Machine Design

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Machine Design

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MODULE 1.

LESSON 1. INTRODUCTION TO MACHINE DESIGN

1.1 What is Design?

The word 'design' can either be a noun or a verb. As a noun, 'Design' can be referred to an object's aesthetic appearance as well as its function. For example, 'a beautifully designed dress' refers to its appeal to our visual perception, and 'a nicely designed car engine that gives very good mileage and performance' refers to the concept of function. As a verb, 'Design' is the act of formalizing an idea or concept into concrete information. It includes the processes of conception, invention, visualization, calculation, refinement and specification of details that determine the form of a product. Design may be defined as an iterative decision making process to formulate a plan by which resources are optimally converted into systems, processes or devices in order to solve a specific problem or to fulfill a specific need. It begins with a need, a problem, an idea or a concept and ends with concrete information, in the form of drawings, computer representation or in any other form, which helps in manufacturing and utilization of a product.

1.2 Phases of Design Process

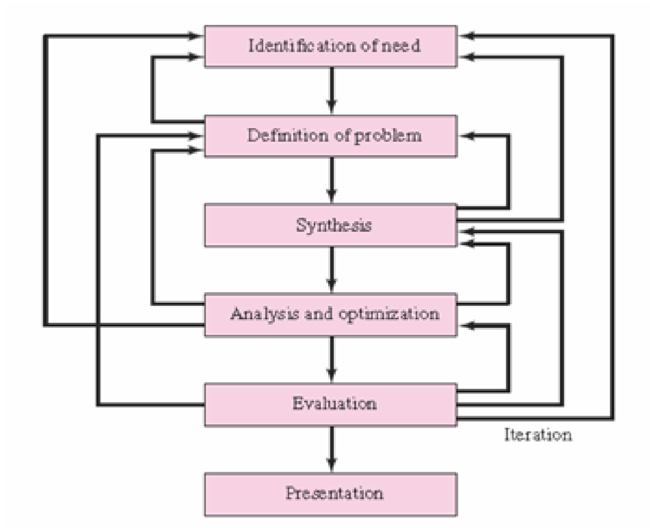


Figure 1.1 Phases of Design Process

The design process begins with identification of a need and a decision to do something about it. After a number of iterations, the process ends with the presentation of the plans for satisfying the need. Depending on the nature of the design task, several phases may be repeated before the design is finalized. Complete design process can be explained with the help of Figure1.1, which shows the phases of design process and iterations.

1.2.1 Identification of Need

Design process generally starts with the identification of need, a problem or with the recognition of a potential market for a product, device or process. Recognition of the need requires a lot of imagination and creativity, because the need is generally not clear and evident and can be vague. It can be just a feeling of uneasiness or a sensing that something is wrong. For example, there is a need to do something about the design of a motor vehicle may be indicated by its low efficiency, lesser maximum speed or high noise and vibration levels. Then the need statement may say that the design of the motor vehicle needs an improvement.

1.2.2 Definition of Problem

Need statement is generally brief and doesn't include the details. The definition of problem is more specific and includes all the specifications for the object to be designed, which include the input i.e. resources to be utilized, expected output, cost limitations, quantity required, expected life and reliability, temperature limitations, maximum range and expected variations in the variables, dimensional & weight limitations, manufacturing limitations etc. Anything that limits the designer's freedom of choice is a constraint and all the constraints should be clearly defined in the problem definition.

Background information on all the relevant aspects of the problem should also be gathered. It includes the study to find out whether this, or a similar problem, has been solved before. If a ready-made solution is available in the market, it can be used. Or even if something new is to be done, the study of existing similar technologies, products, relevant patent literature and technical publications can be very helpful.

1.2.3 Synthsis

Synthesis is the process of concept generation, the purpose of which is to develop as many ideas as possible to offer potential solutions to the problem defined in previous phase. Primary need is identified for initial focus and then all feasible design alternatives are developed. Different proposed alternatives should not be compared with each other, in this phase, and each alternative should be separately evaluated on the basis of established criteria.

1.2.4 Analysis and Optimization

After synthesis, different proposed solutions are analyzed to assess their performance and can be ranked. Alternatives having unsatisfactory performance can either be revised and improved or discarded. It is always advisable to select two or three design concepts and develop them instead of selecting presumably the best one only. Concepts, with potential, are optimized to further increase their performance. Optimization is the repetitive process of refining a set of criteria, which are often conflicting, to achieve the best compromise. These competing design concepts can finally be compared again so that the path leading to the most competitive product can be chosen. For analysis and optimization, mathematical models are prepared to simulate the real physical system.

Synthesis, analysis and optimization go hand in hand. Identification of any kind of deficiency or inadequacy in the proposed solution, in the analysis and optimization phase, may require synthesis of some new solution and process keeps on repeating until an adequate and optimum solution is obtained.

1.2.5 Evaluation

In this phase, prototype model of the design is generally made and tested, which is the final evaluation of the design to make sure that it satisfies the original requirements. In addition to this, design is evaluated for its reliability, competitiveness, economic viability, maintenance requirements, profitability and so on.

1.2.6 Presentation

Final phase of the design process is to communicate the design to others. This presentation can be in the form of drawings, computer models on in any other form that would help in manufacturing and utilization of the product.

1.3 Machine Design

Machine is a combination of linkages having definite motion and capable of performing useful work. Machine Design is creation of plans for the machine to perform the desired functions. Machine design can be defined as creation of right combination of correctly proportioned moving and stationary components so constructed and joined as to enable the liberation, transformation and utilization of energy. Scientific principles, technical knowledge and imagination are used to develop a machine or mechanical system to perform specific functions with maximum economy and efficiency. It includes the creation of new better machines or improving the existing ones. Machine Design requires the knowledge of basic and engineering sciences such as Physics, Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Thermodynamics and Heat Transfer, Vibrations, Fluid Mechanics, Metallurgy, Manufacturing Processes and Engineering Drawing.

In machine design, the designer’s task is to determine the motion, forces and energy transfer involved so as to determine the sizes, shapes and materials for each element of the machine. When any component of machine ceases to perform its intended function, machine element or machine is said to have failed. Generally the machine elements are designed on the basis of strength and rigidity so that they are able to withstand the applied load with permissible deformation or stress. In addition to strength and stiffness, other factors that are considered in design of machine element are weight, cost, wear, safety, reliability etc. Design work may involve concentrating on one component at a time, but it is very important to simultaneously consider its relation with the other components and the product as a whole.

In machine design, as in any other kind of design, no standard procedure or rigid rules can be specified for the designer, but the following steps are generally followed:

1. Problem to be solved or desired purpose of the machine is completely and clearly stated.
2. Possible mechanisms that will provide the desired motion or set of motions are selected.
3. Forces acting on and energy transmitted by each element of the machine are determined.
4. Best suitable material is selected for each of the machine element.
5. Allowable values of stress and deflection are determined for each machine element, depending upon its material and functional requirements.
6. Size and shape of each machine element is determined so that it can withstand the applied loads without failure.
7. Dimensions of the machine elements are modified considering manufacturing aspects.
8. Assembly and detailed drawings of the machine are made with complete specification of materials and manufacturing methods.

1.4 Design Considerations

As discussed in the previous article, strength and stiffness are very important factors that are considered in machine design but there are a number of other considerations that the designer has to keep in mind while designing any product. Some of the important factors considered in design are listed below:

1. Functionality
2. Strength
3. Stiffness
4. Wear
5. Corrosion
6. Thermal Properties
7. Surface Finish
8. Lubrication
9. Friction
10. Weight
11. Noise
12. Shape
13. Size
14. Safety
15. Reliability
16. Manufacturability
17. Utility
18. Maintenance

19. Liability
20. Cost
21. Life
22. Styling
23. Control
24. Volume
25. Marketability
26. Remanufacturing

LESSON 2. ENGINEERING MATERIALS

2.1 Selection of Material

Material selection is a very vital step in the process of machine design. Selection of material depends upon following aspects:

1. Performance Requirements: Material is selected considering the design constraints and performance requirements e.g. loads acting on the member, size & weight constraints, environmental conditions, desired reliability & durability etc.

2. Material Properties: Performance requirements are compared with properties of materials to select the best suitable material. For example stress level estimated for any machine member is compared with the strength of the available materials. Properties can be Physical (melting point, co-efficient of thermal expansion, thermal conductivity, specific heat, specific gravity, electrical conductivity, magnetic properties etc.), Chemical (corrosion resistance, reactivity with acids, bases, water etc.), Mechanical (hardness, toughness, ductility, malleability etc.) or Manufacturing (castability, weldability, formability, machineability etc.). Any of these properties can become important depending upon the design requirements and environmental conditions.

3. Manufacturing Aspects: Along with the selection of material designer also has to decide about the manufacturing processes to be used to give it desired shape. Therefore in addition to the manufacturing properties of the material, manufacturing constraints are also to be taken care of, while selecting a particular material.

4. Availability & Cost: Material selected should be easily available at an acceptable cost. In addition to the material cost, total cost of fabrication is also considered as the desired shape has to be given to the material with best quality and least cost.

Availability of a large number of materials with varying properties makes the job of material selection very difficult. Also due to dynamic nature of the market, cost keeps on varying, desiring the designer to remain updated about the available materials and their cost.

2.2 Stress-Strain Curve

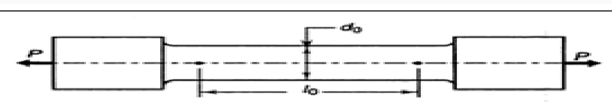


Figure 3.1 Standard Specimen of Tension Test

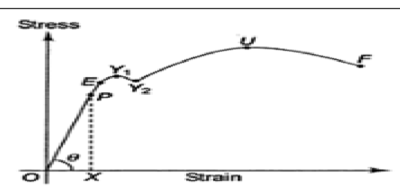


Figure 3.2 Stress-Strain Curves for Ductile Material

As discussed above, to design any component performance requirements are compared with the properties of the material. These properties are obtained experimentally. Tension test is the simplest and basic test that gives very important properties related to the mechanical behavior of the material. In this test, a standard specimen, shown in Figure 3.1, is subjected to gradually increasing axial tensile force. Before starting the test gauge length is marked on the specimen and initial diameter (d_0) and gauge length (l_0) are measured. Axial tensile load is applied on the specimen, which is increased gradually till the fracture of the component takes place. Load and deformation values are measured and stress (=Force/Area) & strain (=Deformation/Original Length) are calculated at each step. This data is then plotted in the form of stress-strain curve. Typical Stress-Strain curve for ductile material is shown in Figure 3.2. Stress-Strain curve provides the following information:

1. Proportional Limit (P): Stress-strain curve is linear upto point P and Hook's Law (Stress $\propto$ Strain) is obeyed in region OP. Proportional Limit (P) is the stress at which the stress-strain curve begins to deviate from the straight line.

2. Modulus of Elasticity: It is the ratio of stress to strain upto point P and is given by slope of line OP. $\text{YoungsModulus}, E = \tan\theta = PX/OX = stress/strain$.

3. Elastic Limit: Upto point E, when the load is removed, specimen regains its original size and shape i.e. it remains in the elastic stage upto point E (elastic limit). The elastic limit is the maximum stress without any permanent deformation. If the specimen is loaded beyond this point, plastic deformation takes place and the material takes a permanent set when the load is removed. Proportional Limit (P) and Elastic Limit (E) are very close to each other and are often taken to be equal.

4. Yield Strength (S_{yt}) : Point on the stress-strain curve at which the strain begins to increase very rapidly without a corresponding increase in stress is called yield point. Yield Strength is the maximum stress at which a marked increase in elongation occurs without increase in load. All materials don't have a well defined yield point. In such cases, Yield Strength is defined as the stress corresponding to a permanent set of 0.2% of gauge length and is determined with the help of offset method by drawing a line parallel to OP passing through A, with OA=0.002 mm/mm strain, which intersects with stress-strain curve at Y, called the Yield Point and corresponding stress is called 0.2% Yield Strength. Proof Strength is also similar to Yield Strength with offset of 0.001 mm/mm, called as 0.1% Proof Strength.

5. Ultimate Tensile Strength (S_{ut}) : As the material begins to deform plastically, it becomes stronger due to strain hardening and higher and higher load is required for its deformation, leading to increase in the stress and after point U, it begins to fall. Ultimate tensile strength is the maximum stress reached in the stress-strain curve, corresponding to point U.

6. Breaking Strength: After U, the cross-sectional area of specimen begins to decrease rapidly and a localized decrease in area called 'necking' takes place and ultimately the fracture takes place. F is called Fracture Point and corresponding stress is called Breaking Strengh.

7. Percentage Elongation: It is the ratio of increase in the gauge length of the specimen, at the time of fracture, to its original length, expressed in percent. It is a measure of ductility of the material and is given by, $$PercentageElongation = \left({l - {l_0}} \right)/{l_0} \times 100$$

8. Percentage Reduction in Area: It is the ratio of decrease in cross-sectional area of the specimen after fracture to the original cross-sectional area, expressed in percent. It also is a measure of ductility and is given by, $$PercentageReductioninArea = \left({{A_0} - A} \right)/{A_0} \times 100$$

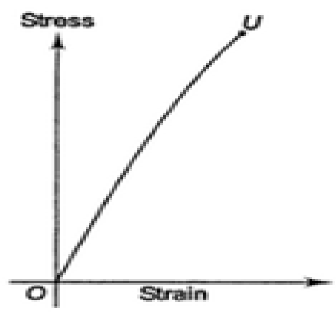


Figure 3.3 Stress-Strain Curves for Brittle Material

Brittle Materials don't exhibit Yield Point, deviation of stress-strain curve from straight line begins very early, there is very small plastic deformation, no necking occurs and fracture takes place suddenly. Stress-strain curve for a brittle material is shown in Figure 3.3.

2.3 Mechanical Properties of Materials

Mechanical properties of materials describe their behavior under the action of external forces and are very important in the determination of shape and size of the components. Following are some important mechanical properties:

Strength	Ability of the material to withstand external forces without yielding or fracture.
Stiffness	Ability of the material to resist deformation under the action of external forces.
Elasticity	Ability of the material to regain its original shape and size when the external load is removed.
Plasticity	Ability of the material to permanently retain the deformation produced due to external load. Ability to have large plastic deformation without fracture is very important property in certain operations like stamping.
Ductility	Ability of the material to have large plastic deformation without fracture when subjected to tensile force. It is measured by %age elongation and %age reduction in area. Ductility decreases with increase in temperature.
Malleability	Ability of the material to have large plastic deformation without fracture when subjected to compressive force. Malleability increases with increase in temperature.
Brittleness	Property of the material to show negligible plastic deformation before fracture.

Hardness	Ability of the material to resist penetration, plastic indentation, abrasion or scratching. Wear resistance increases with increase in hardness and processes like case hardening are used to increase to increase the hardness of surfaces rubbing against each other.
Resiliance	Ability of the material to absorb energy when deformed elastically and release this energy when unloaded. It is measured by Modulus of Resiliance which is strain energy per unit volume upto the elastic limit and is given by the area under the stress-strain curve, from origin to elastic limit.
Toughness	Ability of the material to absorb energy before the fracture takes place. Tough materials have ability to bend, twist or stretch before it gets fractured. It is an important property for components subjected to shock loads. It is measured by Modulus of Toughness which is the total work done or energy absorbed by the component upto the fracture point and is given by the area under the stress-strain curve upto the fracture point.

2.4 Engineering Materials

Engineering Materials can be classified as metals (ferrous & non-ferrous) and non-metals, which are discussed in the following articles:

2.4.1 Ferrous Materials

Ferrous materials can be classified into Wrought Iron, Cast Iron and Steel.

2.4.1.1 Wrought Iron and Cast Iron

Wrought Iron and Cast Iron along with its types are discussed in the table given below

	Introduction	Properties	Applications
Wrought Iron	Purest form of iron with more than 99.5% Fe	Tough, malleable, ductile, weldable, forgeable & corrosion resistant Poor castability, high melting point (1510°C)low impact strength, cannot be hardened or tempered	Bolts, nuts, railway couplings, chains, crain hooks, oil rigs, pipes, pipe fittings, plates, sheets etc.
Cast Iron	An alloy of Fe & C with C > 2%, other ingradients – Si, Mn, S, Ph etc, hard and brittle	Low cost, good castability & machinability, high compressive strength, wear resistant, good vibration damping capacity Very brittle, low plasticity, low malleability, cannot be forged	Automobile engine blocks, machine tool structures
Grey Cast Iron	Contains 2.5 to 3.75 % C present in the form of graphite flakes, giving gray color and hence its name. Examples: FG150, FG250, FG350Si12 (no. indicates UTS,	Low cost, good castability & machinability, high compressive strength, graphite acts as lubricant making it suitable for sliding parts Low tensile and impact	Machine tool structures, gas/ water pipeselectric motor frames, piston rings, flywheels, cylinder block, heads, housings

	Si12 means 12% Si)	strength, less ductility, poor weldability	
White Cast Iron	Contains 1.75 to 2.30% C present in the form of cementite (Fe_3C)	Very hard and brittle, good abrasion resistance, poor mechanical properties, low machinability	Rail/car wheels, valve seats, cams, small pulleys, rollers, gears
Malleable Cast Iron	Obtained by annealing of white cast iron, contains 2.2 to 3.6% C. Examples: BM 300 WM200 PM400 BM, WM & PM indicate Black hearth, White hearth & Pearlitic Malleable CI resp. numbers indicate UTS	Low cost, malleable, ductile, forgeable, good wear resistance, impact strength and vibration damping capaity	Crank case, pump bodies, conveyer chain links, crankshafts, levers etc.
Spheroidal Cast Iron	Also known as nodular or ductile CI, C (graphite) is present in the spheroidal/nodular form	Stronger, more ductile, tougher, good fluidity, castability, machinabilty, weldability and wear resistance	Cylinders, cylinder heads, valves, pipes, pipe fittings, power transmission equipment, earth moving machinery
Alloy Cast Iron	Improved properties by adding alloying elements like Ni, Cr, Mo, Cu, Si, Mn etc.	Increased strength, high wear and corrosion resistance	Automobile parts like cylinders, pistons, piston rings, crank case, brake drums, crushing and grinding machine parts

2.4.1.2 Steel

It is an alloy of Fe and C with C < 1.5%. C is present in the form of iron carbide (Fe_3C), which imparts hardness and strength. No free carbon (graphite) is present. Steel is used for most of the engineering applications. Its properties can be modified using heat treatment. It can be classified as Plain Carbon Steels and Alloy Steels.

Plain Carbon Steel: It contains 0.5 to 1.0 % of C. It is cheap, easily available, has wide range of mechanical properties that can be controlled with the help of heat treatment and alloying elements, has good machinability and weldability. It can be classified as low, medium and high carbon steel.

Type	Carbon %age	Properties	Applications
Low Carbon Steel (Mild Steel)	< 0.3 %	Very soft and ductile, good machinability and weldability	Small forgings, machined, welded and cold formed parts
Medium Carbon Steel	0.3 to 0.7 %	High strength , good weldability	Most machine components
High Carbon Steel	> 0.7 %	High yield strength, tough, hard and brittle, low weldability	Cutting tools, springs, bearings

Alloy Steels: When certain alloying elements are added in sufficient quantity to impart some desired property, these are called alloy steels. For example, Ni provides hardness, strength & toughness without compromising ductility, Cr provides high hardness, strength, wear & corrosion resistance, Mo & W increases hardenabilty & wear resistance, V improves fatigue resistance and so on. Some examples of alloy steels are 40Cr1Mo28, 40Ni3, 37Mn2, 31Ni3Cr65Mo55.Table gives the list of some alloying elements along with the properties they impart.

2.4.2 Non-ferrous Materials

A variety of non-ferrous materials are also used in engineering applications. They are soft, have low melting point, low strength, high corrosion resistance, can be cold worked and have good manufacturing properties. Some important non-ferrous materials are Al Alloys (Duralumin, Y alloy, Magnalium, Hindalium), Cu Alloys (Brasses, Bronzes, Gun metal, Babbits), Ni Alloys (Monel Metal, Inconel, Nichrome, Nimonics) etc.

2.4.3 Non-Metallic Materials

Non-metals have low cost, flexibility and resistance to heat & electricity. Examples are timber, leather, rubber, plastics etc.

MODULE 2.

LESSON 3. DESIGN FOR STATIC LOADING

3.1 Load & Its Determination

All the machine members are subjected to different types of loads that may be acting because of energy, torque or power transmission, their self weight, frictional resistance, inertia or centrifugal forces or due to temperature gradient. Load may be classified as static or dynamic.

Static load is the load which does not change in magnitude or direction and gradually increases to a steady value e.g. dead weight of machine elements. Dynamic load is the load which changes in magnitude or direction or both with respect to time e.g. load acting on the connecting rod of an internal combustion engine. Impact load (load applied with certain velocity) and shock load (suddenly applied load) are also types of dynamic load.

Determination of appropriate loads acting on a machine member is a critical and challenging task. All the stress and deflection analysis is useless and the component cannot function satisfactorily if the operating loads are not calculated or predicted correctly. Sometimes the operating loads are easily determinable e.g. load on a shaft running at known speed and transmitting a known value of torque. But often the loads are difficult to determine e.g. the load on vehicle chassis which depends on road condition and driving practices. Loads acting on a machine member may be directly known or may have to be calculated using basic concepts of engineering mechanics etc. Sometimes experimental methods are used to obtain a statistical definition of the load. Also sometimes the service loads are estimated with the help of record of service failures and strength analysis. After the determination or estimation of applied load, load acting on different members of the machine are determined with the help of free body diagrams and basic equilibrium equations of forces and moments.

3.2 Failure Criteria

A machine element is said to have failed when it ceases to perform its intended function. It may happen if its stress or deflection crosses the acceptable limit. Excessive deformation of a particular element may lead to unwanted interference between the machine elements or jamming of the machine and therefore deformation is considered as a failure criterion. Same way, excessive stresses may result in yielding or fracture of a machine element making it unable to perform its desired function. When stress developed in a ductile material reaches the yield strength, it starts yielding and excessive plastic deformation occurs, therefore Yield strength is taken as failure criterion for ductile materials. In brittle material, very small plastic deformation occurs and fracture takes place once the stress developed reaches Ultimate Tensile Strength. Therefore Ultimate Tensile Strength is considered as failure criterion for brittle materials. Bearing Pressure (for components rubbing against each other with appreciable relative velocity e.g. bearings, clutches, brakes etc.) and wear (for components having sliding or rolling motion e.g. gears, bearings, bushes, piston-cylinders etc.) are examples of other failure criteria.

3.3 Factor of Safety & Allowable Stresses

The factor of safety is a measure of reserve strength provided to take care of any unexpected or unpredicted conditions that may arise due to uncertainties in the properties of the material, magnitude & direction of the load and operating conditions. Value of factor of safety thus depends upon effect of failure (level of severity, cost & danger involved) , type of load (static or dynamic), accuracy in load calculations, material selected (ductile, brittle, homogeneity), desired reliability, service conditions (normal, corrosive, temperature level), manufacturing quality (variation in desired dimensions, quality) and cost etc. Depending upon the criteria of failure decided, strength of the material is divided by factor of safety to obtain allowable stress or design stress which is then used to determine the dimensions of the components as discussed in the next article.

Allowable stress,
$$[\sigma] = S_y/fos \text{ for ductile materials}$$
$$= S_{ut}/fos \text{ for brittle materials}$$

where, \[{S_y}\] is yield strength, \[{S_{ut}}\] is ultimate tensile strength and \[fos\] is factor of safety.

3.4 Design for Simple Stresses

When a mechanical component is subjected to an external load, a resisting force is set up within the component. This resisting force per unit area of the component is called stress. The maximum stress developed in a member should not exceed the allowable value as obtained from the material strength considering certain value of factor of safety i.e any stress 'σ ' should always be ≤ [σ] . Limiting values of dimensions desired can be calculated by equating σ and [σ]. Equation σ = [σ] is called design equation and its use for simple stresses is discussed here.

3.4.1 Direct Tensile & Compressive Stress

When the fibers of the component tend to elongate under the external load, stress developed in the component is called tensile stress. On the other hand, when the fibers tend to shorten under the external load, stress developed in the component is called compressive stress.

Tensile stress is given by, $\sigma_t = \dfrac{P}{A} \le [\sigma]$ Compressive stress is given by, $\sigma_c = \dfrac{P}{A} \le [\sigma_c]$

where, P is external load, A is cross-sectional area of the component and [σ] and $[\sigma_c]$ are allowable tensile and compressive stress of the material. From P / A = [σ] or P/A = $[\sigma_c]$, minimum cross-sectional area required to withstand a known load, P can be determined for given allowable stresses.

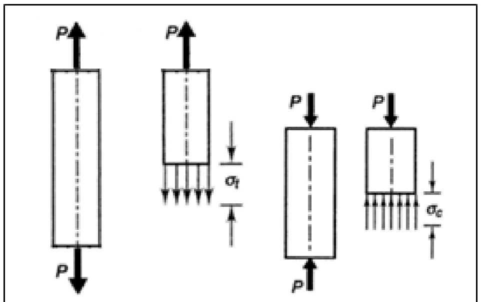


Figure 3.1 Tensile & Compressive Stresses

Figure 3.1 shows tensile and compressive stress developed in members subjected to load P.

Tensile or compressive strain is the deformation per unit length and is given by,

$$\varepsilon = \frac{change\ in\ length}{original\ length} = \frac{\delta l}{l}$$

According to Hook's Law, within the elastic limit, stress is directly proportional to strain. Therefore, $\sigma_t \propto \varepsilon$ or $\sigma_t = E\varepsilon$

where, constant of proportionality E is known as Young's Modulus or Modulus of Elasticity.

E = 207000 N/mm^2 for Carbon Steels, 100000 N/mm^2 for Grey Cast Iron

3.4.2 Direct Shear Stress

When the external load acting on the component tends to slide the adjacent planes with respect to each other, the resulting stresses on these planes are called direct shear stresses.

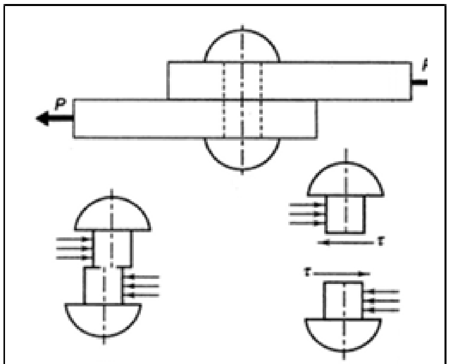


Figure 3.2 Shear Stress developed in a Rivet

Figure 3.2 shows two plates joined together with the help of a rivet and subjected to load P. In this case the rivet is subjected to direct shear stress. Average shear stress is given by,

$$\tau = P/A \leq [\tau]$$

where, P is external load, A is cross-sectional area of the component and \[\left[\tau\right]\] is allowable shear stress.

Shear strain is defined as the change in right angle of a shear element.

Within the elastic limit, $\tau= G\gamma$, where, γ is Shear Strain and G is Modulus of Rigidity

G = 80000 N/mm^2 for Carbon Steels, 40000 N/mm^2 for Grey Cast Iron.

The relation between modulus of elasticity, modulus of rigidity and poisson's ratio (μ) is given by, $E = 2G\left({1 + \mu } \right)$

3.4.3 Bending Stress

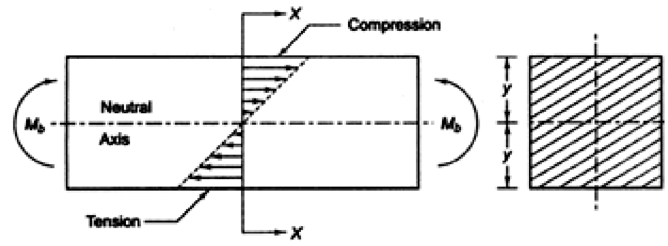


Figure 3.3 Bending Stress Distribution

When a machine member is subjected to bending moment, tensile stress develops on one side of the neutral axis and compressive stress on the other. Therefore, the outside fibers are in tension and the inside fibers are in compression. The bending stress at any fiber is given by,

$$\sigma_b = My/I \leq [\sigma]$$

where, M is Applied bending moment, I is Moment of inertia of the cross-section about the neutral axis and y is the distance of the fiber from the neutral axis. Distribution of bending stress is linear as shown in Figure 3.3. Stress is proportional to the distance of the fiber from neutral axis and is maximum in the farthest fiber.

3.4.4 Torsional Shear Stress

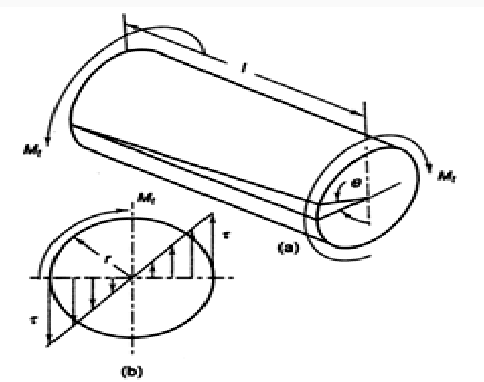


Figure 3.4 a) Shaft subjected to Torque

b) Torsional Shear Stress Distribution

Figure 3.4a shows a shaft subjected to torque. Stress induced in a machine member to resist the action of twist is called torsional shear stress. It is given by, $\tau = Tr/J \leq [\tau]$, where, T is applied torque, r is radial distance of the fibre from the axis of rotation and J is Polar moment of inertia of the shaft about the axis of rotation. Distribution of torsional shear stress is shown in Figure 3.4b. Stress is maximum at the outer fiber and zero at the axis of rotation. Angle of twist for a given value of applied torque, T and length of shaft, l can be calculated using the relation, $T/J = G\theta /l$, where, θ is angle of twist (radians) and G is Modulus of rigidity.

3.4.5 Bearing or Crushing Stress

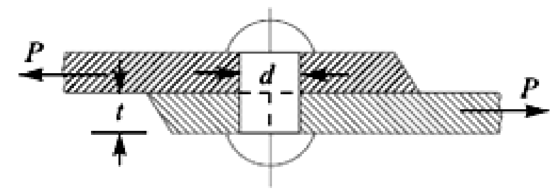


Figure 3.5 Crushing Stress in Riveted Joint

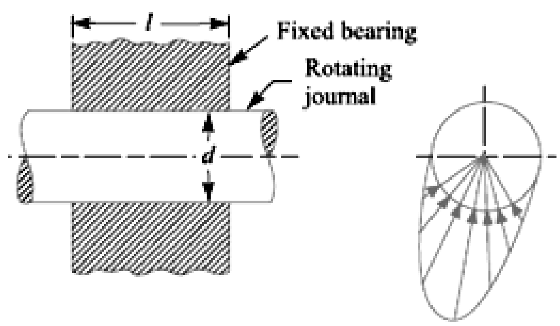


Figure 3.6 Bearing Pressure in Journal

Crushing means to press or squeeze with a force that destroys or deforms or to squeeze into small fragments. Crushing or Bearing stress is defined as the compressive stress developed at the surface of contact between two interacting members that are relatively at rest. Crushing stress is assumed to act uniformly on the projected area. Consider a riveted joint as shown in figure 3.5. If d is diameter of the rivet and tis thickness of the plate, crushing stress is given

by, $\sigma_{crushing} = \frac{load}{projected\ area} = \frac{P}{d\ t} \leq [\sigma_c]$

If n is the total number of rivets used, total projected area will become n.d.t.

