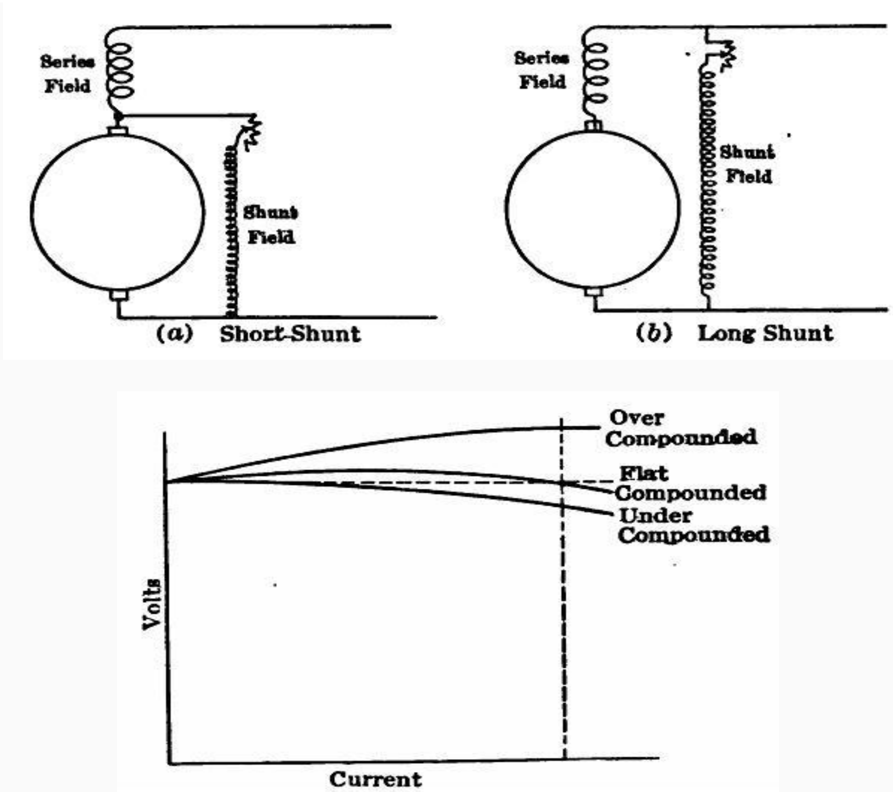


Fig. 7.16 Compound generator Characteristics

If the effect of the series turns is to produce the same voltage at rated load as at no load, the machine is said to be flat compounded. It is seldom possible to maintain a constant voltage for all values of current from no load to rated load. The tendency is for the voltage first to rise and then to drop again, reaching the same voltage at rated load as was obtained at no load. The particular shape of the characteristic is due to the iron becoming saturated, so that the added series ampere-turns do not increase the flux at full load as much as they do at light load. When the rated-load voltage is greater than the no-load voltage, the machine is said to be over compounded. When the rated-load voltage is less than the no-load voltage, the machine is said to be under compounded. Generators are seldom under compounded. Flat-compounded generators are used principally in isolated plants, such as hotels and office buildings. Over-compounded generators are used where the load is located at some distance from the generator. As the load increases, the voltage at the load tends to decrease, due to the voltage drop in the feeder. If, however, the generator voltage rises just enough to offset this feeder drop, the voltage at the load remains constant.

In a compound generator the induced voltage in the armature is:

. . $E = V + I_a R_a + I_f R_f$

where V is the terminal voltage, I_f the series field current, I_a the armature current, and R_f and R_a the series field and armature resistance respectively. In a long shunt generator $I_f = I_a$.

Example.—A compound generator, connected short shunt, has a terminal voltage of 230 volts when it is delivering a current of 150 amp. The shunt field current is 4 amp, the armature resistance 0.03 ohm and the series field resistance 0.01 ohm. Determine the induced voltage in the armature, the total power generated in the armature.

The series field current I_f = 150 amp., and the armature current I_a = 154 amp.

E = 230 + (150 × 0.01) + (154 × 0.03) = 236.1 volts.

Total power generated

P_a = 236.1 × 154 = 36,400 watts = 36.4 kw.

Armature loss

P_a = 154^2 × 0.03 = 711 watts.

Series field loss

P_f = 150^2 × 0.01 = 225 watts.

Shunt field loss

P_{sh} = (230 +1.5)4 = 926 watts.

Power delivered

P = 230 × 150 = 34,500 watts.

Effect of Speed

Fig. 7.17 shows the saturation curve of a 230-volt, compound generator, taken at

900 r.p.m

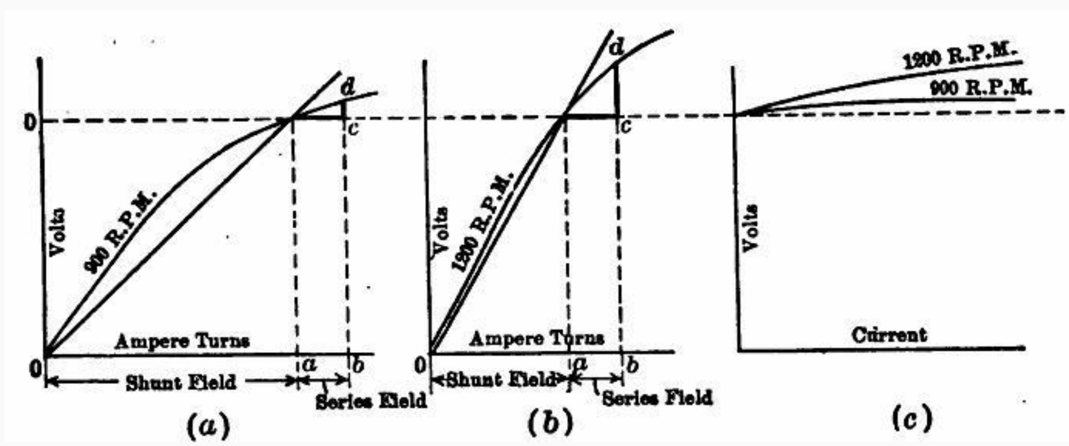


Fig. 7.17 Effect of speed upon compound characteristic.

The shunt field rheostat is so adjusted that the machine builds up to a no-load voltage of 230 volts. To produce this result a certain number of shunt field ampere-turns are necessary, as indicated by the distance 'oa'. When load is applied to the machine a certain number of series ampere-turns are added. Let the number of series ampere-turns be represented by the distance 'ab'. Neglecting armature reaction, the induced voltage will be increased by a value 'cd' shown in heavy lines. Let this same machine be speeded up to 1,200 r.p.m (Fig. 7.17 (b)), and let the no-load terminal voltage still be 230 volts. The distance 'oa' will now be less than it was in (Fig. 7.17 (a)), owing to the increased speed. But the distance 'ab' will be the same in each case, as the increase of series turns depends solely on the load. The increase of voltage 'cd' is much greater in (Fig. 7.17 (b)) than in (Fig. 7.17 (a)), owing to the lesser saturation of the iron. Therefore, the higher speed machine will have the more rising characteristic, as is shown in (Fig. 7.17 (c)). It will be noted that the effect of speed upon the compound characteristic is just opposite to the effect of speed upon the shunt characteristic. This is due to the fact that saturation opposes change of the flux in each case.

The Series Generator:

In the series generator the field winding is connected in series with the armature and the external circuit. It must consist necessarily of a comparatively few turns of wire having a sufficiently large cross-section to carry the rated current of the generator. The series generator in most instances is used for constant current work, in distinction to the shunt generator which maintains constant potential. Fig.7.18 shows the saturation curve of a series generator and also it's characteristic. The saturation curve differs in no way from that of the shunt generator. The external characteristic is similar in shape to the saturation curve for low saturation. The voltage at each point is less than that shown

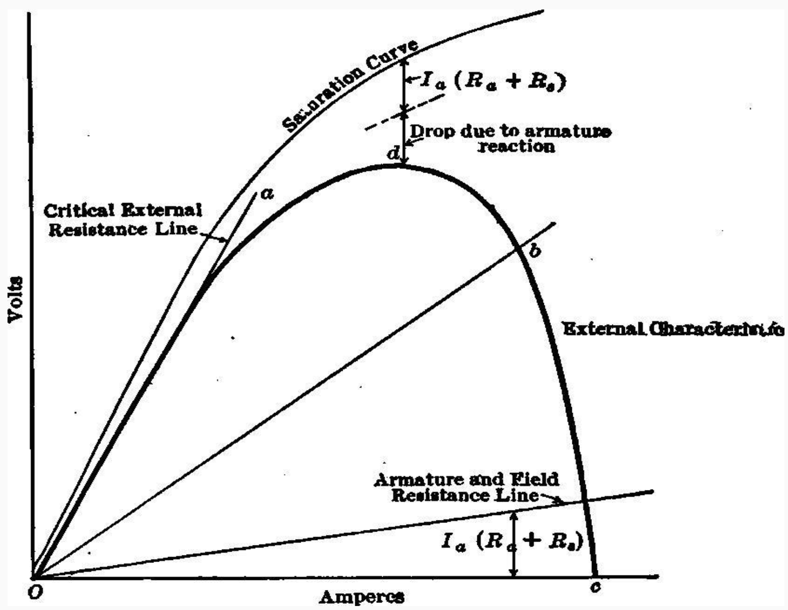


Fig. 7.18 Series generator characteristic.

by the saturation curve by the amount due to the drop through the armature and field $I_a (R_a + R_f)$ and the drop due to armature reaction. The curve reaches a maximum beyond which armature reaction becomes so great as to cause the curve to droop sharply and the voltage drops rapidly to zero. These machines are designed to have a very high value of armature reaction.

The machine builds up as follows: If the series field is connected in such a manner that the current due to the residual magnetism aids this residual magnetism, the generator will build up, provided the external resistance equals or is less than that indicated by the external resistance line 'Oa'. The line 'Oa' is therefore called the critical external resistance line. As the external resistance decreases, the external resistance line swings down to the right. The line 'ob' is such a line. It would be practically impossible to operate with an external resistance corresponding to the line 'Oa', or to any line cutting the curve to the left of 'd', as a small increase in external resistance would swing the resistance line away from the curve resulting in the generator's dropping its load. The machine is designed to operate along the portion 'bc' of the curve, which corresponds to substantially constant current. The current is not affected by a considerable change in external resistance, corresponding to the line 'Ob' swinging up or down. To obtain close regulation the series field is shunted by a rheostat. The resistance of this rheostat is controlled by a solenoid connected in series with the line. In this way the current delivered by the generator may be held substantially constant. In the past, the series generator was much used in arc lighting.

MODULE 8. DC motor characteristics, starting of shunt and series motor, starters, speed control methods-field and armature control

LESSON 16. DC motors

THE MOTOR

A generator is a machine for converting mechanical energy into electrical energy and the motor is a machine for converting electrical energy into mechanical energy. The same machine however, may be used either as a motor or as a generator.

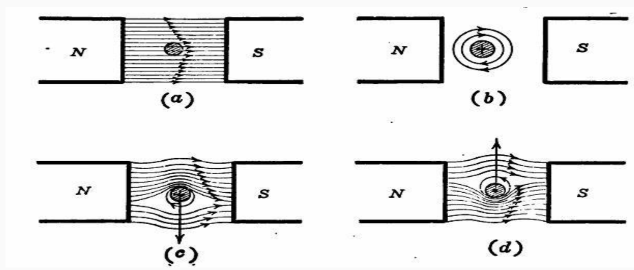


Fig. 8. 1 Force acting on a conductor carrying current in a magnetic field.

Fig. 8.1 (a) above shows a magnetic field of constant strength in which is placed a conductor that carries no current. In Fig. 8. 1 (b) the conductor is shown as carrying a current into the paper, but the field due to the N and S poles has been removed. A cylindrical magnetic field now exists about the conductor due to the current in it (R.H cork screw rule). Figure Fig. 8. 1 (c) shows the resultant field obtained by combining the main field and that due to the current. The field due to the current in the conductor acts in conjunction with the main field above the conductor, whereas it opposes the main field below the conductor. The result is to crowd the flux above the conductor and to reduce the flux density in the region below the conductor. It will be found that a force acts on the conductor, trying to push the conductor down, as shown by the arrow. If the current in the conductor is reversed, the force will tend to move it upward, as shown in Fig. 8. 1 (d). The electric motor works upon this fact that a conductor carrying current in a magnetic field tends to move at right angles to the field.

Force Developed with Conductor Carrying Current.— The force acting on a conductor is expressed as,

$$F = \frac{BlI}{10} \text{ dynes}$$

Where, B is the flux density in lines per cm^2 (gauss), l the active length of the conductor in cm and I the current in amperes.

Fleming's Left hand Rule (Fig. 8. 2) gives the relation between the direction of a magnetic field, the direction of a current in that field and the direction of the resulting motion of the conductor.

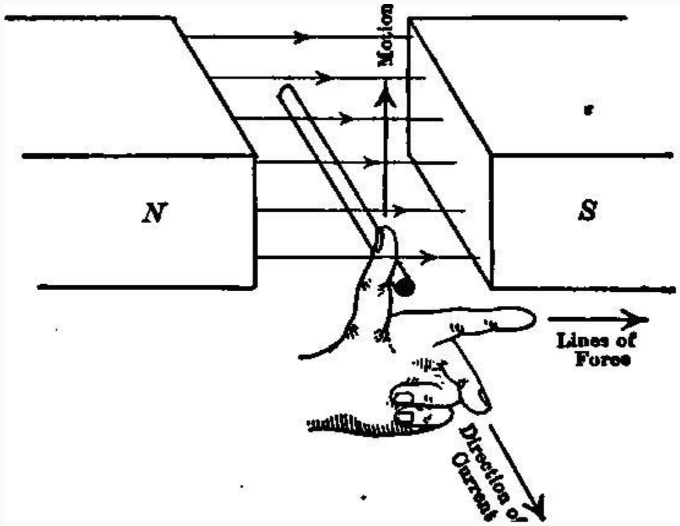


Fig. 8. 2 Fleming's left-hand rule.

"Point the forefinger in the direction of the field or flux, the middle finger in the direction of the current in the conductor, and the thumb will point in the direction in which the conductor tends to move."

Example. A coil consisting of 20 turns lies with its plane parallel to a magnetic field, the flux density in the field being 3,000 lines per cm^2. The axial length of the coil is 8 inch. The current per conductor is 30 A. Determine the force which acts on each side of the coil.

$B = 3{,}000$

$l = 8 \times 2.54 = 20.32$ cm.

$I = 30$ A

$F_1 = 3{,}000 \times 20.32 \times 30/10 = 182{,}900$ dynes.

As there are 20 turns

$F = 20 \times 182{,}900 =$ **3,658,000 dynes.**

Torque: When an armature, a fly wheel or any other device is revolving about its center, a tangential force (Fig. 8. 3) is necessary to produce and maintain rotation. This force may be developed within the machine itself as in a motor or steam engine, or it may be applied to a driven device such as

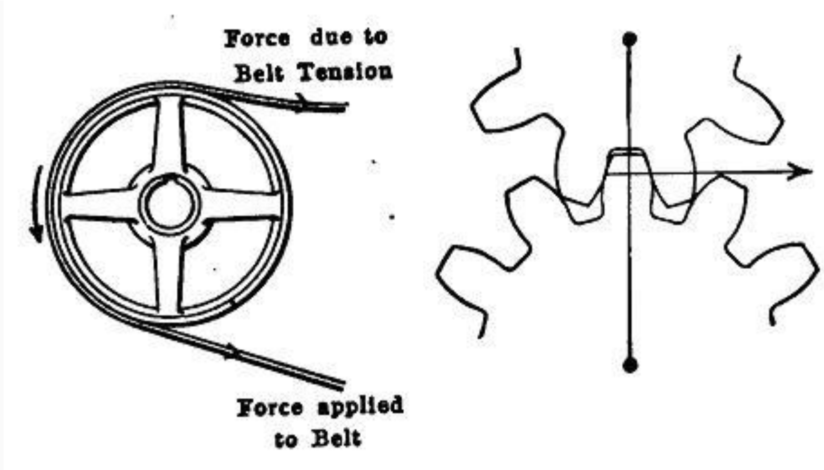


Fig. 8. 3 Torque developed by a belt and by gears.

a pulley, a shaft, a generator, the driving gears on the wheels of car, etc. The product of this force and its perpendicular distance from the axis is called torque. Torque is a mechanical couple tending to produce rotation. In the SI, unit of torque is the Nm and in the metric system the unit is the kilogramforce-meter.

LESSON 17. DC motor – Torque Equation

Torque Developed by a Motor. Figure 8. 4 (a) below shows a coil of a single turn, whose plane lies parallel to a magnetic field. Current flows into the paper in the left-hand side of the coil and out of the paper in the right-hand side of the coil. Therefore, the left-hand conductor tends to move downward with a force F_1 and the right-hand conductor tends to move upward with a force

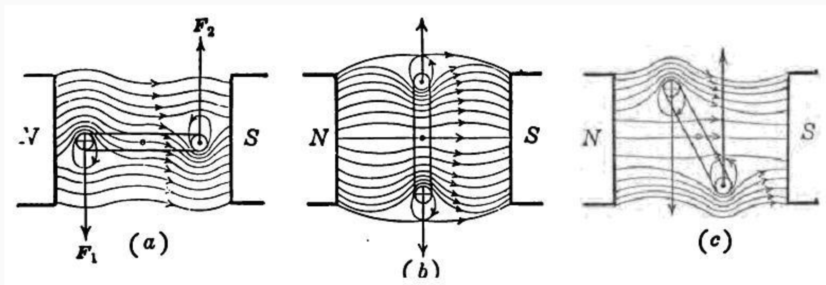


Fig. 8. 4 Torque developed at different positions of a coil.

F_2.These two forces tend to rotate the coil about its axis. Both act to turn it in a counter-clockwise direction and so develop a torque. As the current in each of these conductors is the same and they lie in magnetic fields of the same strength, force $F_1 = F_2$. In Fig. 8. 4 (a) the coil is in the position of maximum torque because the perpendicular distance from the coil axis to the forces acting is a maximum. When the coil reaches the position Fig. 8. 4 (b) this is a position of zero torque because the perpendicular distance from the coil axis to the forces is zero. If, however, the current in the coil be reversed when the coil reaches position Fig. 8. 4 (b) and the coil be carried slightly beyond the dead center, as shown in Fig. 8. 4 (c) a torque is developed which tends to turn the coil in the counter-clockwise direction.

To develop a continuous torque in a motor, the current in each coil on the armature must be reversed just as it is passing through the neutral plane or plane of zero torque and a commutator is therefore necessary. This is analogous to using a commutator in connection with a generator in order that the current delivered to the external circuit may be uni-directional.

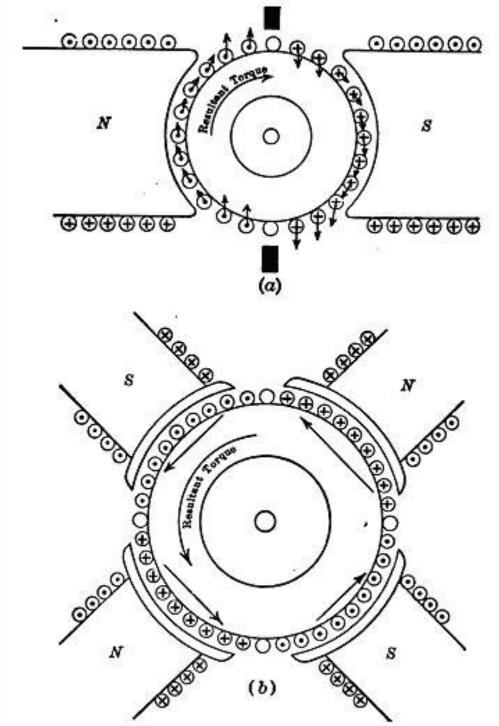


Fig. 8. 5 Torque developed by belt conductors in motor armatures

A single-coil motor as explained now would be impracticable as it has dead centers and the torque which it develops is pulsating. A two-coil armature would eliminate the dead centers, but the torque developed would still be more or less pulsating in character.

The best conditions are obtained when a large number of coils is used, just as in the armature of a generator. In fact there is no difference in the construction of a motor armature and a generator armature. In the figure 8. 5 *(a)*, an armature and a field are shown for a 2-pole machine and the torque developed by each individual conductor is indicated. Figure 8. 5 (b) shows an armature and a field for a 4-pole machine. The direction of the torque developed by each belt of conductors is indicated by the arrow at that belt.

In armatures of this type a very small proportion of the total number of coils is undergoing commutation at any one instant. Therefore, the variation in the number of active conductors is so slight that the torque developed is substantially constant, for constant values of armature current and main flux.

The torque developed by any armature can be shown to be

$T = K'_t ZIF$

where K'_t = a constant of proportionality, involving the diameter of the armature, the parallel paths through the armature, the choice of units, etc.

Z = number of conductors on the surface of the armature.

I = current supplied to the armature, in A.

F = flux from one north pole entering the armature.

For any particular machine Z is a fixed quantity, so that the torque

$T = K_t I F$

where K_t is a new constant of proportionality, i.e., in a given motor, the torque is proportional to the armature current and to the strength of the magnetic field.

Example.- When a certain motor is drawing 50 A from the line it develops 60 Nm torque. If the field strength is reduced to 75 percent of its original value and the current increased to 80 A, what is the new value of the torque developed?

If the current remained constant the new value of torque, due to the weakening of the field, would be

$0.75 \times 60 = 45$ Nm.