Also, the local compression that exists at the surface of contact between two members that are in relative motion is called bearing pressure. For example, bearing pressure exists between the contact surfaces of a journal rotating in a fixed bearing as shown in figure 3.6. For a journal of diameter, d and contact length, l, bearing pressure is given by,

$$P_{bearing} = load/(projected\ area) = P/(d\ l)$$

3.4.6 Thermal Stresses

Materials expand with increase in temperature and contract with decrease in the temperature. Stresses develop in a component, if it is prevented from freely expanding or contracting under the effect of temperature change. Theses stresses are called thermal stresses. Change in length of any machine member as a result of temperature change is given by, $$\delta l = l\alpha t$$

where, l is original length of the member, a is Coefficient of thermal expansion and t is rise or fall of temperature. If this change in length is prevented i.e. the member is not allowed to freely expand or contract, strain induced in the body is given by,

Thermal Strain, $\varepsilon_{th} = \frac{\delta l}{l} = \frac{l\,\alpha\,t}{l} = \alpha\,t$ and Thermal Stress, $\sigma_{th} = \varepsilon_{th}\,E = \alpha\,t\,E \leq [\sigma]$

where, E = Modulus of Elasticity of material of the member

LESSON 4. DESIGN FOR COMBINED LOADING & THEORIES OF FAILURE

4.1 Combined Loading & Principal Stresses

When a machine component is subjected to only axial load, bending moment or torque, uniaxial state of stress is produced, which was discussed in the previous lesson. But in actual practice, the components are mostly subjected to combination of loads e.g. transmission shaft is subjected to bending and torsion at the same time. Combined loading leads to complex state of stress. Infinite number of stress vectors act at any point inside the member subjected to combined loads as infinite number of planes can pass through a point. Each stress vector is characterized by the corresponding plane on which it is acting. State of stress at a point is the totality of all stress vectors acting on it. For design purpose, it is very important to know the state of stress so as to determine the critical planes, the respective critical stresses and relate them to the strength of the material.

If the stress vectors acting on three mutually perpendicular planes passing through the point are known, stress vector acting on any other arbitrary plane at that point can be determined. Let us consider three mutually perpendicular planes (x-plane, y-plane and z-plane) passing through a point as shown in figure 4.1a. Normal and shear stress components acting on these planes are:

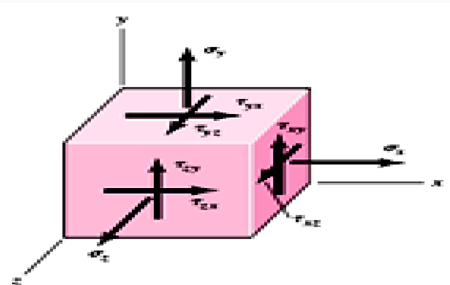


Figure 4.1 a. State of Stress at a Point

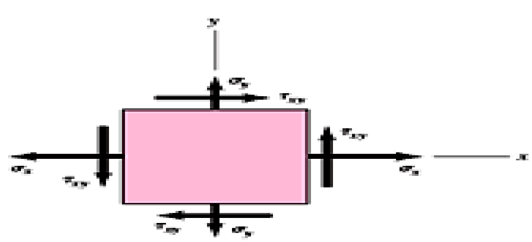


Figure 4.1 b. State of Plane Stress

Plane	Normal Stress Component	Shear Stress Components
x-plane	s_x	τ_{xy}, τ_{xz}
y-plane	s_y	τ_{yx}, τ_{yz}
z-plane	s_z	τ_{zx}, τ_{zy}

For equilibrium, in most cases, cross shears are equal i.e. $\tau_{xy} = \tau_{yx}$, $\tau_{yz} = \tau_{zy}$ and $\tau_{xz} = \tau_{zx}$. Therefore, six components of stress are required to completely define thestate of stress. State of stress, when the stresses on one surface are zero, is called plane stress. Figure 4.1b shows the state of plane stress, with normal and shear stress components on the z-plane to be zero ($s_z = \tau_{zx} = \tau_{zy} = 0$). Stress acting on any oblique plane, whose normal makes an angle \[\theta \] with the x-axis, for state of plane stress, can be determined with the help of figure 4.2a. Considering equilibrium of forces, normal (σ) and shear stress (\[\tau \]) components acting on this arbitrary oblique plane are given by,

$$\sigma = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right) cos2\theta + \tau_{xy}\, sin2\theta \quad and \quad \tau = -\left(\frac{\sigma_x - \sigma_y}{2}\right) sin2\theta + \tau_{xy}\, cos2\theta$$

From strength consideration, it is important to find the plane of maximum normal stress and plane of maximum shear stress and their magnitudes. Differentiating the expression for normal stress (s) with respect to q and equating it to zero, we get:

$$tan2\theta = \frac{2\tau_{xy}}{\sigma_{x-}\sigma_y}$$

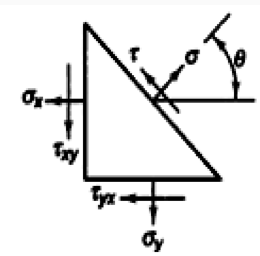


Figure 4.2 a. State of Stress at a Point

This gives two values of \[\theta \] i.e. two planes, which are perpendicular to each other. One has maximum value of normal stress and the other has minimum value. These two planes are called principal planes as the shear stress component along these planes is zero. Normal stress components on these planes are called Principal Stresses and are given by,

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2} \quad and \quad \sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

Differentiating the expression for shear stress (\[\tau \]) with respect to q and equating it to zero, we get

$$tan2\theta = -\left(\frac{\sigma_{x-}\sigma_y}{2\tau_{xy}}\right)$$

This also gives two values of \[\theta \] i.e. two mutually perpendicular planes, which make an angle of 45° with the principal planes. Shear stress component along these planes, also called principal shear stress is given by,

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

State of stress at any point or maximum principal stresses and maximum principal shear

stress at any point can be determined from the abov

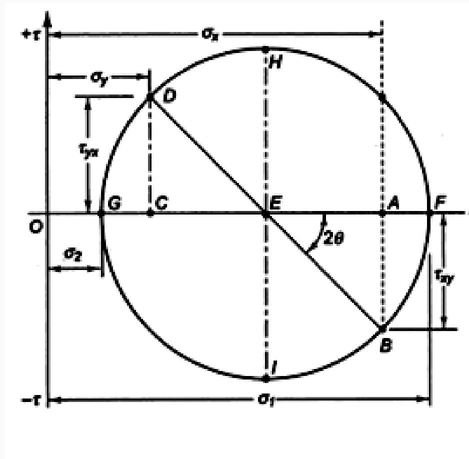


Figure 4.2 b. Mohr's Circle

e relations or graphically with the help of Mohr's Circle as shown in figure 4.2b. Normal stresses are plotted along abscissa and shear stresses are plotted along the ordinate. Tensile normal and clockwise shear stresses are considered to be positive and compressive normal and anticlockwise shear stresses are considered to be negative. Mohr's Circle can be constructed and used to find maximum principal stresses and maximum principal shear stress as given below:

1. Draw OA = σ_x, OC = σ_y, AB = τ_{xy} and CD = τ_{yx}.
2. Join BD to get point E, intersection of AC and BD.
3. Construct Mohr's Circle with E as centre and BD as diameter.
4. OF gives maximum principal stress σ_1 and OG represents minimum principal stress σ_2.
5. EH and EI give the maximum principal shear stress $\pm\tau_{max}$.

4.2 Theories of Failure

In the previous article, it has been discussed that how the state of stress can be determined for any point in a component subjected to combined loads and how to get the value of maximum principal stresses and maximum principal shear stress developed in the component. Now for designing components subjected to combined load, it is important to relate this complex state of stress to the properties of material (yield strength, ultimate tensile strength, percentage elongation etc.), which are obtained from the simple tension test, so that the failure in such conditions can be predicted. This relationship is provided by 'Theories of Elastic Failure', which are discussed below.

4.2.1 Maximum Principal Stress Theory

This theory is credited to W.A. J. Rankine. It states that failure of any mechanical component subjected to complex state of stress occurs whenever

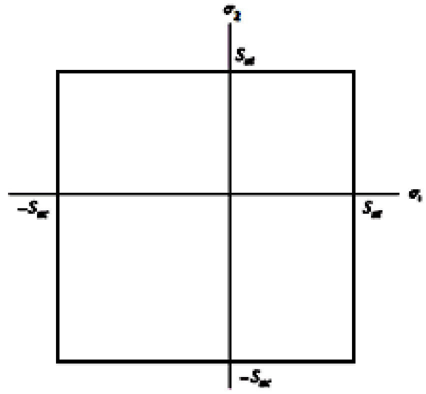


Figure 4.3 Failure Envelop based on Maximum Principal Stress Theory

one of the three principal stresses equals or exceeds the strength i.e. whenever maximum tensile stress exceeds the uniaxial tensile strength or maximum compressive stress exceeds the uniaxial compressive strength. If σ_1,σ_2 and σ_3 are the three principal stresses with

$\sigma_1 \geq \sigma_2 \geq \sigma_3$, theory says that to avoid failure,

$$\sigma_1 \leq S_{ut} \text{ and } \sigma_3 \geq -S_{uc}$$

Therefore, design equations based on maximum principal stress theory can be written as:

$$\sigma_1 = \frac{S_{ut}}{fos} \text{ and } \sigma_3 = -\frac{S_{uc}}{fos}$$

Figure 4.3 shows the failure envelop based on maximum principal stress theory, for plane stress case ($\sigma_2 = 0$) . Area is bounded by lines ${\sigma _1} = {S_{ut}},{\sigma _1} = - {S_{uc}},{\sigma _2} = {S_{ut}} \text{ and } {\sigma _2} = - {S_{uc}}$. Test data shows that this theory is suitable for predicting failure of brittle materials and is not recommended for ductile materials.

4.2.2 Maximum Shear Stress Theory

This theory is credited to C.A. Coulumb, H Tresca and J.J. Guest. It states that any mechanical component subjected to any combination of loads will fail whenever maximum shear stress exceeds shear strength of the material (i.e. shear stress at the time of yielding in the standard specimen of tension test). Figure 4.4 shows Mohr's Circle for 3-dimensional stress with σ_1,σ_2 and σ_3 as principal stresses such that $\sigma_1 \geq \sigma_2 \geq \sigma_3$. Three principal shear

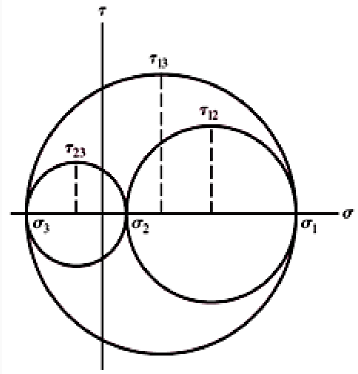


Figure 4.4 Mohr's Circle for 3-D Stress

stresses are then given by,

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2}, \tau_{23} = \frac{\sigma_2 - \sigma_3}{2} \text{ and } \tau_{13} = \frac{\sigma_1 - \sigma_3}{2}$$

As evident from the figure 4.4, maximum shear stress \[{\tau _{max}} = {\tau _{13}} = \left({{\sigma _1} - {\sigma _3}} \right)/2\] .

$$\tau_{max} = \tau_{13} = (\sigma_1 - \sigma_3)/2$$

Now, for simple tension test, $\sigma_1 = \sigma_x$ & $\sigma_2 = \sigma_3 = 0$, giving $\tau_{max} = \sigma_1/2$. Also yielding in simple tension test starts when $\sigma_1 = S_{yt}$. Therefore, maximum shear stress at the time of yielding in simple tension test is \[{\tau _{max}}\] = S_{yt} / 2. Thus, design equation based on maximum shear stress theory can be written as:

$$\tau_{max} = \frac{S_{yt}}{2\,.\,fos} \quad or \quad \sigma_1 - \sigma_3 = \frac{S_{yt}}{fos}$$

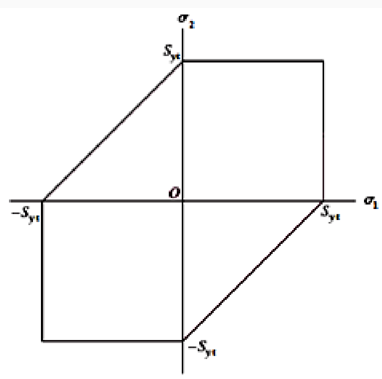


Figure 4.5 Failure Envelop based on Maximum Shear Stress Theory

Note that for plane stress case, $\sigma_3 = 0$ and. $\tau_{max} = (\sigma_1 - \sigma_2)/2.$

Graphical representation of maximum shear stress theory, giving failure envelop for state of plane stress (σ_3=0), is shown in Figure 4.5. According to this theory, for state of plane stress, yielding starts when $\sigma_1 - \sigma_2 = S_{yt}$. But in 1st and 3rd quadrant of this ($\sigma_1 - \sigma_2$) plot, σ_1 and σ_2 are of same nature with ($\sigma_1 - \sigma_2$) > σ_2 and yielding may start when reaches the yield strength, S_{yt} . Therefore, in 1st and 3rd quadrant, area is bounded by lines $\sigma_1 = \pm S_{yt}$ and $\sigma_2 = \pm S_{yt}$. Whereas in

2nd and 4th quadrant, area is bounded by lines, which represent the condition where maximum shear stress reaches the shear strength of the material i.e. \[{\tau _{max}}\] = S_{sy} = $S_{yt}/2$. This theory is suitable for predicting failure of ductile materials but is a little conservative.

4.2.3 Maximum Distortion Energy Theory

This theory is credited to M.T. Hueber, R. von-Mises and H. Hencky. It states that a mechanical component subjected to combined loads fails when the distortion strain energy per unit volume at any point in the component reaches or exceeds the distortion strain energy per unit volume of standard specimen of simple tension test, at the time of yielding.

This theory gives a value of equivalent stress, called von-Mises Stress, which is defined as the value of uniaxial tensile stress that would produce the same level of distortion energy as the actual stress involved. It is given by,

$$\sigma_{vonM} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

To avoid failure of the component, this von-Mises stress should not exceed yield strength of the material. Therefore design equation based on maximum distortion energy theory can be written as,

$$\sqrt{\left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}\right]} = \frac{S_{yt}}{fos}$$

Figure 4.6 shows the failure envelop, based on maximum distortion energy theory, for state of plane stress. Experimental results have proved that this theory is the best suitable for predicting failure of ductile materials. It can be proved that according to maximum distortion energy theory, yield strength in shear is 0.577 times yield strength in tension i.e. S_{ys} = 0.577 S_{yt}.

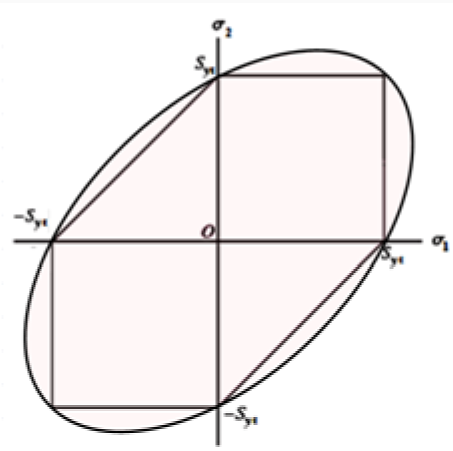


Figure 4.6 Failure Envelop based on Maximum Distortion Energy Theory

LESSON 5. STRESS CONCENTRATION AND CREEP

5.1 Stress Concentration

The basic stress equations for tension, compression, bending, and torsion are based on a number of assumptions. One of the assumptions is that there are no geometric irregularities or abrupt change in the cross-section of the member. But these irregularities and changes in the cross-section of members are unavoidable. There will be holes, oil grooves, notches, keyways, splines, screw threads etc.

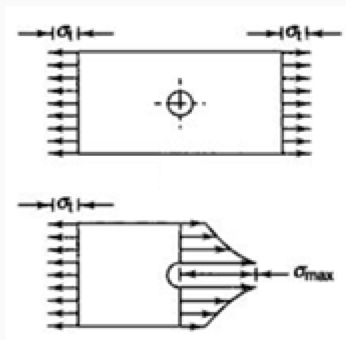


Figure 5.1 Stress Concentration

Any of the discontinuities in a machine part alters the distribution of stress in its vicinity and localized stress much higher than those calculated with the elementary stress equations are observed. This localization of high stresses due to geometrical irregularities or abrupt changes of the cross-section is called 'stress concentration' and the discontinuities are called stress raisers. Stress concentrations can also arise from some irregularity. Stress distribution in a plate with a small circular hole, subjected to tensile load, is shown in figure 5.1. In addition to the abrupt changes in the cross-section, other causes of stress concentration can be variation in material properties e.g. internal cracks, flaws, air holes, foreign inclusions etc. and surface irregularities like scratches or stamp marks.

To consider the effect of stress concentration, stress concentration factor is used, which is the ratio of maximum stress to nominal stress.

Stress concentration factor for normal stress, Stress concentration factor for shear stress,

$$K_t = \frac{\sigma_{max}}{\sigma_0} \qquad\qquad K_{ts} = \frac{\tau_{max}}{\tau_0}$$

Where, σ_{max} , τ_{max} = localized stresses near the discontinuities

σ_0, τ_0 = Nominal stresses as determined by elementary equations for minimum cross-section

For the plate shown in figure 5.1, if 'w' and 't' are width and thickness of the plate, 'd' is diameter of the hole and 'P' is the applied load, nominal tensile stress is given by,

$$\sigma_0 = \frac{P}{(w-d)t}$$

Value of stress-concentration factor depends on the geometry of the part only and is independent of the material used. For this reason, it is called theoretical or geometric stress-concentration factor. Stress-concentration factors for different geometric shapes are found by using experimental techniques like photo-elasticity, grid methods, brittle-coating methods, and electrical strain-gauge methods. The finite-element method has also been used. Theoretical stress concentration factors for different configurations are available in handbooks, few of which are shown in figures below [charts for stress concentration factors are to be provided here].

5.2 Methods to Reduce Stress Concentration

Effect of stress concentration cannot be completely eliminated but its effect can be reduced by slightly altering the geometry of the components. Flow analogy is helpful in understanding how a particular discontinuity affects the stress distribution around it and how its effect can be reduced. Figure 5.2 shows the stress distribution in an axially loaded plate which is similar to the velocity distribution in fluid flow in a channel. For a channel having uniform cross-section, velocities are uniform and streamlines are equally spaced. If the cross-section of the channel is suddenly reduced, velocity increases to maintain same flow and the stream lines become narrower. Similarly, with reduction in cross-section, to transmit same force, the stress lines come closer. Location where the cross-section changes, stress lines bend as the stream lines do. Sudden change in the cross-section leads to very sharp bending of stress lines which results in stress concentration. Therefore by avoiding severe bending of the stress lines, effect of stress concentration can be reduced. Figure 5.3 shows certain methods to reduce stress concentration.

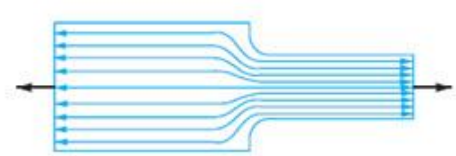


Figure 5.2 Stress Distributions – Flow Analogy

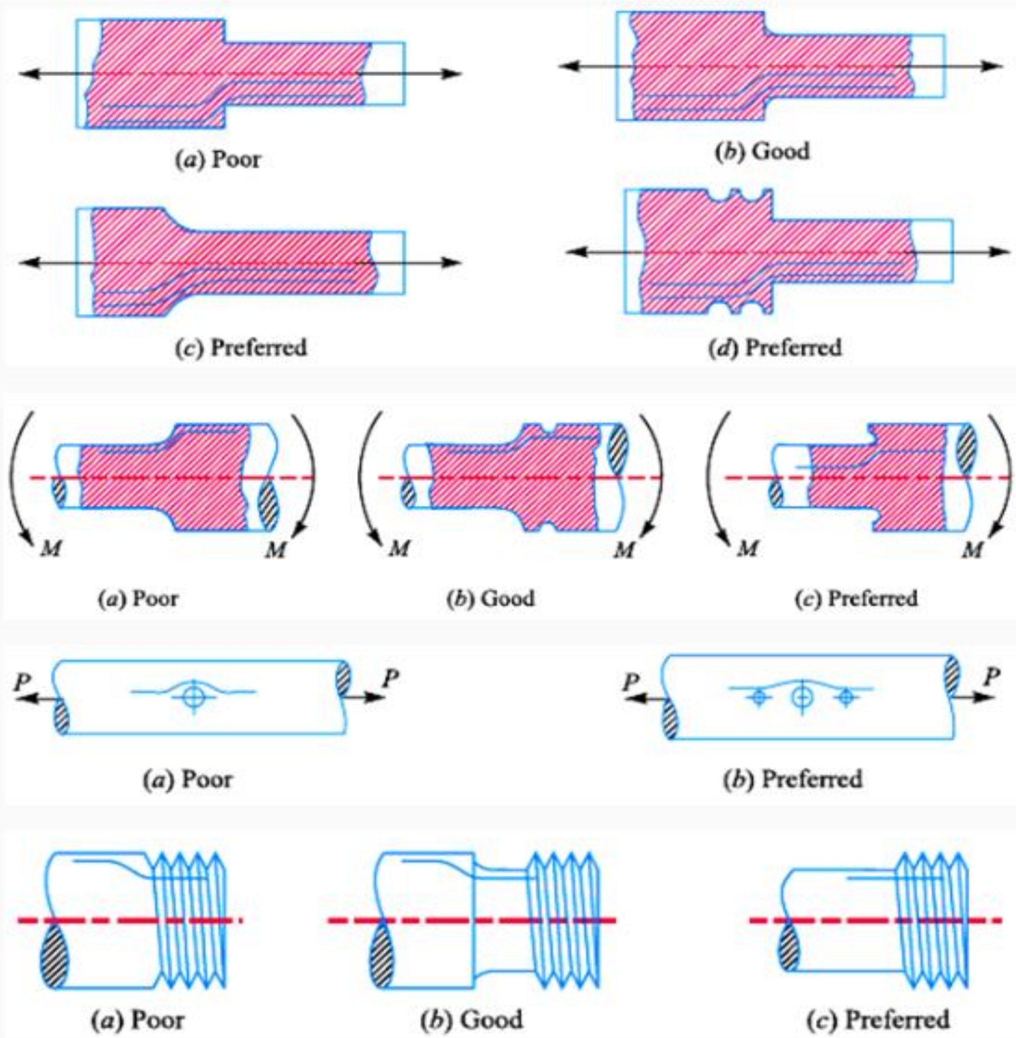


Figure 5.3 Methods to Reduce Stress Concentration

5.3 Effect of Ductility & Brittleness on Stress Concentration

Stress concentration has negligible effect on the ductile materials subjected to static loads. Under the static load, when the stress near the discontinuity reaches the yield point, local plastic deformation takes place and the stresses get redistributed, relieving the stress concentration. Therefore, ductile materials, subjected to static loads, are not affected by stress concentration and there is no need to apply stress concentration factor to statically loaded ductile materials.

But for the ductile materials, subjected to dynamic load, stress at the discontinuity may reach its endurance limit leading to fatigue failure. Therefore, stress concentration reduces the endurance limit of ductile materials and stress concentration factor must be used for dynamically loaded ductile materials. This aspect will be discussed in the next lesson.

Stress concentration has more severe effect on the brittle materials due to their inability to plastically deform. As there is no local yielding, stresses don't get redistributed and local stress due to discontinuity increases highly. Therefore, stress concentration factor must be used for components made of brittle materials subjected to static or dynamic loads.

5.4 Creep

When components are subjected to constant loads continuously for a longer period, they may undergo progressive elastic deformation over time. This time-dependent strain is called creep. Creep can be defined as slow and progressive deformation of material with time under constant stress. Creep deformation is a function of stress and temperature. It generally occurs at absolute temperatures above half the melting point of the material and is even present at room temperature for materials like aluminium, copper and some plastics. Creep increases with the increase in temperature, and therefore becomes very important for components operating at higher temperatures like those of furnaces, steam & gas turbines, internal combustion engines, nuclear reactors, rocket engines etc. The stress can be tensile, compressive, bending or shear.

Components operating at higher temperatures must be designed in such a way that deformation due to creep remains within the allowable limit and also creep deformation doesn't lead to rupture so that the product performs satisfactorily over time. Creep strength and creep rupture strength are the two important properties of the material related to creep design.

Creep Strength is the maximum stress that the material can withstand for a specified length of time without excessive deformation. Creep Rupture Strength is the maximum stress that the material can withstand for a specified length of time without rupture.

Figure 5.4 shows a typical strain vs time plot known as creep curve. To obtain creep curve, specimen is loaded with a constant force and variation of its length is observed at a constant elevated temperature. Units of strain are mm/mm and time is measured in hours. When the load is applied in the beginning of the test, some instantaneous elastic deformation or initial strain occurs. Then with the passage of time, creep strain occurs. Creep curve is divided into three stages, primary, secondary and tertiary creep, after which the rupture takes place. In the start, strain increases rapidly, known as primary. In the secondary creep, strain increases slowly at a constant rate. Again during the tertiary stage, strain rate increases and necking of the specimen takes place followed by rupture. The creep test is repeated over a range of temperatures to obtain a number of creep curves that are useful for design.

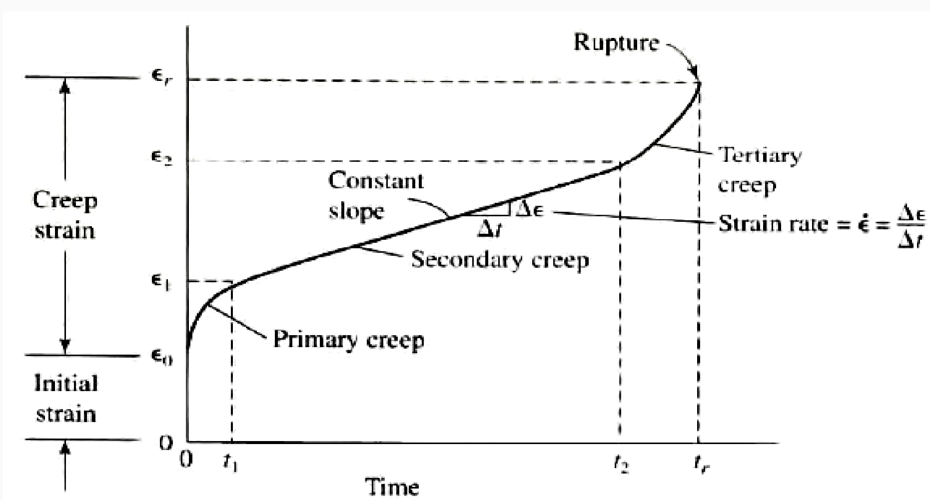


Figure 5.4 Creep Curve

LESSON 6. DESIGN FOR DYNAMIC LOADING - I

6.1 Introduction

Load which changes in magnitude or direction or both with respect to time is known as dynamic load. Cyclic load and impact load are types of dynamic loads. In case of static loading, load is gradually applied and remains stable after reaching the maximum value giving a steady value of stress. But there are a number of machine members which are subjected to cyclic loads resulting in variable stresses that fluctuate between different levels. For example, a particular fiber on the surface of a rotating shaft, subjected to bending load, undergoes both tension and compression for each revolution of the shaft.

It was discussed that the failure of members, subjected to static load, occurs when the induced stress reaches yield strength or ultimate tensile strength. But it has been observed that machine members subjected to repeated or fluctuating stresses often fail at a maximum value of induced stress well below the yield or ultimate tensile strength of the material. Such failure is known as fatigue failure as it occurs after a large number of stress cycles.

A fatigue failure resembles to a brittle fracture and occurs without any noticeable plastic deformation or necking. Its sudden occurrence, without any noticeable warning, makes it dangerous. The fracture surfaces are flat and perpendicular to the stress axis. Fatigue failure begins with a microscopic crack that occurs due to some discontinuity (oil holes, keyways, screw threads etc.), surface irregularities due to machining (scratches, stamp marks, inspection marks etc.) or material defects. This crack propagates due to fluctuating stresses, grows continually and finally sudden fracture takes place.

6.2 Types of Cyclic Stress

A number of different regular and irregular patterns are followed by cyclic stresses in machinery but generally it follows sinusoidal pattern because of the nature of some rotating machinery. Also for design purpose, only maximum and minimum value of stress is important and not the wave form. Therefore, sine wave can be conveniently used to represent any kind of variation of stress between the minimum and maximum values of stress. Following are some important types of cyclic stresses depending upon the level of minimum and maximum stress between which the stress fluctuates:

Completely Reversed Stress	extreme values of stress are of equal magnitude and opposite nature with mean equal to zero	Refer Figure 6.1 a
Repeated Stress	stress varies from zero to certain maximum value (nature of stress does not change)	Refer Figure 6.1 b
Fluctuating Stress	minimum value and maximum value of stress is of same nature (tensile or compressive)	Refer Figure 6.1 c

Alternating Stress	stress changes its nature and magnitude of extreme values of tensile and compressive stress is not same	

All variable stresses can be considered to be made of two components – static component called mean stress (s_m) and a variable component called stress amplitude (s_a).

Mean Stress =	$\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2}$	Stress Amplitude =	$\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2}$
Stress Range,	$\sigma_r = \sigma_{max} - \sigma_{min}$		
Stress Ratio,	$R = \dfrac{\sigma_{max}}{\sigma_{min}}$	Amplitude Ratio,	$A = \dfrac{\sigma_a}{\sigma_m}$

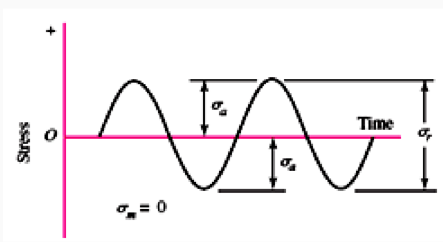


Figure 7.1 a. Completely Reversed Stress

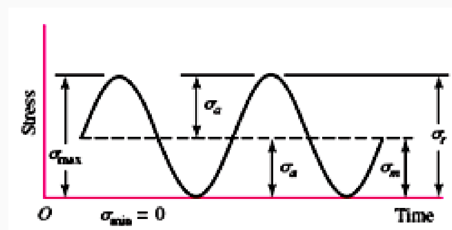


Figure 7.1 b. Repeated Stress

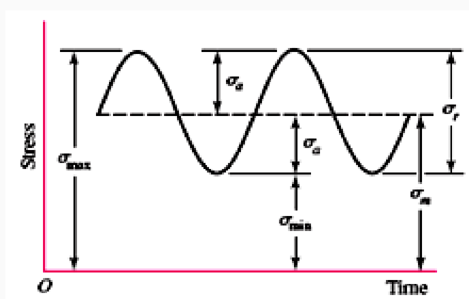
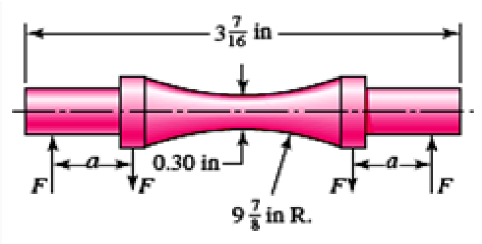


Figure 7.1 c. Fluctuating Stress

6.3 Fatigue Curve (S-N Curve) & Endurance Limit

Fatigue properties of the materials are obtained with the help of standard rotating beam test, in which a highly polished circular section specimen, shown in figure 6.2, is subjected to cyclic loads. Specimen subjected to constant bending moment is rotated at a very high speed due to which fibers of the specimen (except those on neutral axis) undergo repeated stress reversals (maximum tensile stress to maximum compressive stress). Stress-time plot is shown in figure 6.3. Test is repeated for a number of similar specimens, subjecting them to different values of stress and number of stress reversals that the specimen survives before fracture are counted. First test is performed by subjecting the specimen to stress, below ultimate tensile strength and subsequent tests are performed at decreased levels of stress.



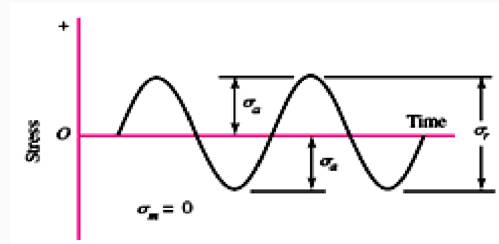


Figure 7.2 Completely Reversed Stress

Figure 7.3 Stress-time Plot

Results of the tests are plotted between stress (S) and number of cycles (N), generally on a log-log scale. S-N Curve is shown in figure 6.4.