Due to the increase in the value of the current, however, the final value of torque will be

$(80/50) \times 45 =$ **72 Nm**

Back Electromotive Force. The resistance of the armature of the ordinary 10-horsepower, 220-volt motor is about 0.05 ohm. If this armature were connected directly across 110-volt mains, the current, by Ohm's Law, would be 110/0.05 = 4400 A. This value of current is not only excessive but unreasonable, especially when we know that its rated current of that motor is 90 A. So when a motor is in operation, the current through the armature is evidently not determined by its ohmic resistance alone. The armature of a motor is in every way similar to that of a generator. The conductors on the armature surface are cutting flux and therefore must be generating an electromotive force. If the right-hand rule is applied to determine the direction of this induced electromotive force, it will be found that it is always in opposition to the current as shown below.

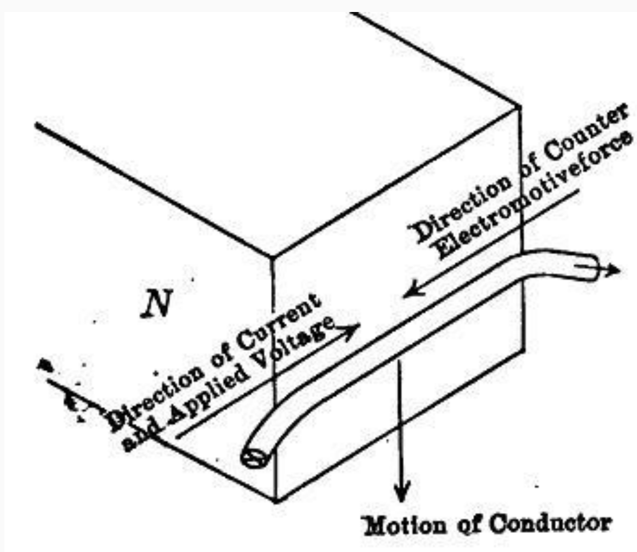


Fig. 8. 6 Torque developed by belt conductors in motor armatures

That is, it opposes the current entering the armature. This induced emf is called the back electromotive force or back emf. As the back emf opposes the current it must also oppose the line voltage. Therefore, the net emf acting in the armature circuit is the difference of the line voltage and the back electromotive force. Let *V be* the line voltage and *E* the back emf, then the net voltage acting in the armature circuit is *V – E*. The armature current follows Ohm's Law and is $I_a = \frac{V - E}{R_a}$, where R_a is the armature resistance.

This equation may now be written as $E = V - I_a R_a$

In a generator, the induced emf (E) is equal to the terminal voltage (V) plus the armature resistance drop ($I_a R_a$). But here in a motor, the induced emf. is equal to the terminal voltage minus the armature resistance drop. The back emf must always be less than the terminal or impressed voltage if current is to flow in the armature.

Example.—Determine the back emf of a 10-hp motor, when the terminal voltage is 110 volts and its armature is taking 90 A. The armature resistance is 0.05 ohm.

E = 110 - (90 X 0.05) = 110 - 4.5 = **105.5 volts**.

The equation developed earlier for induced electromotive force in a generator will obviously apply to a motor. That is, the back emf (induced emf in motor)

$$E = \frac{\Phi \, sPZ}{p \, 10^8} \text{ volts}$$

where *j* is the total flux entering the armature from one north pole, s the speed of the armature in revolutions per second, *P* the number of poles, *Z* the number of conductors on the surface of the armature, and *p* the parallel paths through the armature. As Z, P and *p* are all constants for any given motor, the back emf becomes

$E = K_1 \varphi S$

where S being given in R. P. M.S

i.e. where $S = \frac{KE}{\varphi}$ $K = 1/K_1$

"The speed of a motor is directly proportional to the counter electromotive force and inversely proportional to the field"

Substituting'for *E*, $S = K (V - I_a R_a) / \varphi$

Example.—A certain motor has an' armature resistance of 0.1 ohm. When connected across 110-volt mains and taking 20 amp. its speed is 1,200 r.p.m. What is its speed when taking 50 amp. from these same mains, with the field increased 10 per cent. ? .

$$\frac{S_2}{S_1} = \frac{K\,\dfrac{110-50 \times 0.1}{\Phi_2}}{K\,\dfrac{110-20 \times 0.1}{\Phi_1}} = \frac{\dfrac{105}{\Phi_2}}{\dfrac{108}{\Phi_1}} = \frac{105}{\Phi_2} \cdot \frac{\Phi_1}{108}$$

^S_1 = 1200

Therefore, $S_2 = 1{,}200\,(105 / 108)\,(\varphi_1 / \varphi_2)$

But $\varphi_2 / \varphi_1 = 1.10$

Therefore, S_2 = **1,060 r.p.m.**

LESSON 18. DC shunt motor characteristics

The Shunt Motor

The shunt motor is connected in the same manner as a shunt generator, that is, its field is connected directly across the line in parallel with the armature. A field rheostat is usually connected in series with the field. If mechanical (braking) load is applied to any motor, it immediately tends to slow down. In the case of the shunt motor this decrease of speed lowers the back electromotive force, as the flux remains substantially constant. If the back emf is decreased, more current flows into the armature according to $E = V - I_a R_a$. This continues until the increased armature current produces sufficient torque to meet the demands of the increased load. The suitability of a motor for any particular duty is determined almost entirely by two factors, the variation of its torque with load and the variation of its speed with load.

In the shunt motor the flux is substantially constant. Therefore, from torque equation, the torque will vary almost directly with the armature current. For example, in figure below, when the armature current is 30 amp, the motor develops 40 lbft torque, and when the current is 60 amp, the motor develops 80 lbft torque. That is, when the current doubles the torque doubles.

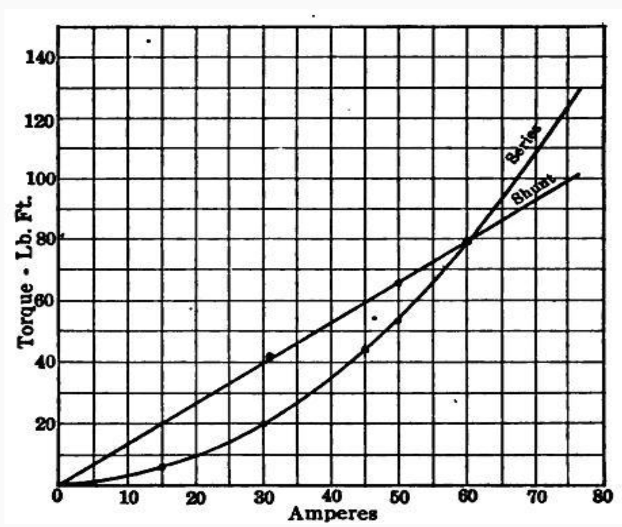


Fig. 8.7 Shunt and series motors; torque-current curves.

The speed of a motor varies according to equation $S = K(\frac{V - I_a R_a}{\varphi})$ In the case of the shunt motor, *K, V, R_a,* and φ are all substantially constant. Therefore, the only variable is I_a. As the load on the motor increases, I_a increases and the numerator of this equation decreases. As a rule the denominator changes only a small amount. The speed of the motor will then drop with increase of load, as shown in the next figure. As $I_a R_a$ is ordinarily from 2 to 6 per cent, of *V*, the percentage drop in speed of the motor is also of same magnitude. For this reason the shunt motor is considered a constant speed motor, even though its speed does drop slightly with increase of load. Owing to armature reaction, φ ordinarily decreases slightly with increase of load and this tends to maintain the speed constant. Occasionally the armature reaction is sufficiently great to give a rising speed characteristic with increase of load.

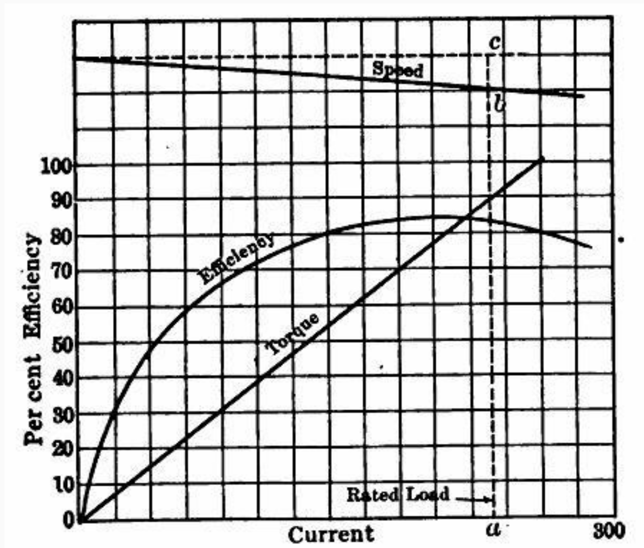


Fig. 8.8 Typical shunt motor characteristics

Speed Regulation: The speed regulation of a shunt motor is almost identical with the voltage regulation of a shunt generator. It is the difference in the no-load and the rated-load speed divided by the no-load speed. In the figure, the percentage speed regulation is hence

$$\frac{ca - \mathrm{ba}}{\mathrm{ca}} 100 = \frac{\mathrm{cb}}{\mathrm{ca}} 100$$

The figure 8.8 shows the three essential characteristics of a shunt motor, the torque, the speed, and the efficiency, each plotted against current. It will be noted that the shunt motor has a definite no-load speed. Therefore it does not run away when the load is removed, provided the field circuit remains intact. Shunt motors are used where a constant speed is required, as in machine shop drives, spinning frames, blowers, etc. There is an erroneous impression that shunt motors have a low starting torque and therefore, should not be started under load. Starters are usually designed to allow 125 per cent of full load current to flow through the armature on the first notch. Therefore, the motor develops 125 per cent, of full-load torque at starting. By decreasing the starting resistance, the motor could be made to develop 150 per cent, of full-load torque without trouble.

LESSON 19. DC series motor characteristics

The Series Motor.

In the series motor the field is connected in series with the armature, as shown in figure 8.9.

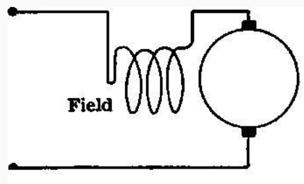


Fig. 8.9 Series motor

The field has comparatively few turns of wire and this wire must be of sufficient cross-section to carry the rated armature current of the motor. In the series motor the flux, φ depends entirely on the armature current. If the iron of motor is operated at moderate saturation, the flux will be almost directly proportional to the armature current. Therefore, in the expression for torque,

$T = K_t I \varphi$, if φ is assumed to be proportional to I,

the expression becomes

$T = K'_t I^2$ where K'_t is a constant.

The torque is proportional to the square of the armature current, as shown in figure below. The doubling of the armature current results in the quadrupling of the torque. It will be noted that as the current increases above 60 A, the torque rises very rapidly. This characteristic of the series motor makes its use desirable where large increases of torque are desired with moderate increases in current. In practice, saturation and armature reaction both tend to prevent the torque increasing as rapidly as the square of the current. When speed equation already seen for shunt motor, is applied to the series motor, the speed

$$S = K\left(\frac{V - I_a(R_a + R_s)}{\varphi}\right)$$

where K is a constant, V the terminal voltage, I_a the motor curtrent, R_a the armature resistance including brushes, R_s the series field resistance and φ the flux entering the armature from a north pole. R_s the resistance of the series field, is now added to the armature resistance in order to obtain the total motor resistance. Both I_a and φ vary with the load.

As the load increases, the voltage drop in the field resistance and the armature resistance increases because this voltage drop is proportional to the current. Therefore, the back emf. becomes less, which causes the motor to run more slowly, although this effect is only of the magnitude of a few per cent. The flux φ, however, increases almost directly with the load.

Therefore the speed must drop, such that the back emf is less than the terminal voltage. Both effects tend to slow down the motor. The resistance drop is ordinarily from 2 to 6 per cent, of the terminal voltage V, so its effect on the speed is only of this magnitude. The speed is, however, inversely proportional to the flux φ and a given percentage change in φ produces the same percentage change in the speed.

When the load is decreased, the flux φ correspondingly decreases and the armature must speed up in order to develop the required back emf. If the load be removed altogether, φ becomes extremely small, resulting in a very high speed. **It is dangerous to remove the load from series motors, as their armatures are almost certain to reach speeds where centrifugal action will wreck them.**

Figure 8.10 shows the characteristic curves of a series motor plotted with current as abscissas. The torque curve concaves upward for the reasons which have just been stated. The speed is inversely proportional to the current. The characteristics cannot be determined for small values of current because the speed becomes dangerously high. The efficiency increases rapidly at first, reaches a maximum at about half load and then decreases. This is due to the fact that at light loads the friction and iron losses are large as compared with the load. The effect of these becomes less as the load increases. The field and armature loss varies as the square of the current , so these losses increase rapidly with the load. The maximum efficiency occurs when the friction and iron losses are practically equal to the copper losses.

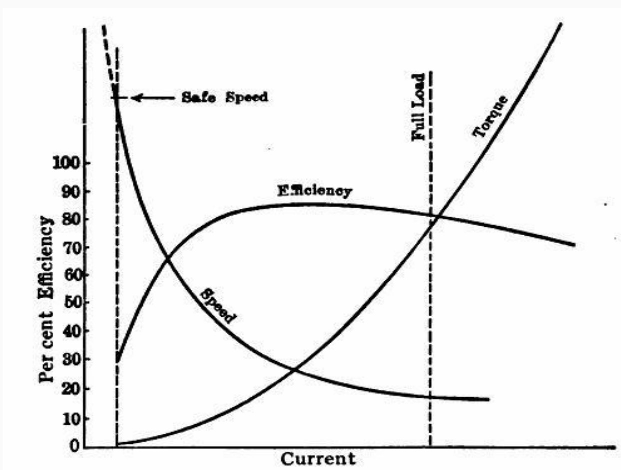


Fig. 8.10 Typical series motor characteristics.

Series motors are used for work demanding large starting torque, such as locomotives, cranes, etc. In addition to the large starting torque, there is another characteristic of series motors which makes them especially desirable for traction purposes. Assume that a shunt motor is used to drive a locomotive. When the vehicle ascends a grade, the shunt motor maintains the speed at approximately the same value that it has on level ground. The motor therefore tends to take an excessive current. A series motor, on the other hand, automatically slows down upon reaching such a grade, because of the increased current. It therefore develops more torque at reduced speed. The drop in speed allows the motor to develop a large torque with but a moderate increase of power. Hence, a series motor could be made smaller than a shunt motor operating under the same conditions.

LESSON 20. DC compound motor characteristics and DC motor starters

The Compound Motor

A shunt motor if added with an additional series winding becomes a compound motor. This winding may be connected to aid the shunt winding, in which case the motor is said to be cumulative compound; or the series winding may oppose the shunt winding, in which case the motor is said to be differential compound. The characteristics of the cumulative compound motor are a combination of the shunt and series characteristics. As the load is applied the series turns increase the flux, causing the torque for any given current to be greater than it would be for the simple shunt motor.

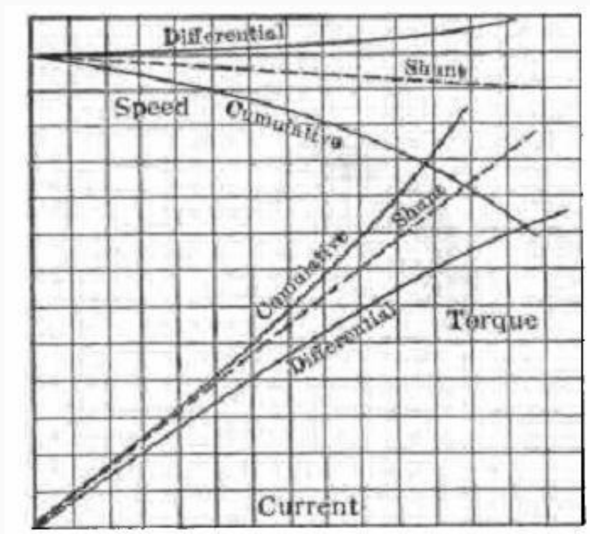


Fig. 8.11 Torque and speed characteristics of shunt and compound motors

On the other hand, this increase of flux causes the speed to decrease more rapidly than it does in the shunt motor. These characteristics are shown in the figure 8.11. The cumulative compound motor develops a high torque with sudden increase of load. It also has a definite no-load speed, so does not run away when the load is removed.

Its field of application lies principally in driving machines which are subject to sudden applications of heavy load, such as in rolling mills, shears, punches, etc. This type of motor is used also where a large starting torque is desirable but where a straight series motor cannot be conveniently used. Cranes and elevators are representative of such loads.

In the differential compound motor, the series field opposes the shunt field so that the flux is decreased as the load is applied. This results in the speed remaining substantially constant or even increasing with increase of load. This speed characteristic is obtained with a corresponding decrease in the rate at which the torque increases with load. Such motors are used where a very constant speed is desired. Because of the substantially constant speed of the shunt motor there is little occasion to use the differential motor. In starting a differential compound motor the series field should be short-circuited, as the large starting current passing through the series field may be sufficiently large to overbalance the shunt field ampere-turns and cause the motor to start in the wrong direction.

To reverse the direction of rotation in any DC motor, either the armature alone or the field alone must be reversed. If both are reversed the direction of rotation remains unchanged.

DC Motor Starters

It was shown that if a 10-hp 220 volt DC motor were connected directly across DC mains, the resulting current (since there wont be back emf at staring time) would be 4400 A. Such a current would not be permissible and hence, resistance should be connected in series with the motor armature when starting. This resistance may be gradually cut out as the armature comes up to speed and develops a back electromotive force. Figure 8.12 shows the use of a simple resistance R for starting a motor. It will be noted that this resistance is in the armature circuit and that the field is connected directly across the line and outside the resistance. If the field were connected across the armature terminals, putting the resistance R in series with the whole motor, there would be little or no voltage across the field. There would be little torque developed and difficulty in starting would be experienced.

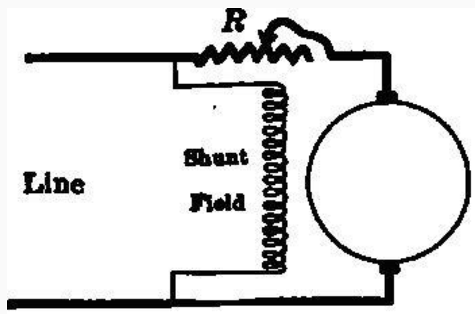


Fig. 8.12 Resistance used for starting purposes

Figure 8.13 shows a 3-point starter, which does not differ fundamentally from the connections shown above. One line connects directly to an armature and a field terminal tied together. It makes no connection whatever with the starter. The other line goes to the line terminal of the starter which is connected directly to the starting arm. The starting arm moves over contacts set in the insulator front board of the starter. These contacts connect with taps distributed along the starting resistance. The armature terminal of the starting box, which is the right-hand end of the starting resistance, is connected to the other armature terminal of the motor. The field connection in the starter is connected from the first starting contact, through the hold up magnet, to the field terminal of the box. This field terminal is connected directly to the other terminal of the shunt field. When the starting arm makes connection with the first contact, the field is put directly across the line and at the same time all the starting resistance is in series with the armature. As this arm is moved slowly, the starting resistance is gradually cut out. When the arm reaches the running position, the starting resistance is all cut out. The field current now feeds back through the starting resistance. This resistance is so low compared with the resistance of the field itself that it has no material effect upon the value of the field current. A spring tends to pull the starting arm back to the starting position.

When the arm reaches the running position, it is held against the action of this spring by a soft-iron magnet (hold-up magnet), connected in series with the shunt field. (A soft-iron armature is often attached to the starting arm as shown in the figure.) If for any reason the line has no voltage, the starting arm will spring back to the starting position. Otherwise, if the voltage again came on the line after a temporary shut-down, the stationary motor armature would be thrown directly across the line and a short-circuit would result.

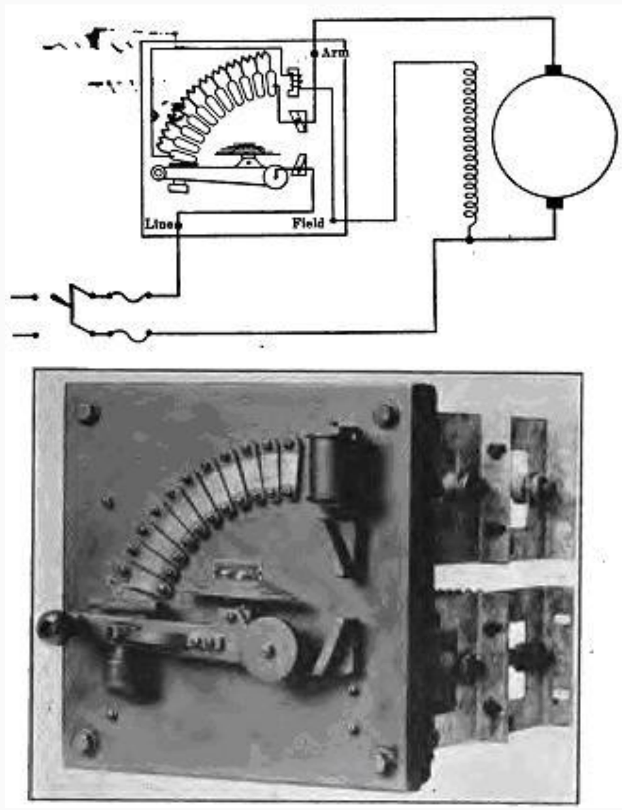


Fig. 8.13 Three point DC motor starter

LESSON 21. DC motor speed control methods-field and armature control

Speed control of shunt motor

We know that the speed of shunt motor is given by:

$$n = \frac{V_a - I_a R_a}{k\varphi}$$

where, V_a is the voltage applied across the armature and j is the flux per pole and is proportional to the field current I_f. Armature current I_a is decided by the mechanical load present on the shaft. Therefore, by varying V_a and I_f we can vary n. For fixed supply voltage and the motor connected as shunt we can vary V_a by controlling an external resistance connected in series with the armature. I_f of course can be varied by controlling external field resistance R_f connected with the field circuit. Thus for .shunt motor we have essentially two methods for controlling speed, namely by:

1. varying armature resistance.
2. varying field resistance.

Speed control by varying armature resistance

The inherent armature resistance r_a being small, speed n versus armature current I_a characteristic will be a straight line with a small negative slope as shown in figure 8.14 . At no load (i.e., I_a = 0) speed is highest and $n = \frac{V_a}{k\varphi}$. Note that for shunt motor, voltage applied to the field and armature circuit are same and equal to the supply voltage V. However, as the motor is loaded, $I_a r_a$ drop increases making speed a little less than the no load speed n_o. For a well designed shunt motor this drop in speed is small and about 3 to 5% with respect to no load speed. This drop in speed from no load to full load condition expressed as a percentage of no load speed is called the inherent speed regulation of the motor. It is for this reason, a d.c shunt motor is said to be practically a constant speed motor (with no external armature resistance connected) since speed drops by a small amount from no load to full load condition.

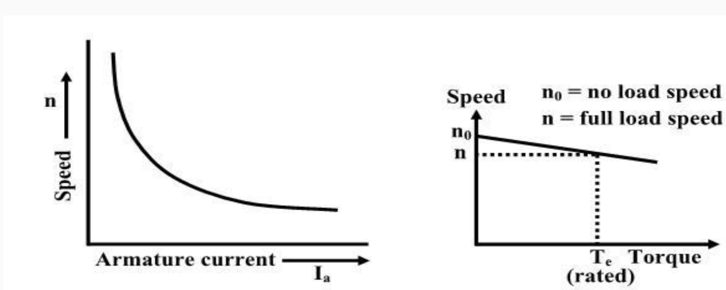


Fig. 8.14 Characteristics of shunt motor

Since $T_e = kjI_a$, for constant j operation, T_e becomes simply proportional to I_a. Therefore, speed vs. torque characteristic is also similar to speed vs. armature current characteristic as shown in figure.

The slope of the n vs I_a or n vs T_e characteristic can be modified by deliberately connecting external resistance r_{ext} in the armature circuit. One can get a family of speed vs. armature curves as shown in figures 8.15. for various values of r_{ext}. From these characteristic it can be explained how speed control is achieved. Let us assume that the load torque T_L is constant and field current is also kept constant. Therefore, since steady state operation demands $T_e = T_L$,and $T_e = kjI_a$ too will remain constant; which means I_a will not change. Suppose $r_{ext} = 0$, then at rated load torque, operating point will be at C and motor speed will be n. If additional resistance r_{ext1} is introduced in the armature circuit, new steady state operating speed will be n_1 corresponding to the operating point D. In this way one can get a speed of n_2 corresponding to the operating point E, when r_{ex2t} is introduced in the armature circuit. This same load torque is supplied at various speeds. Variation of the speed is smooth and speed will decrease smoothly if r_{ext} is increased. Obviously, this method is suitable for controlling speed below the base speed and for supplying constant rated load torque which ensures rated armature current always. Although, this method provides smooth wide range speed control (from base speed down to zero speed), it has a serious draw back since energy loss takes place in the external resistance r_{ext} reducing the efficiency of the motor.

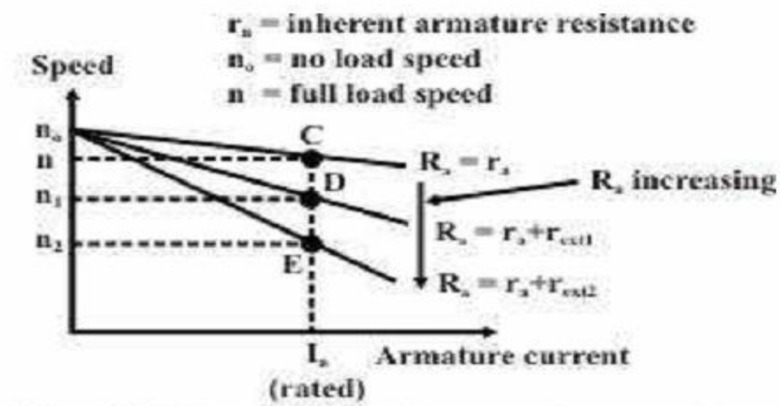


Figure 39.6: Family of speed vs. armature current characteristic.

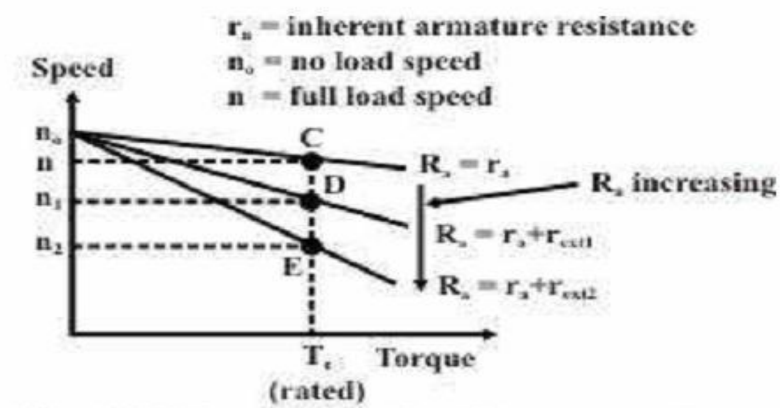


Fig. 8.15 Family of speed vs Torque and speed vs current characteristic,

Speed control by varying field current

In this method field circuit resistance is varied to control the speed of a d.c shunt motor. Let us rewrite .the basic equation to understand the method.

$$n = \frac{V_a - I_a R_a}{k\varphi}$$

If we vary I_f, flux j will change, hence speed will vary. To change I_f, an external resistance is connected in series with the field windings. The field coil produces rated flux when no external resistance is connected and rated voltage is applied across field coil. It should be understood that we can only decrease flux from its rated value by adding external resistance. Thus the speed of the motor will rise as we decrease the field current and speed control above the base speed will be achieved. Speed versus armature current characteristic is shown in figure 8.15 for two flux values j and j_1. Since $j_1 < j$, the no load speed n_0' for flux value j_1 is more than the no load speed n_0 corresponding to j. However, this method will not be suitable for constant load torque. To make this point clear, let us assume that the load torque is constant at rated value. So from the initial steady condition, we have $T_{Lrated} = T_{el} = kjI_{arated}$. If load torque remains constant and flux is reduced to j_1, new armature current in the steady state is obtained from $kj_1 I_{al} = T_{Lrated}$. Therefore new armature current is

$$I_{a1} - \left(\frac{\varphi}{\varphi_1}\right) I_{a\ rated}$$

But the fraction $\left(\frac{\varphi}{\varphi_1}\right) > 1$; hence new armature current will be greater than the rated armature current and the motor will be overloaded. This method therefore, will be suitable for a load whose torque demand decreases with the rise in speed keeping the output power constant as shown in figure 8.16. Obviously this method is based on flux weakening of the main field. Therefore at higher speed main flux may become so weakened, that armature reaction effect will be more pronounced causing problem in commutation.

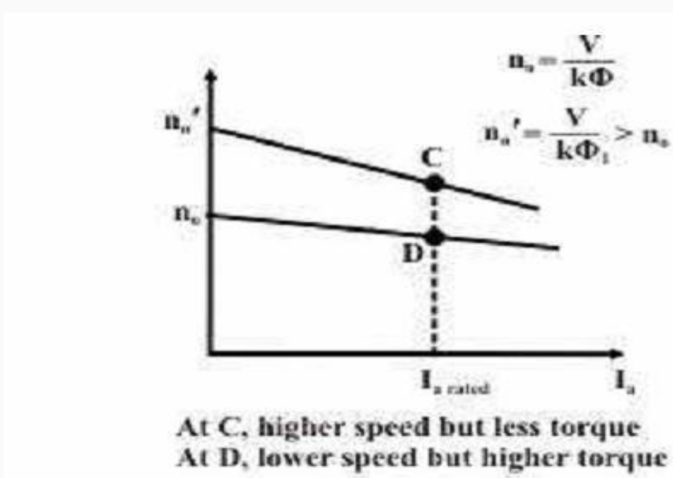


Fig. 8. 16 Family of speed vs armature current characteristics

MODULE 9. Polyphase systems, generation - three phase load connections

LESSON 22. Polyphase systems - generation

Reasons for the Use of Polyphase Currents.

In many industrial applications of alternating current, there are objections to the use of single phase power. In a single phase circuit, the power delivered is pulsating. Even when the current and voltage are in phase, the power is zero twice in each cycle. When the power factor is less than unity, the power is not only zero four times in each cycle, but it is also negative twice in each cycle. This means that the circuit returns power to the generator for a part of the time. This is analogous to a single cylinder gasoline engine in which the fly wheel returns energy to the cylinder during the compression part of the cycle. Over the complete cycle, both the single phase circuit and the fly wheel receive an excess of energy over that which they return to the source. The pulsating nature of the power in single phase circuits makes such circuits objectionable in many instances. A polyphase circuit is somewhat like a multi cylinder gasoline engine. With the engine, the power delivered to the fly wheel is practically steady, as one or more cylinders are firing when the others are compressing. This same condition exists in polyphase electrical systems. Although the power of any one phase may be negative at times, the total power is constant if the loads are balanced. This makes polyphase systems highly desirable for power purposes.

The rating of a given motor, or generator, increases with the number of phases, an important consideration. Below are the approximate capacities of a given machine for different numbers of phases, assuming the single phase capacity as 100.

Single phase... 100

Three phase... 148

Direct current... 154

The same machine operating three phase has about 50 per cent, greater capacity than when operating single phase. A minor consideration in favor of three phase power is the fact that with a fixed voltage between conductors, the three phase system requires but three fourths the weight of copper of a single phase system, other conditions such as distance, power loss, etc., being fixed.

Symbolic Notation.

The solutions of problems involving circuits and systems containing a number of currents and voltages are simplified and are less susceptible to error if the current and voltage vectors are designated by some systematic notation, of which the following is one type. If a voltage is acting to send current from point a to point b, in figure 9.1 (a), it shall be denoted by E_{ab}. On the other hand, if the voltage tends to send current from b to a it shall be denoted by E_{ba} Obviously, $E_{ab} = - E_{ba}$. It may seem as if alternating currents cannot be considered as having direction since they are undergoing continual reversal in direction. The assumed direction of a current, however, is determined by the actual direction of the flow of energy. In an alternator the energy comes out of the armature and the current is considered as flowing out of the armature, even although it is actually flowing into the armature for half the time.

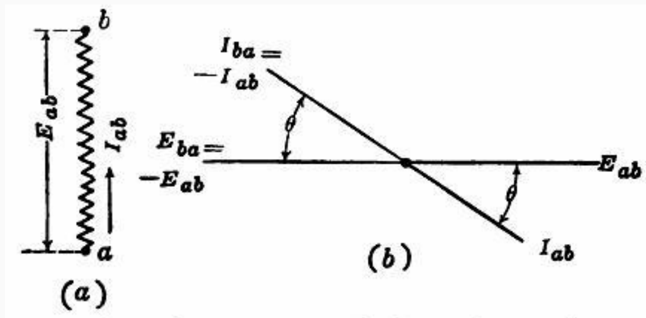


Fig. 9.1 Symbolic Notation

Corresponding to the voltage E_{ab}, in figure 9.1 (a), the current I_{ab} flows from a to b in virtue of this voltage. The current flowing from b to a must be opposite in direction to that flowing from ato b. Therefore $I_{ab} = - I_{ba}$. This relation is illustrated in figure (b), in which I_{ab} differs in phase from I_{ba} by 180°. E_{ba} is 180° from E_{ah}.

Figure 9.2 represents a circuit network abcde. The parts of the network, ab, bc, etc., may be either resistances, inductances, capacitances, or sources of emfs. It is obvious that the voltage from a to c is equal to the voltage from a to b plus the voltage from b to c. That is, $E_{ac} = E_{ab} + E_{bc}$. It is to be noted that when several voltages in series are being considered, the first letter of each subscript must be the same as the last letter of the preceding subscript.

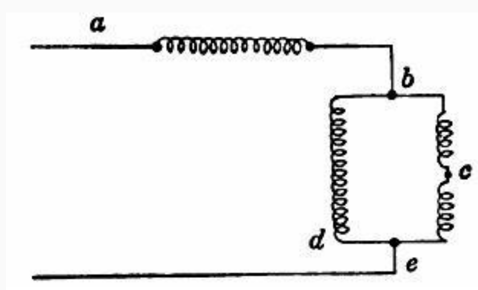


Fig. 9.3 Circuit network.

The figure 9.4 (a) shows vectorially the voltage Eab and the voltage Ecb. To obtain the voltage Eac , Ebc is necessary. Therefore Ecb is reversed giving Ebc. Ebc is now added vectorially to Eab gives Eac.

Currents may be treated in a similar manner, the principle involved being Kirchhof's first law as shown in figure (b). As may be understood, remember that Kirchoff's voltage and current laws are applicable here also, except that the quantities are vectorially dealt with since they are AC quantities and have magnitude and direction. Hence these double subscript notations not only distinguishes the various currents and voltages, but the directions in which they act as well. It is to be noted that the use of arrows is not necessary, the subscripts denoting the directions of the vectors.

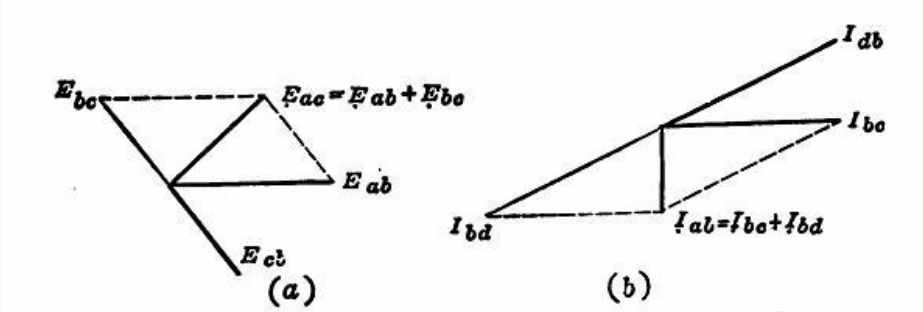


Fig. 9.4 Examples of symbolic notation.

Generation of a three phase voltage.

As three phase is now the most common of the polyphase systems, it will be considered. Figure 9.5 shows three simple coils, 120° apart and fastened rigidly together. These coils are mounted on an axis which can be rotated. The coils are shown rotating in a counter clockwise direction in a uniform magnetic field. The current can be conducted from each of these three coils by means of slip rings, as shown in the figure 9.5 (b). The terminals of coil 'a' are connected to rings a', those of b to rings b', etc., making six slip rings in all. Figure 9.6 (a) shows E_{oa} as the voltage in coil a. E_{oa} is zero and is increasing in a positive direction when the time t is 0. Obviously the voltage induced in coil b will be 120 electrical time degrees behind E_{oa} and that induced in coil c will be 240 electrical time degrees behind E_{oai} as shown in Figs. 9.6 (a) and (b). These three voltages constitute the elementary voltages generated in a three phase system.

An examination of Fig. 9.6 (a) shows that for any particular instant of time, the algebraic sum of these three voltages is zero. When one voltage is zero, the other two are 86.6 per cent, of their maximum values and have opposite signs. When any one voltage wave is at its maximum, each of the others has the opposite sign to this maximum and each is 50 per cent, of its maximum value.

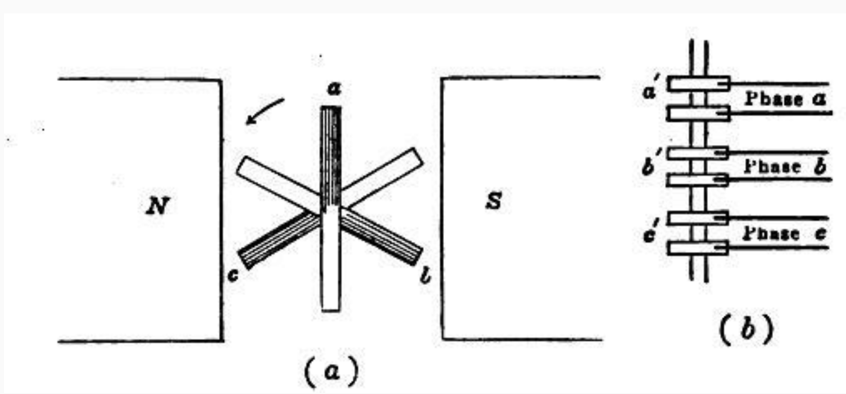


Fig. 9.5 Generation of 3 phase current.

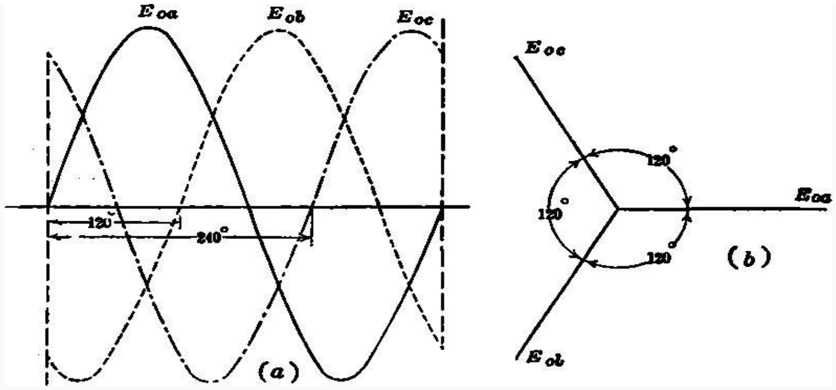


Fig. 9.6. Three phase voltage waves and vector diagram.

Figure 9.6 (b) shows the vectors representing these three voltages, the vectors being 120° apart. Each of the coils of can be connected through its two slip rings to a single phase circuit. This gives six slip rings and three independent single phase circuits. With a rotating field and stationary armature type of generator, which is the most common type in practice, the six slip rings would not be necessary, but six leads would be taken directly from the armature. In practice, however, a machine never supplies three independent circuits by the use of six wires.

Star (Y) connection.

The three coils of Fig. 9.5 are shown in simple diagrammatic form in Fig. 9.7. The three corresponding ends, one for each coil, are tied together at the common point o. This is called the Y connection of the coils. Ordinarily only three wires, aa', bb′ and cc', lead to the external circuit, although the neutral wire oo' is sometimes carried along, making a three phase, four wire system.

Figure 9.8 (a) again shows the three coils and Fig. 9.8 (b) the three corresponding voltage vectors, E_{oa}, E_{ob}, and E_{oc}. These three voltages are called the coil or star (Y) voltages. Let it be required to find the three line voltages E_{ab}, E_{bc} and E_{ca}. The line voltage $E_{ab} = E_{ao} + E_{ob}$. E_{ao} is not on the original diagram but is obtained by reversing E_{oa}. E_{ao} is then added vectorially to E_{ob} giving E_{ab}. From geometry, E_{ab} lags the coil voltage E_{ob} by 30° and is 150° behind E_{oa}. Also, E_{ab} is numerically (magnitude) equal to Ö3 E_{ob}. In a similar manner $E_{bc} = E_{bo} + E_{oc}$ and $E_{ca} = E_{co} + E_{oa}$. These three line voltages are shown in Fig. 9.9.

It is to be noted that in a balanced star (Y) system, the three line voltages are all equal and are 120° apart. Each line voltage is 30° out of phase with one of its respective coil voltages. The three line voltages are each Ö3, or 1.732, times the coil voltage.

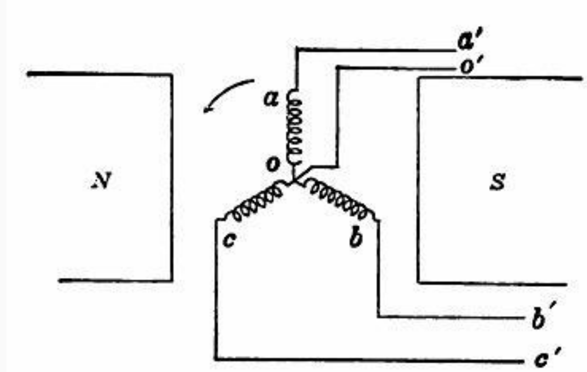


Fig. 9.7 Y (star) connection of generator coils.

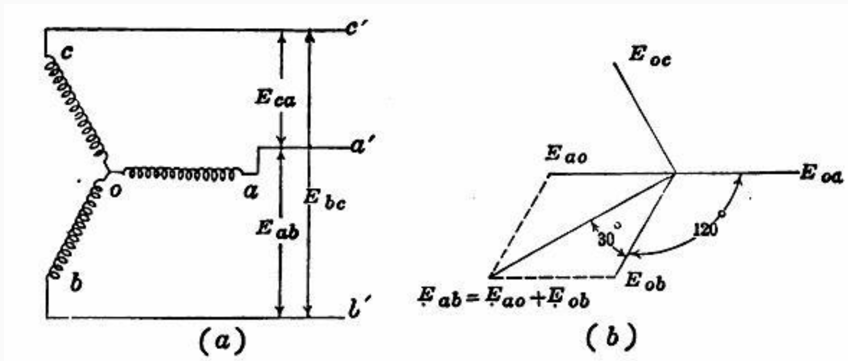


Fig. 9.8 Y connection and corresponding voltage vector diagram.

It is obvious from Fig. 9.8 (a) that the three coil currents I_{oa}, I_{ob} and I_{oc} are respectively equal to the three line currents $I_{aa'}$, $I_{bb'}$ and $I_{cc'}$, as the coil and line is in series.

Therefore, in a star (Y) system the line currents and the respective coil currents are equal. Moreover, as the three coils meet at a common point, the vector sum of the three currents must be zero by Kirchhoff's first law, provided there is no neutral conductor and current. That is

I_{oa},+ I_{ob} + I_{oc} = 0

Power in star (Y) system Figure 9.9 (a) shows the three currents I_{oa}, I_{ob} and I_{oc} of coils oa, ob, and oc respectively. Unity power factor is assumed and the three currents are therefore in phase with their respective coil voltages. A balanced system is assumed and the three currents are therefore equal in magnitude.