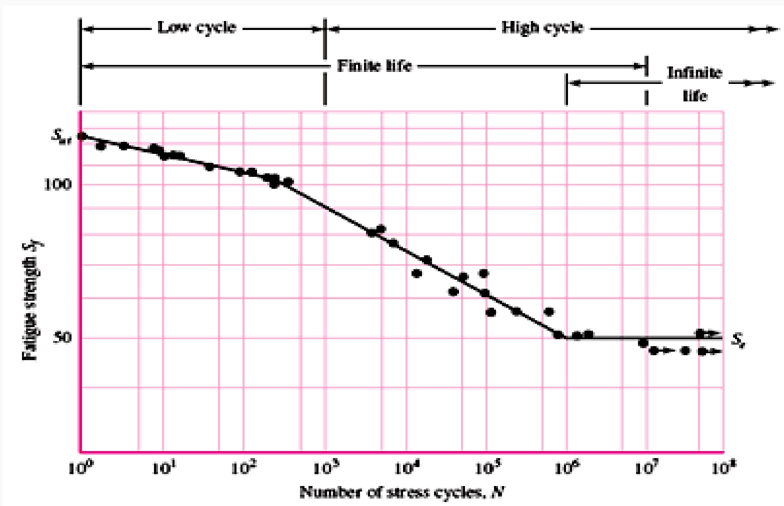


Figure 7.4 S-N Curve

The ordinate of the S-N curve is called Fatigue Strength (S_f), which can be defined as the maximum stress that the material can withstand for a specified number of stress reversals. For ferrous metals and their alloys, S-N curve becomes horizontal after 10^6 to 10^7 cycles, which means that the material can survive infinite number of stress reversals, if the induced stress is below this level. Stress corresponding to this horizontal line is called endurance limit or fatigue limit.

Fatigue or Endurance Limit (S'_e) can be defined as maximum amplitude of completely reversed stress that the standard specimen can sustain for an unlimited number of cycles without fatigue failure. Study of fatigue in which failure takes place before 1000 cycles is called Low Cycle Fatigue. High Cycle Fatigue is concerned with failure corresponding to stress cycles greater than 1000 cycles.

In the absence of experimental fatigue data, following relations are sometimes used:

For Steel, $S'_e = 0.5\ S_{ut}$

For Cast Iron, $S'_e = 0.4\ S_{ut}$

6.4 Endurance Limit Modifying Factors

Rotating-beam specimen used to determine endurance limits is very carefully prepared and is tested under closely controlled conditions. The endurance limit of any machine element cannot match the values obtained from test due to variation in material, quality of manufacture, environmental conditions and design. Therefore, the endurance limit obtained by the test is modified using some factors to obtain more reasonable results. Endurance Limit of a particular machine part can then be estimated using following relation:

$$S_e = \frac{K_{surf}\ K_{size}\ K_{load}\ K_{rel}\ K_{temp}}{K_f}\ S_e'$$

Where, $S_e^{'}$ = Endurance Limit of the specimen

K_{surf} = Surface Finish Factor

K_{size} = Size Factor

K_{load} = Load Factor

K_{rel} = Reliability Factor

K_{temp} = Temperature Factor

K_f = Fatigue Stress Concentration Factor

6.4.1 Surface Finish Factor (K_{surf})

Surface of the rotating beam specimen is highly polished but most of the machine members don't have that kind of surface finish requiring a modification in the endurance limit obtained by rotating beam experiment. Surface finish factor depends upon the manufacturing process used and ultimate tensile strength of the material. Its value can be selected with the help of chart shown in figure 6.5.

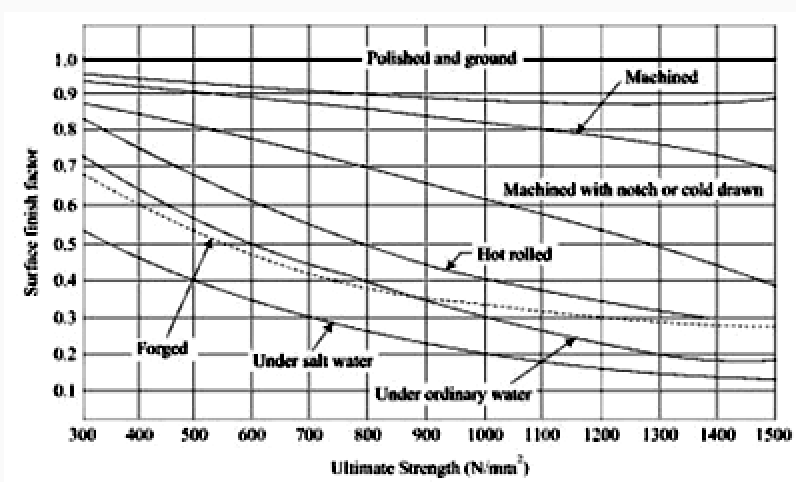


Figure 7.5 Surface Finish Factor

6.4.2 Size Factor (K_{size})

Standard rotating beam specimen has a diameter of 7.6 mm and modification factor, K_{size} must be applied for machine elements of different sizes. Its value may be taken as:

Size Factor, K_{size}	1.0	0.85	0.75
Diameter (d) of Machine Element (mm)	d ≤ 7.6 mm	7.6 ≤ d ≥ 50 mm	d ≥ 50 mm

6.4.3 Load Factor (K_{load})

In rotating beam test, specimen is subjected to bending load and completely reversed stress cycles but many machine members are subjected to different types of loads and stress cycles. To account for this, endurance limit is modified using load factor, K_{load} and its value can be taken as:

Load Factor, K_{load}	1.0	0.85	0.59
Type of Completely Reversed Load	bending	tensile	torsional

6.4.4 Reliability Factor (K_{rel})

Endurance limit obtained experimentally is the mean value and it varies even for same material and conditions. Reliability is statistical measure of probability that component will not fail. Reliability factor to modify endurance limit can be taken as:

Reliability (%)	50	90	95	99	99.9	99.99	99.999
Reliability Factor, K_{rel}	1.000	0.897	0.868	0.814	0.753	0.702	0.659

6.4.5 Temperature Factor (K_{temp})

Incease in temperature accelerates the effect of fatigue. Therefore Temperature factor, K_{temp} is used to modify the experimentally obtained endurance limit. It can be taken as:

K_{temp} = 1.0 for temperature ≤ 300°C

= 0.5 for temperature > 300°C

6.4.6 Fatigue Stress Concentration Factor (K_f)

To account for the effect of stress concentration in case of cyclic loading, endurance limit is modified by dividing it with the fatigue stress concentration factor, which is given by,

$$K_f = \frac{endurance\ limit\ of\ notch\ free\ specimen}{endurance\ limit\ of\ the\ notched\ specimen}$$

Value of this factor is less than the value of theoretical stress concentration factor, K_t, since all the materials are not equally sensitive to the notches. K_f gives the reduced value of stress concentration factor for less sensitive materials. For these materials, the effective maximum stress in fatigue is, \[{\sigma _{max}}={K_f}{\sigma _0}\]

Notch sensitivity is defined as the ratio of actual stress over nominal to the increase in theoretical stress value over the nominal stress. It is given by,

$$q = \frac{K_f \sigma_0 - \sigma_0}{K_t \sigma_0 - \sigma_0} = \frac{K_f - 1}{K_t - 1} \qquad \therefore K_f = 1 + q(K_t - 1)$$

Value of K_f is always greater than one and endurance limit of standard specimen is divided by it, unlike all the other modification factors. For q = 0, K_f =1 and for q=1, K_f = K_t. q can be estimated form figure 6.6, which is based on the experimental data.

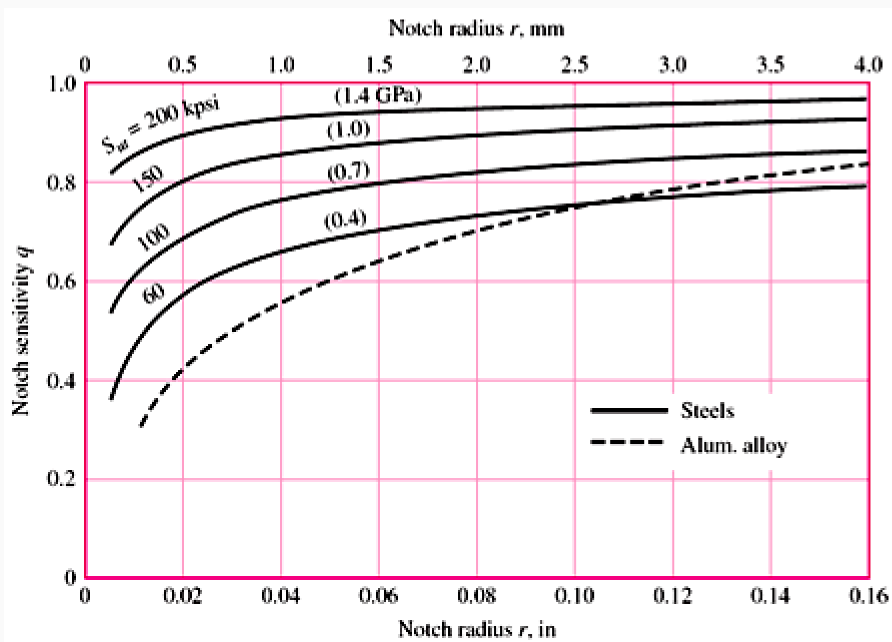


Figure 7.6 Notch Sensitivity

LESSON 7. DESIGN FOR DYNAMIC LOADING - II

7.1 Design for Completely Reversed Stresses

As discussed earlier, in case of completely reversed stress cycles, extreme values of stress are of equal magnitude and opposite nature with mean equal to zero. Design problems for completely reversed stresses can be divided into two groups:

i. Design for Infinite Life ii. Design for Finite Life

7.1.1 Design for Infinite Life

If the stress developed in a component is kept below the endurance limit, it can survive for infinite number of cycles or can have infinite life. Thus endurance limit is the design criteria in this case and the design equation can be written as:

$$\sigma_a \leq [\sigma] = \frac{S_e}{fos}$$

7.1.2 Design for Finite Life

When components are designed to survive for 10^3 to 10^6 number of cycles, it is called design for finite life. For S-N curve of steel shown in figure 7.1, line AB represents this region.

To design for finite life, fatigue strength is taken as design criteria. Fatigue strength for required number of stress cycles can be determined graphically from the S-N curve or mathematically by using the equation of line AB. Or alternatively, for a known value of induced stress, number of cycles that the component will survive can be calculated. S-N curve is a log-log plot and co-ordinates of points A and B are:

$$A(3,\log_{10}\left(0.9S_{ut} \right) \text{ and } B\left(6,\log_{10}S_e \right)$$

Ordinate of point A is approximately 90% of the ultimate tensile strength.

Equation of line AB can be written as,

$$\log_{10}\left(S_f \right) = b\log_{10}N + c \quad \text{or} \quad S_f = N^b 10^c$$

Where, b and c are constants that can be determined using co-ordinates of A and B.

$$b = \frac{1}{3}\log_{10}\left(\frac{S_e}{0.9S_{ut}} \right) \text{ and } c = \log_{10}\left(\frac{(0.9S_{ut})^2}{S_e} \right)$$

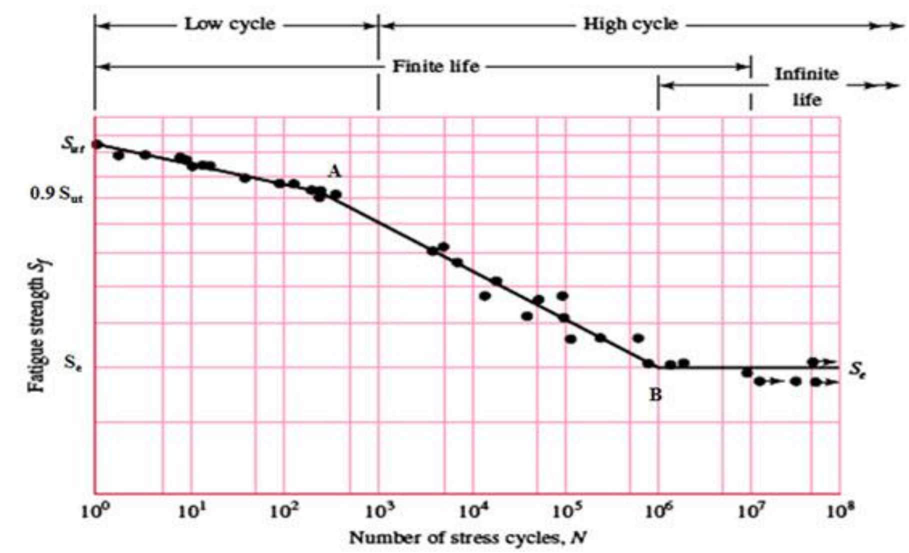


Figure 7.1 S-N Curve of Steel

7.2 Design for Fluctuating Stresses

In this case, mean value of stress has a non-zero value and both static and variable components of stress contribute to the failure. Figure 7.2 shows the scatter of failure points obtained from various experiments performed with different combinations of σ_m and σ_a. Observing these results, Soderberg, Goodman and Gerber have proposed three different theories, defining the boundary line between the safe and unsafe region on the σ_m vs σ_a plot. Soderberg line, Goodman Line and Gerber Parabola are described in table 7.1 and are shown in figure 7.2. When stress amplitude (σ_a) is zero, stress is purely static and S_{yt} or S_{ut} are the criteria of failure. These are plotted on the abscissa, along which s_m is plotted. When the mean stress (σ_m) is zero, stress is completely reversing and S_e is the criteria of failure. It is plotted on the ordinate, along which σ_a is plotted.

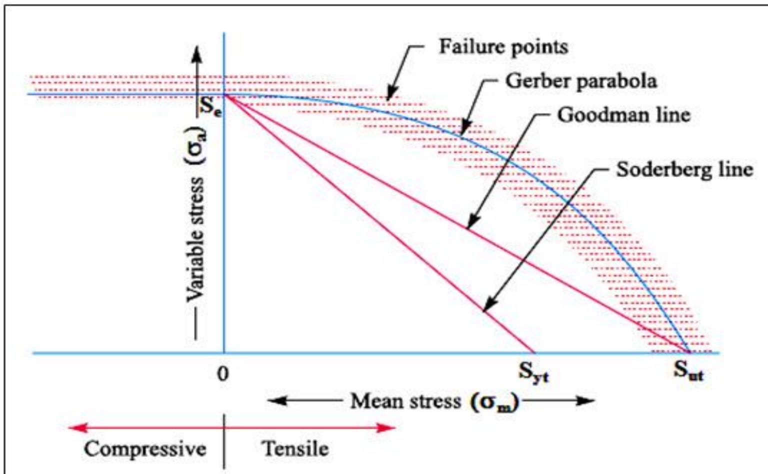


Figure 7.2 Design for Fluctuating Stresses

Table 7.1 Design for Fluctuating Stresses

Criteria	Soderberg Line	Goodman Line	Gerber Parbola
Description	Line joining S_e on the ordinate to S_{yt} on the abscissa.	Line joining S_e on the ordinate to S_{ut} on the abscissa.	Parabolic curve joining S_e on the ordinate to S_{ut} on the abscissa.
Equations	\[\frac{{{\sigma _m}}}{{{S_{yt}}}} + \frac{{{\sigma _a}}}{{{S_e}}} = 1\]	\[\frac{{{\sigma _m}}}{{{S_{ut}}}} + \frac{{{\sigma _a}}}{{{S_e}}} = 1\]	\[{\left({\frac{{{\sigma _m}}}{{{S_{ut}}}}} \right)^2} + \frac{{{\sigma _a}}}{{{S_e}}} = 1\]
Considering Factor of Safety	\[\frac{{{\sigma _m}}}{{{S_{yt}}/fos}} + \frac{{{\sigma _a}}}{{{S_e}/fos}} = 1\]	\[\frac{{{\sigma _m}}}{{{S_{ut}}/fos}} + \frac{{{\sigma _a}}}{{{S_e}/fos}} = 1\]	\[{\left({\frac{{{\sigma _m}}}{{{S_{ut}}/fos}}} \right)^2} + \frac{{{\sigma _a}}}{{{S_e}/fos}} = 1\]
Final Design Equation	\[\frac{{{\sigma _m}}}{{{S_{yt}}}} + \frac{{{\sigma _a}}}{{{S_e}}} = \frac{1}{{fos}}\]	\[\frac{{{\sigma _m}}}{{{S_{ut}}}} + \frac{{{\sigma _a}}}{{{S_e}}} = \frac{1}{{fos}}\]	\[{\left({\frac{{{\sigma _m}}}{{{S_{ut}}}}} \right)^2}.fos + \frac{{{\sigma _a}}}{{{S_e}}} = \frac{1}{{fos}}\]

As discussed in the case of static loading, allowable stress or design stress is obtained by dividing S_{ut} or S_{yt} by factor safety. Similarly, in the present case, taking factor of safety reduces the safe region as shown in figure 7.3 for Soderberg line.

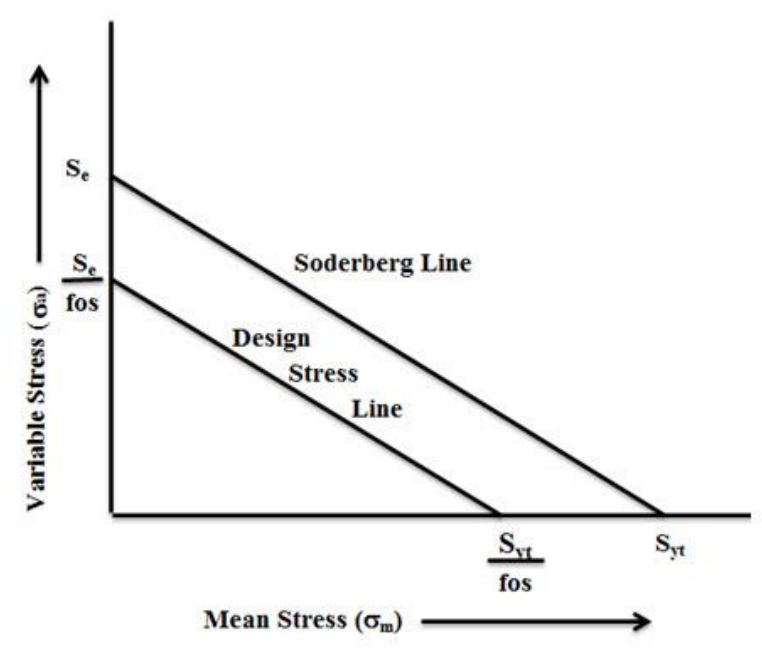


Figure 7.3 Design Stress Line for Soderberg Criteria

MODULE 3.

LESSON 8. DESIGN OF KNUCKLE JOINT

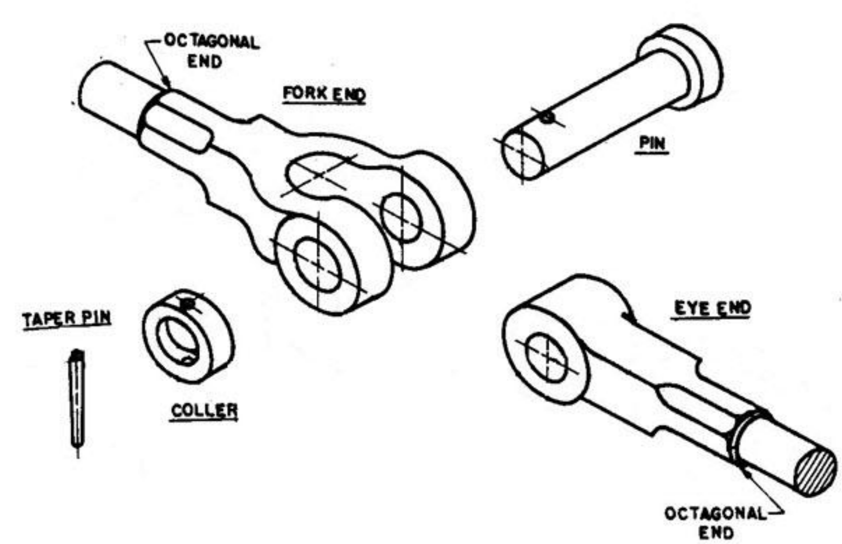
8.1 Introduction

Knuckle joint is used to connect two rods subjected to axial tensile loads. It may also be used to support the compressive load if the joint is guided. It is not suitable to connect rotating shafts which transmit torque. Axes of the shafts to be joined should lie in the same plane and may coincide or intersect. Its construction permits limited relative angular movement between rods, about the axis of the pin. Knuckle joint is widely used to connect valve rod and eccentric rod, in the link of a cycle chain, levers, tie rod joint for roof truss and many other links.

Knuckle Joint has mainly three components – eye, fork and pin as shown in Figure 8.1. Eye is formed on one of the rods and fork is formed on the other. Eye fits inside the fork and the pin is passed through both the fork and the eye. This pin is secured in its place by means of a split-pin. The ends of the rods are made octagonal to some distance for better grip and are made square for some portion before it is forged to make the eye and fork shapes.

Advantages of Knuckle Joint are:

- Simple to design and manufacture.
- Fewer parts – less cost more reliability.
- Simple to assemble and dismantle.



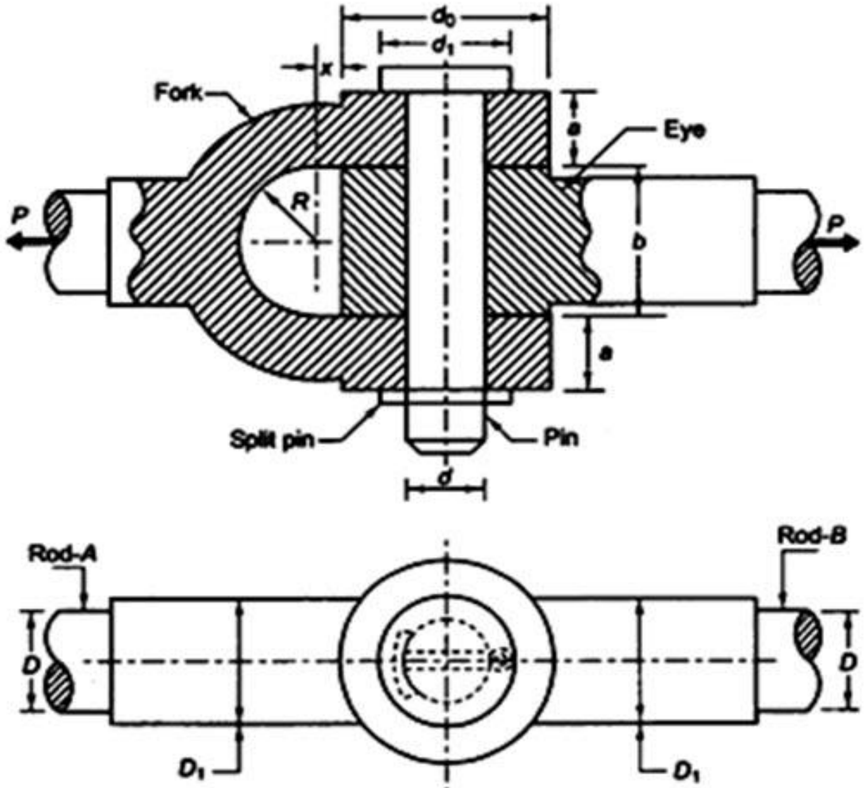


Figure 8.1 Knucle Joint

8.2 Design of Knuckle Joint

Notations Used

D = diameter of each rod (mm)

D_1 = enlarged diameter of each rod (mm)

d = diameter of knuckle pin (mm)

d_0 = outside diameter of eye or fork (mm)

d_1 = diameter of pin head (mm)

a = thickness of each eye of fork (mm)

b = thickness of eye end of rod B (mm)

x = distance of the centre of fork radius R from the eye (mm)

Assumption for stress analysis of Knuckle Joint

- The rods are subjected to axial tensile force.
- The effect of stress concentration due to holes is neglected

- The force is uniformly distributed in different parts.

Figure 8.2 shows the free body diagrams of the three main components of knuckle joint subjected to a tensile force P.

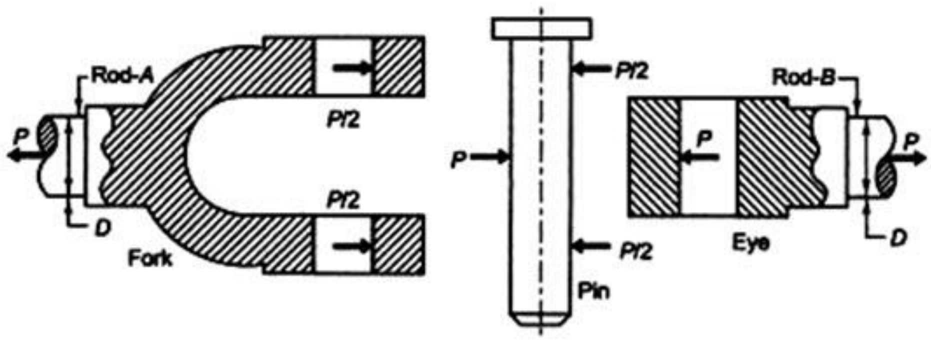


Figure 8.2 Free Body Diagrams of Different Components of Knucle Joint, subjected to Tensile Load

In order to find out various dimensions of the parts of a knuckle joint, failures in different parts and at different x-sections are considered. The stresses developed in the components should be less than the corresponding permissible values of stress. So, for each type of failure, one strength equation is written and these strength equations are then used to find various dimensions of the knuckle joint. Some empirical relations are also used to find the dimensions.

8.2.1 Possible Failure Modes of Knuckle Joint

Tensile Failure of Rods :

Each rod is subjected to a tensile force P.

$$\text{Tensile stress in the rods} = \sigma_t = \frac{P}{\frac{\pi}{4}D^2} \leq [\sigma]$$

where [σ] = allowable tensile stress for the material selected.

Shear Failure of Pin :

The pin is subjected to double shear as shown in Figure 8.2

Total Area that resiststhe shear failure = $2\left(\frac{\pi}{4}d^2 \right)$

$$\text{Shear Stress in the pin,} \quad \tau = \frac{P}{2\left(\frac{\pi}{4}d^2\right)} \leq [\tau]$$

where $[\tau]$ = allowable shear stress for the material selected.

Crushing Failure of Pin in Eye :

Projected Area of Pin in the eye = b d

Crushing Stress, $$\sigma_{crushing} = \frac{P}{b\,d} \le [\sigma_c]$$

where $[\sigma_c]$ = allowable compressive stress for the material selected.

Crushing Failure of Pin in Fork :

Projected Area of Pin in the fork = 2 a d

Crushing Stress, $$\sigma_{crushing} = \frac{P}{2\,a\,d} \le [\sigma_c]$$

Bending Failure of Pin :

When the pin is tight in the eye and fork, failure occurs due to shear, but when it is loose, it is subjected to bending moment as shown in Figure 8.5. It is assumed that: Load acting on the pin is uniformly distributed in the eye and uniformly varying in the two parts of the fork.

Maximum Bending Moment (at centre) =

$$M = \frac{P}{2}\left({\frac{b}{2} + \frac{a}{3}} \right) - \frac{P}{2}\left({\frac{b}{4}} \right)$$

Therefore, $$M = \frac{P}{2}\left({\frac{b}{4} + \frac{a}{3}} \right)$$

Maximum Bending Stress in the pin,

$$\sigma_b = \frac{My}{I} \le [\sigma]$$

where,

$$I = \frac{{\pi {d^4}}}{{64}} \text{ and } y = \frac{d}{2}$$

Tensile Failure of Eye :

Area of the weakest section of eye resisting tensile failure = $b\,(d_0 - d)$

Maximum Tensile Stress in eye,

$$\sigma_t = \frac{P}{b\,(d_0 - d)} \le [\sigma]$$

Shear Failure of Eye :

The eye is subjected to double shear

Total area resisting shear =

$$2\left[\frac{b\left(d_0 - d\right)}{2}\right] = b\left(d_0 - d\right)$$

Maximum Shear Stress in eye =

$$\tau = \frac{P}{b\,(d_0 - d)} \le [\tau]$$

Tensile Failure of Fork :

Area of the weakest section of fork resisting tensile failure = 2a (d_0 – d)

Maximum Tensile Stress in fork =

$$\sigma_t = \frac{P}{2a\,(d_0 - d)} \le [\sigma]$$

Shear Failure of Fork :

Each of the two parts of the fork is subjected to double shear.

Total area resisting shear = $2\left[\frac{2a\left(d_0 - d\right)}{2}\right] = 2a\left(d_0 - d\right)$

Maximum Shear Stress in fork = $\tau = \frac{P}{2a\,(d_0 - d)} \le [\tau]$

8.2.2 Design Procedure for Knuckle Joint

Some standard proportions for dimensions of the knuckle joint are taken as:

D_1 = 1.1 D, d = D, d_0 = 2d, a= 0.75 D, b = 1.25 D , d_1 = 1.5 d & x = 10 mm

Dimensions can be determined using these empirical relations and the strength equations can be then used as a check. By doing so the standard proportions of the joint can be maintained. The other method, of designing it, can be making the use of above strength equations to find the dimensions mathematically.

Procedure to determine various dimensions of knuckle joint is as follows:

i) Calculate the Diameter of each rod using $\frac{P}{\frac{\pi}{4}D^2} = \left[\sigma\right]$

ii) Calculate D_1 for each rod using empirical relation ${D_1} = 1.1D$

iii) Calculate dimensions a and b also using empirical relations a = 0.75 D & b = 1.25 D

iv) Calculate diameter of the pin by shear and bending consideration and select the diameter which is maximum. $\frac{P}{2\left\{ \frac{\pi}{4} d^2 \right\}} = \left[\tau \right]$ and $\frac{My}{I} = \left[\sigma \right]$

v) Calculate dimensions d_0 and d_1 using empirical relations $d_0 = 2d$ and $d_1 = 1.5d$

vi) Check the tensile, crushing and shear stresses in the eye

$$\sigma_t = \frac{P}{b\,(d_0 - d)} \leq [\sigma] \qquad \sigma_{crushing} = \frac{P}{bd} \leq [\sigma_c] \qquad \tau = \frac{P}{b\,(d_0 - d)} \leq [\tau]$$

vii) Check the tensile, crushing and shear stresses in the fork

$$\sigma_t = \frac{P}{2a\,(d_0 - d)} \leq [\sigma] \qquad \sigma_{crushing} = \frac{P}{2ad} \leq [\sigma_c] \qquad \tau = \frac{P}{2a\,(d_0 - d)} \leq [\tau]$$

Example *It is required to design a knuckle joint to connect two circular rods subjected to an axial tensile force of 50 kN. The rods are co-axial and a small amount of angular movement between their axes is permissible. Design the joint and specify the dimensions of its components. Select suitable materials for the parts.*

Solution

Given $P = (50 \times 10^3)$ N

Part I *Selection of material*

The rods are subjected to tensile force. Therefore, yield strength is the criterion for the selection of material for the rods. The pin is subjected to shear stress and bending stresses. Therefore, strength is also the criterion of material selection for the pin. On strength basis, the material for two rods and pin is selected as plain carbon steel of Grade 30C8 (S_{yt} = 400 N/mm^2). It is further assumed that the yield strength in compression is equal to yield strength in tension. In practice, the compressive strength of steel is much higher than its tensile strength.

Part II *Selection of factor of safety*

In stress analysis of knuckle joint, the effect of stress concentration is neglected. To account for this effect, a higher factor of safety of 5 is assumed in the present design.

Part III *Calculation of permissible stresses*

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{5} = 80 \text{ N/mm}^2$$

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{S_{yt}}{(fs)} = \frac{400}{5} = 80 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5\,S_{yt}}{(fs)} = \frac{0.5(400)}{5} = 40 \text{ N/mm}^2$$

Part IV *Calculation of dimensions*

The dimensions of the knuckle joint are calculated by the procedure outlined in Section 4.10.