The coil current I_{oa} and the line current $I_{aa'}$ are the same. Therefore, the line current $I_{aa'}$ is 30° out of phase with the line voltage E_{ca} when the power factor is unity. This is true for each phase.

The power delivered by each coil is

$P' = E_{oa} I_{oa}$ (unity power factor; q = 0; cos q = 1)

and the total power delivered by the generator is three times this; $P = 3E_{coil} I_{coil}$

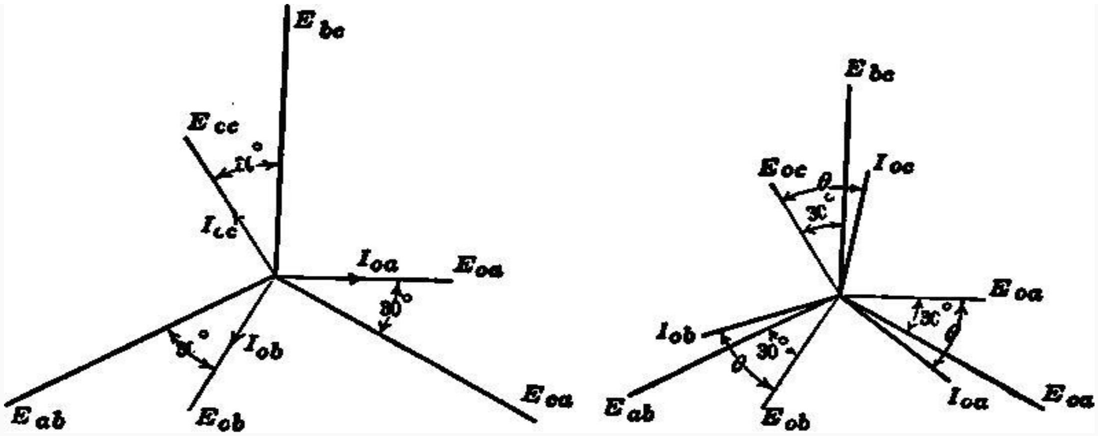


Fig. 9.9 Relation of line to coil voltages and currents in a Y system,

(a) Unity power factor. b. Power factor = cos q.

As the power in the line is the same as that delivered by the generator, substituting E_{line}/Ö3 for the value of E_{coil} ;

P = (3/Ö3) E_{line} I_{coil} = Ö3 E_{line} I_{line} with the coil current and the line current being equal.

In a balanced three phase system, the line power at unity power factor is equal to Ö3 times the line voltage times the line current.

Figure 9.9 (b) shows this same three phase system when the power factor is no longer unity. Each coil current now lags its respective coil voltage by the angle q.

Each coil power is now

P = 3 E_{coil} I_{coil} cos q_{coil}

The total power is

P = Ö3 E_{line} I_{line} cos q_{coil}

Therefore, in a balanced three phase system, the system power factor is the cosine of the angle between the coil current and the coil voltage.

The angles between the line currents and the line voltages are not power factor angles, for they involve the factors (q — 30°) [Refer Fig. 9.9 (b)] and also (q + 30°), q being the coil power factor angle.

Obviously the system power factor, which is the coil power factor, is

P F. = P / Ö3 E_{line} I_{line}

where P is the total system power.

Example.—A three phase alternator has three coils each rated at 1,330 volts and 150 amp. What is the voltage, kva., and current rating of this generator if the three coils are connected in star (Y)?

E_{line} = Ö3 (1330) = 2,300 volts.

Rating = Ö3 (2300) 150 = 600 kva.

Current rating = 150 amp.

LESSON 23. Polyphase systems – delta in generation - three phase load connections

Delta connection

The three coils of Fig. 9.5 can be connected as shown in Fig. 9.10 (a), the diagram being simplified in Fig. 9.10 (b). The end of each coil, which was connected to the neutral in star (Y) earlier, is now connected to the outer end of the next coil, as shown in Fig. 9.10 (a). As points o and a are now connected directly together, the o's are now superfluous and are dropped. Fig. 9.11 (a) shows vectorially the three voltages E_{ab}, E_{bc} and E_{ca}, acting from a to b, b to c, and c to a, respectively. At first sight Fig. 9.10 looks like a short circuit, the three coils, each containing a source of voltage, being short circuited on themselves. The actual conditions existing in this closed circuit may be demonstrated by the use of the subscript notation. Assume that the coil bc is broken at c', Fig. 9.12 (a). The voltage $E_{bc} = E_{ba} + E_{ac}$. The vector sum of these two voltages, shown in Fig. 9.12 (b), lies along voltage $E_{bc'}$, and is equal to it. Therefore, the voltage $E_{cc'} = 0$ and points c and c' can be connected without any resulting flow of current. This is the same condition which exists when two direct current generators having equal voltages are connected in parallel. No current flows between the two if the proper polarity is observed.

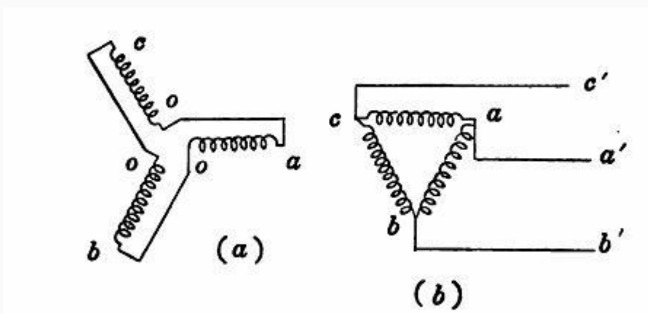


Fig. 9. 10 Delta connection of alternator coils.

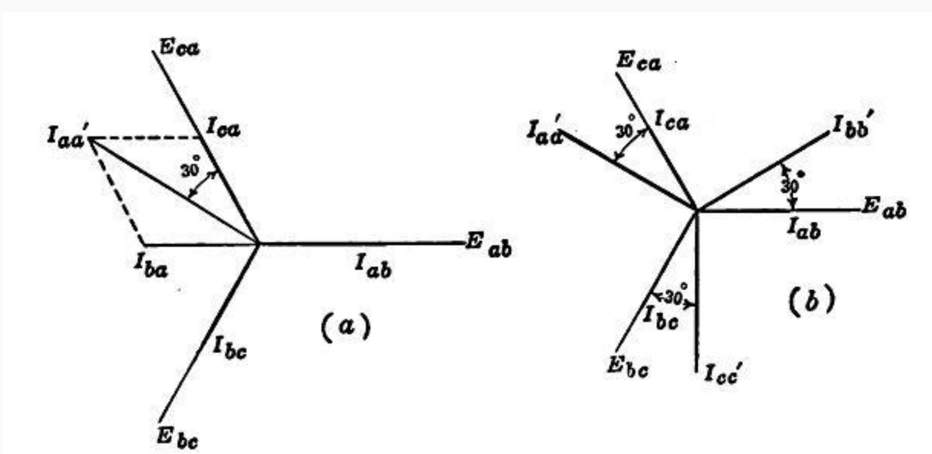


Fig. 9.11 Relation of coil voltages and currents in a delta system at unity power factor.

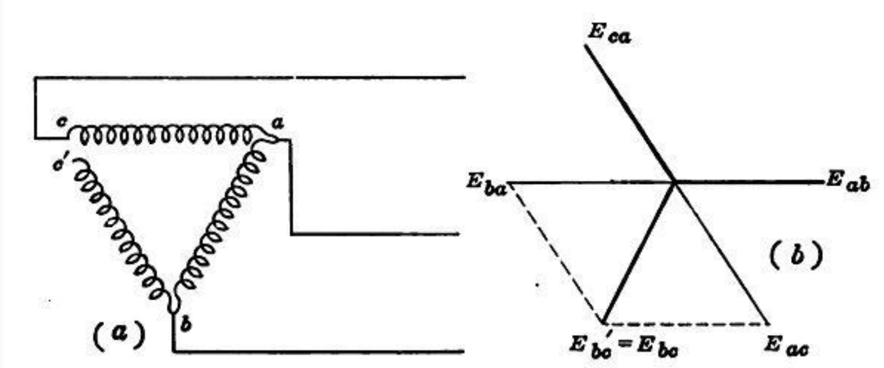


Fig. 9.12 Showing that Delta connection is not a short circuit

The coil currents of Fig. 9.10 are shown in Fig. 9.11 in phase with their respective voltages, balanced conditions being assumed. The line current

$I_{ac'} = I_{ba} + I_{ca}$

This addition is made vectorially in Fig. 9.11 (a), giving $I_{ac'}$ 30° from E_{ca}. It will be observed that I_{ac}' is Ö3 times the coil current. Line currents $I_{bb'}$ and $I_{cc'}$ may be found in a similar manner, with the result shown in Fig. 9.11 (b). Therefore, in the delta system there is a phase difference of 30° between the line currents and the line voltages at unity power factor, just as in the star (Y) system. It is obvious that the line voltage is equal to the coil voltage in a delta system. Moreover, the sum of the three voltages acting around the delta must be zero by Kirchhoff's second law.

In a balanced delta system, the line voltage is equal to the coil voltage, but the line current is $\sqrt{3}$ times the coil current.

Figure 9.13 shows three lamp loads, each requiring 10 amp at 115 volts. They are first connected in star (Y) and then in delta. In order to supply the proper voltage in each case, there are 199 volts across lines in the Y system and 115 volts in the delta system. There are 10 amp. per line in the Y system and 17.3 amp per line in the delta system. The power supplied is the same in each system.

Power in Delta system The total power in a delta system is

$P = 3\, E_{coil}\, I_{coil} \cos\theta_{coil}$

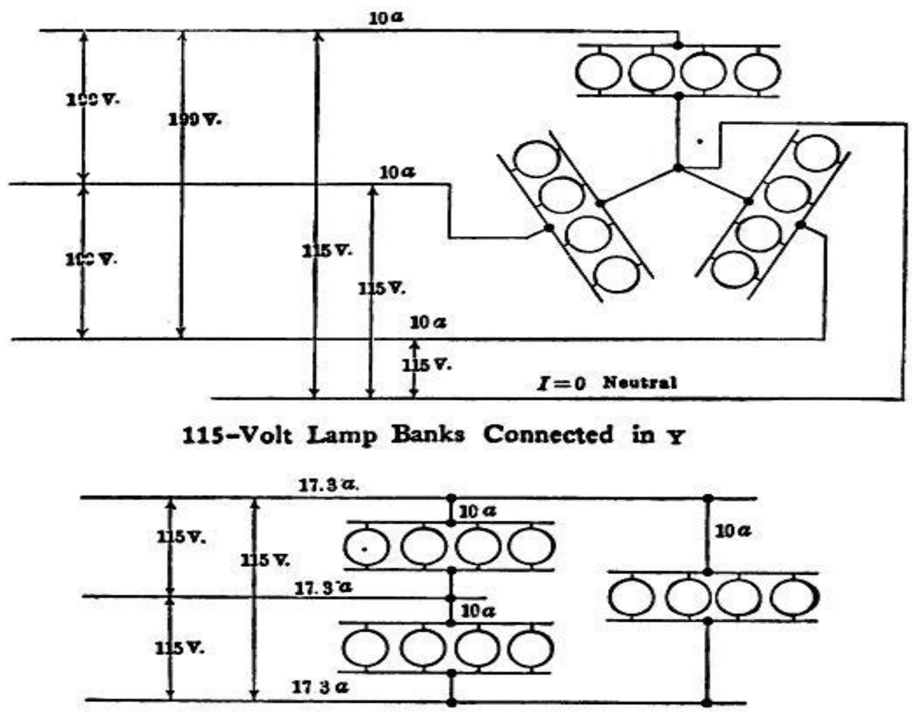


Fig. 9.13 Lamp loads in Y and in delta

This power is equal to that in the line, as there is no intervening loss. Also, the line current

$I_{line} = \sqrt{3}\ I_{coil}$ and $E_{line} = E_{coil}$

Hence, substituting in the above equation $P = \sqrt{3}\ E_{line}\ I_{line}\ \cos\theta_{load}$

This equation is the same as that developed for the star (Y) system. This should be so, for the relations in a three phase line are the same whether the power originates in a delta or in a Y connected generator. The power factor of the delta system is the same as that for a Y system.

P F. $= P / \sqrt{3}$ EI, where P is the total power of the system, and E and I are the line voltage and line current respectively.

The denominator, $\sqrt{3}$ EI, gives the volt amperes of the three phase system.

MODULE 10. Polyphase induction motor: construction, operation, equivalent circuit

LESSON 24. Polyphase induction motor – theory of operation

The induction motor is the most widely used type of alternating-current motor. This is due to its ruggedness and simplicity, the absence of a commutator, and the fact that its operating characteristics are suitable for constant speed work.

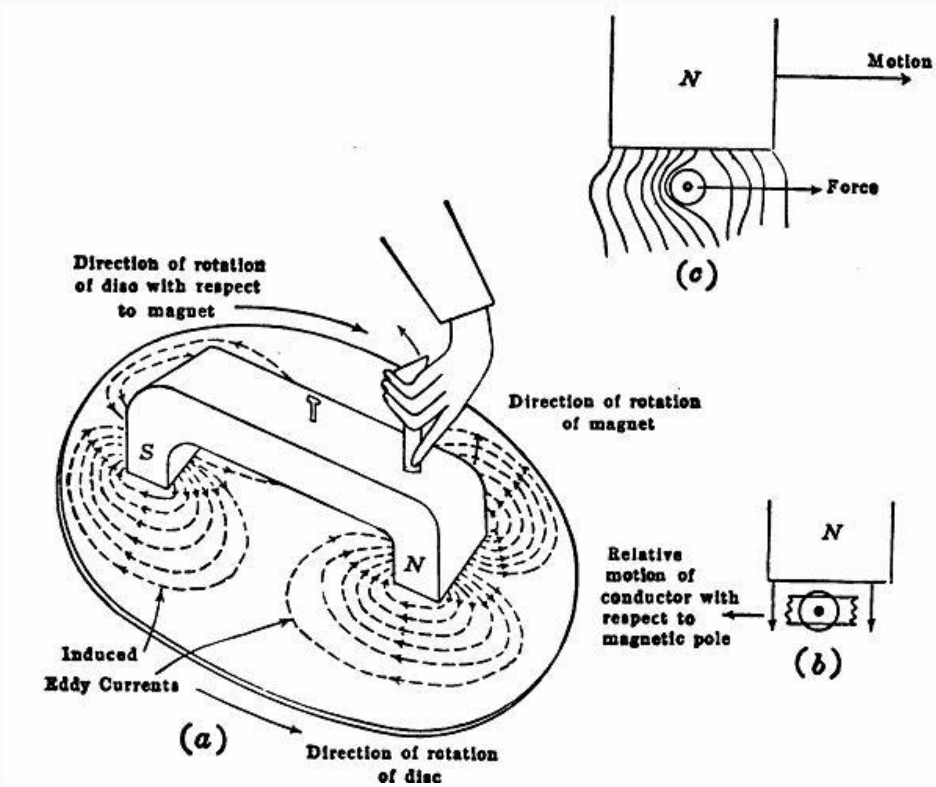


Fig. 10.1 Rotation of metal disc produced by rotating magnet.

The principle of the motor may be illustrated as follows: A metal disc in Fig. 10.1 (a), is free to turn upon a vertical axis. The disc may be of any conducting material, such as iron, copper, or aluminum. A magnet, free to rotate on the same axis as the disc, is placed above the disc and its ends are bent down so that its magnetic flux cuts through the disc. When this magnet is rotated, the magnetic lines cut the disc and induce currents in it, as shown in the figure Fig. 10.1. As these currents find themselves in a magnetic field, they tend to move across this field, just as the currents in the conductors of a direct-current motor tend to move across its magnetic field. By Lena's law, the direction of the force developed between these currents in the disc and the magnetic field producing them will be such that the disc tends to follow the magnet, as shown in the figure. In Fig. 10.1 (a), the north pole of the rotating magnet is shown as moving in a counter-clockwise direction. The conductor beneath the magnet also moves in a counter-clockwise direction, but more slowly than the magnet. Therefore, the relative motion between the magnet and the conductor is the same as if the magnet were stationary and the conductor moved in the clockwise direction. This relative motion of the magnet and the conductor is illustrated in (b), where the north pole is shown as being stationary and the conductor is moving from right to left. Applying Fleming's right-hand rule, the direction of the induced current is toward the observer. The lines of force about the conductor, due to its own current, are therefore counter-clockwise and the resultant field is found by combining the conductor field and the field produced by the magnet. The appearance of this resultant field is shown in (c). As the magnetic field is increased in intensity to the left of the conductor and reduced in intensity to the right of the conductor, there is a force developed which urges this conductor from left to right. That is, the conductor tends to follow the magnet. Actually, the magnet rotates in a counter-clockwise direction. Therefore, the disc rotates in the same direction but at a speed less than that of the magnet.

The disc can never attain the speed of the magnet, for were it to attain this speed, there would be no relative motion of the disc and the magnet and, therefore, no cutting of the disc by the magnetic flux. The disc current would then become zero and no torque would be developed, which would result in the disc speed becoming less than that of the magnet. So there must always exist a difference of speed between the two. This difference of speed is called the slip.

Alternating current Rotating Field

The rotating fields described in the previous paragraph were produced by rotating the magnetic poles mechanically. Rotating magnetic fields are, however, produced by sending polyphase currents through polyphase windings, such as alternator windings. Such rotating fields are produced entirely by electrical means, there being **no** mechanical rotation of the pole pieces themselves.

A cylinder may be used instead of the disc used earlier as shown in Fig. 10.2. Four poles are shown, the magnetic lines of which cut the cylinder. If the frame carrying these poles be revolved by mechanical means, the currents induced in the cylinder will cause the cylinder to rotate in the same direction as that of the rotating frame.

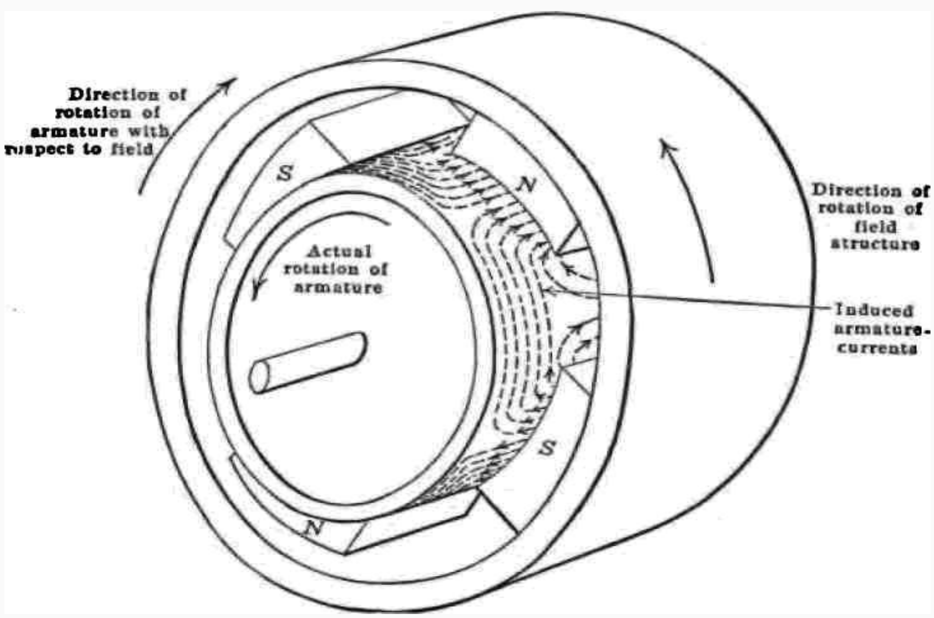


Fig. 10.2 Rotation of conducting cylinder due to induced currents.

The simplest type of rotating field is that produced by the gramme ring winding illustrated next. A gramme ring, wound for two-phase currents, has two separate windings, one for each phase. Each winding consists of two sections located diametrically opposite each other and each section occupies approximately one-fourth the winding space of the ring. The two windings are called the A-phase and the B-phase, respectively. Care must be taken to connect the two sections of each winding correctly as shown. Curves I_A and I_B show the variation with time of the currents in phases A and B, respectively. As these are two-phase currents, they differ in time-phase by 90° or one-fourth of a cycle. At the instant marked (1) in Fig. 10.3, the current in phase A is zero and that in B is negative maximum. With the method of connecting the windings, and the direction of the currents as shown, two South poles are formed on the upper ends of the B-windings and two N-poles on the lower ends.

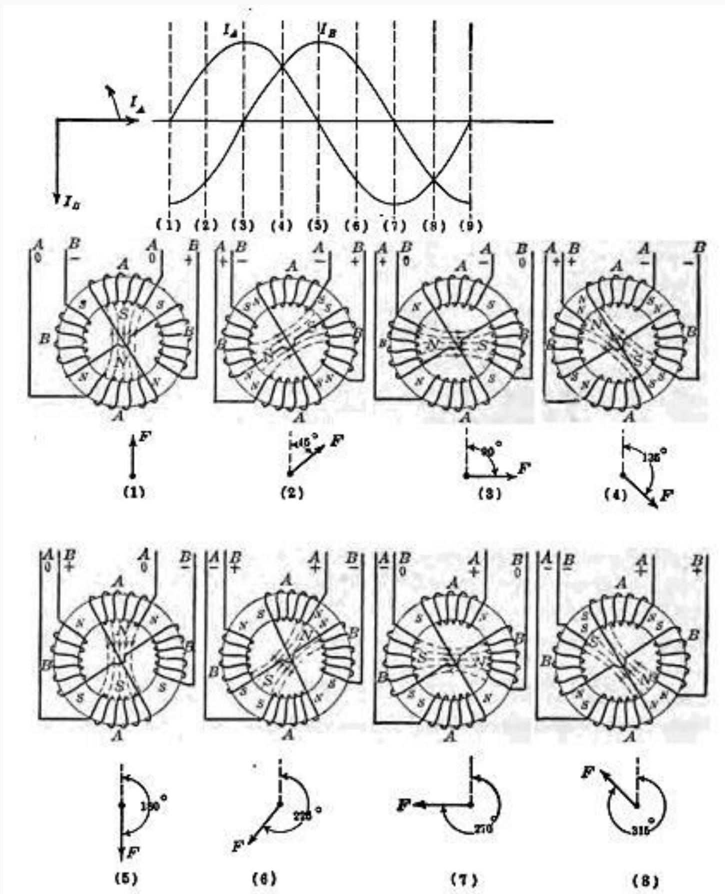


Fig. 10.3 Rotating field produced by 2-phase currents in gramme-ring winding.