Step I *Diameter of rods*

$$D = \sqrt{\frac{4P}{\pi \sigma_t}} = \sqrt{\frac{4(50 \times 10^3)}{\pi\,(80)}} = 28.21 \text{ or } 30 \text{ mm}$$

Step II *Enlarged diameter of rods (D_1)*

$$D_1 = 1.1\,D = 1.1(30) = 33 \text{ or } 35 \text{ mm}$$

Step III *Dimensions a and b*

$$a = 0.75\,D = 0.75(30) = 22.5 \text{ or } 25 \text{ mm}$$
$$b = 1.25\,D = 1.25(30) = 37.5 \text{ or } 40 \text{ mm}$$

Step IV *Diameter of pin*

$$d = \sqrt{\frac{2P}{\pi \tau}} = \sqrt{\frac{2\,(50 \times 10^3)}{\pi\,(40)}} = 28.21 \text{ or } 30 \text{ mm}$$

Also,

$$d = \sqrt[3]{\frac{32}{\pi \sigma_b} \times \frac{P}{2}\left[\frac{b}{4} + \frac{a}{3}\right]}$$

$$= \sqrt[3]{\frac{32}{\pi\,(80)} \times \frac{(50 \times 10^3)}{2}\left[\frac{40}{4} + \frac{25}{3}\right]}$$

$$= 38.79 \text{ or } 40 \text{ mm}$$

$$\therefore \quad d = 40 \text{ mm}$$

Step V *Dimensions d_0 and d_1*

$$d_0 = 2d = 2(40) = 80 \text{ mm}$$
$$d_1 = 1.5d = 1.5(40) = 60 \text{ mm}$$

Step VI *Check for stresses in eye*

$$\sigma_t = \frac{P}{b\,(d_0 - d)} = \frac{(50 \times 10^3)}{40\,(80 - 40)} = 31.25 \text{ N/mm}^2$$

$$\therefore \quad \sigma_t < 80 \text{ N/mm}^2$$

$$\sigma_c = \frac{P}{b\,d} = \frac{(50 \times 10^3)}{40\,(40)} = 31.25 \text{ N/mm}^2$$

$$\therefore \quad \sigma_c < 80 \text{ N/mm}^2$$

$$\tau = \frac{P}{b\,(d_0 - d)} = \frac{(50 \times 10^3)}{40(80 - 40)} = 31.25 \text{ N/mm}^2$$

$$\therefore \quad \tau < 40 \text{ N/mm}^2$$

Step VII *Check for stresses in fork*

$$\sigma_t = \frac{P}{2a\,(d_0 - d)} = \frac{(50 \times 10^3)}{2(25)(80 - 40)} = 25 \text{ N/mm}^2$$

$$\therefore \quad \sigma_t < 80 \text{ N/mm}^2$$

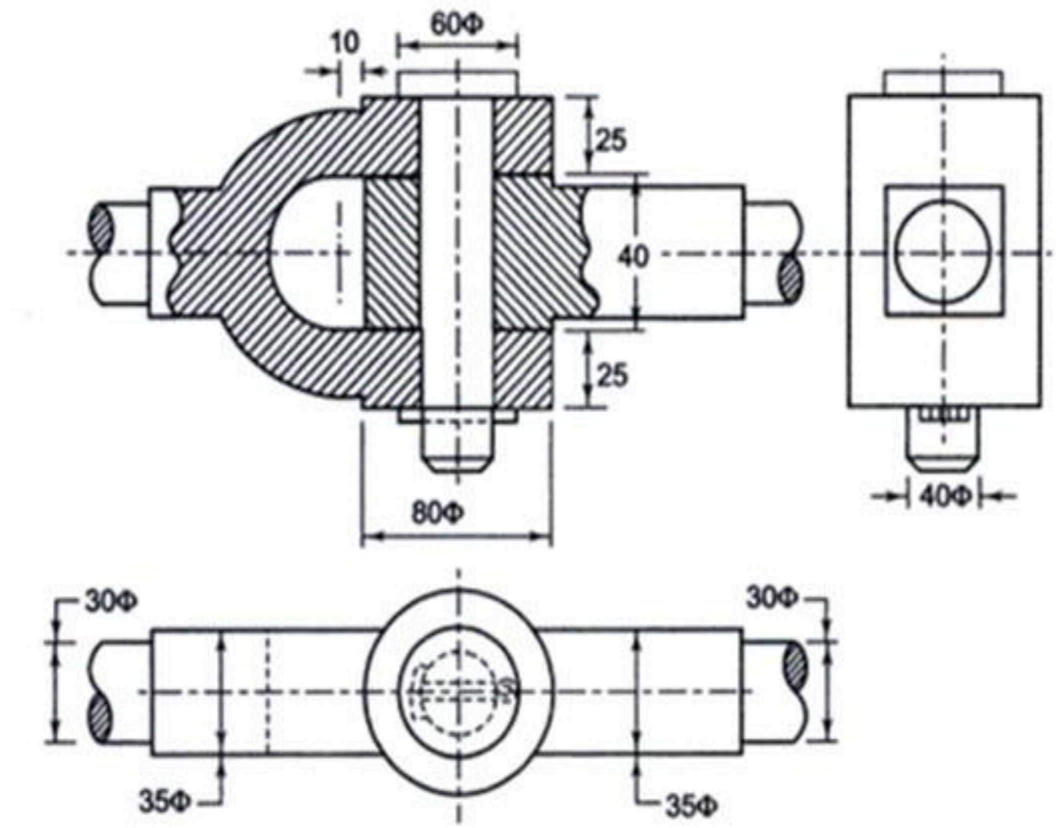
$$\sigma_c = \frac{P}{2ad} = \frac{(50 \times 10^3)}{2(25)\,(40)} = 25 \text{N/mm}^2$$

$$\therefore \quad \sigma_c < 80 \text{ N/mm}^2$$

$$\tau = \frac{P}{2a\,(d_0 - d)}$$

$$= \frac{(50 \times 10^3)}{2(25)\,(80 - 40)} = 25 \text{N/mm}^2$$

$$\therefore \quad \tau < 40 \text{ N/mm}^2$$


10
60Φ
25
40
25
80Φ
40Φ
30Φ
30Φ
35Φ
35Φ

LESSON 9. DESIGN OF COTTER JOINT

9.1 Introduction

Cotter joint is used to connect two rods subjected to axial tensile or compressive loads. It is not suitable to connect rotating shafts which transmit torque. Axes of the rods to be joined should be collinear. There is no relative angular movement between rods. Cotter joint is widely used to connect the piston rod and crosshead of a steam engine, as a joint between the piston rod and the tailor pump rod, foundation bolt etc.

Cotter Joint has mainly three components – spigot, socket and cotter as shown in Figure 9.1. Spigot is formed on one of the rods and socket is formed on the other. The socket and the spigot are provided with a narrow rectangular slot. The cotter is tightly fitted in this slot. Spigot fits inside the socket and the cotter is passed through both the socket and the spigot. A cotter is a wedge shaped piece made of a steel plate. It has uniform thickness and the width dimension is given a slight taper. Taper is usually 1 in 24 and provides mainly two benefits: i) cotter becomes tight in the slot due to wedge action. This ensures tightness of the joint in operating conditions and prevents loosening of the parts. ii) Due to its taper shape, it is easy to remove the cotter and dismantle the joint.

The construction of cotter joint, used to connect two rods subjected to tensile force P is shown in the figure. When the cotter is inserted into the slot, the central portion of cotter comes in contact with spigot and the spigot gets pushed into the socket till the collar of the spigot comes in contact with the collar of socket. As shown in the figure, finally the cotter is in contact with the spigot on one side having some clearance with the socket slot and is in contact with the socket on the other side having some clearance with the spigot slot. Clearance provided is generally 1.5 to 3 mm. Cotter gets locked because of the frictional force of the contacting surfaces.

Advantages of Cotter Joint:

- Simple to design and manufacture.

- Simple to assemble and dismantle.

- Very high tightening force due to wedge action, which prevents loosening of parts in service.

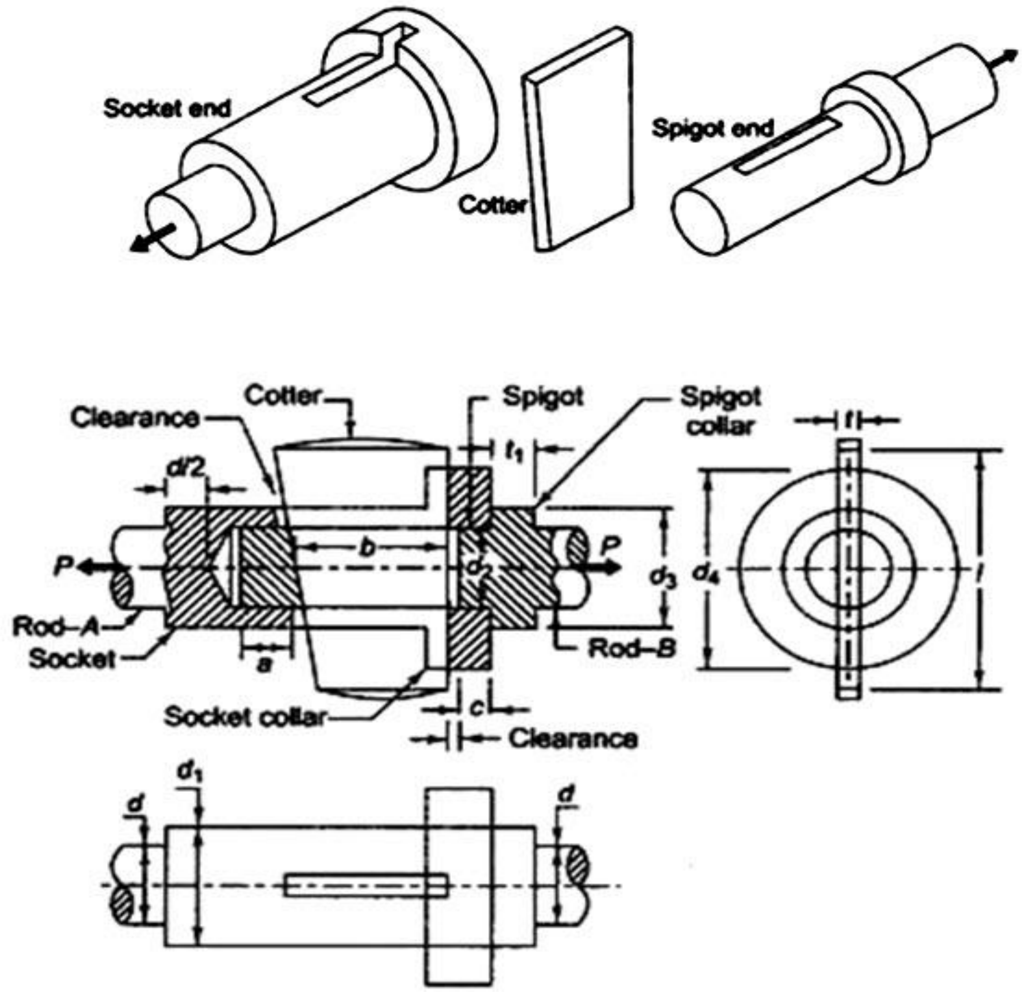


Figure 9.1 Cotter Joint

9.2 Design of Knuckle Joint

Notations Used :

d = diameter of each rod (mm)

d_1 = outside diameter of socket (mm)

d_2 = diameter of spigot or inside diameter of socket (mm)

d_3 = diameter of spigot-collar (mm)

d_4 = diameter of socket-collar (mm)

a = distance from end of slot to the end of spigot on rod-B (mm)

b = mean width of cotter (mm)

c = axial distance from slot to end of socket collar (mm)

t = thickness of cotter (mm)

t_1 = thickness of spigot collar (mm)

l = length of cotter (mm)

Assumption for stress analysis of Cotter Joint :

- The rods are subjected to axial tensile force.
- The effect of stress concentration due to holes is neglected
- The force is uniformly distributed in different parts.

Free body diagram of forces acting on three components of the cotter joint (socket, cotter and spigot) are shown in Figure 9.2.

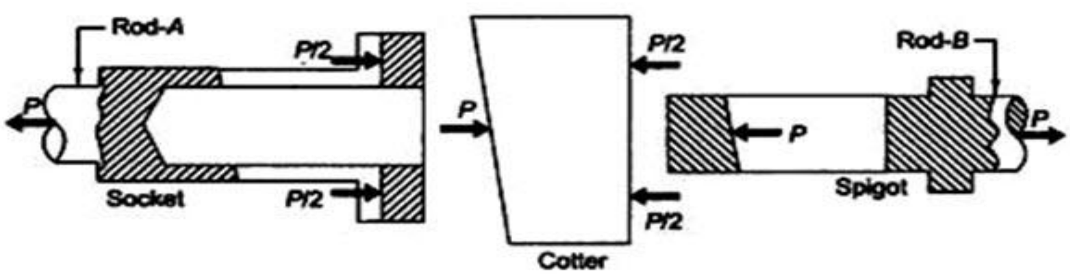


Figure 9.2 Free Body Diagrams of Different Components of Knucle Joint, subjected to Tensile Load

In order to find out various dimensions of the parts of a cotter joint, failures in different parts and at different x-sections are considered. The stresses developed in the components should be less than the corresponding permissible values of stress. So, for each type of failure, one strength equation is written and these strength equations are then used to find various dimensions of the cotter joint. Some empirical relations are also used to find the dimensions.

9.2.1 Possible Modes of Failure of Cotter Joint

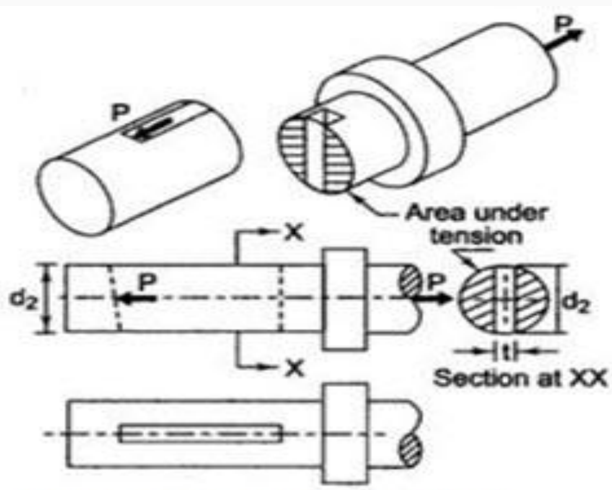


Figure 9.3 Tensile Failure of Spigot

Tensile Failure of Rods :

Each rod is subjected to a tensile force P.

Tensile stress in the rods = $\sigma_t = \dfrac{P}{\frac{\pi}{4}d^2} \leq [\sigma]$

where [σ] = allowable tensile stress for the material selected.

<u>Tensile Failure of Spigot :</u>

Area of the weakest section of spigot resisting tensile failure =

$$\left[{\frac{\pi }{4}d_2^2 - {d_2}t} \right]$$

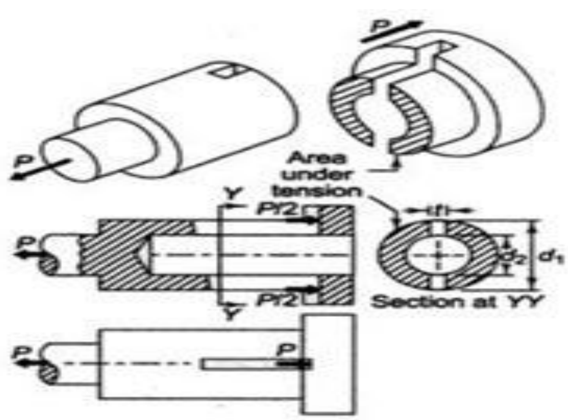


Figure 9.4 Tensile Failure of Socket

Tensile Stress in the Spigot = $\sigma_t = \dfrac{P}{\left[\frac{\pi}{4}d_2^2 - d_2 t\right]} \leq [\sigma]$

<u>Tensile Failure of Socket :</u>

Area of the weakest section of socket resisting tensile failure =

$$\left[{\frac{\pi }{4}(d_1^2 - d_2^2) - \left({{d_1} - {d_2}} \right)t} \right]$$

Tensile Stress in the Socket = $\sigma_t = \dfrac{P}{\left[\frac{\pi}{4}(d_1^2 - d_2^2) - (d_1 - d_2)t\right]} \leq [\sigma]$

<u>Shear Failure of Spigot End :</u>

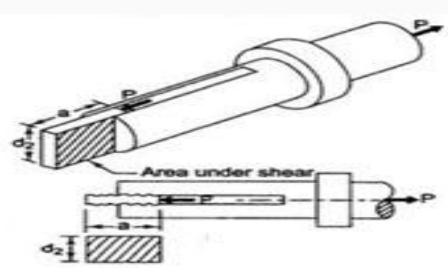


Figure 9.5 Shear Failure of Spigot

The spigot end is subjected to double shear.

Total area that resists the shear failure = $2ad_2$

Shear Stress in the socket = $\tau = \dfrac{P}{2(ad_2)} \leq [\tau]$

where $[\tau]$ = allowable shear stress for the material selected.

<u>Shear Failure of Socket End :</u>

The socket end is subjected to double shear.

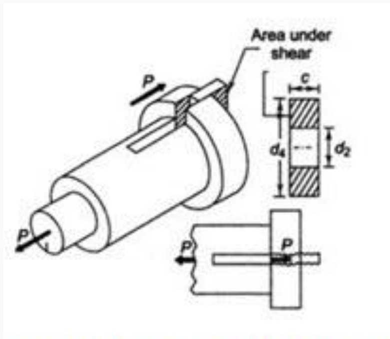


Figure 9.6 Shear Failure of Socket

Total area that resists the shear failure = 2 $\left({d_4} - {d_2} \right)c$

Shear Stress in the socket = $\tau = \dfrac{P}{2(d_4 - d_2)c} \leq [\tau]$

<u>Crushing Failure of Spigot End :</u>

Area under crushing $= {\text{t}}{\text{d}}_2$

Crushing Stress = $\sigma_{crushing} = \dfrac{P}{td_2} \leq [\sigma_c]$

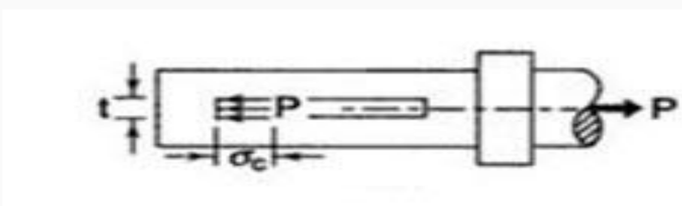


Figure 9.7 Crushing Failure of Spigot

where $[\sigma_c]$ = allowable compressive stress for the material selected.

<u>Crushing Failure of Socket End</u> :

Area under crushing $= \left({d_4} - {d_2} \right)t$

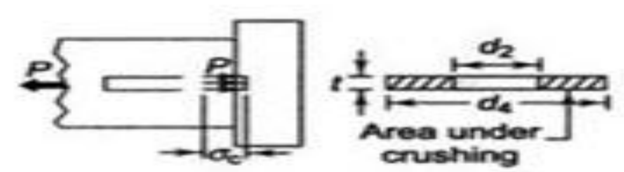


Figure 9.8 Crushing Failure of Socket

Crushing Stress, $$\sigma_{crushing} = \frac{P}{(d_4 - d_2)t} \leq [\sigma_c]$$

<u>Shear Failure of Spigot Collar</u> :

Area of spigot collar that resists the shear failure = $\pi\, d_2 t_1$

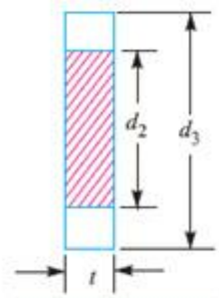


Figure 9.9 Area of Spigot Collar Resisting Shear

Shear Stress in the socket = $$\tau = \frac{P}{\pi\, d_2 t_1} \leq [\tau]$$

<u>Crushing Failure of Spigot Collar</u> :

Area of spigot collar under crushing = $\frac{\pi}{4}(d_3^2 - d_2^2)$

Crushing Stress = $$\sigma_{crushing} = \frac{P}{\frac{\pi}{4}(d_3^2 - d_2^2)} \leq [\sigma_c]$$

<u>Shear Failure of Cotter</u> :

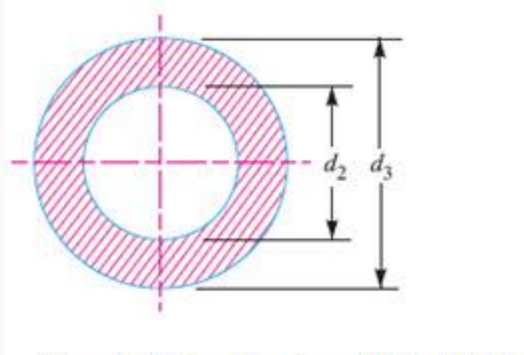


Figure 9.10 Crushing Area of Spigot Collar

The cotter is subjected to double shear.

Total area of cotter that resists the shear failure = 2bt

Shear Stress in the pin = $\tau = \frac{P}{2bt} \leq [\tau]$

<u>Bending Failure of Cotter</u> :

When the cotter is tight in the socket and spigot, failure occurs due to shear, but when it is loose, it is subjected to bending moment as shown in figure

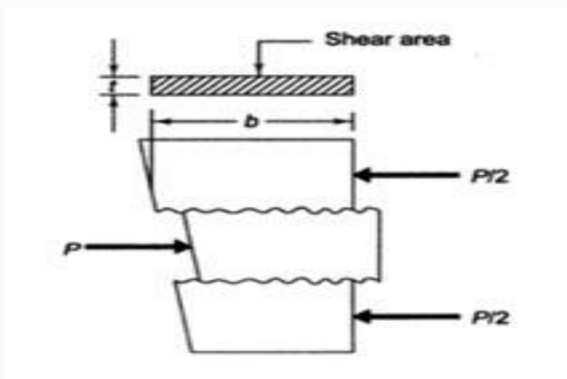


Figure 9.11 Shear Failure of Cotter e.

It is assumed that: the force between cotter and spigot end is uniformly distributed and uniformly varying between the socket end and cotter.

Maximum Bending Moment (at centre) =

$$M = \frac{P}{2}\left({\frac{{{d_2}}}{2} + \frac{{{d_4} - {d_2}}}{6}} \right) - \frac{P}{2}\left({\frac{{{d_2}}}{4}} \right)$$

$$= \frac{P}{2}\left({\frac{{{d_2}}}{4} + \frac{{{d_4} - {d_2}}}{6}} \right)$$

Maximum Bending Stress in the cotter =

$$\sigma_b = \frac{My}{I} \leq [\sigma]$$

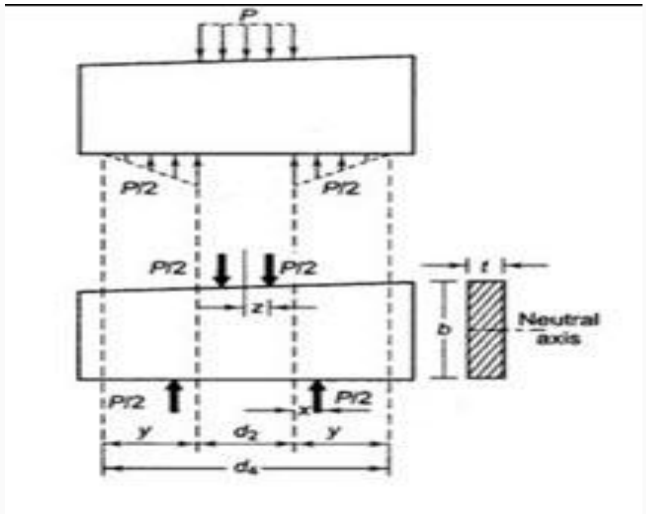


Figure 9.12 Bending of Cotter

where,

$$I = \frac{tb^3}{12} \text{ and } y = \frac{b}{2}$$

9.2.2 Design Procedure for Cotter Joint

Procedure to determine various dimensions of cotter joint is as follows:

i) Calculate the Diameter of each rod using $\frac{\text{P}}{\frac{\pi}{4}d^2} = \left[\sigma\right]$

ii) Calculate thickness of cotter using empirical relation $t = 0.31d$

iii) Calculate diameter of the spigot on the basis of tensile stress $\frac{\text{P}}{\left[\frac{\pi}{4}d_2^2 - d_2 t\right]} = \left[\sigma\right]$

iv) Calculate outside diameter of the socket on the basis of tensile stress

$$\frac{\text{P}}{\left[\frac{\pi}{4}(d_1^2 - d_2^2) - \left(d_1 - d_2\right)t\right]} = \left[\sigma\right]$$

v) Diameter of spigot collar, d_3 and diameter of socket collar, d_4 are determined using empirical relations $d_3 = 1.5$ d and $d_4 = 2.4$ d

vi) Dimensions a and c are also determined using empirical relations a = c = 0.75 d.

vii) Calculate width of cotter by shear and bending consideration and select the width which is maximum, $\frac{\text{P}}{2bt} = \left[\tau\right]$

and $\frac{My}{I} = \left[\sigma\right]$

viii) Check the crushing and shear stresses in the spigot end.

$$\sigma_{crushing} = \frac{P}{td_2} \leq [\sigma_c] \quad \text{and} \quad \tau = \frac{P}{2a\,d_2} \leq [\tau]$$

ix) Check the crushing and shear stresses in the socket end

$$\sigma_{crushing} = \frac{P}{(d_4 - d_2)t} \leq [\sigma_c] \quad and \quad \tau = \frac{P}{2(d_4 - d_2)c} \leq [\tau]$$

x) Calculate thickness t_1 of spigot collar by the following empirical relationship \[{t_1} = 0.45d\]

xi) Check the crushing and shear stresses in the socket collar

$$\sigma_{crushing} = \frac{P}{\frac{\pi}{4}(d_3^2 - d_2^2)} \leq [\sigma_c] \quad \text{and} \quad \tau = \frac{P}{\pi\, d_2 t_1} \leq [\tau]$$

LESSON 10 INTRODUCTION TO WELDED JOINTS

10.1 Introduction

Welding is a process for joining two similar or dissimilar metals by fusion and provides a permanent joint. In welding, the parts are coalesced at their contacting surfaces by a suitable application of heat and/or pressure, with or without the addition of a filler metal. Welding provides a permanent joint but it normally affects the metallurgy of the components. It is therefore usually accompanied by post weld heat treatment for most of the critical components. The welding is widely used as a fabrication and repairing process in industries. Some of the typical applications of welding include the fabrication of ships, pressure vessels, automobile bodies, bridges, welded pipes, sealing of nuclear fuel and explosives, etc.

Advantages

1. Welding is more economical and is much faster process as compared to other processes (riveting, bolting, casting etc.)
2. Welding, if properly controlled results permanent joints having strength equal or sometimes more than base metal.
3. Large number of metals and alloys both similar and dissimilar can be joined by welding.
4. General welding equipment is not very costly.
5. Portable welding equipments can be easily made available.
6. Welding permits considerable freedom in design.
7. Welding can join welding jobs through spots, as continuous pressure tight seams, end-to-end and in a number of other configurations.
8. Welding can also be mechanized.

Disadvantages

1. It results in residual stresses and distortion of the work pieces.
2. Welded joint needs stress relieving and heat treatment.
3. Welding gives out harmful radiations (light), fumes and spatter.
4. Jigs and fixtures may also be needed to hold and position the parts to be welded
5. Edges preparation of the welding jobs are required before welding
6. Skilled welder is required for production of good welding
7. Heat during welding produces metallurgical changes as the structure of the welded joint is not same as that of the parent metal.

10.2 Types of Welding

Welding processes can be broadly classified in two groups: fusion welding and solid-state welding.

10.2.1 Fusion Welding Processes

Fusion Welding processes use heat to melt the base metals. In fusion welding operations, a filler metal is generally added to the molten pool. Fusion welding processes can further be subdivided into following types:

Arc Welding: Arc welding refers to a group of welding processes in which heating of the metals is accomplished by an electric arc.

Resistance welding: Resistance welding achieves coalescence using heat from electrical resistance to the flow of a current passing between the faying surfaces of two parts held together under pressure.

Oxyfuel Gas Welding: These joining processes use an oxyfuel gas, such as a mixture of oxygen and acetylene, to produce a hot flame for melting the base metal.

Other welding processes that produce fusion of the metals joined include electron beam welding and laser beam welding.

10.2.2 Solid-State Welding

Solid-state welding refers to joining processes in which coalescence results from application of pressure alone or a combination of heat and pressure. If heat is used, the temperature in the process is below the melting point of the metals being welded. No filler metal is utilized. Some welding processes in this group are:

Diffusion welding: Two surfaces are held together under pressure at an elevated temperature and the parts coalesce by solid-state fusion.

Friction welding: Coalescence is achieved by the heat of friction between two surfaces.

Ultrasonic welding: Moderate pressure is applied between the two parts and an oscillating motion at ultrasonic frequencies is used in a direction parallel to the contacting surfaces. The combination of normal and vibratory forces results in shear stresses that remove surface films and achieve atomic bonding of the surfaces.

10.3 Types of Welded Joints

Welded joints are primarily of two types: 1. Lap joint or fillet joint, and 2. Butt joint.

10.3.1 Lap Joint

The lap joint or the fillet joint is obtained by overlapping the plates and then welding the edges of the plates. The cross-section of the fillet is approximately triangular. The fillet joints are of three types: - Single transverse fillet, Double transverse fillet and Parallel fillet joints. These are shown in Figure 10.1.

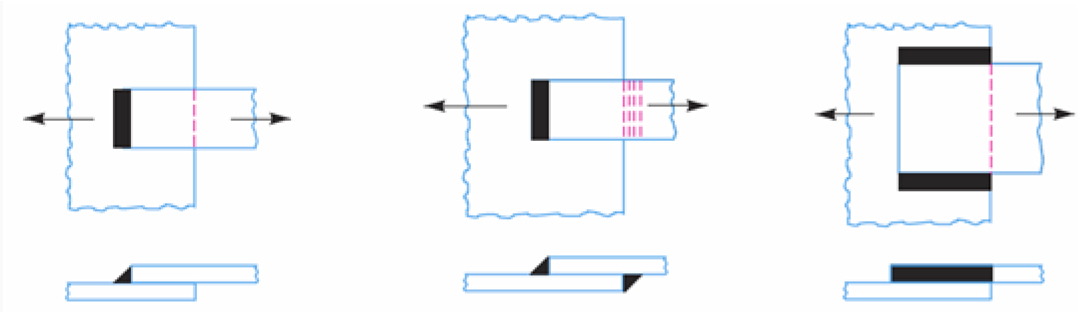


Figure 10.1 Types of Lap or Fillet Joint

10.3.2 Butt Joint

The butt joint is obtained by placing the plates edge to edge as shown in figure. In butt welds, the plate edges do not require bevelling if the thickness of plate is less than 5 mm. On the other hand, if the plate thickness is 5 mm to 12.5 mm, the edges should be bevelled to V or U-groove on both sides. Figure 10.2 shows the types of butt joints.

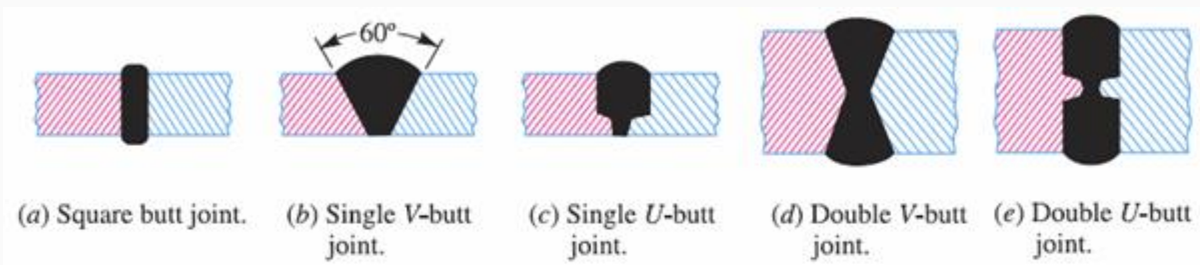


Figure 10.2 Types of Butt Joint

Corner joint, edge joint and T-joint (shown in Figure 10.3) are some other types of welded joints.

(a) Corner joint (b) Edge joint (c) T - joint

Figure 10.3 Some Other Types of Welded Joints

LESSON 11 DESIGN OF WELDED JOINTS

11.1 Design of a Butt Joint

The butt joints are designed for tension or compression. Average Tensile Stress in a butt welded joint subjected to tensile load, P is given by, $\sigma_t = \frac{P}{A} = \frac{P}{tl}$

where, A is throat area, t is throat thickness and l is length of the weld.

σ_t must be ≤ [σ_t] for the joint to be safe.