These four poles combine into two poles, a single S-pole and a single N-pole, each of these last being twice the magnitude of the individual poles which combined to form them. The resultant field is vertical and is directed upwards, as indicated by the arrow F beneath diagram Fig. 10.3 (1). In Fig. 10.3 (2) the current in B is still negative, but of lesser magnitude than in Fig. 10.3 (1). The current in A has increased positively until its magnitude is equal to that of B. Two S-poles and two N-poles again combine to form a single S-pole and a single N-pole, each of double the magnitude of the individual poles forming them. The direction of the resulting field is 45° clockwise from its position in Fig. 10.3 (1). It is to be noted that while the two currents are passing through 45 electrical time-degrees, the resulting field in the gramme ring advances 45 space-degrees. Diagrams Fig. 10.3 (3), (4), (5), (6), (7), and (8) show at different instants the positions of the gramme ring field resulting from the combined magnetic effects of phases A and B. The diagram for Fig. 10.3 (9) would be identical with that for Fig. 10.3 (1). The rotating magnetic field has passed through 360 space-degrees while the two-phase currents have gone through 360 electrical time-degrees or one cycle. This constitutes a two pole rotating field and its speed in revolutions per second is the same as the frequency, or the cycles per second, of the currents. For example, if the currents had a frequency of 60 cycles per second, the field would make 60 revolutions per second, or 3,600 r.p.m.

The commercial polyphase induction motor consists of a fixed member called the stator, carrying a polyphase drum winding (wrapped inside the stator from one side to another like on a drum), and a rotating member called the armature or rotor. As the stator usually receives the power from the line it is called the primary, and as induced currents flow in the rotor, it is called the secondary, just as in the transformer. The figure Fig. 10.4 shows four successive positions of the rotating field, for corresponding values of the polyphase currents in the stator of a three-phase induction motor.

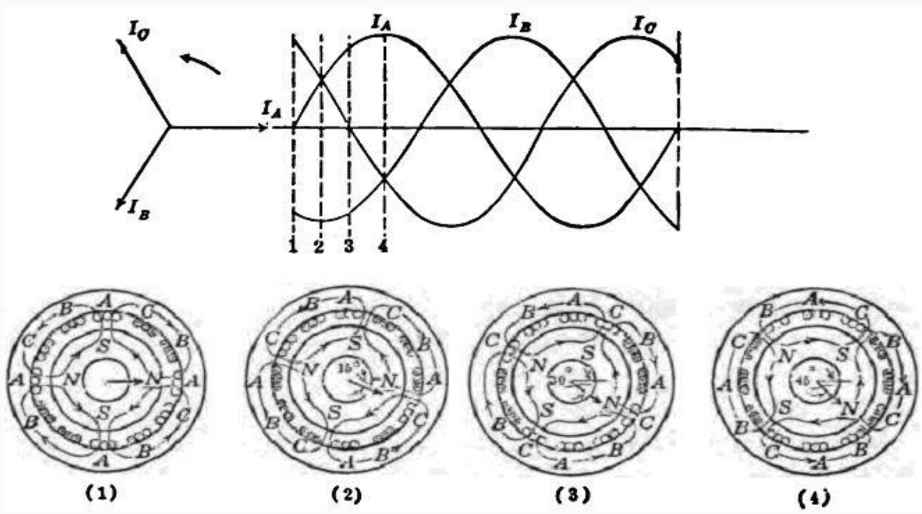


Fig. 10.4 Rotating field produced by 3-phase currents in a 4-pole, induction-motor winding.

In Fig. 10.4 (1) the current I_A is zero, so that I_B and I_c are opposite and equal. The position of the field is shown at this instant. In Fig. 10.4 (2) the currents I_A and I_c are but half their maximum positive values and their positions on the stator are such that their phase belts are on each side of B-belt in which the current is a maximum. Therefore, the field is symmetrical at this position.

It will be noted also that the time-angle between successive values of current in 1-2-3 is 30 electrical-degrees, whereas the field advances but 15 space-degrees between Fig. 10.4 (1) and (2) and also between Fig. 10.4 (2) and (3). Between positions Fig. 10.4 (1) and (4) the currents have advanced 90 electrical time-degrees, but the rotating field has advanced only 45 space-degrees. That is, the advance of the rotating field in space-degrees is equal to one-half the advance of the currents in electrical time-degrees. Therefore, the speed of such a field in revolutions per second is equal to one-half the circuit frequency in cycles per second.

In general it may be stated that in order to produce a two-pole rotating field, the angular space-degrees between the phase belts of the winding must be the same as the electrical time-degrees between their respective currents. If the machine has p poles, the angular space-degrees between phase belts is 2/p times the electrical time degrees between their respective currents. For example, in a six-pole, three-phase machine, the successive phase belts start 40° from each other, that is, (2/6) × 120° or 40°.

Synchronous Speed.

It was just shown that the angular speed of an alternating current rotating field depends upon the frequency of the current and the number of poles for which the machine is wound. The relation between speed, frequency and poles is given by the following equation:

N = 120 f /p

where N is the speed of the rotating magnetic field in r.p.m, f the frequency in hertz and p the number of poles. This speed the rotating magnetic field,is called the synchronous speed of the motor. The common synchronous speeds for commercial motors running on a supply of 50 hz are as follows:

Poles	RPM
2	3000
4	1500
6	1000
8	750
12	500

LESSON 25. Polyphase induction motor – construction – Equivalent circuit

Slip.

If an armature whose conductors form closed circuits be placed in a rotating field, it will develop torque because of the induced currents, acting in conjunction with the rotating magnetic field. As seen already, the armature can never attain the speed of the rotating field, for if it did, the cutting of conductors by flux would cease, there would be no rotor current and, therefore, no torque. The difference between the speed of the rotating field and that of the rotor is called the revolutions slip of the motor. For example, if the rotor of a four-pole, 50-cycle motor has a speed of 1440 rpm., its revolutions slip is 1500 — 1440 = 60 rpm., where 1500 r.p.m. is its synchronous speed. It is more convenient to express the slip as a fraction of the synchronous speed as

$$s = \frac{N - N_2}{N}$$ where N_2 is actual rotor speed

The slip of the above motor would be (1500-1440)/1500 = 0.04 = 4%. The full-load slip in commercial motors varies from 1 to 10 per cent., depending upon the size and the type of motor.

Rotor Frequency and Induced Emf.

If the rotor of a two-pole, 50 cycle motor is at standstill and voltage is applied to the stator, each rotor conductor will be cut by a north pole 50 times per second and by a south pole 50 times per second, as this is the speed of the rotating field. If the stator be wound for four poles, the speed of the rotating field is halved, but each conductor is then cut by two north and two south poles per revolution of the field and therefore by 50 north and 50 south poles per second, the same as in the two-pole motor. Consequently, the frequency of the rotor currents at standstill will be the same as the stator frequency. This holds true for any number of poles. At standstill the motor is a simple static transformer, the stator being the primary and the rotor being the secondary. If the rotor of the above 50 cycle motor revolves at half speed in the direction of the rotating field (s = 0.5), the rotor conductors are cut by just one-half as many north and south poles per second as when standing still and the frequency of the rotor currents is therefore 25 cycles per second. Generalizing, $f_2 = sf$, where f_2 is the rotor frequency, s the slip, and f the stator frequency.

Example. What is the frequency of the rotor current of a 50 cycle, six-pole induction motor, if the rotor speed is 900 rpm.

The synchronous speed = 1000

Rotor current frequency = 50 × (1000-900)/1000 = 5 hz

Squirrel Cage Motors

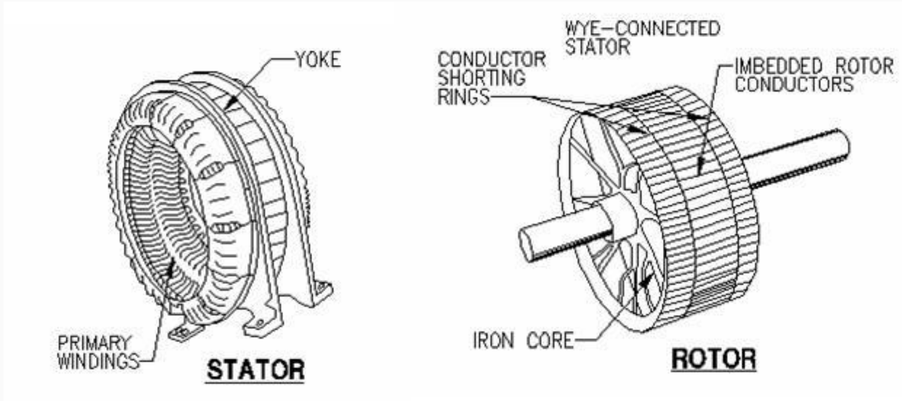
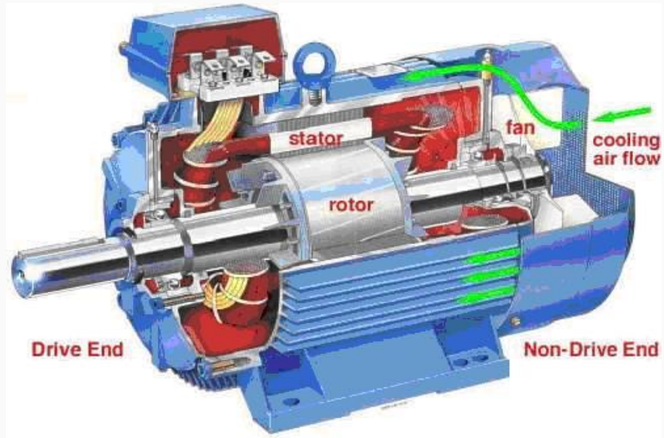


Fig. 10.5 Construction of a squirrel cage motor

A squirrel-cage motor consists essentially of two units, namely Rotor and Stator. The rotor (or secondary) is also constructed of steel laminations, but the windings consist of conductor bars placed approximately parallel to the shaft and close to the rotor surface. These windings are connected at each end of the rotor, by a solid ring. The rotors of large motors have bars and rings of copper connected at each end by a conducting end ring made of copper or brass. The joints between the bars and end rings are usually electrically welded into one unit, with blowers mounted on each end of the rotor. In small squirrel-cage rotors, the bars, end rings, and blowers are of aluminum cast in one piece instead of welded together.

The stator (or primary) consists of a laminated sheet-steel core with slots where the insulated coils are placed. The coils are grouped and connected to form a definite polar area and to produce a rotating magnetic field when connected to a polyphase alternating current. The air gap between the rotor and stator must be very small in order for the best power factor to be obtained. The shaft must, therefore, be very rigid and furnished with quality bearings of the sleeve or ball-bearing type. In a squirrel-cage motor, the secondary winding takes the place of the field winding in a synchronous motor. As in a synchronous motor, the currents in the stator set up a rotating magnetic field. This field is produced by the increasing and decreasing currents in the windings. When the current increases in the first phase, only the first winding produces a magnetic field. As the current decreases in this winding and increases in the second, the magnetic field shifts slightly, until it is all produced by the second winding. When the third winding has maximum current flowing in it, the field is shifted a little more. The windings are so distributed that this shifting is uniform and continuous. It is this action that produces a rotating magnetic field. As this field rotates, it cuts the squirrel-cage conductors, and voltages are set up in these just as though the conductors were cutting the field in a DC generator. These voltages cause currents to flow in the squirrel-cage circuit – through the bars under the adjacent south poles into the other end ring, and back to the original bars under the north poles to complete the circuit. The current flowing in the squirrel-cage, down one group of bars and back in the adjacent group, makes a loop that establishes magnetic fields in the rotor core with north and south poles. This loop consists of one turn, but there are several conductors in parallel and the currents may be large. The poles in the rotor are attracted by the poles of the rotating field set up by the currents in the armature winding and follow them around in a manner similar to the way in which the field poles follow the armature poles in a synchronous motor.



EQUIVALENT CIRCUIT

At standstill (slip, s = 1), the rotating magnetic field produced by the stator has the same speed with respect to the rotor windings as with respect to the stator windings. Thus, the frequency of the rotor currents, f_2, is the same as the frequency of the stator currents f. At synchronous speed (s = 0), there is no relative motion between the rotating field and the rotor, and the frequency of rotor current is zero. At intermediate speeds the rotor current frequency is proportional to the slip, as already seen earlier. Noting that the rotor currents are of slip frequency, we have the rotor equivalent circuit (on a per-phase basis) in figure below (a), which gives the rotor current, I_2, as

$$I_2 = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Here, E_2 is the induced rotor emf at standstill; X_2 is the rotor leakage reactance per phase at standstill; and R_2 is the rotor resistance per phase. This may also be written as

$$I_2 = \frac{E_2}{\sqrt{(R_2/s)^2 + X_2^2}}$$

So we can redraw the circuit in figure Fig 10.6 (a) as figure (b).

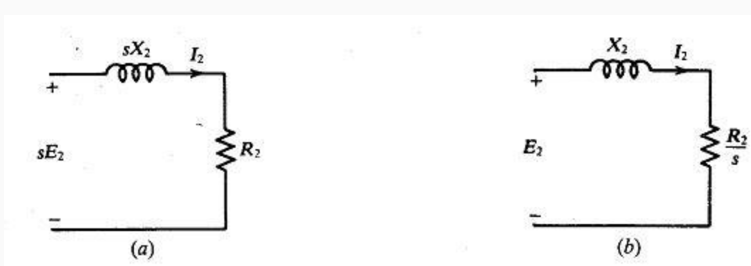


Fig. 10.6 Equivalent circuit of rotor

In order to include the stator circuit, the induction motor may be viewed as a transformer with an air gap, having a variable resistance in the secondary. Thus the primary of the transformer corresponds to the stator of the induction motor, whereas the secondary corresponds to the rotor on a per-phase basis. Because of the air gap, however, the value of the magnetizing reactance, X, tends to be low as compared to that of a true transformer. As in a transformer, we have a mutual flux linking both the stator and rotor, represented by the magnetizing reactance and various leakage fluxes. For instance, the total rotor leakage flux is denoted by X_2. Considering the rotor as being coupled to the stator as the secondary of a transformer is coupled to its primary, we may draw the circuit as shown in Fig 10.7. To develop this circuit further, we need to express the rotor quantities as referred to the stator. For this purpose we must know the transformation ratio, as in a transformer.

The voltage transformation ratio in the induction motor must include the effect of the stator and rotor winding distributions. It can be shown that, for a cage-type rotor, the rotor resistance per phase, R'_2 referred to the stator, is

$$R_2' = a^2 R_2 \quad \text{where} \quad a^2 \equiv \frac{m_1}{m_2}\left(\frac{k_{w1}N_1}{k_{w2}N_2}\right)^2$$

Here k_{w1} = winding factor of the stator having N_1, series-connected turns per phase

k_{w2} = winding factor of the rotor having N_2 =/4 series-connected turns per phase, for a cage rotor,

where p is the number of poles

m_1 = number of phase on the stator

m_2 = number of bars per pole pair

R_2 = resistance of one bar

Similarly,

$X_2' = a^2X_2$

where X_2' is the rotor leakage reactance per phase, referred to the stator.

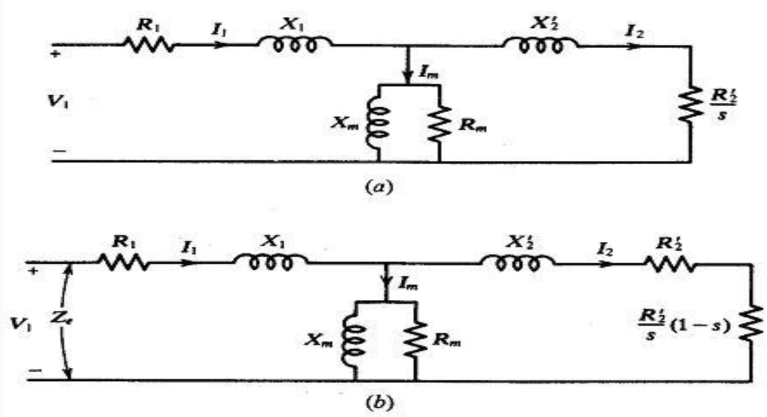


Fig. 10.7 Equivalent circuit of motor

Bearing in mind both the similarities and the differences between an induction motor and a transformer, we now refer the rotor quantities to the stator to obtain the exact equivalent circuit (per phase) shown above in figure (a). For reasons that will become immediately clear,

$$\frac{R_2'}{s} \equiv R_2' + \frac{R_2'}{s}(1 - s)$$

we split R_2' / s as to obtain the circuit shown in figure (b) above. Here, R'_2 is simply the per-phase standstill rotor resistance referred to the stator and $R'_2(1- s)/s$ is a per-phase dynamic resistance that depends on the rotor speed and corresponds to the load on the motor.

MODULE 11. Phase diagram, effect of rotor resistance, torque equation, starting and speed control methods

LESSON 26. Polyphase motor - Phase diagram, torque equation, starting and speed control methods

Alternating-current Torque

In DC motors, we learnt that torque is proportional to the current and to the density of the magnetic field in which the current finds itself. This is true for alternating current motors also, provided the instantaneous values of current and flux are considered.

If the slip is small, the reactance of the rotor conductors is low because f_2 = sf and $X_2' = 2\pi f_2 L_2$, where f is the stator frequency, X_2' is the rotor reactance at slip s, and L_2 is the rotor inductance. Because of the rotor reactance the rotor current lags the induced emf. of the rotor by an angle a. At low values of slip, this angle a is very small, since tan a= $2\pi\ fsL_2/R_2$, where R_2 is the rotor resistance.

The induced emf in any single conductor, l centimeters in length, in a field having a density of B gausses, the conductor moving at a velocity of v cm/s with respect to the field, is e = $Blv10^{-8}$ volts, the flux, the conductor and the velocity being mutually perpendicular Therefore, when a conductor is cutting flux at a uniform velocity, the flux being sinusoidally or otherwise distributed in space, the emf in the conductor is zero when it is moving in a region where B, the flux density, is zero; the emf is a maximum when the conductor is moving in a region where B the flux density, is a maximum.

Alternating-current torque when current and flux are in space-phase. It may be said that e, the emf per conductor, is in phase with the flux. It further follows that

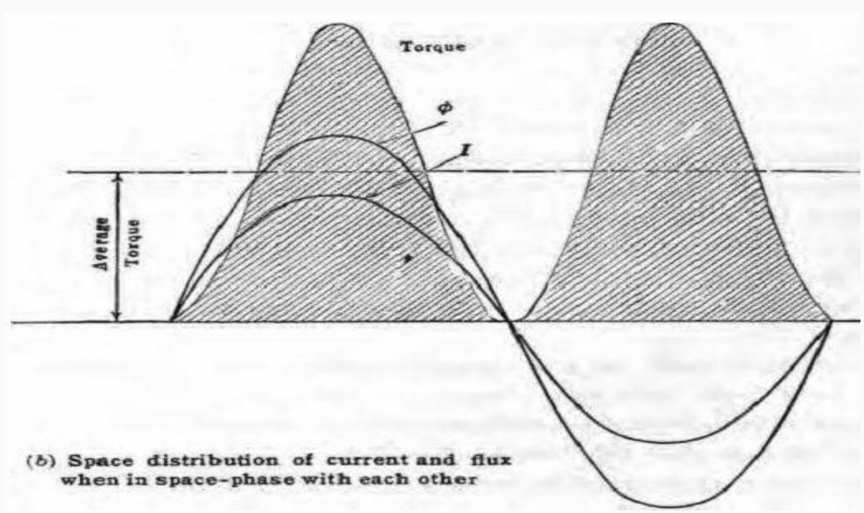


Fig. 11.1 Torque developed

the wave shape of the emf. in a single conductor is the same as the shape of the space-distribution curve of the flux. At small values of slip, the angle a, between the induced emf in each conductor and the current in the conductor, is small and therefore the current in each of the conductors, is practically in phase with its induced emf. As the induced emf is a maximum when the conductor is in the field of greatest flux density, the current will be a maximum at practically the same instant. The current is then in time-phase with the emf and hence in space-phase with the flux. Under these conditions the current in the particular conductor which is under the center of the pole, is a maximum, and that in the other conductors is less, decreasing sinusoidally as indicated.

The force acting on each conductor is proportional to its current and to the flux density of that part of the field in which the conductor finds itself. The torque curve (Fig. 11.1) is obtained by taking the product of the current and flux at each point at each point, multiplied by a constant. It will be noted that this torque curve is of double frequency, it is always positive and reaches zero twice every cycle.

As the value of the slip increases, the reactance of the rotor increases, the reactance being proportional to the rotor frequency and hence to the slip, and the angle a by which the current lags its induced emf increases, since $\tan a = 2\pi fsL_2/R_2$. The current in any conductor will not reach its maximum value until a time-degrees after the induced emf has reached its maximum value.

Operating Characteristics of the Squirrel-cage Motor

The squirrel-cage motor, like the direct-current shunt motor, operates at substantially constant speed. As the rotor cannot reach the speed of the rotating magnetic field, it must at all times operate with a certain amount of slip. At no load the slip is very small. As load is applied to the rotor, more rotor current is required to develop the necessary torque in order to carry the increased load. Consequently, the rotating magnetic field must cut the rotor conductors at an increased rate, in order to produce the necessary increase of current. The slip of the rotor must accordingly increase, so that the rotor speed drops. The ratio of the slip to the total power delivered to the rotor is proportional to the I^2R loss in the rotor. As the resistance of the squirrel cage is very low, the I^2R loss is low and, therefore, the slip for ordinary loads is small. In large motors, 50 hp. or greater, the slip is of the order of 1 to 2 per cent, at full load. In the smaller sizes of motor, the slip may be as high as 8 to 10 per cent at full load.

Figure 11.2 shows the characteristic curves of a 10-hp. squirrel-cage motor. It will be noted that the torque, speed and efficiency curves are very similar to those of a shunt motor.

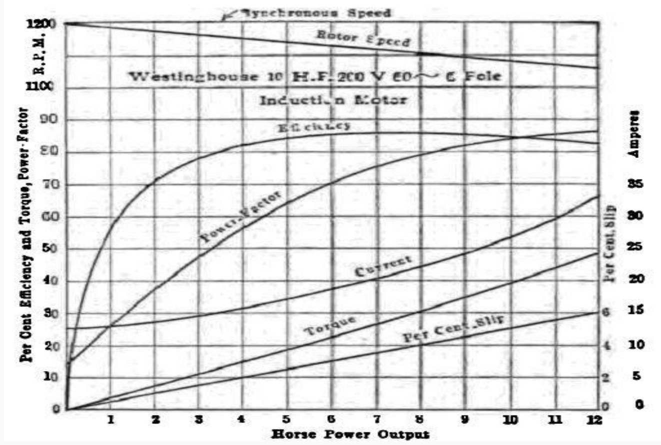


Fig. 11.2 Operating characteristics of a squirrel-cage induction motor.

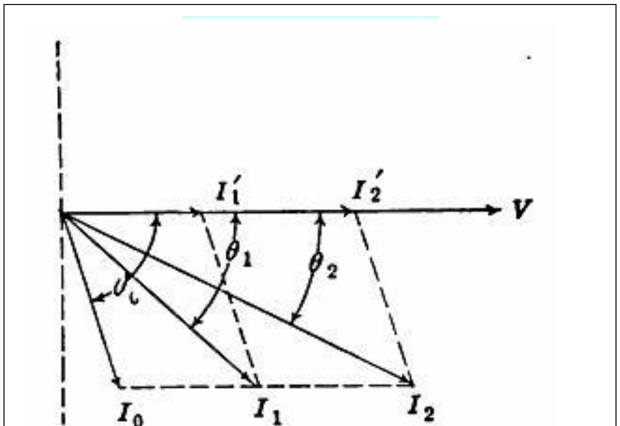


Fig. 11.3 Increase of power factor with load

The power factor increases with the load for the following reason: At no load the motor takes a current I_0, in figure 11.3, which is mostly magnetizing current, although there is a small energy component necessary to supply the no-load losses. The power factor at no load is cos q_0, the value of which may be as low as 0.10 to 0.15. The back emf of the motor remains nearly constant from no load to full load. Therefore, the flux must remain substantially constant, just as it does in the transformer, so that the magnetizing current changes but slightly from no load to full load. As load is applied to the motor, an energy current I_1' is required to carry the load. This current, when combined with I_0, gives the total current I_1 at this load, and the resulting power-factor is cos q_i. As the load increases, an energy current I_2' is required. The total current then becomes I_2 and the corresponding power-factor becomes cos q_2. It will be observed that the power factor angle decreases and therefore the power factor increases as the load on the motor increases. The increased reactance drops in the stator and in the rotor with increase of load tend to oppose this increase of power-factor and when the load exceeds a certain value may even bring about a decrease of power-factor.

As the power-factor increases, a smaller increase of current is required for a given increase of load than would be necessary if the power factor were constant. Therefore, the current increases more slowly than the load as shown. At first the efficiency increases rapidly and reaches a maximum value for the same reason that it does in other electrical apparatus. At all loads there are certain fixed losses, such as core loss and friction. In addition there are the load losses (I^2R) which increase nearly as the square of the load. Therefore, at light loads the efficiency is low because the fixed losses are large as compared with the input. As the load increases, the efficiency increases to a maximum, the fixed and variable losses being equal at this point. Beyond this point the I^2R losses become relatively large, causing the efficiency to decrease.

One disadvantage of the squirrel-cage motor lies in the fact that it takes a very large current at low power-factor on starting, and in spite of this large current it develops little torque. When the motor is at standstill, the squirrel cage acts as the short-circuited secondary of a transformer, causing the motor to take an excessive current on starting, if full voltage is applied.

Figure 11.4 shows the variation of torque with slip for two different values of line voltage. For small values of slip up to and beyond full load, which is the ordinary range of operation, the torque is substantially proportional to the slip. At higher values of slip, however, the torque curve bends over and finally reaches a maximum. This maximum is called the break-dawn torque.

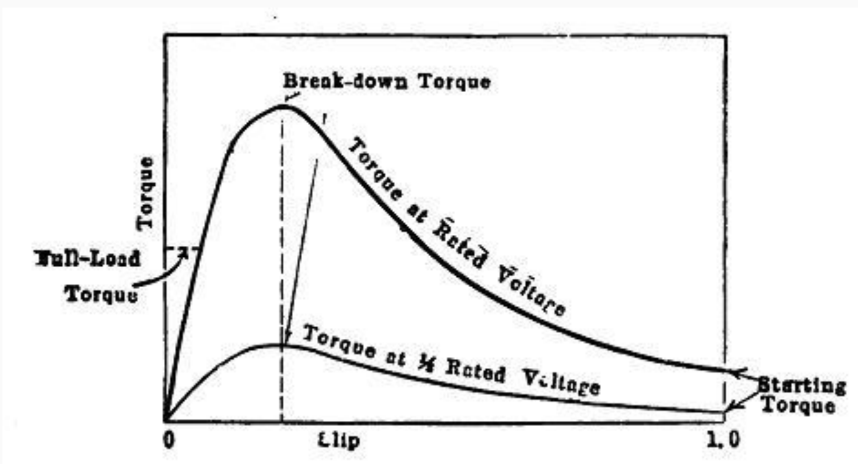


Fig. 11.4 Slip-torque curves for squirrel-cage motor.

It is also true that the torque of an induction motor for a given slip is proportional to the square of the line voltage. For this reason a 10 per cent drop in voltage (Fig. 11.4), may cause a 19 per cent reduction in the break-down and starting torques.

The stator impedance also reduces the break-down torque. A high stator impedance means a comparatively large impedance drop in the stator for a given current. This decreases the back emf, E, hence the air-gap flux becomes less, and therefore the value of the rotor current at any given slip is reduced. This results in a reduction of torque for each value of slip.

The effect of each of these various factors upon the break-down torque is shown in the following equation: The break-down torque

$$T_{max} = \frac{KV^2}{r_1 + \sqrt{r_1^2 + (x_1 + x_2)^2}}$$

where K is constant, V is the terminal voltage, r_1 is the stator resistance, x_1 is the stator reactance, and x_2 is the rotor reactance at standstill. As the squirrel-cage motor is ordinarily started at low voltage, it develops but little starting torque, because the flux is small.

Small induction motors are directly connected through simple 3 pole switches and the higher starting current is not minded much. Because the magnitude of current is relatively less. Howver for motors of more than 3 hp, the higher starting current should reduced to save the motor. As the squirrel cage motor at starting is equivalent to a short-circuited transformer, it is necessary to reduce the starting current in the larger sizes. One simple method as shown in figure 11.5 is to use a delta-connected motor. By means of a triple-pole, double-throw (T.P.D.T) switch the windings are first thrown in star across the line, thus applying only l/3 of the normal voltage to each coil. This makes the line current one-third the value if the motor were directly across the line. When the motor has attained sufficient speed, the switch is thrown over, connecting the motor in delta across the line.

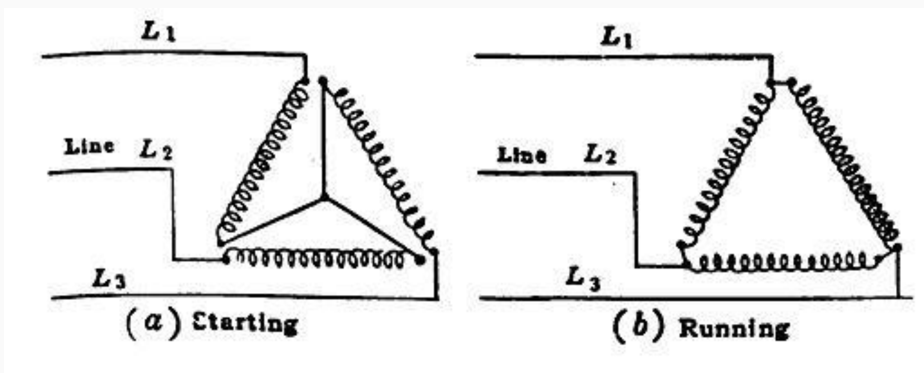


Fig. 11.5 Star delta method of starting an induction motor.

MODULE 12. Single phase induction motor: double field revolving theory, equivalent circuit, Phase split, shaded pole motors

LESSON 27. Single phase motor – series motor

Series Motor.

It will be remembered that the direction of rotation of either the direct current shunt motor or the direct current series motor is the same irrespective of the polarity of the line voltage. If the line terminals be reversed, both the field current and the armature current are reversed and the direction of rotation remains unchanged. If such motors be supplied with alternating current, the net torque developed acts in one direction only.

With alternating current, the shunt motor develops but little torque. The high inductance of the shunt field causes the field current and therefore the main flux to lag nearly 90° in time phase with respect to the line voltage. The armature current cannot lag the line voltage by a large angle if the motor is to operate at a reasonable power factor. Therefore, there will be considerable phase difference between the main flux and the armature current. Consequently, such a motor will develop but little torque per ampere. This particular type of alternating current shunt motor is therefore not practicable.

In the series motor (Fig. 12.1), the armature current and the field current are in phase with each other. The main flux is practically in phase with the field current. Therefore, the armature current is substantially in phase with the flux, and the torque curve has no negative loops. Consequently, the series motor develops approximately the same torque per ampere with alternating current as it does with direct current. Fundamentally, the series motor has possibilities as an alternating current motor.

The ordinary direct current series motor does not operate satisfactorily with alternating current for the following reasons:

(a) The alternating field flux sets up eddy currents in the solid parts of the field structure, such as the yoke, cores, etc., causing excessive heating and a lowering of efficiency,

In the alternating current series motor this difficulty is eliminated by laminating the field structure. Even with laminated field cores, however, losses in the iron occur with alternating current which do not occur with direct current.

(b) There is a relatively large voltage drop across the series fields, due to their high reactance. This limits the current and also reduces the output and power factor to such low values as to make the motor impracticable.

In the alternating current motor this difficulty is partially overcome as follows:

A low frequency is used, since reactance, X, is 2r/L, where / is the frequency and L the inductance. Even when the field

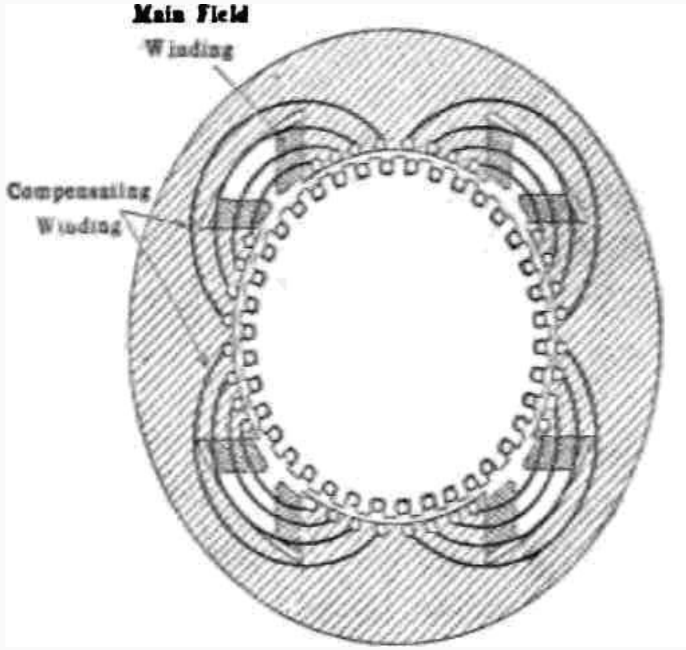


Fig. 12.1 Windings of an alternating current series motor.

inductance, L, is made as low as is practicable, the field reactance, X, will be considerably too high unless the frequency/is made low. The usual lighting frequency of 50 cycles is much too high, except for motors of fractional horsepower rating. Difficulty is experienced in designing a series motor for a frequency of 25 cycles, even. To obtain satisfactory operation, frequencies of 12 and 15 cycles are commonly used for this type of motor.

(c) The armature of an alternating current series motor of a given rating has an unusually large number of conductors. A motor of fixed horsepower and speed must develop a corresponding torque. The torque developed by a motor is proportional to the product of the field flux and the armature ampere conductors. Therefore, if the total flux of the alternating current motor is less than the total flux of a direct current motor of the same rating, the armature ampere conductors of the alternating current motor must be correspondingly increased in order to obtain the required torque. This is one reason why the armature of the alternating current motor is larger than that of the direct current motor of the same rating.

(d) The alternating current motor has a lesser number of field ampere turns and a greater number of armature ampere turns than the corresponding direct current motor. That is, the motor has a strong armature and a weak field. This means that the armature reaction is unduly large. Therefore, the effect of the armature cross magnetizing turns, unless compensated, is to produce unusually great field distortion. As this distortion of the field by the cross magnetizing armature ampere turns would make commutation practically impossible, this cross magnetizing action must be neutralized. This is accomplished by means of a compensating winding placed between the main poles, this winding being embedded in the pole faces.

(e) In the allernating current series motor a commutating difficulty occurs which is not present in the direct current motor.

The single phase series motor has practically the same operating characteristics as the direct current series motor. This is illustrated below (Fig. 12.2), which gives the operating characteristics of a typical railway motor. The torque or tractive effort varies nearly as the square of the current and the speed varies inversely as the current, or nearly so. If conductively compensated, the motor operates satisfactorily with direct current and at increased output and efficiency. When the motor is operated with alternating current, the speed may be efficiently controlled by taps on a transformer. This efficient speed control is not possible with direct current.

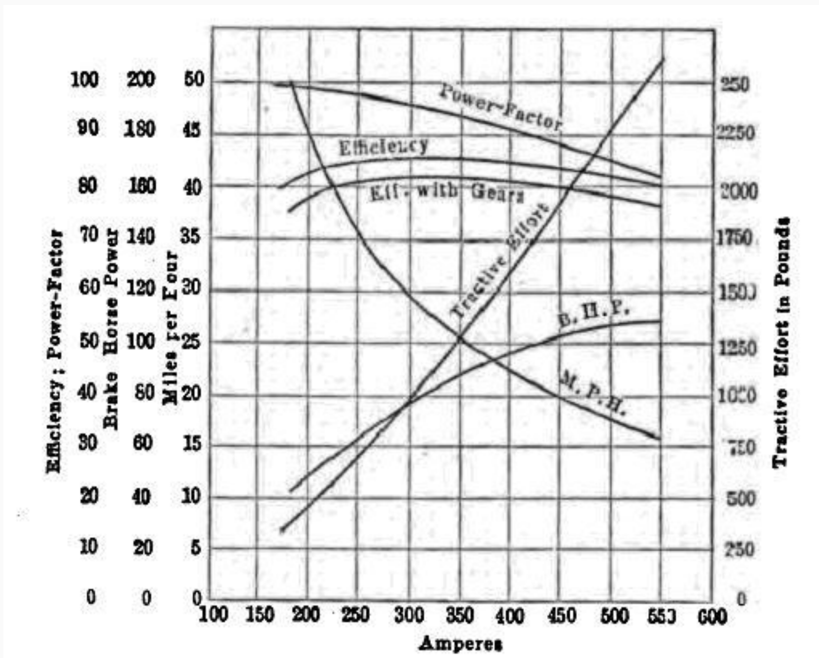


Fig. 12.2 Characteristic curves of a series motor

LESSON 28. Single phase motor – Repulsion motor

The Repulsion Motor

If an ordinary direct current armature be placed in a single phase magnetic field and the brushes be short circuited, a simple repulsion motor is obtained. In order to develop torque, however, the brush axis must be displaced from the axis of the main field by about 18 or 20 electrical space degrees, as will be shown.

The armature is same as the DC motor's which operates in a bipolar magnetic field which is excited by a single phase AC supply voltage. At the instant shown in Fig. 12.3, upper line is positive and current increasing in positive direction. By corkscrew rule, the magnetic field is upward and increasing with current magnitude. This flux divides half going through each side of the shorted armature. Each side of armature hence acts as the secondary of a transformer.

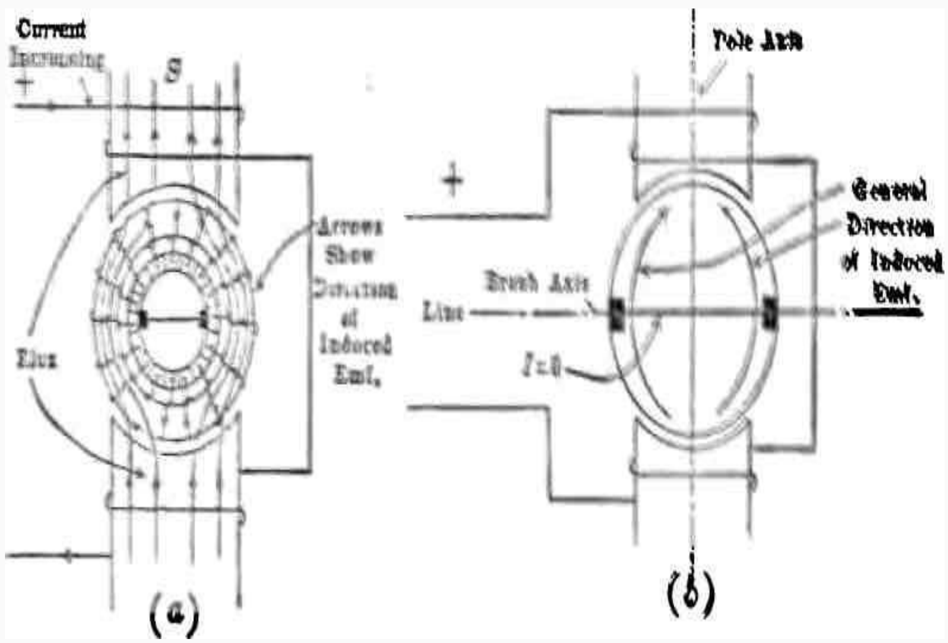


Fig. 12.3 Currents in the windings of a repulsion motor (brushes in geometrical neutral).

Therefore, the alternating flux produced by the field winding, as primary, induces an emf in each half of the armature. By Lenz's law, this induced emf has such a direction as to oppose the inducing flux. The direction of this induced emf at the instant indicated in figure Fig. 12.3 (a) is given by the arrows on the windings. It will be noted, by following through the winding, that the resultant direction of this induced emf is upward on each side of the armature. Were there no brushes, it is evident that no current would flow in the armature winding, as the emf in one half of the winding is equal and in phase opposition to that in the other half. The brushes are shown as being in the geometrical neutral and short circuited. Each brush is at the mid point of its transformer winding. As the total emfs. in each winding are the same and the windings are connected in parallel, each midpoint must be at the same potential.

Therefore, the brushes short circuit two points at the same potential and no current flows between brushes. It is clear that without brushes there is no armature current, and even with brushes there is no armature current, provided the brush axis is at right angles to the pole axis. Therefore, under both these conditions there is no armature current and, hence, no torque.

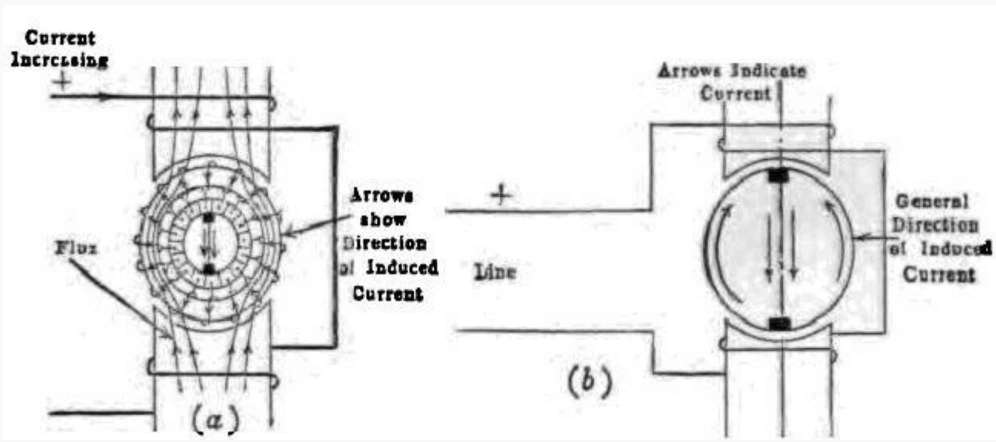


Fig. 12.4 Currents in the windings of a repulsion motor, brushes along pole axis

Figure 12.4 shows the same condition existing in the field and armature as before, except that the brushes now lie along the pole axis. As the general direction of the induced emfs have not changed, the brushes are now short circuiting the points of the armature winding across which the maximum potential difference exists. Therefore, current flows between the brushes from both sides of the armature, and in this brush position, the current in the armature is a maximum. But the motor develops no torque since two conditions are necessary for the development of torque.