Similarly Average Compressive Stress in a butt welded joint subjected to compressive load, P is given by, $\sigma_c = \frac{P}{tl}$, which must be ≤ [σ_c].

i. Single V-Butt Joint ii. Double V-Butt Joint

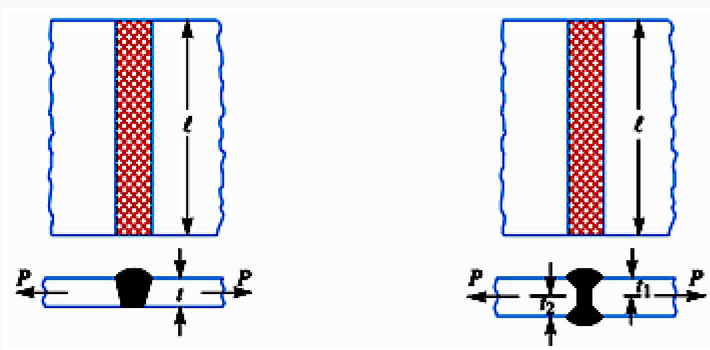


Figure 11.1 Butt Joint

For double V-butt joint, throat area is (t_1+t_2) l, where t_1 and t_2 are throat thickness at top and bottom.

11.2 Design of a Fillet Joint

11.2.1 Transverse Fillet Weld

Transverse Fillet welds are designed for tensile strength. For strength calculations, the section of fillet is assumed to be a right angled triangle, with hypotenuse making equal angles with the two sides as shown in Figure 11.2.

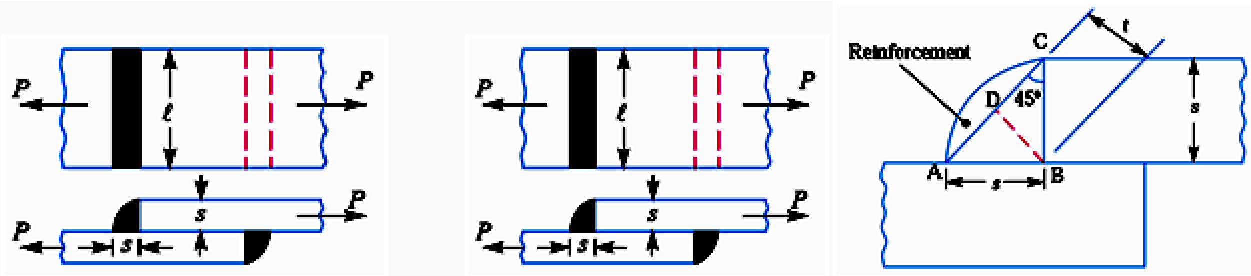


Figure 11.2 Single & Double Transverse Fillet Weld

Length of each side (AB=BC) is known as size or leg of the weld (s) and the distance of the hypotenuse from the intersection of two legs (BD) is known as throat thickness (t). Minimum area is obtained at the throat. If l is the length of the weld,

Throat area, $A = t\,l = s.\sin 45°.\,l = 0.707\,s\,l$

Tensile Stress of single transverse fillet weld subjected to tensile load, P is given by,

$$\sigma_t = \frac{P}{A} = \frac{P}{0.707\ s\ l} \leq [\sigma_t]$$

And that for a double transverse fillet weld is given by,

$$\sigma_t = \frac{P}{2\ A} = \frac{P}{1.414\ s\ l} \leq [\sigma_t]$$

11.2.2 Parallel Fillet Weld

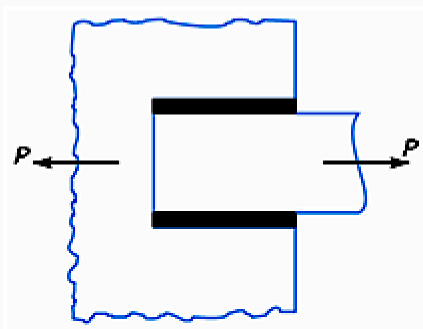


Figure 11.3 Parallel Fillet Weld

Parallel fillet welds are designed for shear strength. Consider a parallel fillet weld as shown in Figure 11.3. Throat Area, A = 0.707 s l, where s and l are size and length of the weld.For a parallel fillet weld subjected to tensile load, P, shear stress is given by,

$$\tau = \frac{P}{2\ A} = \frac{P}{1.414\ s\ l} \leq [\tau]$$

11.2.3 Combination of Transverse and Parallel Fillet Welds

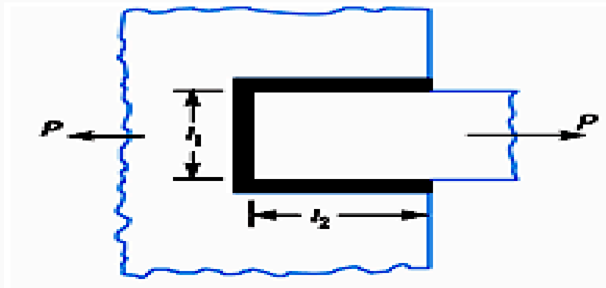


Figure 11.4 Combination of Transverse & Parallel Fillet Weld

If a tensile load, P is applied on a combination of transverse and parallel fillet weld, shear stress will develop in the parallel fillet welds and tensile stress will develop in the transverse fillet weld such that the maximum load that the weld can withstand is given by,

$P_{max} = 1.414\ s\ l_1\ [\sigma_t] + 1.414\ s\ l_2\ [t]$

$= 1.414\ s\ (l_1\ [\sigma_t] + l_2\ [t])$

l_1 and l_2 are weld lengths on two sides, as shown in Figure 11.4.While designing any fillet weld, 11.5 mm length must be left on each side of the weld to allow for the start and stop of the bead.

11.3 Unsymmetrical Welded Sections

For unsymmetrical welded sections subjected to tensile loads as shown in Figure 11.5, the length of welds should be so proportioned that the resisting moment of the welds about the gravity axis is zero.

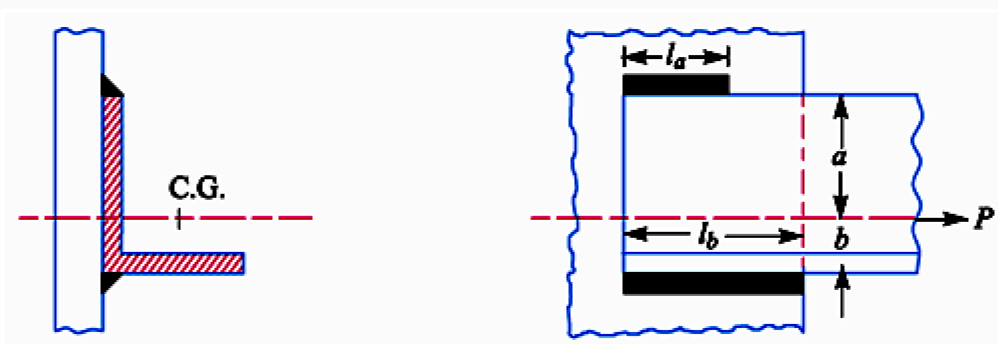


Figure 11.5 Axially Loaded Unsymmetrical Welded Sections

Let

l_a, l_b = Length of welds on two sides

a , b = Distance of welds from gravity axis

l = Total length of weld = $l_a + l_b$

P = Axial load,

f = Resistance offered by the weld per unit length.

Moment of resistance offered by weld on side A about gravity axis = $l_a \times f \times a$

Moment of resistance offered by weld on side B about gravity axis = $l_b \times f \times b$

For the moments about the gravity axis to be zero,

$l_a \times f \times a = l_b \times f \times b \Rightarrow l_a \times a = l_b \times b$

Also, $l = l_a + l_b$

Therefore, $l_a = \dfrac{l\,b}{a+b}$ and $l_b = \dfrac{l\,a}{a+b}$

11.4 Eccentrically Loaded Welded Joints

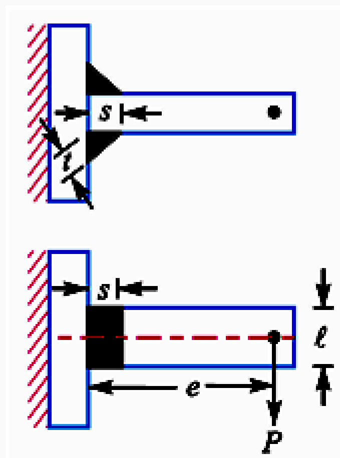


Figure 11.6 Bending Stress due to Eccentricity

In many cases the welded joints are eccentrically loaded. Different stresses may get induced depending upon the type of joint and loading. if the stresses are of same nature , those may be vectorially added but for those of different nature, resultant maximum tensile and shear stresses may be calculated. Depending upon the type of joint, eccentricity may lead to bending stress or torsional shear stress in the joint in addition to the direct shear stress induced by applied load.

11.4.1 Eccentricity leading to Bending Stress

Consider a T-joint subjected to loading as shown in figure. Let s and l be the size and length of the weld and t be the throat thickness.

Throat area = A = 2 t l

This applied load may be considered as a load P directly acting on the joint through the CG and a bending moment of magnitude P.e acting on the joint. 1st one will lead to direct shear stress and the 2nd will lead to a bending stress.

Direct Shear Stress, $\tau = \frac{P}{A} = \frac{P}{2tl}$ and $\sigma_b = \frac{My}{I}$ Bending Stress,

where y = distance of the point on the weld from the neutral axis

I = Moment of inertia of the weld section

Maximum tensile and shear stress may be calculated as:

$\sigma_{t_{max.}} = \left(\frac{\sigma_b}{2}\right)^2 + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2}$ and $\tau_{max.} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2}$

1.4.2 Eccentricity Leading to Torsional Shear Stress

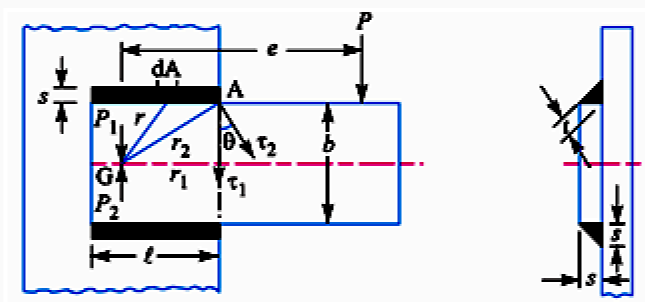


Figure 11.7 Shear Stress due to Eccentricity

Let us consider a double parallel fillet weld subjected to an eccentric load P acting at a distance e from the CG of the welds as shown in Figure 11.7.

Eccentric force P may be considered as a force P acting on the CG of the joint and a torque equivalent to Pe acting on the joint. The force P through the CG leads to direct shear stress, called primary shear stress and is assumed to be uniformly distributed over the throat area of all welds. The torque Pe causes torsional shear stress called secondary shear stress.

Primary Shear Stress, $\tau_1 = \frac{P}{A} = \frac{P}{2tl}$ and Secondary Shear Stress, $\tau_2 = \frac{Mr}{J}$

where r = distance of the point on the weld from the CG

J = Polar moment of inertia of the weld section

r is calculated from the geometry for the farthest point of the weld from the CG.

LESSON 12 INTRODUCTION TO THREADED FASTNERS

12.1 Introduction

In threaded joints two or more machine members are joined together with the help of threaded fastening e.g. a nut and bolt. These are non-permanent type joints i.e. members can be disassembled without damaging the component parts for the purpose of maintenance, checking and replacement. Threads are formed by cutting a helical groove on the surface of a cylindrical rod or cylindrical hole. Threaded fasteners are standardized and a wide variety is available for different operating conditions and applications. These are easy to manufacture and a high accuracy can be maintained. Holes are required in the machine parts to be assembled by threaded joints, which lead to stress concentration. Another disadvantage is that, threaded joints tend to loosen when subjected to vibrations.

12.2 Terminology of Screw Threads

Figure 12.1 shows some important terms used in screw threads

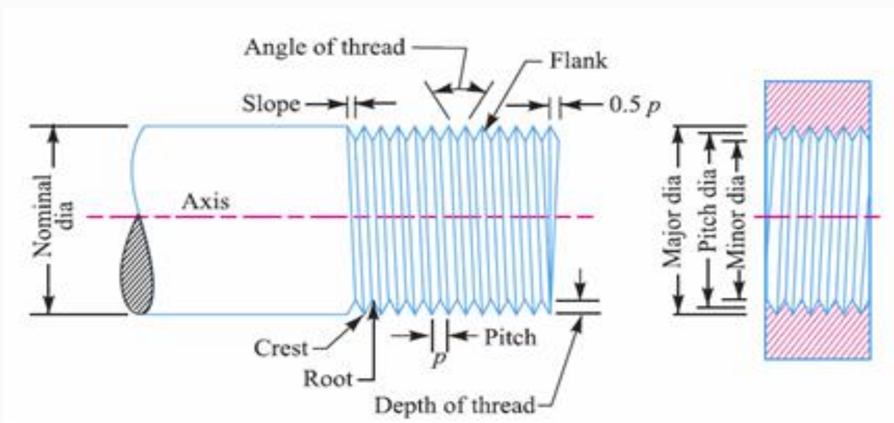


Figure 12.1 Terms used in Screw Threads

Major diameter: It is the largest diameter of an external or internal screw thread. The screw is specified by this diameter. It is also known as outside or nominal diameter.

Minor diameter: It is the smallest diameter of an external or internal screw thread. It is also known as core or root diameter.

Pitch diameter: It is the diameter of an imaginary cylinder, on a cylindrical screw thread, the surface of which would pass through the thread at such points as to make equal the width of the thread and the width of the spaces between the threads. It is also called an effective diameter. In a nut and bolt assembly, it is the diameter at which the ridges on the bolt are in complete touch with the ridges of the corresponding nut.

Pitch: It is the distance from a point on one thread to the corresponding point on the next. This is measured in an axial direction between corresponding points in the same axial plane.

Mathematically, Pitch = 1/ No. of threads per unit length of screw

Lead: It is the distance between two corresponding points on the same helix. It may also be defined as the distance which a screw thread advances axially in one rotation of the nut. Lead is equal to the pitch in case of single start threads; it is twice the pitch in double start, thrice the pitch in triple start and so on.

Crest: It is the top surface of the thread.

Root: It is the bottom surface created by the two adjacent flanks of the thread.

Depth of thread: It is the perpendicular distance between the crest and root.

Flank: It is the surface joining the crest and root.

Angle of thread: It is the angle included by the flanks of the thread.

Slope: It is half the pitch of the thread.

12.3 ISO Metric Screw Threads

In screws used for fastening, generally V-threads are used. They provide higher friction thus reducing the chances of loosening. They have higher strength because of higher thickness at the core and also are convenient to manufacture. Profile of an ISO metric screw thread is shown in Figure 12.2. It consists of an equilateral triangle with side equal to pitch and internal angle 60° as thread angle. Crests and roots of the threads are rounded, which reduces stress concentration in the threads and also increases the life of thread cutting tool.

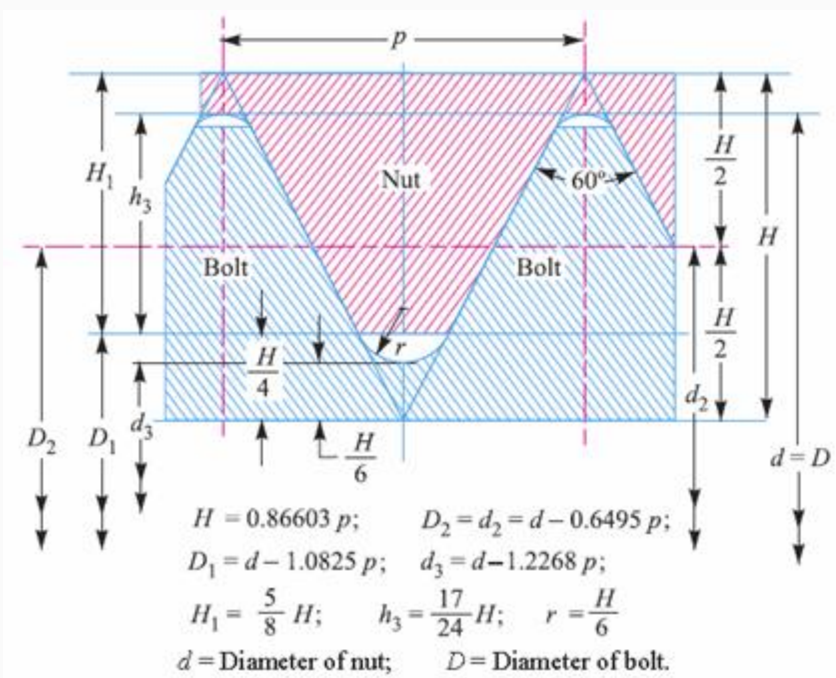


Figure 12.2 Profile of an ISO Metric Screw Thread

Metric threads are divided into coarse and fine series. Thread profiles in both the series is similar. Coarse threads, considered the basic series, have higher static load carrying capacity, are easier to cut, have less effect on strength because of manufacturing errors and wear and have more even stress distribution. On the other hand fine threads have greater strength against fluctuating loads and have greater resistance to unscrewing because of its lower helix angle. Therefore fine series threads are more dependable in terms of self loosening. Coarse threads are used in members, which are free from vibrations and fine threads are used in parts subjected to dynamic loads and hollow thin walled parts as the coarse threads will weaken the members considerably. Fine threads are also used in the parts where the threads are used for the purpose of adjustment.

A screw thread of coarse series is designated by the letter 'M' followed by the value of the nominal diameter in mm. For example ‘M 12’. A screw thread of fine series is specified by the letter ‘M’ followed by the values of the nominal diameter and the pitch in mm and separated by the symbol ‘×’. For example. M 12 × 1.25.

12.4 Material

Threads are produced by rolling or machining. Because of cold work, the rolled threads are stronger and have better fatigue properties. Threads can also be produced using casting. Selection of material for threaded fasteners depends upon type of loading, operating environment and temperature etc. Plain Carbon Steel is used for common applications and Alloy Steels are used in high temperature applications and where high strength, better fatigue and corrosion resistance is required. Aluminium, Brass and Bronze are also used in specific applications. Generally a factor of safety of 2 to 3 on the basis of yield strength is considered in case of carbon steels and 1.5 to 3 for alloy steels.

12.5 Types of Screw Fasteners

Bolt (Through Bolt): It is a cylindrical bar with threads for the nut at one end and head at the other end. The cylindrical part of the bolt is known as shank. It is passed through drilled holes in the two parts to be fastened together and clamped them securely to each other as the nut is screwed on to the threaded end. Bolts have hexagonal or square heads.

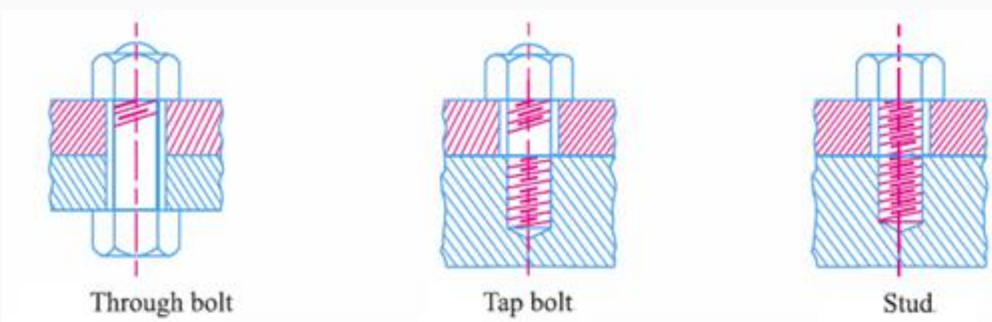


Figure 12.3 Through Bolt, Tap Bolt and Stud

Tap bolts: Tap bolt is screwed into a tapped hole of one of the parts to be fastened and nut is not used with it.

Studs: A stud is a round bar threaded at both ends. One end is screwed into a tapped hole of the parts to be fastened, while the other end receives a nut on it.

Cap screws: The cap screws are similar to tap bolts except that they are of small size and a variety of shapes of heads are available.

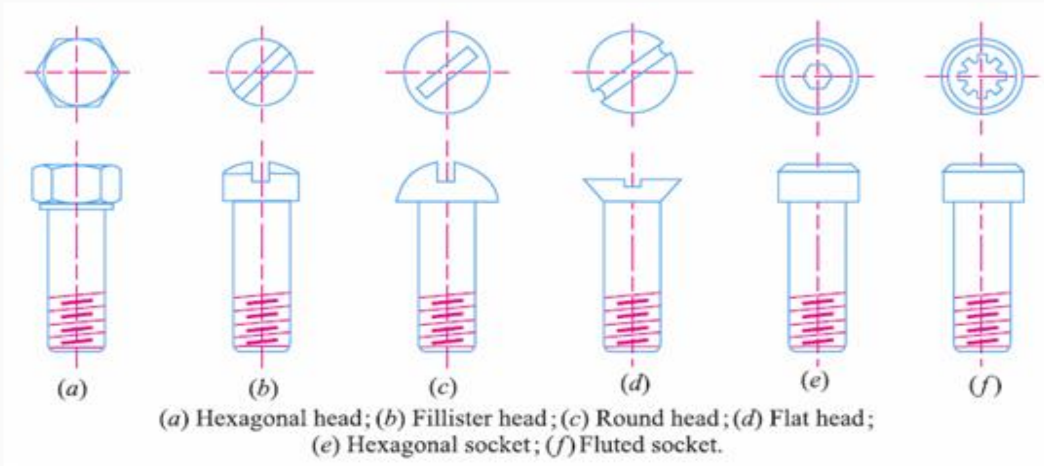


Figure 12.4 Types of Cap Screws

Machine screws: These are similar to cap screws with the head slotted for a screw driver and are generally used with a nut.

Set screws: Set screws are used to prevent relative motion between the two parts. A set screw is screwed through a threaded hole in one part so that its point (i.e. end of the screw) presses against the other part. This resists the relative motion between the two parts by means of friction between the point of the screw and one of the parts.

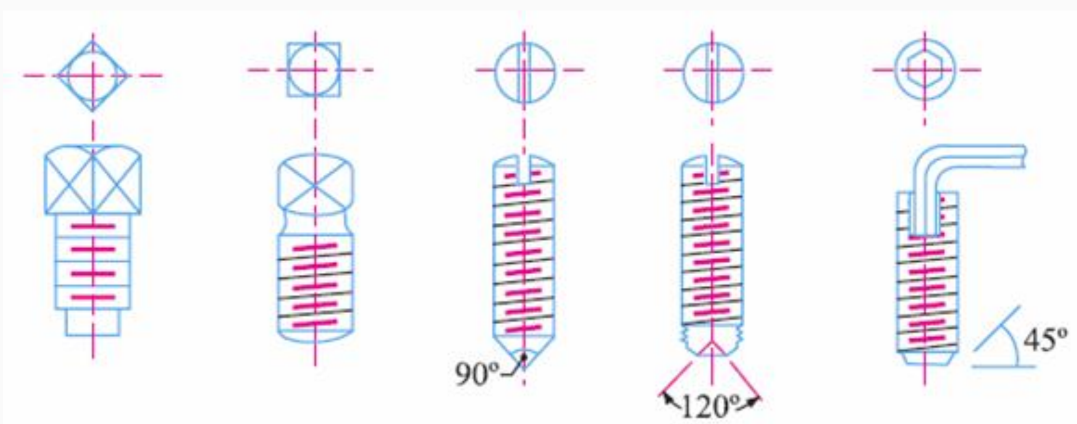


Figure 12.5 Types of Set Screws

LESSON 13 DESIGN OF THREADED FASTNERS

13.1 Modes of Failure

Consider a bolted joint subjected to tensile load as shown in Figure 13.1. Possible modes of failure of bolt under this loading are as follows:

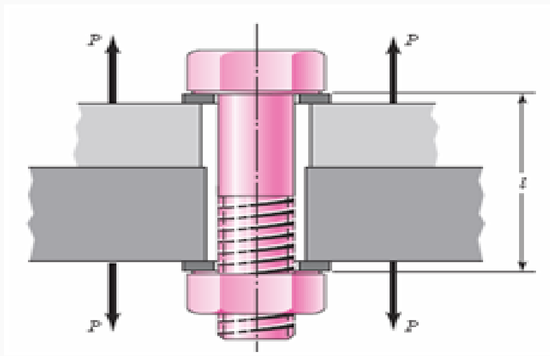


Figure 13.1 Bolt subjected to tensile load

13.1.1 Tensile Failure of Bolt

Maximum tensile stress induced in the bolt is given by,

$$\sigma_t = \frac{P}{A} = \frac{P}{\frac{\pi}{4} d_c^2} \text{ should be } \leq [\sigma_t]$$

where, d_c = core diameter of the bolt. Cross-section at core diameter is the weakest section.

In addition to this, threads of the bolt can also fail in shear and crushing. For analysing that, it is assumed that each turn of the thread supports equal load and failure occurs in the threads of the bolt and not in the threads of the nut. Also stress concentration is neglected in the analysis of the bolts.

13.1.2 Shear Failure of Threads

Maximum shear stress developed in the threads is given by,

$$\tau = \frac{P/n}{\pi d_c b} = \frac{P}{\pi d_c h} \text{ should be } \leq [\tau]$$

where n = no. of turns, b = width of thread section at the root

h = n×b = height of the bolt

13.1.3 Crushing Failure of Threads

Maximum crushing stress developed in the threads is given by,

$$\sigma_{crushing} = \frac{P/n}{\frac{\pi}{4}(d^2 - d_c^2)} \ should\ be\ \leq [\sigma_c]$$

where n = no. of turns, d = outer diameter of the bolt

13.1.4 Shear Failure of Bolt

In addition to the above case of tensile loading, bolts may also be subjected to shear loads. In such case the maximum shear stress is given by,

$$\tau = \frac{P}{\frac{\pi}{4}d_c^2} \ should\ be\ \leq [\tau]$$

where P, in this case, is the force acting on the joint perpendicular to the axis of bolt.

Using above relations, core diameter of the bolt can be calculated for a given material and type of loading. If the standard tables of bolts are available, a suitable bolt can be selected and other dimensions can be taken from the table. If tables are not available approximate relation $d_c = 0.8\ d$ is generally used to find the nominal diameter of the bolt. (Exact relation for ISO metric threads is $d_c = d - 1.22687\ p$, where p is the pitch).

13.2 Pre-stress in Bolts

Stress develops in the threaded joint because of initial tightening torque. Stress developed is compressive in the members and tensile in nature in the bolts. Value of initial tension in the bolts is calculated using an empirical relation.

Initial Tension, $F_i = 2840\ d$ (N) (for fluid tight joints)

$= 1420\ d$ (N) (for other joints)

where, d is nominal diameter of the bolt in mm. Initial Stress in the bolt can be calculated from F_i.

13.3 Eccentrically Loaded Bolted Joints

13.3.1 Eccentric Load acting in the plane of the Bolts

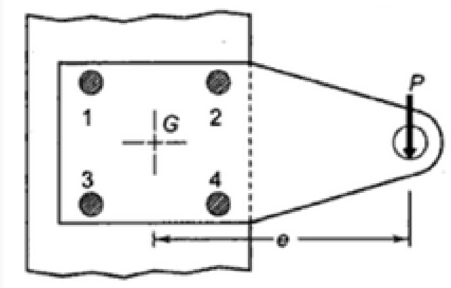


Figure 13.2 Eccentric load acting in plane of bolts

Consider the joint shown in Figure 13.2. Let a force P is acting at a distance e from the centre of gravity. This eccentric force can be considered as equivalent to an imaginary force P acting at the centre of gravity and a moment 'Pe' about the centre of gravity.

Primary shear force developed in the bolts, because of the direct load,

$$P_1' = P_2' = P_3' = P_4' = \frac{P}{No. of\ bolts}$$

Secondary shear force because of the moment can be determined as follows.

$$Pe = P_1''.r_1 + P_2''.r_2 + P_3''.r_3 + P_4''.r_4$$

It is assumed that the secondary shear force in any bolt is proportional to its distance from the centre of gravity.

$$P_1'' \propto r_1, \quad P_2'' \propto r_2, \quad P_3'' \propto r_3 \quad and \quad P_4'' \propto r_4$$

Considering C as a proportionality constant,

$$P_1'' = C\, r_1, \quad P_2'' = Cr_2, \quad P_3'' = Cr_3 \quad and \quad P_4'' = Cr_4$$

$$Pe = C\, r_1^2 + C\, r_2^2 + C\, r_3^2 + C\, r_4^2$$

$$\therefore C = \frac{Pe}{r_1^2 + r_2^2 + r_3^2 + r_4^2}$$

So,

$$\therefore P_1'' = \frac{P\, e\, r_1}{r_1^2 + r_2^2 + r_3^2 + r_4^2}$$

Similarly P"2, P"3 and P"4 the values of , and can be calculated. Primary and secondary shear forces are then vectorially added to get the resultant shear force in each bolt, which can then be used to find the stresses.

13.3.2 Eccentric Load acting perpendicular to the plane of the Bolts

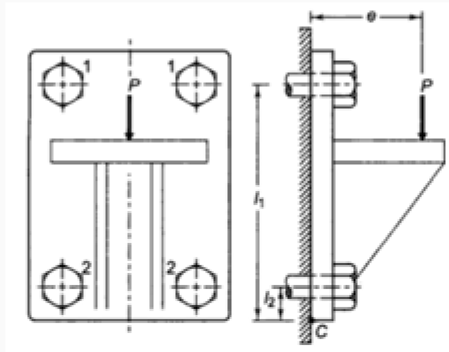


Figure 13.3 Eccentric load acting perpendicular to the plane of bolts

Consider a bracket bolted to a structure as shown in Figure 13.3. Let an eccentric force P be acting at a distance e from the structure. Lower two bolts are denoted by 2 and upper two by 1. P is acting perpendicular to the axes of the bolts and leads to a direct shear load, which can be given by,

$$P_1' = P_2' = \frac{P}{No.\ of\ bolts}$$

Also, because of eccentricity, P leads to a moment Pe, which tends to tilt the bracket about the edge C. This leads to resisting tensile forces in all the bolts, which are proportional to their distance from C. If and are the resisting tensile forces developed in bolts at position 1 and 2 respectively,

$$P_1'' = C\ l_1 \ \ and \ \ P_2'' = C l_2$$

Considering C as a proportionality constant,

Equating the moments due to P and due to the resisting forces,

$$Pe = 2\ P_1''.l_1 + 2\ P_2''.l_2$$

$$Pe = 2\ C\ l_1^2 + 2\ C\ l_2^2$$

Or

$$\therefore C = \frac{Pe}{2(l_1^2 + l_2^2)}$$

$$\therefore P_1'' = \frac{P\ e\ l_1}{2(l_1^2 + l_2^2)}$$ and $$P_2'' = \frac{P\ e\ l_2}{2(l_1^2 + l_2^2)}$$

Bolts farthest from the tilting edge have maximum value of resisting tensile force. Therefore stresses in the bolts denoted by 1 will have maximum stresses which can be determined as follows:

Maximum principal stresses developed in the bolts can then be found and compared with the allowable values or bolt dimensions can be found for a given material.

13.3.3 Eccentric Load acting parallel to the plane of the Bolts

Consider the bracket bolted to structure shown in Figure 13.4. Let an eccentric force P be acting at a distance e from the edge C about which it tends to tilt the bracket. There are two bolts at each position i.e. 1 & 2. As P, in this case, is acting parallel to the axes of the bolts, it leads to a primary tensile force and a secondary tensile force due to the moment.

Primary tensile forces are given by, $P_1' = P_2' = \frac{P}{No.\ of\ bolts}$

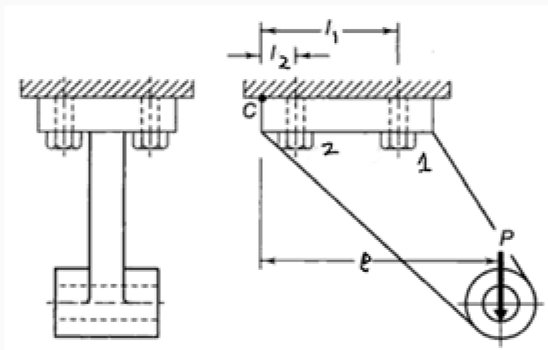


Figure 13.4 Eccentric load acting parallel to the plane of bolts

As discussed in the previous case, secondary tensile forces are given by,

$$P_1'' = \frac{P\,e\,l_1}{2(l_1^2 + l_2^2)} \text{ and } P_2'' = \frac{P\,e\,l_2}{2(l_1^2 + l_2^2)}$$

Total tensile force in each bolt at position 1 & 2 are given by,

$P_1 = P'_1 + P''_2$ and $P2 = P'_2 + P''_2$

As bolts at 1 are farthest from the edge about which the bracket tends to tilt, maximum resisting force is developed in those. Maximum tensile stress in bolts at position 1 is given by, $\sigma_{t_1} = \frac{P_1}{\frac{\pi}{4} d_c^2}$

This can be compared with the allowable values of tensile stress.