(1) The angle between the space position of the flux axis and the brush axis must be greater than zero. For maximum torque this angle should be 90°. For example, in a direct current motor with fixed flux and armature current, the maximum torque occurs when the brushes are in the neutral plane, that is at right angles to the flux. No torque would be developed were the brush axis parallel to that of the flux.

(2) There must be a component of current in time phase with the flux. If there is 90° time lag between the current and the flux, the current is a maximum at the instant the flux is zero, etc., and the average torque is zero. With flux, armature current and brush position all fixed, the maximum torque occurs when the flux and armature current are in time phase with each other.

Under the conditions, the brush axis being parallel to the resultant flux, (the angle between the flux and the brush axis is zero), the current flows in opposite directions in the two equal conductor belts on each side of the brush axis. Hence in this type of motor, no torque is developed when the brush axis is at right angles to the flux, for then there is no current; no torque is developed when the brush axis is parallel to the flux, because the ampere conductors under each pole develop opposite and equal torques.

It is obvious, however, that if the brushes be placed in some intermediate position, they will be short circuiting points of the winding between which a difference of potential exists and therefore currents will flow in the winding, and also the net ampere conductors under each pole cannot be zero. It can also be shown that the armature current is substantially in time phase with the flux. Therefore, under these conditions, the motor develops torque.

Figure 12.4 (a) shows the brush axis making an angle a with the pole axis. The arrows in this figure show the direction of the armature current at the instant when the upper wire is positive and the current is increasing positively.

The figure 12.5 shows diagrammatically the general direction of these currents through the armature and brushes. It will be observed that the current direction in the conductors under each pole is such as to develop torque. Figure 12.5 (c) shows the direction of induced emf in the armature, neglecting the distorting effect of the armature mmf, on the field flux. The emfs in each half of the armature act in conjunction as shown Fig. 12.5 (b). Assume for the time being that angle b equals angle a in Fig. 12.5 (c). The current paths through the winding are abcd and afed. In path abcd, the emfs, E_{cd} and E_{cb} included in angles a and b respectively, each equal to the brush displacement angle, are equal and act in opposition. Therefore, they cancel each other, leaving E_{ab} as the net emf through path abcd. Likewise in path afed, the emfs. E_{fa} and E_{fa} cancel, leaving E_{ad} as the net emf through this path. The net emfs E_{ab} and E_{cd} are effective in sending the current through the armature.

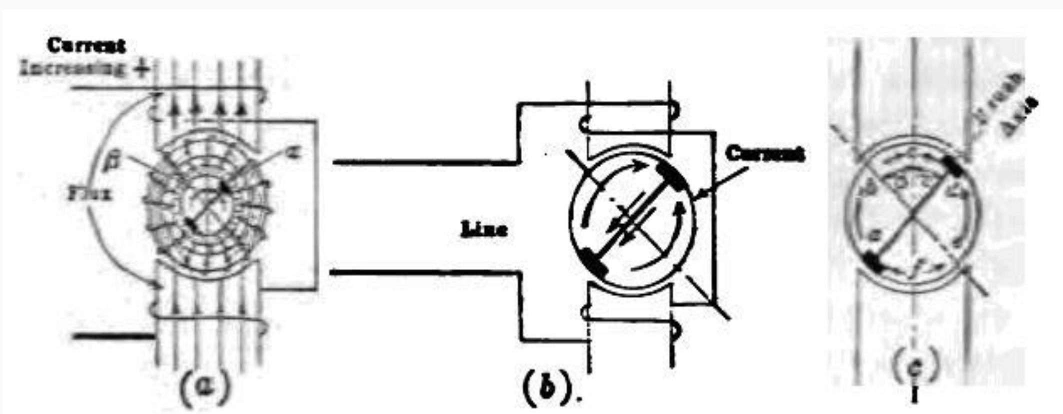


Fig. 12.5 Brush position in a repulsion motor which gives both current and torque

In this type of motor, the direction of rotation depends on the brush position. For example, the direction of rotation may be reversed by moving the brushes so that they cross the pole axis, the brush axis then making an angle b with the pole axis. Angle b must be less than 90°.

Instead of displacing the brushes from the geometrical neutral so that a potential difference exists between them, which results in a current, giving rise to torque, the same effect may be obtained by using two field windings displaced at right angles to each other. Practically all repulsion motors are made with non salient poles, rather than with the salient poles shown in the diagrammatic illustrations. The windings are usually of the distributed type, such as are used for induction motors. Repulsion motors have characteristics similar to those of series motors and have large starting torque. The sparking is very small at synchronous speed (3,600 rpm for a two pole, 50 cycle motor) but at speeds differing greatly from this, the sparking may be excessive. There are several types of repulsion motor on the market which, while differing in detail from the motor just described, involve identical principles.

LESSON 29. Single phase induction motor - double field revolving theory

Single phase Induction Motor

Figure 12.6 shows a two pole motor whose magnetic field is produced by single phase current flowing in a simple field winding.

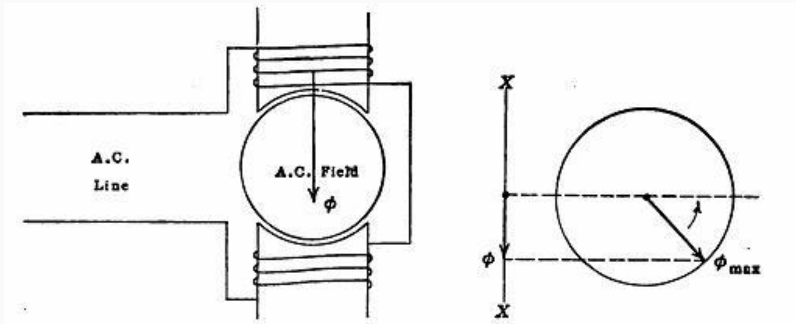


Fig. 12.6 Single phase, alternating field and the Time variation of the AC field.

The current in this field is assumed to vary sinusoidally with time and if the iron be assumed to operate at moderate flux densities, the flux through the armature will vary practically sinusoidally with time. The variation of this field with time may be represented (Fig. 12.6) by the projection of a rotating vector φ_{max} upon a vertical axis XX shown. The vector φ_{max} is equal to the maximum value of the flux and its speed of rotation in revolutions per second is equal to the line frequency in cycles per second.

It may also be assumed that this single phase field is made up of two equal and oppositely rotating fields represented by two equal and oppositely rotating vectors as shown in Fig. 12.7 (a), the maximum value of each of these fields or vectors being equal to one half of j_{max}. The resultant of two such vectors always lies along the vertical axis and is equal in magnitude at any instant to the field actually existing at that instant.

The same thing is represented in Fig 12.7(b), which shows the flux distribution curves of two fields j_1 and j_2, each of which is equal to one half the maximum field. These two fields glide around the air gap in opposite directions and with equal velocities.

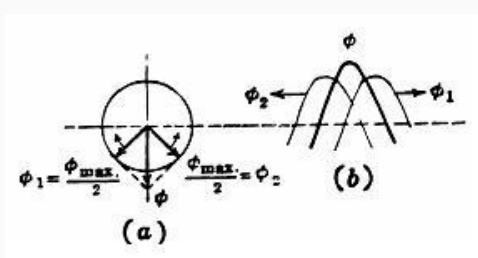


Fig. 12.7 Single phase field as split fields.

Their algebraic sum j at any instant is the value of the resultant field at that instant and this resultant field is stationary in space.The single phase field may be considered therefore as made up of two equal rotating fields, revolving in opposite directions. (Experiment shows that two such fields actually exist.) Each field acts independently upon the rotor and in the same manner as the rotating field of the polyphase induction motor. One field tends to cause rotation in a clockwise direction and the other field tends to cause rotation in a counter clockwise direction. Figure 12.8 shows the slip torque curve due to each of the two fields.

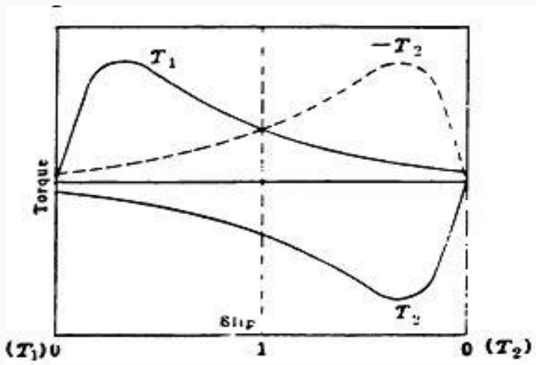


Fig. 12.8 Two opposing torques in a single phase induction motor

The torques act in opposite directions as shown. At standstill (slip = 1) the two torques are opposite and equal, and the rotor has no tendency to start. If the rotor in some manner be caused to rotate in the direction in which the torque T_1\ is acting, T_1\ will immediately exceed the counter torque T_2 and armature will begin to accelerate in the direction of T_1. As the armature speeds up, T_1 predominates more and more over T_2 and the armature approaches synchronous speed without difficulty. The counter torque due to T_2 always exists, however, although it has little effect near the synchronous speed of the field which produces T_1.

When the rotor operates near synchronous speed and rotates in the direction of T_1, its slip is nearly two as regards T_2. Therefore, the rotating field which produces T_1 induces double frequency currents in the rotor at this speed. These double frequency currents, however, produce but little torque because of their high frequency. This frequency is double the stator frequency. Therefore, the rotor reactance is many times its value at slip frequency. Consequently, these currents are small in magnitude and make a considerable space angle with the air gap flux, developing little counter torque.

It is obvious that the single phase induction motor rotates in the direction in which it is started.

Starting Single phase Induction Motors.

As the single phase induction motor is not self starting, auxiliary means must be used to supply initial torque. One method is to split the phase by the use of inductance, resistance or capacitance.

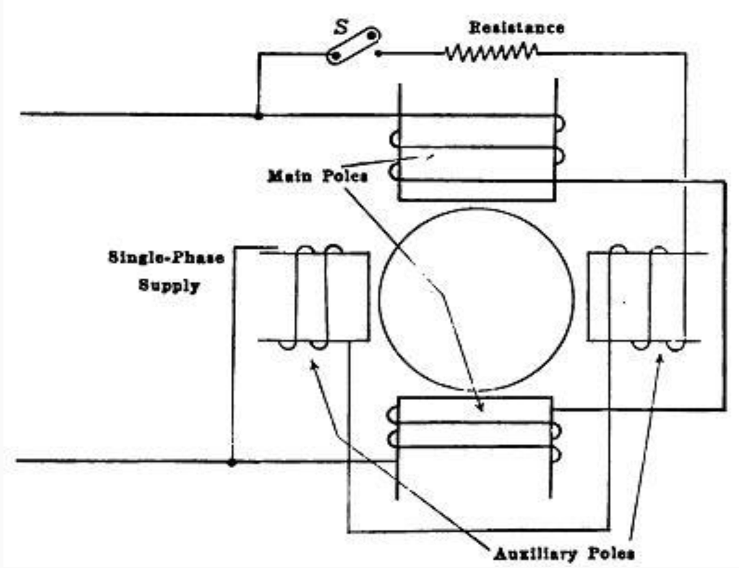


Fig. 12.9 Starting Single phase Induction Motors

Figure 12.9 shows one method of splitting the phase, a two pole motor being shown. The main winding, which is highly inductive, is connected across the line in the usual manner. Between the main poles are auxiliary poles which have a high resistance winding and this winding is also connected across the line. As the auxiliary winding has a high resistance, its current will be more nearly in phase with the voltage than the current in the main winding. For the best conditions, the two currents should differ in phase by 90°, but this condition is not readily obtainable, and in fact is not necessary. These two sets of poles produce a sort of rotating field which starts the motor. When the motor comes up to speed, a centrifugal device in the rotor opens the switch 'S' and disconnects the auxiliary winding.

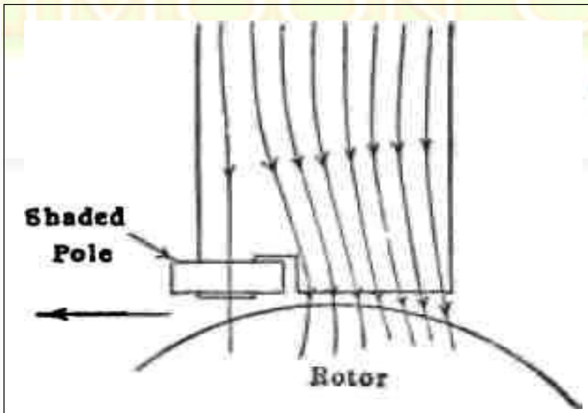


Fig. 12.10 Shaded pole method

The **shaded pole method** is shown figure 12.10. A short circuited coil of low resistance is connected around one pole tip. When the flux is increasing in the pole, a portion of the flux attempts to pass down through this shaded tip. The flux induces a current in the coil which by Lenz's law is in such a direction as to oppose the flux entering the coil. Hence, at first the greater portion of the flux passes down the right hand side of the pole, as shown.

Ultimately, however, the main flux reaches its maximum value, where its rate of change is zero. The opposing emf in the shading coil then becomes zero, and later the opposing mmf of the short circuited coil ceases, the current in this coil lagging its emf. Considerable flux then penetrates the short circuited coil. After the main flux begins to decrease, the induced current in the shading coil tends to prevent the flux then existing in the shaded portion of the pole tip from decreasing. Therefore, the flux first reaches its maximum value at the right hand or non shaded side of the pole, and later reaches its maximum at the left hand or shaded side. The effect of the shading coil is to retard in time phase a portion of the flux, so that there is a sweeping of the flux across the pole face from the right hand to the left hand side in the direction of the shading coil. This flux cutting the rotor conductors induces currents, which in turn produce a torque sufficient to start the motor. The shaded pole is not a common method of starting single phase induction motors and is used only in motors of very small size.

MODULE 13. Disadvantage of low power factor and power factor improvement

LESSON 30. Disadvantage of low power factor and power factor improvement

Reactive Power

In any AC system, a purely resistive load, for example, electrical resistance heating, incandescent lighting , etc., the current and the voltage are in phase. Whereas, in the case of inductive loads, the current is out of phase with the voltage and it lags behind the voltage. Most of the equipment are inductive in nature such as inductive motors. In the case of a capacitive load the current and voltage are again out of phase but now the current leads the voltage. The most common capacitive loads are the capacitors installed for the correction of power factor of the load. Inductive loads require two forms of power - Working/Active power (measured in kW) to perform the actual work of creating heat, light, motion, etc., (consumed in the resistive portion of the load) and Reactive power (measured in kVAr) to sustain the electromagnetic field. The vector combination of these two power components (active and reactive) is termed as Apparent Power (measured in kVA), the value of which varies considerably for the same active power depending upon the reactive power drawn by the equipment. The ratio of the active power (kW) of the load to the apparent power (kVA) of the load is known as the power factor of the load.

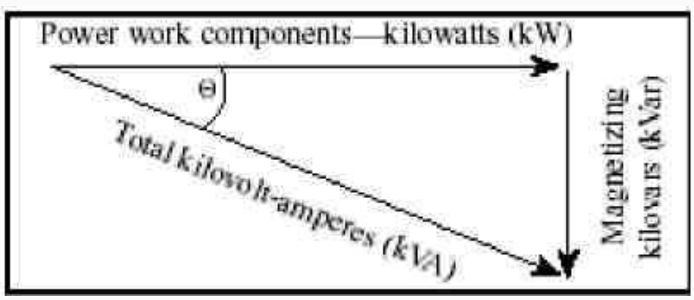


Fig. 13. 1. Apparent power

Power Factor = Active Power (kW) / Apparent Power (kVA)

Thus when the nature of the load is purely resistive the kVAr or the reactive component will be nil and thus the angle φ will be equal to 0 degrees and the power factor will be equal to unity. For a purely inductive load the power factor will be 0.0 lagging and for a purely capacitive load the power factor will be 0.0 leading.

Thus, it is evident from above that, more the power factor departs from unity, the more will be the apparent (kVA) demand for the same real (kW) load. The customers with a low power factor will pay more for their useful electrical power.

The disadvantages of a low power factor are :

1. The load draws greater current for the same value of the useful power.

A simple example showing the current required by a single phase electric motor is given below:

Supplied Voltage –240 Volts Single phase.
Motor input – 10 KW

Power Factor – 0.65

Current (I_1) = Power (kW) /Volts (V) * PF = 10000/ 240*0.65 = 64.1 Amp.

If the power factor of the motor is increased to 0.9 the current drawn by the motor shall be –

Current (I_2) = 10000/ 240*0.9 = 46.3 Amp.

Thus, as the power factor decreases the current required for the same value of active, or useful, power increases. So the sizes of the equipment, like the switchgear, cables, transformers, etc., will have to be increased to cater the higher current in the circuit adding cost. Further, the greater current causes increased power loss or I^2R losses in the circuits. Also due to higher current, the conductor temperature rises and hence the life of the insulation is reduced.

2. With the increased current, the voltage drop increases, thereby the voltage at the supply point.

Example : Let us take an example of a cement industry with initial load condition of 5000 kVA at 60% power factor with a consumption of 19,20,000 units per month, supplied at 33 KV.

Assuming the Tariff as below:

Demand charges Rs. 144/kVA/month

Energy Charges Rs. 4.11 / Unit

PF surcharge for each 1% below 90% 1% of (Demand charges + Energy Charges)

A. Cost saving due to Power Factor improvement

(i) By improving the power factor, there will be a reduction in the kVA demand of the load.

Power Factor= cos φ = kW/ kVA

$\text{Cos}\varphi_1 = 0.6 = \text{kW}/\text{kvA}_1 = \text{kW}/5000$; KW=5000*0.6

$\text{Cos}\varphi_2 = 0.9 = \text{kW}/\text{kVA}_2$; $\text{KW}=\text{kVA}_2*0.9$

Equating, $5000*0.6=\text{kVA}_2*0.9$; $\text{kVA}_2=(5000*0.6)/0.9 = 3333.33$ kVA

Thus, in this case the kVA is dropped from 5000 kVA (at 60%) to 3333.33 kVA (at 90%):

Therefore reduction in energy bill due to reduction in maximum demand due to improved power factor from 0.6 to 0.9 shall be :

Rs. 144.00 * (5000-3333.33)= Rs. 240000.48 per month

(ii) In addition, by increasing the power factor from 60% to 90%,, there shall be no power factor penalty/surcharge on account of low power factor. Thus the savings due to avoidance of the PF surcharge per month would be as below:

Rs. ((5000-3333)*144*(90-60))*1/100= Rs.72014.14

(iii) Thus the total monthly reduction in bill due to P.F improvement from 0.6 to 0.9 would be:

Rs. 240000.48 + 72014.14 = Rs. 312014.88 per month.

Net reduction per annum = 312014.88*12 = 3744178.56 $\simeq$ Rs.37,44,179/-

Cost of investment for Power Factor improvement:

Size of capacitor required to improve the PF from 0.6 to 0.9

Reduction in reactive power

= kVA_1* $\sin\varphi_1$ – kVA_2* $\sin\varphi_2$ =5000*sin(53.1) – 3333.33*Sin(25.84)

=5000*0.8 - 3333.33*0.436 =4000-1453=2547 kVAr say 2550 kAVr

If we take the cost of capacitor bank per kVAr as Rs. 200/- , the cost of the capacitor bank = 2550*200 = Rs. 5,10,000/-

Cost of switching and associated equipment = Rs. 3,00,000/-and installation, etc.

Total cost = Rs. 8,10,000/-

Annual depreciation and interest@ 20% = Rs. 810000*0.2

= Rs. 1,62,000/-

Net Annual saving = 37,44,179 - 1,62,000 = Rs. 35,82,179/-

Net monthly saving = Rs. 2,98,515/-Therefore payback period = 2.7 months

Improving Power Factor :

Improving power factor by reducing the kVAr load requires the use of power factor equipment which operate at a leading power factor such as:

- Synchronous motors which are either over-excited or under loaded with full excitation so they will supply kVAr to the electrical system.

- Static capacitors which are electrical devices without moving parts that have the ability to provide magnetizing current to the load. Their efficiency is high since losses are less than one-half of 1 percent of their kVAC (or kVAR) rating.

The synchronous condenser is a synchronous motor without shaft extensions (so it cannot carry any mechanical load) which idles across the power system. Increasing its field excitation results in its furnishing magnetizing power (kVAr) to the system. Its principal advantage is the ease with which the amount of correction could be adjusted. These machines are automatically controlled and generate or consume reactive power depending on the system requirement. The synchronous converter is a machine with both slip rings and a commutator connected to the armature windings. This could supply direct current in much the same way as a conventional motor-generator set, but with some economy of size, weight, and material. Adjustment of the field excitation changes the amount of magnetizing power it could supply to the alternating current power lines. Both of these machines have been replaced principally by the use of static capacitors.

Static Capacitors:

Static capacitors are the cheapest and the simplest means for reactive power compensation . They are installed by power utilities in the transmission and distribution network and also at the consumers' place on to different loads such as motors, transformers, incoming supply, etc. In present days automatic switching of the capacitors enables keeping a high power factor for heavily fluctuating loads as well.

Compensation or Power Factor Correction

Based on the reactive power requirement at their installations, the consumers have to provide for the necessary reactive compensation at their end to achieve the minimum power factor level prescribed by the utility. The most economical and reliable method of reactive compensation is the installation of power capacitors. Lagging power factor can be corrected by connecting capacitors in parallel with the system. The current in a capacitor produces a leading power factor. Current flows in the opposite direction to that of the inductive device. When the two circuits are combined, the effect of capacitance tends to cancel that of the inductance. Most customer loads (particularly motors, but many lighting circuits also) are inductive.