References:

1. Design of Machine Elements by V B Bhandari
2. Analysis and Design of Machine Elements by Vijay Kumar Jadon
3. A Text Book of Machine Design by RS Khurmi

MODULE 4.

LESSON 14 DESIGN OF SHAFTS

14.1 Introduction

Shaft is a common machine element which is used to transmit rotary motion or torque. It generally has circular cross-section and can be solid or hollow. Shafts are supported on the bearings and transmit torque with the help of gears, belts and pulleys etc. Shafts are generally subjected to bending moment, torsion and axial force or a combination of these three. So the shafts are designed depending upon the combination of loads it is subjected to. Spindle stub and axle are some important types of shaft. Small shaft is called spindle. Shaft integral part of the prime mover is called stub shaft. An axle is a non-rotating member that carries no torque and is used to support rotating wheels, pulleys etc. And therefore is subjected to bending moment only.

14.2 Shaft Materials

Hot-rolled plain carbon steel is the least expensive material used for shafts. These essentially require machining to remove the scales of hot rolling process. Cold rolled plain carbon steel provides better yield strength and endurance strength but the cold working induces residual stresses. Surface is smooth in this case and amount of machining therefore is minimal. It is used for general purpose transmission shafts. When a shaft is to work under severe loading and corrosive conditions and require more strength, alloy steels are used, generally having Ni, Cr, Mo and V as alloying elements. Alloy steels are expensive. Sometimes shafts are heat treated to improve hardness and shock resistance and surface hardening techniques are also used if high wear resistance is the requirement. As the shafts transmitting power are subjected to fatigue loading, therefore higher factor of safety of 3 to 4 is used on the basis of yield strength for static load analysis.

14.3 Design of Shafts

Shafts are designed on the basis of strength or rigidity or both. Design based on strength is to ensure that stress at any location of the shaft does not exceed the material yield stress. Design based on rigidity is to ensure that maximum deflection (because of bending) and maximum twist (due to torsion) of the shaft is within the allowable limits. Rigidity consideration is also very important in some cases for example position of a gear mounted on the shaft will change if the shaft gets deflected and if this value is more than some allowable limit, it may lead to high dynamic loads and noise in the gears.

In designing shafts on the basis of strength, the following cases may be considered:

(a) Shafts subjected to torque

(b) Shafts subjected to bending moment

(c) Shafts subjected to combination of torque and bending moment

(d) Shafts subjected to axial loads in addition to combination of torque and bending moment

14.3.1 Shafts Subjected to Torque

Maximum shear stress developed in a shaft subjected to torque is given by,

$$\tau = \frac{T\,r}{J} \leq [\tau]$$

where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation

$= \frac{\pi d^4}{32}$ for solid shafts with diameter d

$= \frac{\pi (d_o^4 - d_i^4)}{32}$ for hollow shafts with d_o and d_i as outer and inner diameter.

r = Distance from neutral axis to the outer most fibre = d/2 (or $d_o/2$)

So dimensions of the shaft subjected to torque can be determined from above relation for a known value of allowable shear stress, [τ].

14.3.2 Shafts Subjected to Bending Moment

Maximum bending stress developed in a shaft is given by,

$$\sigma_b = \frac{M\,y}{I} \leq [\sigma_t]$$

where M = Bending Moment acting upon the shaft,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation

$= \frac{\pi d^4}{64}$ for solid shafts with diameter d

$= \frac{\pi (d_o^4 - d_i^4)}{64}$ for hollow shafts with d_o and d_i as outer and inner diameter.

y = Distance from neutral axis to the outer most fibre = d / 2 (or $d_o/2$)

So dimensions of the shaft subjected to bending moment can be determined from above relation for a known value of allowable tensile stress, [ ].

14.3.3 Shafts Subjected to Combination of Torque and Bending Moment

When the shaft is subjected to combination of torque and bending moment, principal stresses are calculated and then different theories of failure are used. Bending stress and torsional shear stress can be calculated using the above relations.

$$\tau = \frac{T\,r}{J} = \frac{T\,\frac{d}{2}}{\frac{\pi}{32}d^4} = \frac{16\,T}{\pi d^3}$$

$$\sigma_b = \frac{M\,y}{I} = \frac{M\,\frac{d}{2}}{\frac{\pi}{64}d^4} = \frac{32\,M}{\pi d^3}$$

Maximum Shear Stress Theory

Maximum shear stress is given by,

$$\tau_{max.} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2} = \sqrt{\left(\frac{16\,M}{\pi d^3}\right)^2 + \left(\frac{16\,T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2} \le [\tau]$$

$\sqrt{M^2 + T^2}$ is called equivalent torque, T_e, such that

$$\tau_{max.} = \frac{T_e\,r}{J} \le [\tau]$$

Maximum Principal Stress Theory

Maximum principal stress is given by,

$$\sigma = \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2} = \frac{16\,M}{\pi d^3} + \sqrt{\left(\frac{16\,M}{\pi d^3}\right)^2 + \left(\frac{16\,T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right] \le [\sigma_t]$$

$\left[M + \sqrt{M^2 + T^2}\right]$ is called equivalent bending moment, M_e, such that

$$\sigma = \frac{M_e\,y}{I} \le [\sigma_t]$$

A.S.M.E. Code for Shaft Design

According to A.S.M.E. code, the bending and twisting moment are to be multiplied by factors k_b and k_t respectively, to account for shock and fatigue in operating condition. Therefore, if the shaft is subjected to dynamic loading, equivalent torque and equivalent bending moment will become:

$$T_e = \sqrt{k_b M^2 + k_t T^2} \quad \text{and} \quad M_e = \left[k_b M + \sqrt{k_b M^2 + k_t T^2}\right]$$

Table 14.1 Values of k_b and k_t for different types of loading

	k_b	k_t
Gradually applied load	1.5	1.0
Suddenly applied load (minor shock)	1.5-2.0	1.0-1.5
Suddenly applied load	2.0-3.0	1 .5-3.0

14.3.4 Shafts Subjected to Axial Loads in addition to Combination of Torque and Bending Moment

Tensile Stress due to axial load is given by,

$$\sigma_t = \frac{P}{A}$$

where, P = axial load acting on the shaft

A = cross-sectional area of the shaft

As nature of the bending stress and this axial stress is same, these can be vectorially added for any location on the shaft, so as to get the resultant tensile/compressive stress, which can then be used to find the principal stresses in the shaft.

14.3.5 Design of Shaft on the basis of Rigidity

14.3.5.1 Torsional Rigidity

For a shaft subjected twisting moment, the angle of twist is given by,

$$\theta = \frac{TL}{GJ} \leq [\theta]$$

Where, T = Torqe applied

L = Length of the shaft

J = Polar moment of inertia of the shaft about the axis of rotation

G = Modulus of rigidity of the shaft material

Therefore for the known values of T, L and G and allowable value of angle of twist, diameter of the shaft can be calculated.

15.3.5.2 Lateral Rigidity

Bending moment acting on any shaft is given by,

$$M = EI\frac{d^2y}{dx^2}$$

Integrating this equation twice with respect to x and applying the boundary conditions, y can be calculated. y should be ≤ allowable value of deflection, [y].

14.3.6 A.S.M.E. Code for Shaft Design

According to A.S.M.E. code, the bending and twisting moment are to be multiplied by factors k_b and k_t respectively, to account for shock and fatigue in operating condition. Therefore, if the shaft is subjected to dynamic loading, equivalent torque and equivalent bending moment will become:

$$T_e = \sqrt{k_b M^2 + k_t T^2}$$ and $$M_e = \left[k_b M + \sqrt{k_b M^2 + k_t T^2}\right]$$

Table 14.1 Values of k_b and k_t for different types of loading

	k_b	k_t
Gradually applied load	1.5	1.0
Suddenly applied load (minor shock)	1.5-2.0	1.0-1.5
Suddenly applied load	2.0-3.0	1.5-3.0

LESSON 15 DESIGN OF KEYS

15.1 Introduction

Key is a machine element which is used to connect the transmission shaft to rotating machine elements like pulley, gear, sprocket or flywheel. Keys provide a positive means of transmitting torque between shaft and hub of the mating element. A slot is machined in the shaft or in the hub or both to accommodate the key is called keyway. Keyway reduces the strength of the shaft as it results in stress concentration.

Keys are made of ductile materials. Commonly used materials for a key are hardened and tempered steel of grades C30, C35, C40, C50 and 55Mn75 etc. Brass and stainless keys are used in corrosive environment. Factor of safety of 3 to 4 is generally taken on yield strength.

15.2 Types of Keys

Common types of keys are:

1. Sunk keys 2. Saddle keys 3. Tangent keys 4. Round keys 5. Splines

15.2.1 Sunk Keys

A sunk key is a key in which half of the thickness of key fits into the keyway in the shaft and half in the keyway of the hub. The sunk keys are of the following types:

Rectangular sunk key: It is the simplest type of key and has a rectangular cross-section. A taper of about 1 in 100 is provided on its top side. Rectangular sunk key is shown in Figure 15.1.

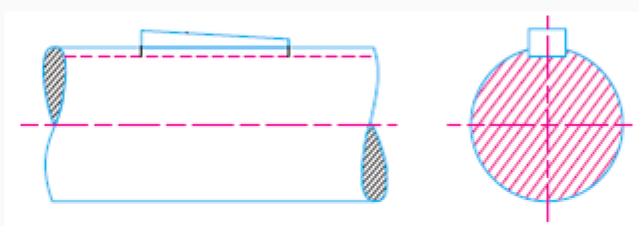


Figure 15.1 Rectangular Sunk Key

Square sunk key: Rectangular sunk key having equal width and thickness is called square sunk key.

Parallel sunk key: If no taper is provided on the rectangular or square sunk key, it is called parallel sunk key i.e. it is uniform in width and thickness throughout. It is used where the pulley, gear or other mating piece is required to slide along the shaft.

Gib-head key: It is a rectangular sunk key with a head at one end known as gib head, which is provided to facilitate the removal of key. Gib Head key is shown in Figure 15.2.

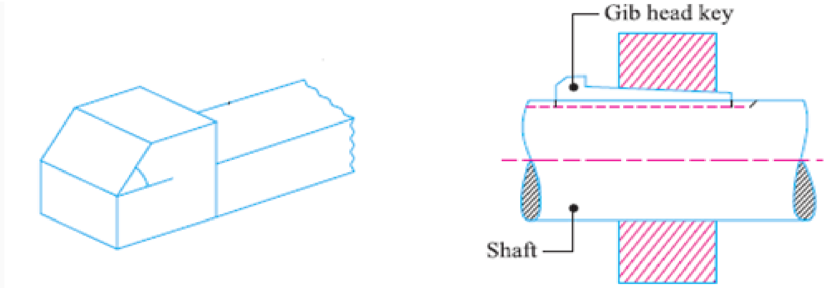


Figure 15.2 Gib Head Key

Feather key: Feather key is a parallel key made as an integral part of the shaft with the help of machining or using set-screws. It permits axial movement and has a sliding fit in the key way of the moving piece. Feather keys are shown in Figure 15.3.

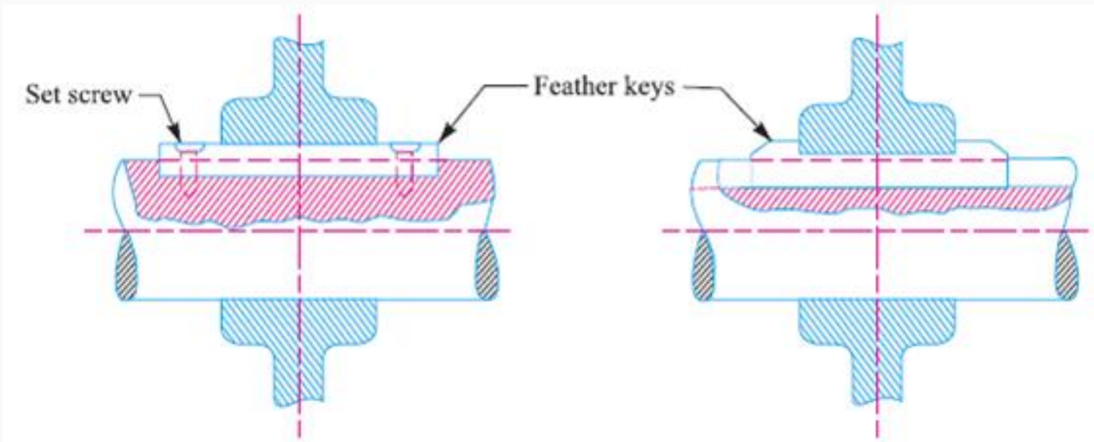


Figure 15.3 Feather Key

Woodruff key: Woodruff key is a sunk key in the form of a semicircular disc of uniform thickness. Lower portion of the key fits into the circular keyway of the shaft. It can be used with tapered shafts as it can tilt and align itself on the shaft. But the extra depth of keyway in the shaft increases stress concentration and reduces strength of the shaft. Woodruff key is shown in Figure 15.4.

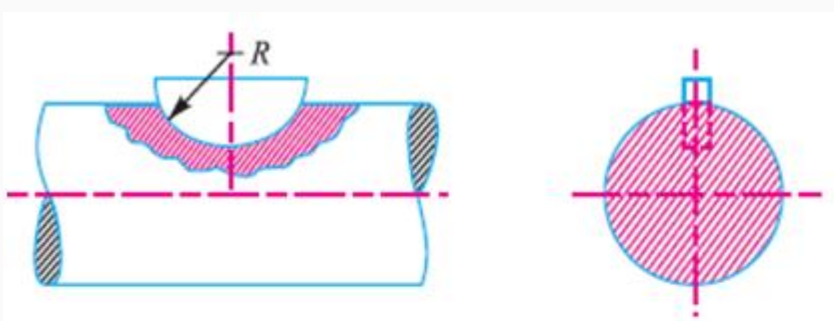
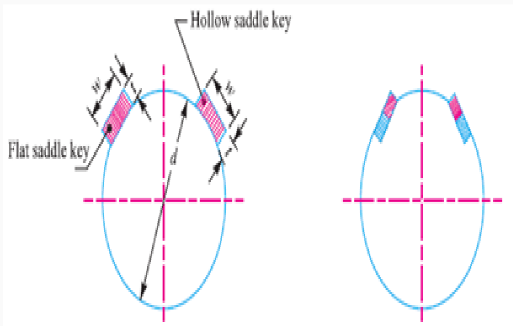


Figure 15.4 Woodruff Key

15.2.2 Saddle Keys

Slot for this type of is provided only in the hub as shown in Figure 15.5. Torque is transmitted by friction only and cannot therefore transmit high torque and is used only for light applications. The saddle keys are of two types: Flat Saddle Key and Hollow Saddle Key. In flat saddle key, the bottom surface touching the shaft is flat and it sits on the flat surface machined on the shaft. Hollow saddle key has a concave surface at the bottom to match the circular surface of the shaft. Chances of slip in case of the flat saddle key are relatively lesser and can transmit more power than the hollow saddle key.



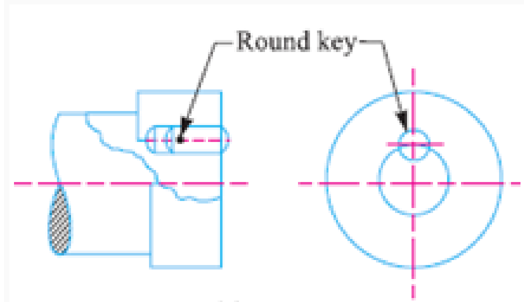


Figure 15.5 Saddle Keys Figure 15.6

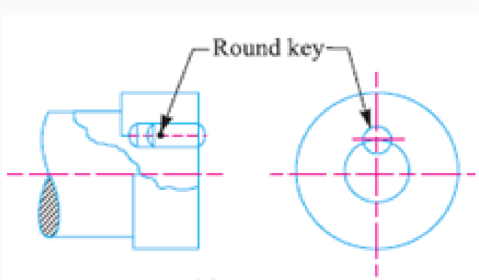
Tangent Keys

15.2.3 Tangent Keys

Tangent keys are shown in Figure 15.6. These are used to transmit high torque. They may be used as a single key or a pair at right angles. Single tangent key can transmit torque only in one direction.

15.2.4 Round Keys

The round keys have a circular cross-section and fit into holes drilled partly in the shaft and partly in the hub. Slot is drilled after the assembly so the shafts can be properly aligned. These are used for low torque transmission. Round keys are shown in Figure 15.7.



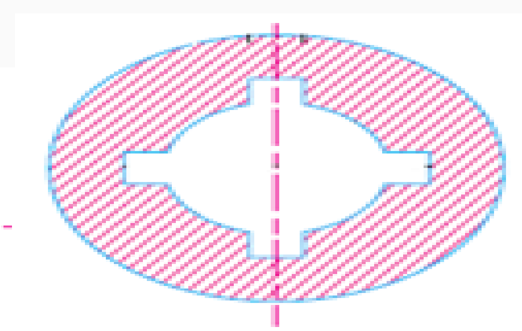


Figure 15.7 Round Key Figure 15.8 Splines

15.2.5 Splines

A number of keys made as an integral part of the shaft are called splines. Keyways are provided in the hub. These are used for high torque transmission e.g. in automobile transmission. Splines also permit the axial movement. Splines are shown in Figure 15.8.

15.3 Design of Sunk Keys

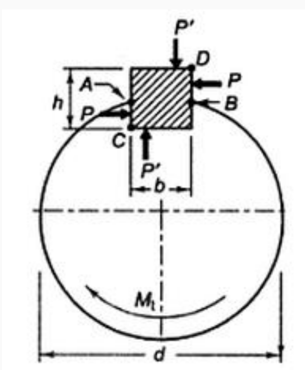
Figure 15.9 shows the forces acting on a rectangular key having width w and height h. Let l be the length of the key. Torque is transmitted from the shaft to the hub through key. Shaft applies a force P on the key and the key applies an equal force on the hub. Therefore the key is acted upon by two equal forces of magnitude P, one applied by the shaft (on the lower portion) and the other because of the reaction of hub (on the upper portion).

As these two forces are not in same plane, they constitute a couple which tries to tilt the key. Therefore equal and opposite forces P' also act on the key, which provide a resisting couple that keeps the key in position.

As the exact location of force P is not known, to simplify the analysis it is assumed that the force P acts tangential to the shaft. If T is the torque transmitted,

$$P = \frac{T}{d/2}$$

where, d = diameter of the shaft



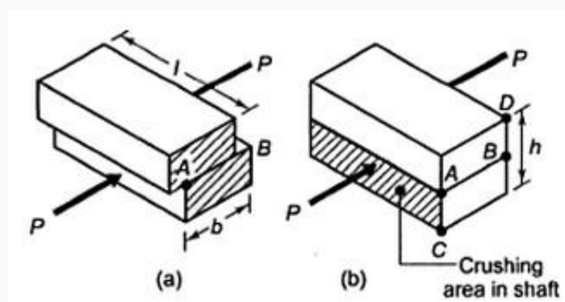


Figure 15.9 Forces Acting on KeyFigure 15.10 Failure of Key a.

Shear Failure b. Crushing Falure

In the design of key two types of failures are considered, shear failure and crushing failure.

Area resisting shear failure = w l

Shear stress, $\tau = \frac{P}{bl} \leq [\tau]$

Crushing Area = $l\,h/2$

Crushing stress, $\sigma_{crushing} = \frac{P}{l\ h/2} \leq [\sigma_c]$

Tables are available which give standard cross-sections for square and rectangular keys corresponding to different shaft diameters. But in the absence of such data, following relations are generally used:

For Rectangular Key: w = d / 4 and h = d / 6

For Square Key: w = h = d /4

For a known diameter of shaft, w and h can be calculated using these relations and then using the above strength equations required length of the key is calculated for given values of allowable stresses. Length is calculated both for shear and crushing and then maximum value out of the two is considered.

LESSON 16 DESIGN OF COUPLINGS

16.1 Introduction

Couplings are used to connect two rotating shafts to transmit torque from one to the other. For example coupling is used to connect the output shaft of an electric motor to the input shaft of a hydraulic pump.

16.2 Types of Shafts Couplings

Rigid Couplings

Rigid Couplings are used to connect two shafts which are perfectly aligned. These are simple and inexpensive.

Rigid Couplings are of following types:

1. Sleeve or Muff Coupling
2. Clamp or Split-muff or Compression Coupling
3. Flange Coupling

Flexible Couplings

Flexible couplings are used to connect two shafts having lateral or angular misalignment. Flexible elements provided in flexible coupling absorb shocks and vibrations.

Flexible Couplings are of following types:

1. Bushed pin type Coupling
2. Universal Coupling
3. Oldham Coupling

16.3 Muff Coupling

16.3.1 Introduction

Assembly of muff coupling is shown in Figure 16.1. Sleeve, a hollow cylinder, is fitted on the ends of input and output shaft with the help of a sunk key. Torque is transmitted from input shaft to the sleeve through key and from the sleeve to the output shaft through the key again. It is simple to design and manufacture but difficult to assemble and dismantle. It requires more axial space and has small radial dimensions. Sleeve is made of cast iron and for it a larger factor of safety of 6-8 is used on the ultimate strength. Standard proportions used for sleeve are:

Outer diameter of the sleeve, $D = 2d + 13$

Length of the sleeve, $L = 3.5d$

where d is the diameter of the shaft.

So the muff coupling has three main components: shafts, sleeve and key.

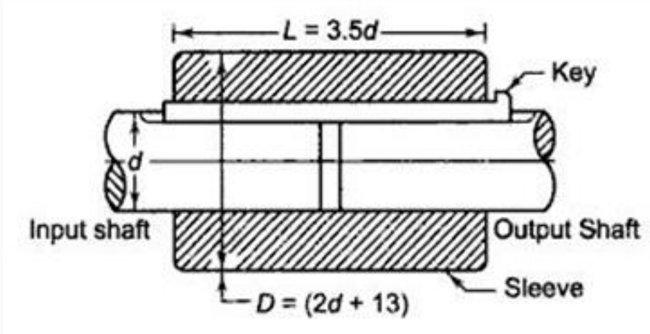


Figure 16.1 Muff Coupling

16.3.2 Design

16.3.2.1 Design of Shafts

Shafts are designed on the basis of torsional shear stress induced because of the torque to be transmitted. Shear stress induced in shaft for transmitting torque, T is given by,

$$\tau = \frac{T\,r}{J} \leq [\tau]$$

Where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation

r = Distance from neutral axis to the outer most fibre = d/2

So dimensions of the shaft can be determined from above relation for a known value of allowable shear stress, [τ].

16.3.2.2 Sleeve Design

As discussed earlier, following relations are used to calculate the dimensions.

D = 2d + 13 L = 3.5d

Then the torsional shear stress in the sleeve is checked considering it as a hollow shaft.

Shear stress, $\tau = \frac{T\,r}{J} \leq [\tau]$

where, T = Twisting moment (or torque) to be transmitted

J = Polar moment of inertia about the axis of rotation

r = Distance from neutral axis to the outer most fibre = D/2

16.3.2.3 Design of Key

Cross-section of the key is taken from the table corresponding to the shaft diameter or relations (square key) or and (for rectangular key) are used to find the cross-section, where w is width and h is the height of the key.

Length of key in each shaft, .

The keys are then checked in shear and crushing.

Shear stress,	$\tau = \dfrac{P}{wl} \leq [\tau]$
Crushing stress,	$\sigma_{crushing} = \dfrac{P}{l\ h/2} \leq [\sigma_c]$

16.4 Clamp Coupling

16.4.1 Introduction

Clamp coupling is also known as split-muff coupling or compression coupling. In this coupling, sleeve or muff is made in two halves, which are split along the plane passing through the axes of the shafts. These two halves are clamped together with the help of bolts, which are placed in recesses made in the sleeve halves. Dynamic balancing of clamp coupling is difficult making it unsuitable for high speed applications. It is also unsuitable for shock loads. Assembly and dismantling is easier for this coupling. Figure 16.2 shows Clamp Coupling assembly.

A small clearance is provided between the two halves of the sleeve along the parting line and the force due to clamping of bolts creates frictional force between the surface of the shafts and inner surface of sleeve halves. Torque is transmitted by means of this frictional force and through the key, from the input shaft to the sleeve and from sleeve to the output shaft. It is not possible to find out the exact percentage of torque transmitted by friction and by the key. Therefore, for designing the bolts it is assumed that whole of the torque is transmitted by friction and while designing the key, it is assumed that whole of the torque is transmitted by it.

Design is similar to the design of muff coupling and an additional calculation is required for designing the bolts.

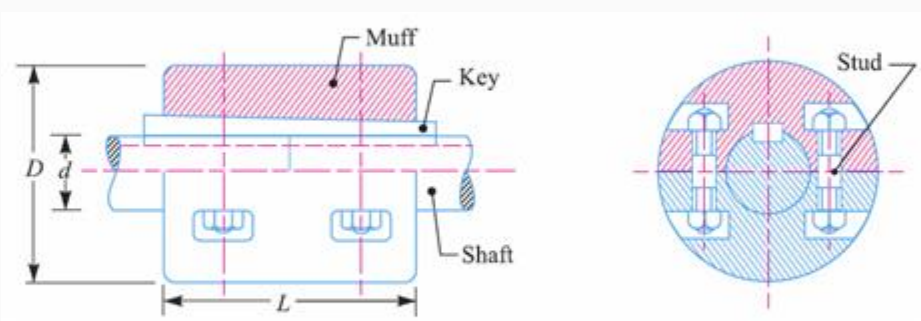


Figure 16.2 Clamp Coupling

16.4.2 Design

Design of Clamp Coupling is similar to the design of muff coupling and an additional calculation is required for designing the bolts.

16.4.2.1 Design of Shafts

Same as discussed in sleeve coupling.

16.4.2.2 Sleeve Design

Same as discussed in Sleeve Coupling

16.4.2.3 Design of Key

Same as discussed in Sleeve Coupling

16.4.2.4 Design of Bolts

Bolts are designed assuming that whole of the torque is transmitted by friction between sleeve and shafts.

Let $[\sigma_t]$ = permissible tensile stress of bolts

d_c = core diameter of bolts

n = number of bolts

Clamping force of each bolt,

$$P_b = \frac{\pi}{4} d_c^2 [\sigma_t]$$

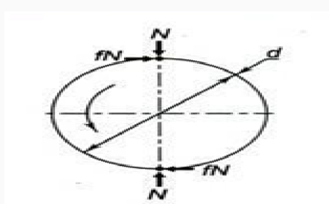


Figure 16.3 Forces acting on Bolts

Assuming that half of the bolts apply clamping force on one shaft and half of the bolts on the other. Clamping force on each shaft, $N = \frac{\pi}{4} d_c^2 [\sigma_t] \frac{n}{2}$

Frictional Torque, $T_f = \mu N d$

where, m = coefficient of friction between shafts and sleeve.

Above two relations can be used to find the core diameter of the bolts by equating to the total torque transmitted.

16.5 Flange Coupling

16.5.1 Introduction

Flange coupling consists of two flanges keyed to the shafts. The flanges are connected together by means of bolts arranged on a circle concentric to shaft. Power is transmitted from driving shaft to flange on driving shaft through key, from flange on driving shaft to the flange on driven shaft through bolts and then to the driven shaft through key again. Projection is provided on one of the flanges and a corresponding recess is provided in the other for proper alignment. Flange coupling is of two types – unprotected and protected. These are shown in Figure 16.3. If in case failure of bolts occurs during the operation, the bolts may hit the operator in case of unprotected flange coupling. To avoid this, protective circumferential flanges are provided in the protected type flange coupling.

Flange of a protected type flange coupling has three distinct regions – inner hub, flanges and protective circumferential flanges. Following standard proportions are used in the design of flange coupling:

Outer diameter of hub, $D = 2\ d$

Pitch circle diameter of bolts, $D_1 = 3\ d$

Outer diameter of flange, $D_2 = 4\ d$

Length of the hub, $L = 1.5\ d$

Thickness of flange, $t_f = 0.5\ d$

Thickness of protective circumferential flange, $t_p = 0.25\ d$

where d is the diameter of shafts to be coupled.

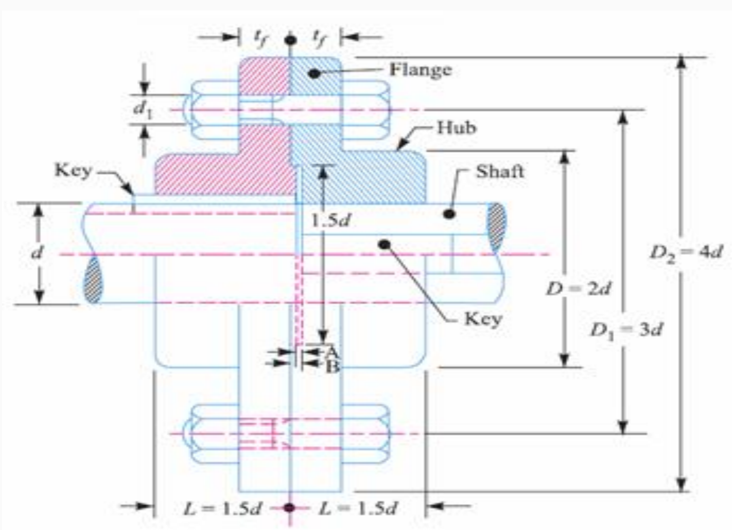


Figure 16.4 Flange Coupling

16.5.2 Design

16.5.2.1 Design of Shafts

Same as discussed in sleeve coupling.

16.5.2.2 Design of Hub

Hub is designed considering it as a hollow shaft, with inner diameter equal to diameter of shafts and outer diameter double of that. It is checked for torsional shear stress.

Shear stress, $\tau = \frac{T\,r}{J} \leq [\tau]$

Where T = Twisting moment (or torque) to be transmitted

J = Polar moment of inertia about the axis of rotation

r = Distance from neutral axis to the outer most fibre = D/2

16.5.2.3 Design of Key

In this case two separate keys are used for the two shafts. Key is designed as discussed earlier. In this case, length of key, (length of the hub)

16.5.2.4 Design of Flange

The flange is subjected to shear at the junction of the hub as it transmits torque through the bolts. Area resisting shear

where, is the thickness of the flange.

If T is the torque to be transmitted, tangential force,		$F = \frac{T}{d/2}$
Shear stress,	$\tau = \frac{F}{\pi D t_f} \leq [\tau]$	

16.5.2.5 Design of Bolts

Due to transmission of torque, force acts perpendicular to the bolt axes and the bolts are subjected to shear and crushing stresses. Let n be the total number of bolts.

Force acting on each bolt,	$F_b = \frac{T}{n\,D_1/2}$

where D_1 is the pitch circle diameter of bolts.

Area resisting shear	$= \frac{\pi}{4} d_c^2$
where, d_c = core diameter of bolts	
Shear stress,	$\tau = \frac{F_b}{\frac{\pi}{4} d_c^2} \leq [\tau]$
Area under crushing	
Crushing stress,	$\sigma_{crushing} = \frac{F_b}{d_c t_f} \leq [\sigma_c]$

MODULE 5.

LESSON 17 SPRINGS

17.1 Introduction

Mechanical spring is an elastic member whose primary function is to deflect under load and to recover its original shape and position when load is released. Springs are used to absorb shocks and vibrations (e.g. vehicle suspension system), store energy (e.g. springs n clocks and toys), measure force (e.g. spring balance) and to apply force and control motion (e.g. cam and follower).

17.2 Types of Springs

Springs are classified based on their shape. Some of the important types of springs are as follows:

17.2.1 Helical Springs

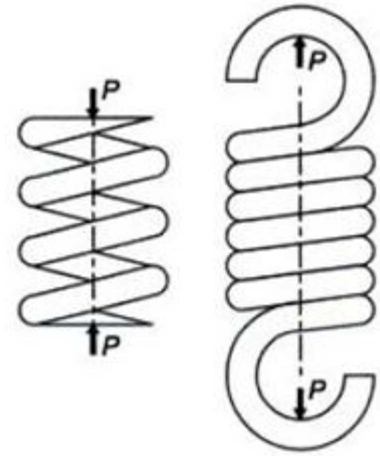


Figure 17.1 Helical Spring

The helical springs are made from a wire coiled in the form of a helix as shown in Figure 1. Cross-section of the wire is generally circular and it can be square or rectangular also. Helical springs are easy to manufacture, reliable and have a constant spring rate i.e spring deflection is directly proportional to the force acting. These are of two types – compression helical springs and tension helical springs. Compression helical springs are designed to take compressive loads and they get compressed under the loading and the tension helical springs are designed to take tensile loads and they get elongated under the external loads. The load acts along the axes of these springs. In helical sprigs, the wire is subjected to torsional shear stress.

Helical springs are also classified as closely-coiled and open-coiled springs. In closely-coiled springs, wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix i.e. the helix angle is very small, usually less than 10°. In open-coiled helical springs, the wire is so coiled that there is a gap between the two consecutive turns i.e. the helix angle is large.

17.2.2 Conical Springs

Conical spring works in compression and is used where variable spring rate is required. Wire is coiled in the form of a cone as shown in Figure 17.2.

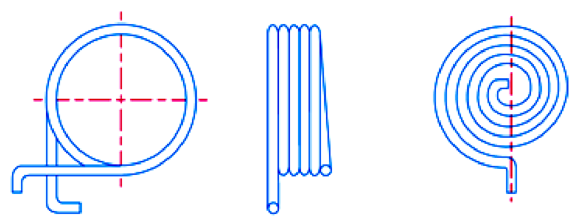
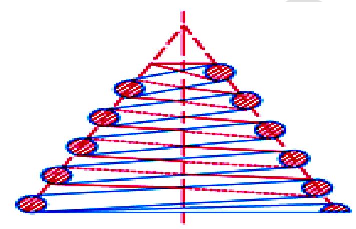


Figure 17.2 Conical Spring Figure



17.3 Torsion Spring

17.2.3 Torsion Springs

Torsion springs are of two types - helical and spiral. These springs are loaded in torsion and the load tends to wind up the spring. Wire is subjected to bending moment in this case. Torsion springs are shown in Figure17.3.

17.2.4 Leaf or Laminated Springs

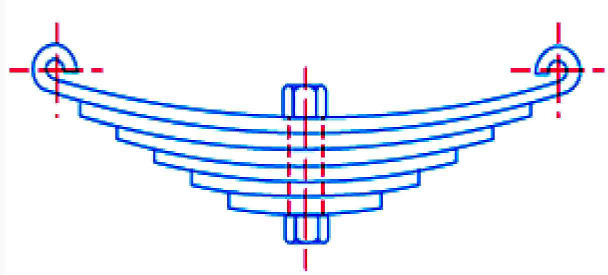


Figure 17.4 Leaf Spring

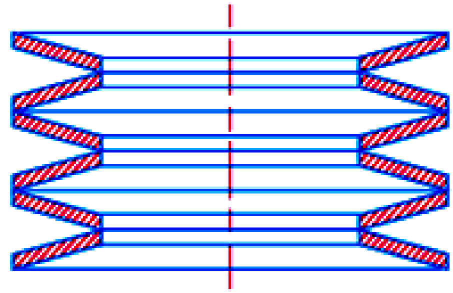


Figure 17.5 Belleville Spring

The laminated or leaf spring consists of a number of flat plates (known as leaves), usually of semi-elliptical shape, of varying lengths held together by means of clamps and bolts. These are mostly used in automobiles. A typical leaf spring is shown in Figure 17.4.

17.2.5 Disc or Belleville Springs

These springs consists of a number of conical discs held together as shown in Figure 17.4. Belleville springs have high spring rate and are compact.

17.3 Spring Materials

Spring materials should have high yield strength and low modulus of elasticity so that they don't permanently deform under the applied loads. Springs are made of the materials that can be formed (rolled or drawn) to high strength and retain enough ductility to form or the alloys that can be heat treated to high strength and ductility before or after forming. Both hot and cold working processes are used for manufacturing springs. The selection of process depends upon size of material, spring index and the properties desired. Winding of the spring induces residual stresses due to bending, which are released with the help of heat treatment. Materials used for springs are plain carbon steels, alloy steels and also the non-ferrous materials like phosphor bronzes, spring brass, beryllium copper and various nickel alloys.

17.4 Terminology of Helical Springs

Figure 17.6 shows a helical spring subjected to compressive force, W.

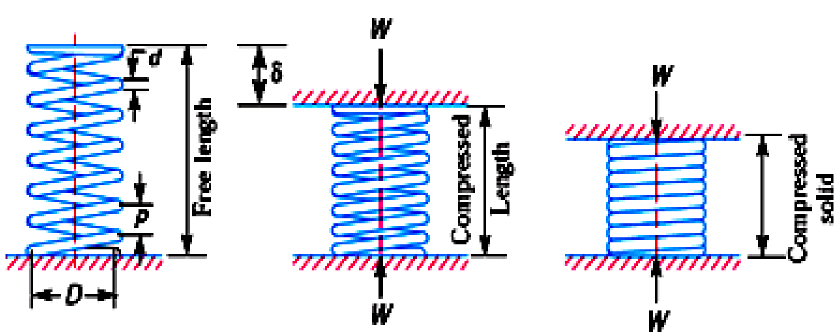


Figure 17.6 Compression Helical Spring

Solid Length: Length of the spring when it is compressed so that the coils touch each other.

Solid Length , Ls = n.d

where n = numbers of coils and d = wire diameter

Compressed Length: Length of the spring, when it is subjected to maximum compressive force.

Even under the worst load, minimum clearance is maintained between the two adjacent coils so that they don't clash with each other. It is called clash allowance and is generally taken as 15% of the maximum deflection.

Free Length: Length of the spring in the free or unloaded condition.

$$\begin{aligned} Free\ Length &= Solid\ length + Maximum\ compression + clash\ allowance \\ &= nd + \delta_{max} + 0.15\delta_{max} \end{aligned}$$

Spring Index: Ratio of the mean diameter of the coil to the diameter of the wire .

$Spring\ index, C = D / d$

Spring Rate/Spring Stiffness/Spring Constant: Force required to produce unit deflection in the spring.

Spring Rate , k = W / σ where, and .

Pitch of the Coil: Axial distance between adjacent coils of the spring in uncompressed state.

Pitch of the Coil = p = Free Length / (n - 1)

17.5 Design of Helical Springs

17.5.1 Stress in Helical Springs

Under the compressive load, W acting on the spring, coil of the spring is subjected to two types of stresses – direct shear stress and torsional shear stress due to twisting of the coil.

Torque = T = W . D / 2

$$Torsional\ shear\ stress = \tau_1 = \frac{Tr}{J} = \frac{\frac{WD}{2} \cdot \frac{d}{2}}{\frac{\pi}{32} d^4} = \frac{8WD}{\pi d^3}$$

$$Direct\ shear\ stress = \tau_2 = \frac{W}{A} = \frac{4W}{\pi d^2}$$

$$Resultant\ shear\ stress = \tau = \tau_1 \pm \tau_2 = \frac{8WD}{\pi d^3} \pm \frac{4W}{\pi d^2} = \frac{8WD}{\pi d^3}\left(1 \pm \frac{d}{2D}\right) = \frac{8WD}{\pi d^3}\left(1 \pm \frac{1}{2C}\right)$$

$$Maximum\ shear\ stress = \tau_{max} = \tau_1 + \tau_2 = \frac{8WD}{\pi d^3}\left(1 + \frac{1}{2C}\right) = K_s \frac{8WD}{\pi d^3}$$

$$\text{where}, K_s = Shear\ stress\ factor = \left(1 + \frac{1}{2C}\right)$$

In the analysis above effect of stress concentration is not considered. AM Wahl introduced a factor called Wahl's Stress Factor, K that takes care of stress concentration also along with the shear stress.

According to Wahl, Maximum shear stress is given by,

Maximum shear stress = $\tau_{max} = K \frac{8WD}{\pi d^3}$

Wahl's Factor is given by,

Wahl's Factor , $K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$

17.5.2 Deflection in Helical Springs

$$Length\ of\ the\ wire, l = length\ of\ one\ coil \times number\ of\ coils = (\pi D).n = \pi Dn$$

If J is polar moment of inertia and G is Modulus of Rigidity of coil, angular deflection of the wire due to twisting is given by,

$$Angular\ Deflection, \theta = \frac{TL}{JG} = \frac{W.\frac{D}{2}.\pi Dn}{\frac{\pi}{32} d^4.G} = \frac{16WD^2 n}{Gd^4}$$

$$\text{Also}, Spring\ Stiffness = \frac{W}{\delta} = \frac{Gd^4}{8d^3 n} = \frac{Gd}{8C^3 n} = Constant$$

LESSON 18 LEAF SPRINGS

18.1 Introduction

A cantilever beam or a simply supported beam can be used as a spring. For example diving board used in a swimming pool is a spring in the form of a cantilever beam. Such springs are called leaf springs. Basic equations of beam can be used to find stress and deflection in the leaf springs.

Let us consider a flat plate with width 'B', thickness 't' and length 'l'. If this plate is used as a leaf spring in the form of a cantilever, subjected to load 'F', as shown in figure 18.1, maximum stress and deflection in the plate is given by:

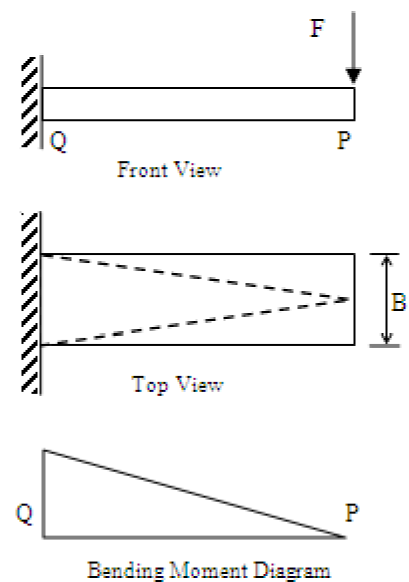


Figure 18.1 Cantilever Beam

$$Maximum\ Bending\ Stress, \sigma_b = \frac{My}{I} = \frac{6Fl}{Bt^2}$$

$$Maximum\ Deflection, y_{max} = \frac{Fl^3}{3EI} = \frac{4Fl^3}{Bt^3 E}$$

As the bending moment varies linearly along the length of the plate as shown by the bending moment diagram, maximum value of bending stress occurs at the point of support of the cantilever (Q) and it decreases linearly as we move away from the point of support and is zero at the point P. Material can be saved if the cross-sectional area is reduced from Q to P in such a way that each section has same binding stress. Beam so formed is called beam of uniform strength.

Cross-section can be reduced linearly by reducing either the width or thickness. Generally the thickness is kept constant and width 'B' is reduced as shown by dotted lines in the top view of plate shown in figure 18.1. If we consider only binding stress, we can have zero width (B = 0) at point P. But practically we cannot have zero width and also at each point in the beam the cross section must be enough to resist the shear force, which also exists in addition to bending. So we need to have some minimum value of B at point P also. Beam of uniform strength so formed has the same value of maximum binding stress as that in case of beam of uniform cross-section but maximum deflection increases. Higher deflection means that they have more resistance and capacity to absorb impact energy and can store more energy in comparison to the spring of uniform cross-section with same value of maximum binding stress.

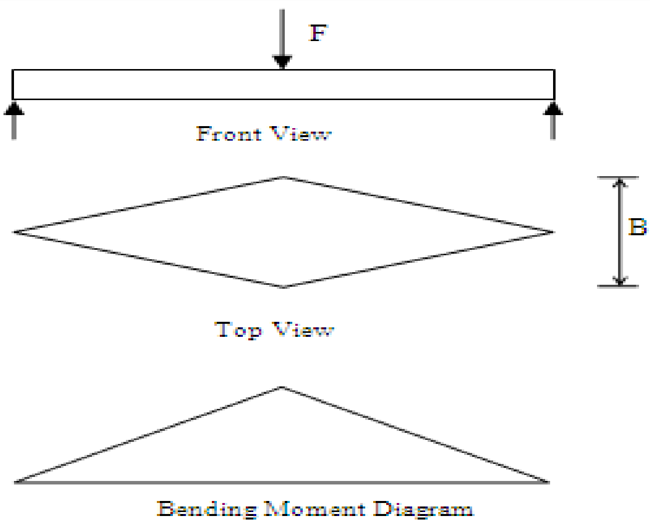


Figure 18.2 Simply Supported Beam

In a similar way simply supported a beam of uniform cross-section and uniform strength can be compared. Figure 18.2 shows simply supported beam of uniform strength. Comparison of stress and maximum deflection is given in table 18.1.

Table 18.1 Stress & Deflection in Beams of Uniform Cross-section & Uniform Strength

		Maximum Bending Stress	Maximum Deflection
Cantilever Beam	Uniform Cross-section	$\frac{6Fl}{Bt^2}$	$\frac{4Fl^3}{EBt^3}$
	Uniform Strength	$\frac{6Fl}{Bt^2}$	$\frac{6Fl^3}{EBt^3}$
Simply Supported Beam	Uniform Cross-section	$\frac{3Fl}{2Bt^2}$	$\frac{Fl^3}{4EBt^3}$
	Uniform Strength	$\frac{3Fl}{2Bt^2}$	$\frac{3Fl^3}{8EBt^3}$

18.2 Laminated Springs

The width 'B' may become too large if only a single leaf spring is used. In order to make the spring compact, the original plate may be cut into a number of strips which can be placed one below the other and assembled using a clamp as shown in figure 18.3.

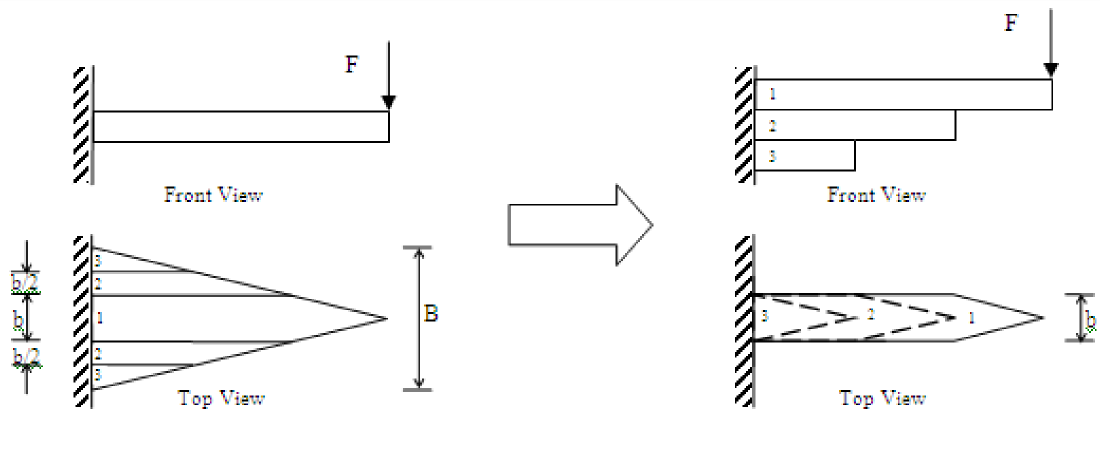


Figure 18.3 Laminated Spring

If friction between the strips (also called leaves) is neglected, stress and deflection equations used for original plate can be used for this arrangement by replacing 'B' with 'nb', where n is number of strips or leaves and b is width of each strip. Relation of maximum bending stress and maximum deflection of cantilever beam and simply supported beam of uniform strength, in the form of a laminated spring, are given in table 18.2.

Table 18.2 Stress & Deflection in Beams Used as Laminated Springs

	Maximum Bending Stress	Maximum Deflection
Cantilever Beam	$\frac{6Fl}{nbt^2}$	$\frac{6Fl^3}{Enbt^3}$
Simply Supported Beam	$\frac{3Fl}{2nbt^2}$	$\frac{3Fl^3}{8Enbt^3}$

Laminated springs are used in automobile suspension, railway carriages, coaches etc.

18.3 Semi-elliptical Laminated Springs

Semi-elliptical leaf spring is the most popular and widely used leaf spring. It consists of a number of flat plates or leaves of semi-elliptical shape. U-bolts and center clip are used to hold these leaves together. To keep the leaves aligned and avoid lateral shifting, rebound clips are used. Ends of the longest leaf are bent to form eyes. This longest leaf is called 'master leaf' and other smaller leaves are called 'graduated leaves'. One or two extra full length leaves are generally provided along with the master leaf, to increase strength against the transverse shear force. Typical semi-elliptical leaf spring is shown in figure 18.4.

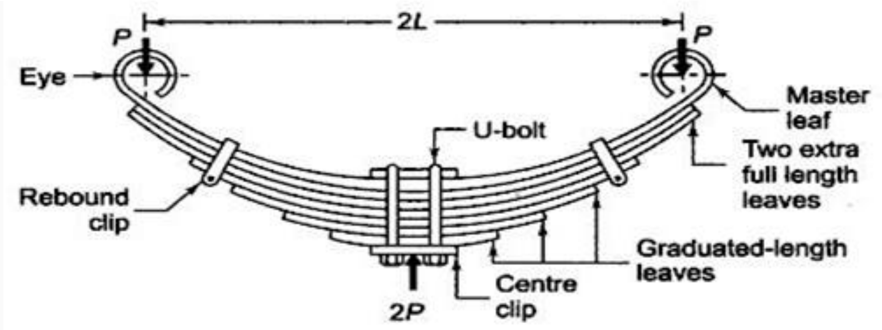


Figure 18.4 Semi-elliptical Leaf Spring

When no external load is acting, the spring is curved or cambered. Camber is the perpendicular distance between the reference line and the master leaf and its magnitude is such that the spring is approximately straight under the max static load. Center of the spring is fixed to the axle of the automobile.

For the analysis purpose, leaves are divided into two groups: i. Master leaf and graduated leaves ii. Extra full length leaves. Let

n_g = number of graduated-length leaves including master leaf

n_f = number of extra full length leaves

n = total number of leaves

b = width of each leaf

t = thickness of each leaf

L = half the length of semi-elliptical spring

F = force applied at the ends of the spring

F_f = part of F taken by extra full length leaves

F_g = part of F taken by graduated leaves and master leaf

Now, this spring can be treated as a simply supported beam of length 2L, with load 2F acting at its centre or for simplification of analysis, half portion of it can be considered as a cantilever of length L, with one end fixed (centre of the spring which is fixed with axle) and load F acting on the other. Now the first group of leaves i.e. master leaf along with the graduated leaves can be considered as a cantilever beam of uniform strength as discussed in previous article. Similarly group of extra full length leaves can be considered as a cantilever beam of uniform cross-section. Therefore the relations given in table 18.1 can be used to write the stress and deflections in these leaves by replacing B with 'n_f b' and 'n_g b' and considering the the share of load taken by them, as given in table 18.3 below.

Table 18.3 Stress & Deflection in Leaves of Semi-elliptical Leaf Spring

Group of Leaves	Can be Treated as	Maximum Bending Stress	Maximum Deflection
Extra Full Length Leaves	Cantilever Beam of Uniform Cross-section	$\sigma_{b_f} = \frac{6F_f L}{n_f b t^2}$	$\delta_f = \frac{4F_f L^3}{E n_f b t^3}$
Master Leaf and Graduated Leaves	Cantilever Beam of Uniform Strength	$\boldsymbol{\sigma_{b_g} = \frac{6F_g L}{n_g b t^2}}$	$\boldsymbol{\delta_g = \frac{6F_g L^3}{E n_g b t^3}}$

As deflection in the full length leaves and graduated leaves is equal,

$$\delta_g = \delta_f$$

$$\frac{6F_g L^3}{E n_g b t^3} = \frac{4F_f L^3}{E n_f b t^3}$$

$$\frac{F_g}{F_f} = \frac{2n_g}{3n_f}$$

Also, $$F_g + F_f = F$$

Solving, $$F_g = \frac{2n_g F}{2n_g + 3n_f} \quad and \quad F_f = \frac{3n_f F}{2n_g + 3n_f}$$

Final relation for stress and deflection can be written as,

$$\sigma_{b_g} = \frac{12FL}{(2n_g + 3n_f)bt^2} \quad and \quad \sigma_{b_f} = \frac{18FL}{(2n_g + 3n_f)bt^2}$$

$$\delta = \delta_g = \delta_f = \frac{12FL^3}{(2n_g + 3n_f)Ebt^3}$$

18.4 Nipping of Leaf Springs

It is clear from the above equations that the stress in the full length leaves is 50% higher then the graduated leaves. In addition to this, the full length leaves als

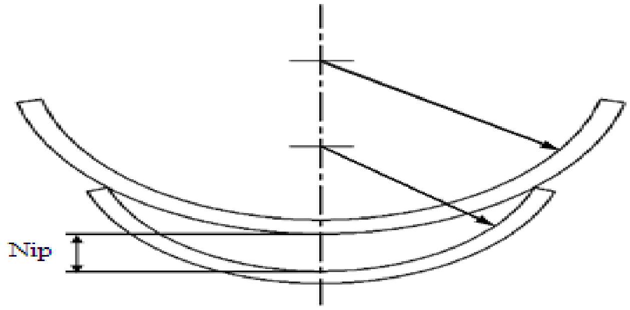


Figure 18.5 Nipping of Leaf Spring

o have to carry transverse load, longitudinal load and also the load due to twisting, which further increases the stress in the full length leaves. In order to reduce the value of maximum stress in the full length leaves and to equalize the stresses in different leaves the spring may be pre-stressed in such a way that the full length leaves are stressed in the opposite direction to the stresses due to external load. This is done by giving different radius of curvature to the full length leaves than the graduated leaves. Full length leaf is given higher radius of curvature than the adjacent leaf and it is reduced further for smaller length leaves. Initial gap between the full length leaf and graduated leaf is called nip and this process is known as nipping. The leaves get pre-stressed when they are clamped together. As discussed earlier, pre-stressing leads to equalization of stresses in graduated leaves and full length leaves, therefore

$$\sigma_{b_g} = \sigma_{b_f}$$

$$\frac{6F_g L}{n_g b t^2} = \frac{6F_f L}{n_f b t^2}$$

$$\frac{F_g}{F_f} = \frac{n_g}{n_f}$$

Also, $$F_g + F_f = F$$

$$F_g + F_f = F$$

$$\therefore F_g = \frac{n_g}{n} F \quad and \quad F_f = \frac{n_f}{n} F$$

$$Nip, C = \delta_g - \delta_f = \frac{2FL^3}{Enbt^3}$$

Initial pre-load required to close the gap C can be determined as follows:

Nip is equal to the sum of initial deflections in the full length leaves and graduated leaves,

$$C = \delta_{g_i} + \delta_{f_i}$$

$$\frac{2FL^3}{Enbt^3} = \frac{6(F_i/2)L^3}{En_g bt^3} + \frac{4(F_i/2)L^3}{En_f bt^3}$$

$$F_i = \frac{2n_g n_f F}{n(2n_g + 3n_f)}$$

Resultant stress in the extra full length leaves is then given by,

$$\sigma_{b_f} = \frac{6(F_f - F_i/2)L}{n_f b t^2}$$

Substituting the values of F_f and F_i

And considering that the stress in all the leaves should now be equal,

$$\sigma_b = \sigma_{b_g} = \sigma_{b_f} = \frac{6FL}{nbt^2}$$

MODULE 6.

LESSON 19 BELTS

19.1 Introduction

To transmit power from the prime mover to the driven machine either flexible or non-flexible drive elements are used. Gears are an example of rigid or non-flexible drives whereas belts, chains and ropes are flexible drive elements. Flexible elements are used when there is larger centre distance between the shafts to be connected. Flexible drives are simple in construction, are less noisy, have low initial and maintenance cost and help in absorbing shock loads and damping vibrations. Low and variable velocity ratio is the main disadvantages of these drives.

19.2 Belt Drive

A belt drive consists of two pulleys attached to each shaft and an endless belt wrapped around them with some initial tension. Power is transmitted from the driver pulley to the belt and from the belt to the driven pulley with the help of friction. Friction between belt and pulley surface limits the maximum power that can be transmitted. If this limiting value is exceeded, belt starts slipping. Belts have limited life and should be periodically inspected for wear, aging, and loss of elasticity and should be replaced at the first sign of deterioration.

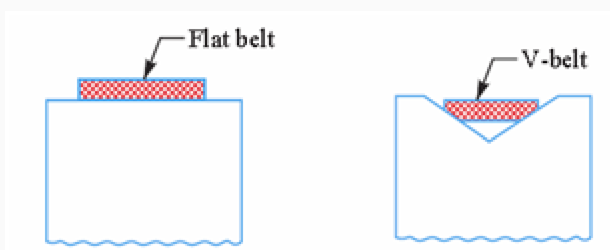


Figure 19.1 Flat and V-Belts

Belts having rectangular cross-section are called flat belts and those with the trapezoidal cross-section are known as V-belts. Both the sections are shown in Figure 19.1.

Flat belts are used to transmit moderate amount of power between shafts less than 6m apart. V-belts are used along with pulleys having similar cross-section as that of the belt. These are used for higher power transmission between parallel shafts having smaller centre distance. Included angle in the groove of the pulley is kept smaller than the included angle of the belt cross-section. Due to this, bottom of the belt doesn't touch the pulley and power is transmitted by friction between sides of the belt and inner walls of the pulley. This helps in larger amount of power transmission and lesser slip. To further increase the power transmission capacity, multiple V-belt system is used, in which power is transmitted from one shaft to the other with the help of more than one V-belts running on pulleys having multiple grooves.

Belts with circular cross-section or ropes are used to transmit large power between shafts more than 10m apart. Grooved pulleys called sheaves are used with the ropes. Rope fits in the groove, gripping its sides, reducing the chances of slip. Sheaves with multiple grooves are used to further increase the power transmission capacity. Pulleys used with flat belts are slightly crowned to keep the belt running centrally on the pulley. V-groove pulleys are used with V-belts, which have a groove deeper than the cross-section of belt so that bottom of the belt doesn't touch the pulley. This leads to wedging action, due to which slip is lesser and it can transmit more power than a flat belt.

19.3 Types of Belt Drives

Commonly used belt arrangements are shown in Figure 19.2

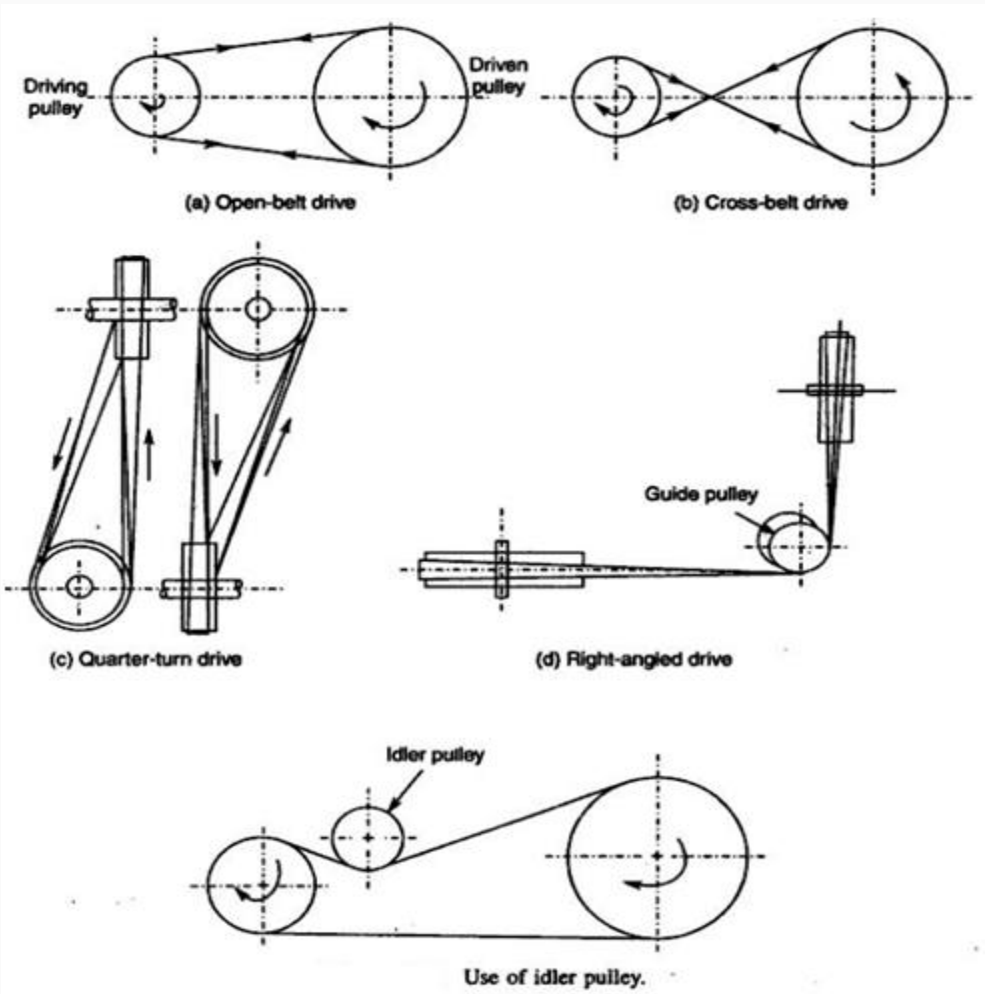


Figure 19.2 Types of Belt Drives

These different arrangements are used depending upon the required direction of rotation of driven shaft, plane of rotation of the driving and driven shafts and the angle between the axes of shafts. When the direction of rotation of driver and driven pulleys is required to be the same open-belt drive is used. If the driven pulley is to be rotated in the opposite direction to that of the driving pulley, cross-belt drive is used. Due to larger angle of contact between the belt and the pulley, cross-belt drive can transmit more power than an open-belt drive but the wear rate is higher for cross-belt drive as the belt bends in two different planes. For the shafts arranged at right angles, quarter turn belt drive is used. Idler pulleys are used to increase the angle of contact, obtain higher velocity ratio and increase the belt tension. Belt is subjected to reversed bending when idler pulley is used which may reduce the life of the belt.

19.4 Belt Material

The belt material should have high coefficient of friction, tensile strength, wear resistance and flexibility and low flexural rigidity. Commonly used belt materials are:

Leather: The most widely used material for belts is oak-tanned and chrome-tanned leather. Leather belts are available in two varieties-oak tanned and chrome tanned. Oak-tanned leather is used for ordinary applications and chrome-tanned is used for applications where belts are exposed to moisture, oil or chemicals. Leather strips are cemented together to increase thickness and life of the belt. Belts are specified according to the number of layers (called plies) e.g. single, double or triple ply belts.

Rubber: Cotton duck or canvas impregnated with rubber is also used as belt material. Cotton duck or canvas provides strength whereas the rubber provides protection and increases coefficient of friction. These are generally available in three to ten plies. They should be vulcanized if they are to be exposed to high temperature, oil or grease; otherwise they get damaged very quickly. Service life of rubber belts is shorter than that of leather belts but these are cheaper.

Balata: These belts are similar to rubber belts except that cotton duck impregnated with balata gum instead of rubber. These belts are acid and water proof and also don't get affected by animal oils and alkalis but get seriously affected by the mineral oils. Balata belts have 20-40% higher strength than the rubber belts. Balata starts softening and becomes sticky at temperatures above 40°C. So these are not recommended for high temperature applications.

Cotton or Fabric: Fabric belts are made from a number of closely woven layers of cotton or canvas ducks. To make the belt waterproof, they are impregnated with linseed oil. Cotton belts are cheaper and are suitable for warm and damp conditions and require less attention. These are generally used in farm machinery; conveyors etc. Mechanical properties of fabric belts are at par with rubber belts.

Plastics: Plastic belts have plastic core of nylon canvas or thin plastic sheets surrounded by a layer of rubber. Plastic-cored belts have high strength almost twice of that of leather belts. These can be wrapped around very small pulleys and can be operated at high speeds.