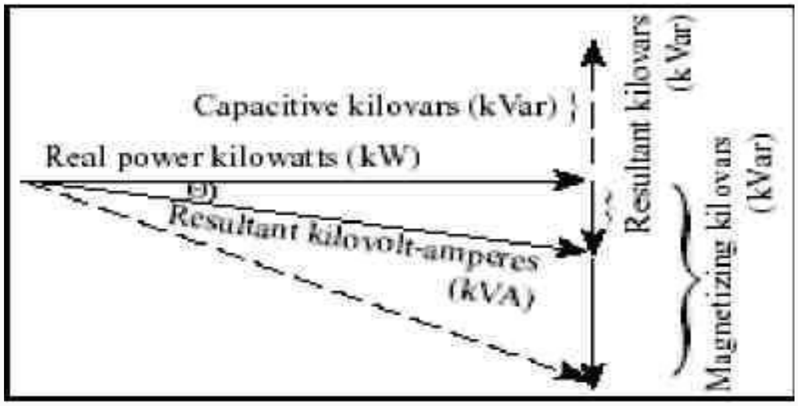


Fig. 13.2. resultant kVAr component after correction.

In figure 13.2, note the smaller resultant with the capacitors added, but real power (kW) does not change. A properly chosen capacitance value will neutralize the inductance and produce unity power factor.

Methods of correction

a. For motors of 50 hp and above, it is best to install power factor correction capacitors at the motor terminals since distribution circuit loading is reduced. When this is done, motor settings that are over current protection relays must be adjusted down accordingly.

b. In the second arrangement capacitor banks are connected at the bus for each motor control centre. This compromise to Method 1 will reduce installation costs.

c. In the least expensive method the capacitor banks are connected at the service entrance. However, the disadvantage is that higher feeder currents still flow from the service entrance to the end of line equipment.

MODULE 14. Various methods of single and three phase power measurement

LESSON 31. Various methods of single phase power measurement

The Wattmeter

Alternating current power is equal to the product of the effective current and the effective volt age only when the power factor is unity. Therefore, the ammeter and voltmeter method, as used with direct currents, can seldom be used to measure alternating-current power. Consequently, a wattmeter is necessary for measuring alternating-current power. The wattmeter shown below operates on the electro-dynamometer principle. M is a moving coil wound with fine wire and is practically identical with the moving coil of the dynamometer voltmeter.

It is connected across the line in series with a high resistance. The current is led into this coil through springs. The two fixed coils FF are wound with a few turns of heavy wire, capable of carrying the load current. As there is no iron present, the field due to the current coils FF is proportional to the load current at every instant. The current in the moving coil M is proportional to the voltage at every instant. 'Therefore, for any given position of the moving coil, the torque is proportional at every instant to the product of the current and voltage or to the instantaneous power of the circuit. If the power factor is other than unity, there is negative torque for part of the cycle. That is, during the periods when there are negative loops in the power curve, the current in the fixed coil and the current in the moving coil reverse their directions with respect to each other, and so produce a negative torque. The moving coil takes a position corresponding to the average torque. The torque is also a function of the angle between the fixed and moving coil axes, but this factor is taken into account by the scale calibration.

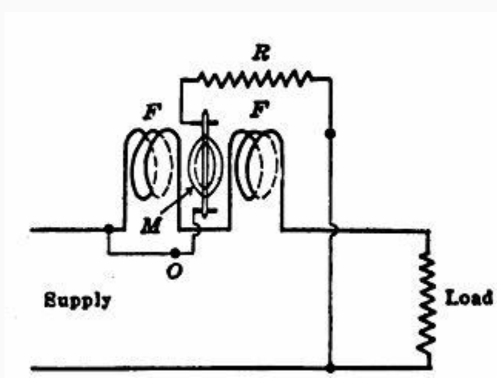


Fig. 14.1 Wattmeter

As the torque acting on the moving coil varies from instant to instant, having a frequency twice that of either the current or the voltage, the coil tends to change its position to correspond with these variations of torque. If the moving system had little inertia, the needle would vibrate so that it would be impossible to obtain a reading. Because of the relatively large moment of inertia of the moving system, the needle assumes a steady deflection for constant values of average power. The position taken by the coil corresponds to the average value of the power, which is the result desired.

Wattmeter Connections Figure 14.2 shows wattmeter W measuring the power taken by a certain load. In order to measure this power correctly, the wattmeter current coil should

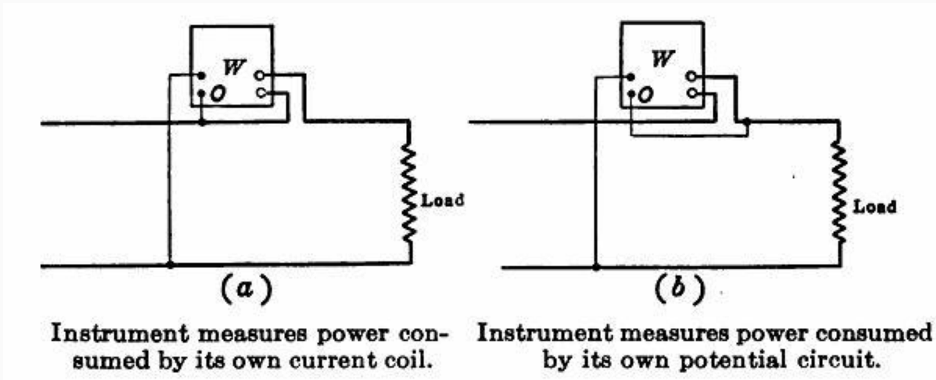


Fig. 14.2 Wattmeter connections

carry the load current, and the wattmeter voltage-coil, in series with its resistance, should be connected directly across the load. The current in the wattmeter current coil is the same as the load current, but the wattmeter potential circuit is not connected directly across the load, but is measuring a potential in excess of the load potential by the amount of the impedance drop in the wattmeter current-coil. Therefore, the wattmeter reads too high by the amount of power consumed in its own current-coil.

Under these conditions the true power

$P = P' - I^2R_c$

where P' is the power indicated by the wattmeter, I is the current in the wattmeter current coil, and R_c is the resistance of this coil. This loss is ordinarily of the magnitude of 1 or 2 watts at the rated current of the instrument, and may often be neglected.

If the wattmeter be connected as shown in (b), the wattmeter potential circuit is connected directly across the load, but the wattmeter current coil carries the potential coil current in addition to the load current. In fact, the wattmeter potential circuit may be considered as being a small load connected in parallel with the actual load whose power is to be measured. Therefore, the power consumed by this potential circuit must be deducted from the wattmeter reading. The true power taken bv the load,

$P = P' - E^2/R_c$

where P' is the wattmeter reading, E the load voltage and R_c the resistance of the wattmeter potential coil circuit. An idea of the magnitude of this correction may be obtained from the following example.

Example.—A certain wattmeter indicates 157 watts when it is connected in the manner shown in figure (b). The line voltage is 120 volts and the resistance of the wattmeter potential circuit is 2,000 ohms. How much power is taken by the load?

$P = 157 - 120^2/2{,}000 = 149.8$ watts.

It will be observed that a considerable percentage error would result in this case if the wattmeter loss were neglected. The instrument is so manufactured which compensates for this loss. A small auxiliary coil, connected in series with the moving-coil system, is inter wound with the fixed coils so that a small counter-torque is exerted, this counter torque being proportional to the power consumed by the potential circuit.

The current and potential circuits of a wattmeter must each have a rating corresponding to the current and voltage of the circuit to which the wattmeter is connected. A wattmeter is rated in amperes and volts, rather than in watts, because the indicated watts show neither the amperes in the current-coil nor the voltage across the potential-circuit.

If the current in an ammeter or the voltage across a voltmeter exceed the rating of the instrument, the pointer goes off scale and so warns the user. A wattmeter may be considerably overloaded and yet the load power factor be so low that the needle is well on the scale. For this reason a voltmeter and an ammeter should ordinarily be used in conjunction with a wattmeter (Fig. 14.3) so that it is possible to determine whether either the voltage or the current exceeds the wattmeter rating.

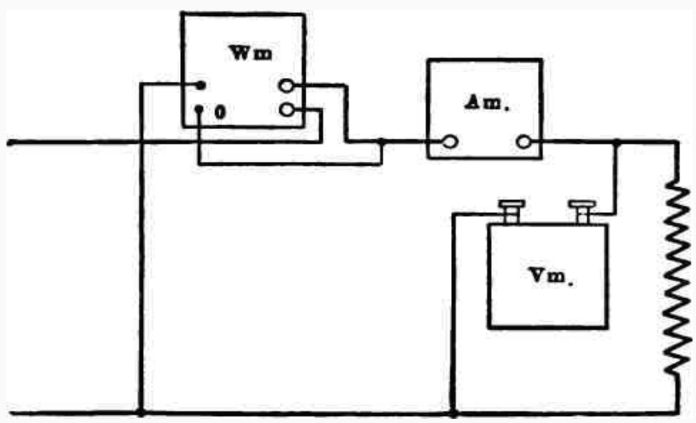


Fig. 14.3 Wattmeter, ammeter and voltmeter connections for measuring power

LESSON 32. Methods of three phase power measurement

Measurement of Power in THREE PHASE SYSTEM

Three-wattmeter Method: Let the load coils in figure 14.4 be the three coils of either a Y-connected alternator or of a Y-connected load.

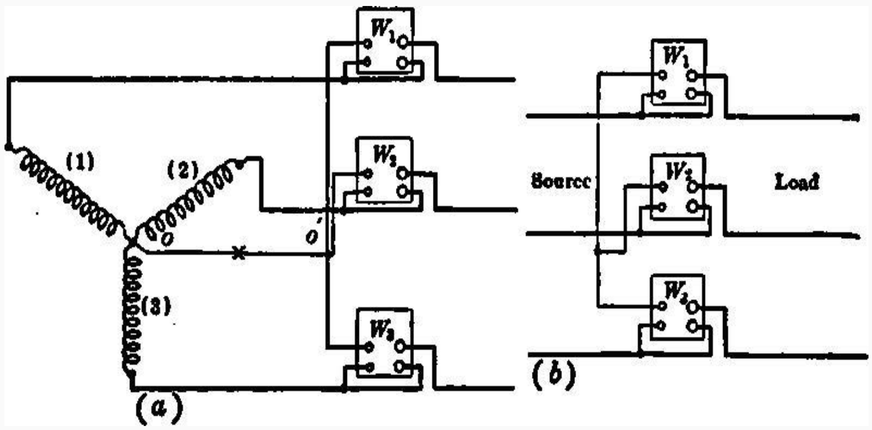


Fig. 14.4 The 3-wattmeter method of measuring 3-phase power.

If the neutral of the Y is accessible, it is possible to measure the power of each phase by connecting the current-coil of a wattmeter in series with the phase and by connecting the wattmeter potential-coil across the phase. Therefore ,W_1, W_2 and W_3 measure the power in load coils 1, 2, and 3 respectively, regardless of power-factor, degree of balance, etc. The total power

$P = W_1 + W_2 + W_3$

If the potential circuits of the three wattmeters have equal resistances, these three potential circuits constitute a balanced Y-load, having a neutral. As coils 1, 2, and 3 and these three wattmeter potential-circuits are both symmetrical systems, neutral of their connection must be at the same potential as 0. Therefore, no current flows between these two neutrals and the line can be cut at without changing existing conditions. Figure below shows the three wattmeter connection for a three phase system. It can be shown that the total power is the sum of the wattmeter readings even though the wattmeter potential circuits have different resistances. Under these conditions, however, the wattmeters may not all have the same reading, even with balanced loads. The three wattmeter method is well adapted to measuring power in a system where the power-factor is continually changing, as in obtaining the phase characteristics of a synchronous motor. If the three instruments have equal potential-circuit resistances, they read alike regardless of power-factor, if the loads are balanced. The three wattmeter method is necessary in a three-phase four-wire system, as a system of n wires ordinarily requires n — 1 wattmeters in order to measure the power correctly.

Two-wattmeter Method.

The power in a three-wire, three-phase system can be measured by two wattmeters connected as shown in Figure 14.5. The current-coils of the two instruments are connected in two of the lines and the potential-coil of each instrument is connected from its respective line to the third line. Two-wattmeter method of measuring 3-phase power.

Under these conditions the total power passing through the system

$p = W_1 \pm W_2$

regardless of power-factor, balance, etc.

One method of proving that these instruments give the correct power is as follows: Let e_1,e_2, e_3 and i_1, i_2 , i_3 be the respective voltages and currents of the three loads at any particular instant. These being instantaneous values, the power at the instant under consideration is equal to their products regardless of power-factor. That is, the instantaneous power

$P = p = e_1 i_1 + e_2 i_2 + e_3 i_3$

But

$$i_1 + i_2 + i_3 = 0 \text{ (Kirchhoff's first law)}$$

$$i_2 = -(i_1 + i_2)$$

Substituting

$$p = e_1 i_1 - e_2(i_1 + i_3) + e_3 i_3$$

$$= (e_1 - e_2) i_1 + (e_3 - e_2) i_3$$

$$W_1 \text{ reads } (e_1 - e_2)\, i_1$$

and

$$W_1 \text{ reads } (e_3 - e_2) i_3$$

The same proof may be used for a delta-load, except that

$ei + e_2 + c_z = 0$

It can be shown that a phase difference of 30° exists between the line voltage and line current at unity power factor.

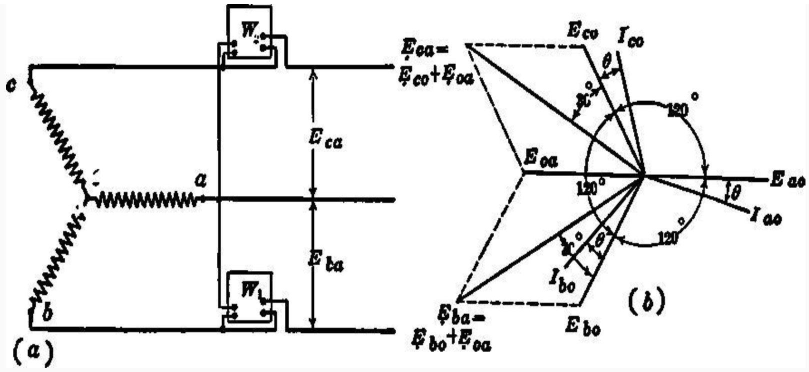


Fig. 14.5 Vector diagram illustrating 2-wattmeter method for measuring 3-phase power, balanced load.

For power-factors other than unity, this phase difference becomes (30° ± φ), where j is the power factor angle of the coil. Figure shows two watt meters, W_1 and W_2, measuring the power taken by a balanced three-phase, Y-connected load. The wattmeter W_1 is so connected that the current i_b flows in its current coil and the voltage E_{ba} is across its potential-circuit. Therefore, the reading of W_1 is equal to the product of i_b , E_{ba} and the cosine of the angle between this current and this voltage. Figure shows the vector diagram of the load also. The three coil voltages E_{ao}, E_{bo}, and E_{co} are all equal and 120° apart. The coil currents $I_{a,}$, I_b and I_c are equal and lag their respective coil voltages by the angle j. The voltage E_{ba} is found by reversing E_{ao}, giving E_{oa}, and then adding E_{bo} and E_{oa} vectorially ($E_{ba} = E_{bo} + E_{oa}$). The current I_{bo} is given. The angle between E_{ba} and I_{bo} is 30°- φ. Therefore, the reading of this wattmeter is

$W_1 = E_{ba} I_{bo} \cos (30° - \varphi) = E_{line} I_{line} \cos (30° - \varphi)$

Likewise, the wattmeter W_2 reads the product of E_{ca}, I_{co} and the cosine of the angle between them. From the vector diagram, the angle between E_{ca} and I_{co} is 30° + j . E_{ca} is found by adding vectorially E_{co} and E_{oa}.

Therefore the reading of this wattmeter is

$W_2 = E_{ca} I_{co} \cos (30° + 0) = E_{Une} I_{line} \cos (30° + \varphi)$

Summarizing

$W_1 = EI \cos (30° - \varphi)$; $W_2 = EI \cos (30° + \varphi)$

where E and I are the line voltage and line current, respectively, the system being balanced.

W_1 and W_2 will read alike when φ = 0 and φ = 180°. Both conditions correspond to unity power factor. When φ equals 180°, however, the power has reversed. The two instruments also read alike at zero power-factor (φ = 90°), although this condition is never realized.

When φ = 60°, corresponding to a power-factor of 0.5, Wt reads zero, as cos (30° + 60°) = cos 90° = 0. In this case, the reading of W_1 gives the total power. For angles greater than 60°, corresponding to power factors less than 0.5, cos (30° +φ) becomes negative, W_2 reads negative and the total power becomes p = $W_1 - W_2$

Therefore, discretion must be used when two single instruments are employed, as the total power may be either the sum or the difference of the readings.

It may also be shown that

$$\tan \theta = \sqrt{3}\,\frac{W_1 - W_2}{\quad}$$

where φ is the load’s power factor angle. Therefore it is possible to obtain the power factor in a balanced three-phase system by means of the wattmeter readings alone.

Example.—In a test of a three phase induction motor, two watt meters are used to measure the input. Their readings are 1,900 and 800 watts respectively. Both instruments are known to be reading positive. What is the power-factor of the motor at this load?

$\tan \varphi = \sqrt{3}\,(1900-800)/(1900+800) = 0.705$

$\varphi = = 35.3°$

$\cos 35.3° = 0.815$.

The two-wattmeter method cannot be used to measure power in a three-phase, four-wire system unless the current in the neutral wire is zero (Fig. 14.6). When the current in the neutral wire of figure below is zero, the power is correctly indicated by $W_1 \pm W_2$.

If we apply load B'O between line B and the neutral, the current to this load will complete its circuit from wire B through the neutral without going through the current coil of either watt meter. As neither wattmeter can indicate this additional load, the two watt meters are not sufficient to measure the power in such a four wire system under all conditions of load.

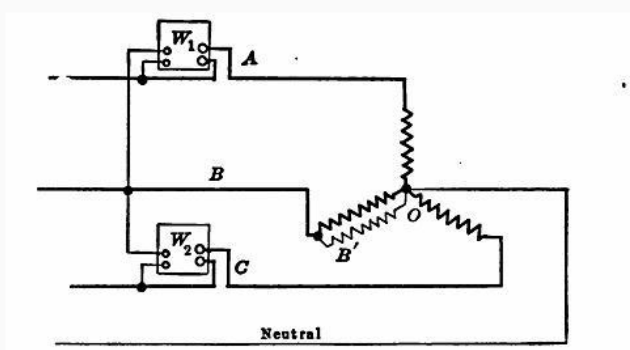


Fig. 14.6 Two-wattmeter method generally not applicable to a 4-wire system.

Poly phase Wattmeter:

Ordinarily, it requires two or more watt meters to measure the total power of a three phase circuit. If the load fluctuates, it is difficult to obtain accurate simultaneous readings of two watt meters. At power factors less than 0.5 in the three phase circuit, one of the wattmeters reverses its reading.

This necessitates reversing the connections of one of the instruments, which is often inconvenient. If both watt meters be combined in one, that is, if both moving coils are mounted on the same spindle (Fig. 14.7), the turning moments for each element add or subtract automatically, and the total power is read on a single scale.

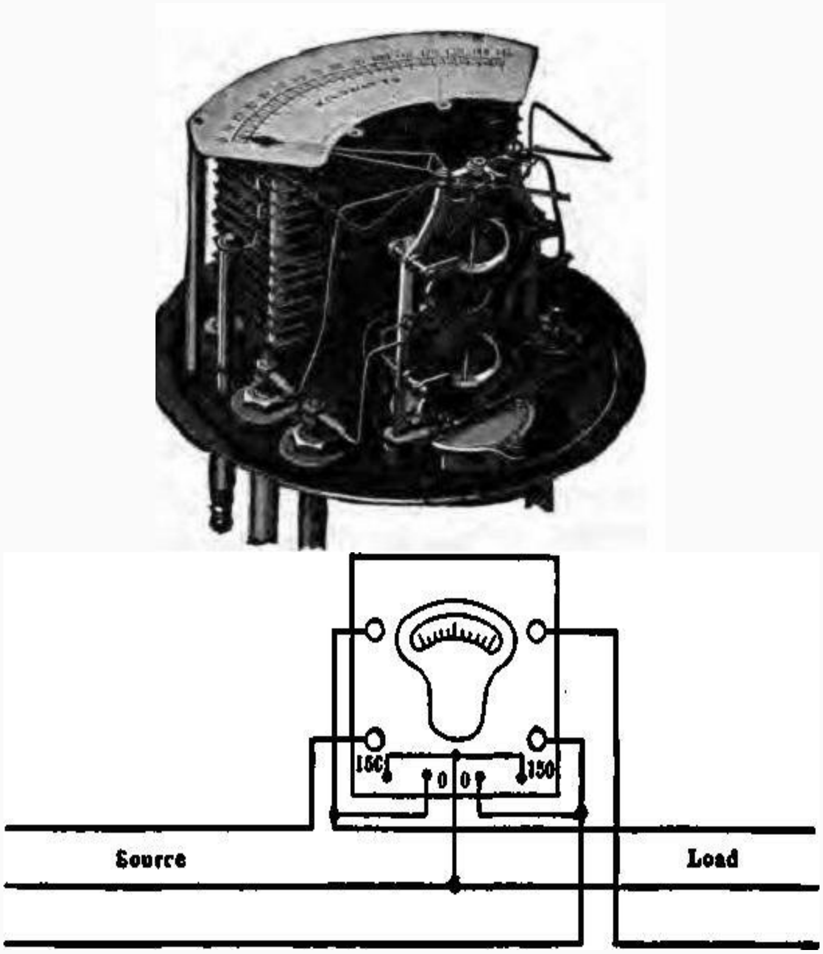


Fig. 14.7 Connections for poly phase wattmeter on 3-phase circuit