19.5 Velocity Ratio

Velocity Ratio is the ratio of speeds of driver and driven pulley. Let

N_1, d_1 = speed (rpm) and diameter of driving pulley

N_2, d_2 = speed (rpm) and diameter of driven pulley t = thickness of belt

$$\text{Velocity Ratio}, \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

If thickness of the belt is considered,

$$\text{Velocity Ratio}, \frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

Slip of belt over the pulleys reduces the velocity ratio. Let

s_1 = percentage slip between driver and belt

s_2 = percentage slip between belt & driven pulley

s = total percentage slip

$$\text{Velocity Ratio}, \frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left[1 - \frac{s}{100}\right]$$

19.6 Length of the Belt

If d & D are diameters of smaller and larger pulleys respectively with C as the centre distance between the axes of the pulleys, length of the belt is given by,

$$L = 2C + \frac{\pi}{2}(d + D) + \frac{(d - D)^2}{4C} \text{ (for open belt drive)}$$

$$L = 2C + \frac{\pi}{2}(d + D) + \frac{(d + D)^2}{4C} \text{ (for cross belt drive)}$$

19.7 Angle of Wrap

Angle of wrap on smaller (a_s) and larger pulleys (a_l) are given by,

$$\alpha_s = 180° - 2\sin^{-1}\left(\frac{D - d}{2C}\right) \text{ (for open belt drive)}$$

$$\alpha_l = 180° + 2\sin^{-1}\left(\frac{D - d}{2C}\right) \text{ (for open belt drive)}$$

$$\alpha_s = \alpha_l = 180° + 2\sin^{-1}\left(\frac{D + d}{2C}\right) \text{ (for cross belt drive)}$$

19.8 Belt Tension

To increase friction and avoid slip, some initial tension is provided in the belt. Even when the belt is not transmitting any power, there is some initial tension (T_i) in it, provided to avoid slipping of the belt. Consider a flat belt drive shown in figure 19.3, with the smaller pulley as driver and the larger pulley as driven. When the driver starts rotating (say clockwise), it applies a tangential force on the belt and tends to rotate it. This leads to increase in tension on one side and decrease in tension on the other side of the belt by same amount, dT, which equals the tangential force applied by the pulley. Side of the belt in which the tension increases is called tight side (lower side in this case) and the side in which tension decreases is called slack side.

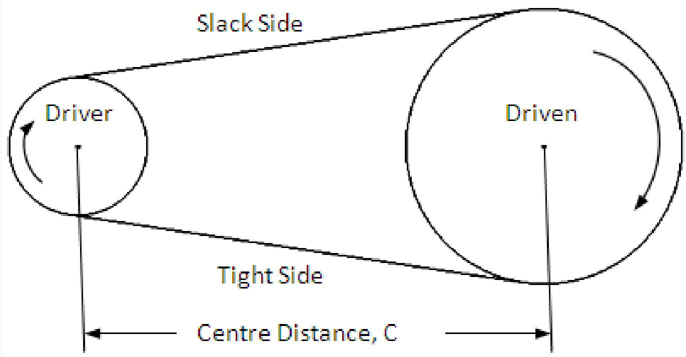


Figure 19.3 Belt Tensions

Tension in tight side, $T_1 = T_i + dT$

Tension in slack side, $T_2 = T_i - dT$

$$\text{Therefore, } T_i = \frac{T_1 + T_2}{2}$$

Ratio of tension in tight side and slack side is given by,

$$\frac{T_1}{T_2} = e^{\mu\theta} \qquad \text{(for flat belt drive)}$$

$$\frac{T_1}{T_2} = e^{\mu\theta}\operatorname{cosec}\beta \qquad \text{(for cross belt drive)}$$

Where, μ = coefficient of friction between belt & pulley

Θ = angle of contact

β = half of the groove angle of v-belt

When the belt operates at higher speeds, centrifugal force acts on it, which increases the tension in the belt. This additional tension in the belt due to the centrifugal force is called centrifugal tension and it can be proved that, centrifugal tension,

$T_c = mv^2$

where, m = mass of the belt per unit length

v = velocity of the belt in m/s

Therefore the tensions in the tight and slack side increase by an amount equal to T_c. The maximum tension in the belt then becomes,

$T = T_1 + T_C$

If b and t are width & thickness of a flat belt and [σ] is the maximum allowable stress in the belt, maximum permissible tension for it can be given by,

$$[T] = \text{maximum stress} \times \text{cross} - \text{sectional area} = [\sigma]\, bt$$

For belt to run safely, maximum tension in the belt should not cross the permissible limit i.e.

19.9 Power Transmitted by Belt Drive

Power transmitted by the belt is given by, $P = (T_1 - T_2)v$

$$\therefore P = \left[T_1 - \frac{T_1}{e^{\mu\theta}}\right]v = T_1 v\left[1 - \frac{1}{e^{\mu\theta}}\right]$$
$$= [T - T_c]v\left[1 - \frac{1}{e^{\mu\theta}}\right] = (T - mv^2)v\left[1 - \frac{1}{e^{\mu\theta}}\right]$$

To obtain the condition for maximum power transmission,

$$\frac{dP}{dv} = 0$$
$$\frac{d}{dv}\left[(T - mv^2)v\left(1 - \frac{1}{e^{\mu\theta}}\right)\right] = 0$$
$$\left(1 - \frac{1}{e^{\mu\theta}}\right)\frac{d}{dv}(Tv - mv^3) = 0$$
$$T - 3mv^2 = 0 \quad \text{or} \quad v = \sqrt{\frac{T}{3m}}$$
$$T - 3T_c = 0 \quad \text{or} \quad T_c = \frac{T}{3}$$

To calculate the limiting values T and [T] can be equated. Therefore, for maximum transmission of power, centrifugal tension in the belt should be 1/3rd of the maximum permissible tension in the belt or velocity of the belt should be be $\sqrt{\frac{[T]}{3m}}$.

LESSON 20 SELECTION OF BELTS

20.1 Selection of Flat Belt

In most of the applications, belts are generally selected by the designer from the manufacturer's catalogue. This helps in the use of standard available sizes. Following input data is required for the selection of belt:

1. Power to be transmitted
2. Transmission ratio
3. Centre distance

Following is the procedure for selection of a flat belt:

1. Select suitable belt material

Two types of belts are generally available – HI-SPEED duck belting and FORT duck belting. Specified transmission capacities of these two types, for an angle of contact of 180°and belt velocity of 5.08 m/s, is as follows:

Type of Belt	Transmission Capacity/ Power Rating (R)
HI-SPEED	0.012 kW / mm width / ply
FORT	0.015 kW / mm width / ply

2. Assume belt velocity and calculate diameters of pulleys

Optimum value of belt velocity lies between 15 m/s to 25 m/s. Assume any value for belt velocity, within this range and calculate diameter of the smaller (driving) pulley using the following relationship:

$$d = \frac{v}{\pi N_1}$$

where, v = velocity of belt

N_1 = input speed (rpm) of smaller pulley

Diameter of larger pulley (D) can be calculated, for required velocity ratio i.e. for required output speed (N_2), using the following relationship:

N1d = N2D

3. Calculate design power

For design purpose, maximum power transmitted by the belt is obtained by multiplying the required power (P) by a load correction factor (K_{load}). Value of K_{load} can be taken from Table 20.1. Also, Power transmission capacity of belts (Step 1) is based on angle of contact of 180°. But in actual conditions, angle of contact varies depending upon diameters of pulleys and centre distance. If angle of contact is less than 180°, there is additional tension in the belt. This is taken care of, by multiplying the required power to be transmitted by angle of contact factor, K_a.. Value of K_a can be taken from Table 20.2. Design power of the belt is thus given by,

$$P_{design} = K_{load} \times K_{\propto} \times P$$

Table 20.1 Load Correction Factor, K_{load}

Type of Load		K_{load}
Normal Load		1.0
Steady Load	(light machine tools, fans, centrifugal pumps, conveyors etc.)	1.2
Intermittent Load	(compressors, blowers, heavy-duty fans, line shafts, reciprocating pumps, heavy-duty machines etc.)	1.3
Shock Load	(hammers, grinders, rolling mills, vacuum pumps etc.)	1.5

Table 20.2 Angle of Contact Factor, K_a

Angle of Contact (a_s)	120°	130°	140°	150°	160°	170°	180°	190°	200°	210°
Angle of Contact Factor, K_a	1.33	1.26	1.19	1.13	1.08	1.04	1.00	0.97	0.94	0.91

4. Determine corrected power rating

Power transmission capacity of belts (Step 1) is based on belt velocity of 5.08 m/s. But in actual conditions, belts run at different speeds. For different velocity, corrected power rating is obtained as follows:

$$Corrected\ Power\ Rating,\quad R_{corrected} = \frac{R \times v}{5.08}$$

where, R is Power Transmission Capacity or Rating of belt and v is actual belt velocity.

5. Calculate the product of width and number of plies by dividing maximum power to be transmitted by corrected power rating.

$$Width \times No.of\ Plies = \frac{P_{design}}{R_{corrected}}$$

6. Select suitable width and number of plies

Suitable width and number of plies can now be selected from the catalogue so that the desired product is obtained. Table 20.3 gives standard width thickness and number of plies for rubber belts.

Table 20.3 Rubber Belt Data

No. of Plies	Thickness (mm)	Belt Width (mm)
3	3.9	25, 32, 40, 50, 63, 71
4	5.2	40, 50, 63, 71, 80, 90, 100, 112, 125, 140
5	6.5	71, 80, 90, 100, 112, 125, 140, 160
6	7.8	112, 125, 140, 160, 180, 200, 224, 250
7	9.1	160, 180, 200, 224, 250, 280, 315
8	10.4	224, 250, 280, 315, 355, 400, 450, 500

7. Calculate desired length of belt and specify all the dimensions.

20.2 Selection of V-Belts

V-belts are also selected from the manufacturer's catalogue. Following is the procedure for selection of a flat belt:

1. Select suitable V-belt section

Five types of standard V-belt sections are available. Dimensions of these are given in Table 20.4. Refer figure 20.1 for basic dimensions of the trapezoidal section of V-belt.

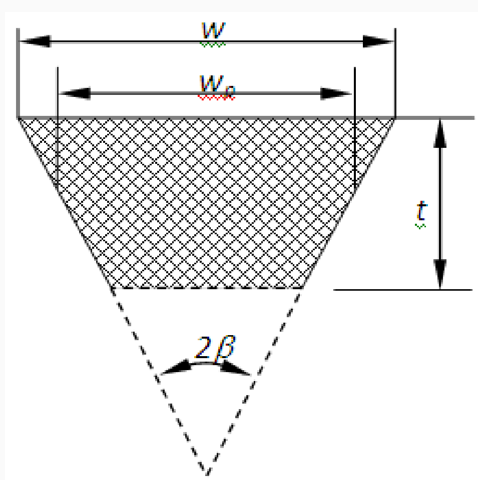


Figure 20.1 V-belt Section

Table 20.4 Dimensions of V-belt Sections

Section	A	B	C	D	E
Pitch Width, w_p (mm)	11	14	19	27	32
Nominal Top Width, w (mm)	13	17	22	32	38
Nominal Thickness, t (mm)	8	11	14	19	23
Recommended Velocity (m/s)	25	25	25	30	30
Recommended Power Range (kW)	0.4 - 4.0	1.5 - 15	10 - 70	35 - 150	70 – 260
Recommended Minimum Pitch Diameter of Pulley (mm)	125	200	315	500	630

2. Determine diameters of pulleys

Recommended diameter of smaller pulley (d) can be taken from table 20.4 for selected cross-section. Diameter of larger pulley (D) can be calculated, for required velocity ratio i.e. for given input speed (N_1) and required output speed (N_2), using the following relationship:

$N_1 d = N_2 D$

3. Calculate design power

For design purpose, maximum power transmitted by the belt is obtained by multiplying the required power (P) by service factor (K_s). Value of K_s can be taken between 1 and 2, depending upon the service conditions i.e. light, medium, heavy or extra-heavy duty, type of driver and driven machinery and operational hours.

$P_{design} = K_s \text{ X } P$

4. Determine Pitch Length and Centre Distance

Calculate length of the belt from its relation with d, D and C. Select the nearest standard value of belt pitch length from table 20.5. Calculate exact centre distance, C from the relation again.

Table 20.5 Belt Pitch Lengths & Pitch Length Correction Factor

Belt Pitch Length (mm)					Pitch Length Correction Factor, K_{length}
Belt Cross-section					
A	B	C	D	E	
630					0.80
	930				0.81

700		1560	2740		0.82
	1000				0.83
790		1760			0.84
	1100				0.85
890			3130		0.86
	1210	1950	3330		0.87
990					0.88
1100	1370	2190	3730	4660	0.90
		2340			0.91
	1560	2490	4080	5040	0.92
1250					0.93
		2720	4620	5420	0.94
	1760	2800			0.95
1430		3080		6100	0.96
	1950		5400		0.97
1550		3310			0.98
1640	2180	3520		6850	0.99
1750	2300		6100		1.00
1940	2500	4060		7650	1.02
			6840		1.03
2050	2700				1.04
2200	2850	4600	7620	9150	1.05
2300					1.06
			8410	9950	1.07
2480	3200	5380			1.08
2570			9140	10710	1.09

2700	3600				1.10
		6100			1.11
2910			10700	12230	1.12
3080	4060				1.13
3290		6860		13750	1.14
	4430				1.15
3540	4820	7600	12200		1.16
	5000		13700	15280	1.17
	5370				1.18
	6070		15200	16800	1.19
		9100			1.20
		10700			1.21

Table 20.6 Angle of Contact Factor, K_a

	Angle of Contact on Smaller Pulley, a_s	Angle of Contact Factor, K_a
0.00	180	1.00
0.05	177	0.99
0.10	174	0.99
0.15	171	0.98
0.20	169	0.97
0.25	166	0.97
0.30	163	0.96
0.35	160	0.95
0.40	157	0.94
0.45	154	0.93

0.50	151	0.93
0.55	148	0.92
0.60	145	0.91
0.65	142	0.90
0.70	139	0.89
0.75	136	0.88
0.80	133	0.87
0.85	130	0.86
0.90	127	0.85
0.95	123	0.83
1.00	120	0.82

5. Determine corrected power rating

Power transmission capacity / rating (R) for a single V-belt, for different types of cross-sections, can be taken from manufacturer's catalogue. Corrected power rating is obtained by multiplying the power rating by Pitch Length Correction Factor (K_{length}) and Angle of Contact Factor, K_a as follows:

$$Corrected\ Power\ Rating,\quad R_{corrected} = R \times K_{length} \times K_{\alpha}$$

Values of K_{length} and K_a can be taken from table 20.5 and table 20.6 respectively.

6. Determine the number of belts required

Required number of belts is determined by dividing the design power with corrected power rating for one belt.

$$No.\,of\ Belts = \frac{P_{design}}{R_{corrected}}$$

LESSON 21 FLAT BELT PULLEYS

21.1 Introduction

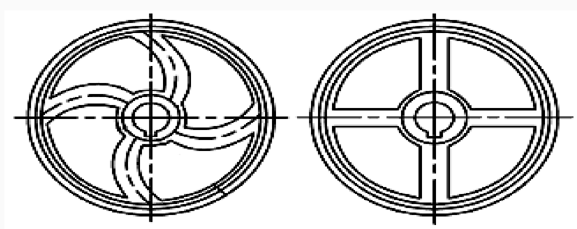
Pulleys are the wheels used to transmit power from one shaft to another, with the help of belts. Proper selection of diameters of pulleys is important as the velocity ratio is the inverse ratio of the diameters of driving and driven pulleys. For the belt to travel in a line normal to the pulley faces, the pulleys must be perfectly aligned with each other. Pulleys should have following important properties:

1. Ability to absorb shocks
2. High heat conductivity
3. High corrosion resistance
4. High coefficient of friction to reduce belt slippage
5. High strength to weight ratio

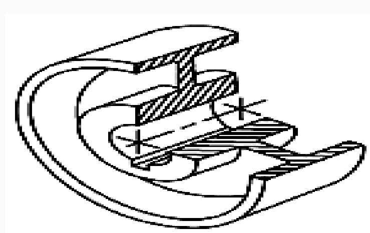
Pulley has three main components: i. Hub ii. Arms/spokes or web iii. Rim

21.1 Pulley Material

Pulleys are generally made of cast iron, forged steel, wood or compressed paper pulp. Because of their low cost, cast iron pulleys are most widely used. Arms or spokes of cast iron pulleys have elliptical cross-section and can have straight or curved shape as shown in figure 21.1a. In some pulleys, instead of arms, web is provided to join hub with rim, as shown in figure 21.1 b. Split cast iron pulleys, shown in figure 21.2, are made in two halves that are bolted together. These are easier to mount on shafts with a range of diameters, by tightening those on the shaft.



a. Pulleys with Arms



b. Pulley with Web

Figure 21.1 Solid Cast Iron Pulleys

Steel pulleys have higher strength, are lighter in weight and can run at higher speeds. But they have lower coefficient of friction in comparison to cast iron pulleys. Steel pulleys are generally made in two parts, which are bolted together on the shaft. Bushings are provided to take care of shafts of different diameters and for normal service; power is transmitted without key, only through frictional force obtained by clamping.

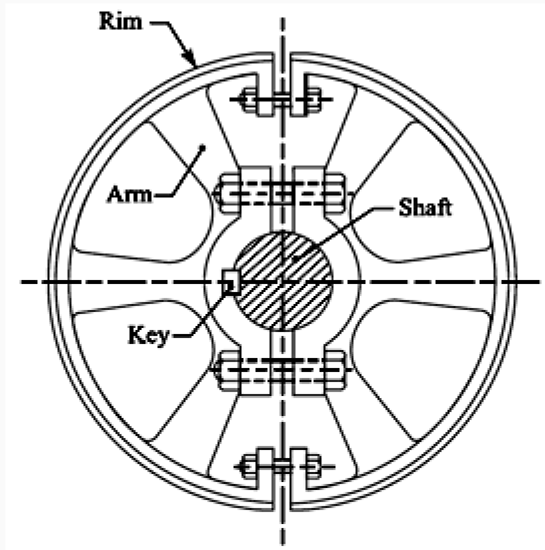
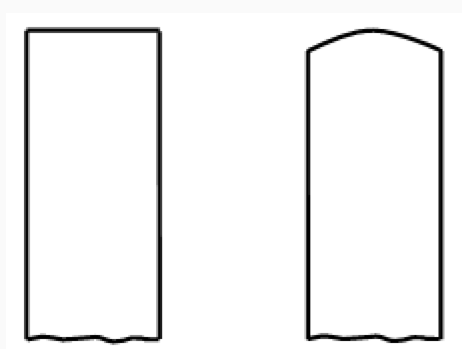


Figure 21.2 Split Cast Iron Pulley

Wooden pulleys are lighter and have higher coefficient of friction. Wooden pulleys are made of segments glued together under heavy pressure. Protective coatings of shellac or varnish are applied to avoid warping due to moisture. Wooden pulleys are also made as solid or split and have cast iron hubs with keyways. Adjustable bushings are also used in some of the wooden pulleys.

Paper pulleys are generally used for smaller centre distances to transmit power from electric motors. These are made from compressed paper fibre and are formed with a metal in the centre.

21.3 Crowning of Pulleys



a. Flat Pulley   b. Crowned Pulley

Figure 21.3 Crowning of Pulleys

In case of flat belts, thickness of the rim is increased from the centre so that it gets convex shape as shown in figure 21.3. This is known as crowning of the pulley. It helps in preventing the belt from running off the pulley by bringing it to the mid-plane of pulley whenever it moves to sides. Thus it helps in keeping the belt running in equilibrium position near the centre of the rim.

21.4 Design of Cast Iron Pulleys

The following procedure may be adopted for the design of cast iron pulleys.

21.4.1 Dimensions of Pulley

Diameter of the pulley (D) is selected depending upon the required velocity ratio, as discussed in the selection of belts.

Width of the pulley or face of the pulley (B) is taken 1.25 times the width of belt (b).

B = 1.25 b

Thickness of the pulley rim (t) is taken between (D/300 +2)mm and (D/300 +3)mm for single belt and for double belt.

21.4.2 Dimensions of Arms

Number of arms can be taken as follows:

Pulley Diameter	No. of Arms
< 200 mm	Web (with thickness = rim thickness)
> 200mm and < 450mm	4
> 500 mm	6

Cross-section of arms is generally elliptical with major axis (b_1) equal to twice the minor axis (a_1). The cross-section of the arm is obtained by considering the arm as cantilever, fixed at the hub end and carrying a concentrated load at the rim end and having a length equal to the radius of the pulley. Also, it is assumed that at any given time, the power is transmitted from the hub to the rim or vice versa, through only half of the total number of arms.

Tangential load per arm is given by, $$F_t = \frac{T}{R \times \frac{n}{2}} = \frac{2T}{R\,n}$$

where, T = Torque transmitted

R = Radius of pulley

n = Number of arms

Maximum bending moment on the arm at the hub end,

$$M = F_t \times R = \frac{2T}{n}$$

Maximum bending stress is given by,

$$\sigma_b = \frac{M\,y}{I} \leq [\sigma_t]$$

where, I = Moment of inertia of cross-sectional area of the arm about the axis of rotation

$= \dfrac{\pi a_1 b_1{}^3}{64} = \dfrac{\pi a_1^4}{64}$ as $b_1 = 2a_1$

y = Distance from neutral axis to the outer most fibre = $b_1 / 2 = a_1$

So, required dimensions of the arm, near the hub, can be determined from above relation for a known value of allowable tensile stress, [ ]. A taper, generally of 1/48 to 1/32, is provided on the arms, from hub to rim.

21.4.3 Dimensions of Hub

For known shaft diameter (d), diameter of the hub (d_1) can be taken as:

$$d_1 = 1.5d + 25\ mm$$

But the diameter of the hub should not be greater than 2d.

Length of the hub can be taken as,

$$L = \frac{\pi}{2} \times d$$

The minimum length of the hub is 2/3rd of the width of the pulley and it should not be more than the width of the pulley.

MODULE 7.

LESSON 22 GEARS

22.1 Introduction

Gears are machine elements used for the transmission of motion and power from one shaft to another by progressive engagement of teeth. Gear drive is a positive drive and the velocity ratio remains constant. Gears can transmit very large power, can be operated at very low speeds, have very high efficiency and require lesser space in comparison to the belt and chain drives. However, the cost of manufacturing is high as special tools and equipment are required for that. Also error or inaccuracy in manufacturing leads to noise and vibrations during operation of the drive. Gears require accurate alignment of shafts and proper lubrication.

22.2 Types of Gears

Gears are broadly classified into following four groups:

Spur Gears: Spur gears have teeth parallel to the axis of rotation and are used for parallel shafts. These are the simplest type of gears. Spur gears impose radial loads on the shafts

Helical Gears: Helical gears are also used for parallel shafts. Teeth of helical gears are inclined to the axis of rotation. Due to gradual engagement of the teeth during meshing, these are less noisy in comparison to the spur gears. Inclined teeth develop axial thrust loads in addition to the radial loads. Sometimes helical gears are used to transmit motion between nonparallel shafts also.

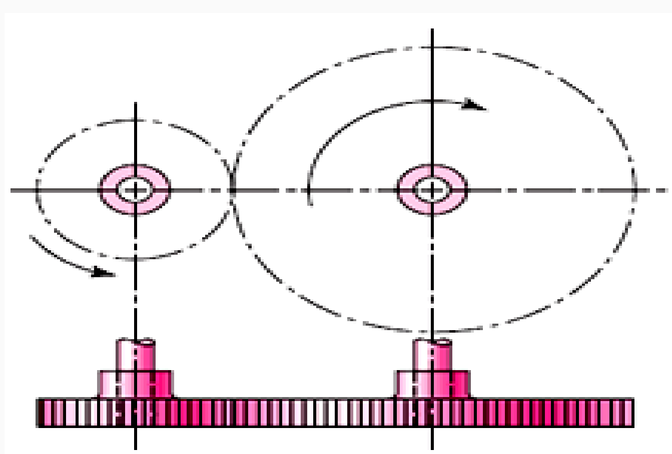


Figure 22.1 Spur Gears

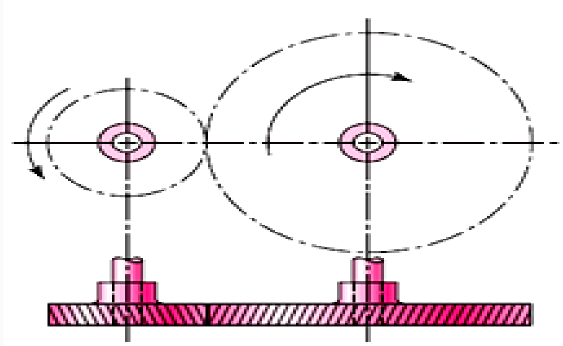


Figure 22.2 Helical Gears

Bevel Gears: have the shape of a truncated cone and are used to transmit power between intersecting shafts. Bevel gears can have straight or spiral teeth.

Worm Gears: Worm is in the form of a threaded screw which engages with a wheel. Axes of the two shafts are neither parallel nor intersecting and are generally at right angles to each other. Worm gears have very high reduction ratio.

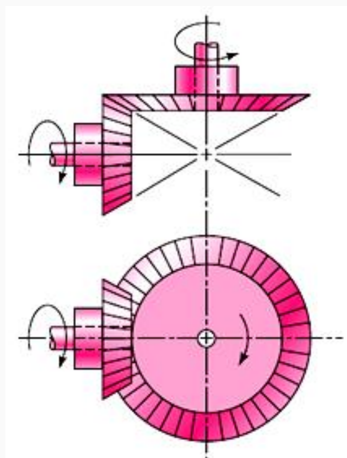


Figure 22.3 Bevel Gears

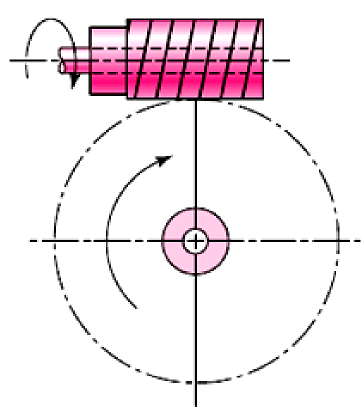


Figure 22.4 Worm Gears

22.3 Gear Terminology

Important terms related to gears are discussed in Table 22.1 and are shown in Figure 22.5.

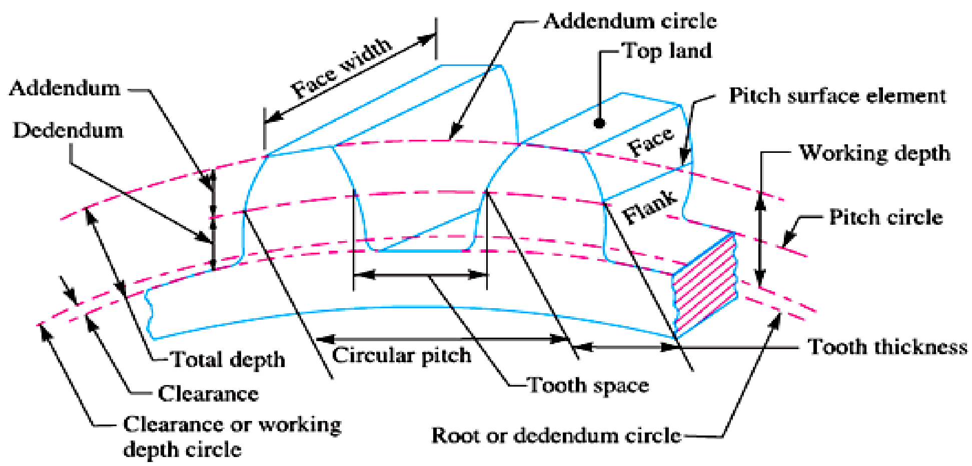


Figure 22.5 Gear Terminology

Table 22.1 Gear Terminology

Pitch Surface	Surface of imaginary cylinders that roll together without slipping to give same motion as the actual gears
Pith Circle	It is the intersection of the pitch surface and a plane perpendicular to the axis of rotation
Pitch Circle Diameter	Diameter of the pitch circle
Pitch Point	Pitch circles of two mating gears are tangent to each other. The point of tangency where the two pitch circles meet is called pitch point
Base Circle	Circle from which the tooth profile curve is generated
Addendum Circle	Circle that bounds the outer ends of the teeth
Dedendum Circle	Circle that bounds the inner ends of the teeth
Addendum	Radial distance between the pitch circle and the addendum circle or height of the tooth above pitch circle
Dedendum	Radial distance between the pitch circle and the dedendum circle or depth of the tooth below pitch circle
Clearance	Amount by which Dedendum of a gear exceeds the addendum of the mating gear
Backlash	Amount by which width of a tooth space exceeds the thickness of mating tooth
Circular Pitch	Distance between two similar points on adjacent teeth measured along the pitch circle. It is given by, where d and Z are pitch circle diameter and number of teeth of the gear.

Diametral Pitch	Number of teeth per unit length of the pitch circle diameter and is given by,
Module	Ratio of pitch circle diameter to the number of teeth. It is given by,

22.4 Gear Profiles

Fundamental Law of Gearing states that common normal to the tooth profile at the point of contact should always pass through the pitch point. This law is satisfied by two curves – involute curve and cycloidal curve.

Cycloid is a curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line. Curve traced by a point on the circumference of a circle which rolls without slipping on the outside of a fixed circle is called epicycloid and curve which rolls without slipping on the inside of a fixed circle is called hypercycloid. Involute is the curve traced by a point on a line that rolls on a circle without slipping.

Cycloidal teeth have epicycloid curve as their profile above the pitch circle and hypercycloid curve below the pitch circle. Combination of two curves makes the accurate manufacturing of cycloid tooth difficult. Also the pressure angle in case of cycloidal teeth doesn't remain constant. Due to these disadvantages, cycloidal gears have become obsolete. Profile of involute teeth is made of a single curve, making it easier to manufacture. Also in case of the involute teeth pressure angle remains constant. Therefore involute teeth profile is mostly used in the gears.

22.5 Interference in Involute Gears

Involute profile is generated by the point on a line that rolls without slipping on a circle. Therefore the profile exists only outside this circle, called the base circle and portion of the gear tooth inside the base circle doesn't follow the involute curve. The portion of the tooth outside base circle can be given any shape. But it is observed that if, for a given pressure angle, number of teeth on pinion are kept below certain value, this portion, which is not involute, comes in contact with the addendum of mating tooth. This is called interference. This can be avoided by providing undercuts on the pinion teeth but that decrease the strength of the gear tooth. Therefore to avoid interference minimum number of teeth are specified as given in the Table 22.2.

Table 22.2 Minimum Number of Teeth on Pinion to Avoid Interference

Gear Tooth System	Minimum Number of Teeth Required on Pinion
$14^1/_2$° Full depth system	32
20° Full depth system	18
20° Stub system	14
$14^1/_2$° Composite system	12

22.6 Backlash

It is the amount by which width of a tooth space exceeds the thickness of mating tooth. Backlash is intentionally provided to avoid jamming of the mating teeth, to compensate for machining errors and thermal expansion of teeth. Backlash can be provided by cutting the gears thinner or by slightly increasing the centre distance. This has no effect on the tooth action or velocity ratio. For gears to be used in precision equipment, backlash should be minimum.

22.7 Lubrication

Gears should be properly lubricated with a suitable lubricant for proper functioning and maximum life of the gears. Lubrication is required following purpose:

1. To reduce friction between rubbing tooth surfaces
2. To develop a fluid film between the tooth surfaces
3. To remove the heat generated due to functioning of gears and avoid overheating
4. To remove the abrasive particles from the tooth surfaces and reduce wear
5. Lubricant should be free from any debris, dirt or foreign particles and should be changed periodically.

LESSON 23 DESIGN OF SPUR GEARS

23.1 Force Analysis

To transmit power from one gear to the other, force is applied by the tooth of the driving gear on the mating tooth of the driven gear. This force, called as Normal Force ( ) acts along the pressure line and is always normal to the tooth surface. This normal force can be resolved into two components:

1. Tangential component (F_t)– helps in transmission of torque and determines its magnitude.
2. Radial component (F_r)– tends to push the gears apart, has no contribution in power transmission.

If torque to be transmitted, T is known, tangential component of force can be calculated as,

$$F_t = \frac{T}{d/2}$$

Where d is pitch circle diameter

F_n acts along the pressure line and F_n along the common tangent, therefore angle between F_n and F_t is φ , the pressure angle. Referring to Figure 23.1, following relations can be written:

$F_r = F_t \tan \varphi$ and $F_n = F_t / \cos \varphi$

Above analysis is based on the following assumptions:

1. F_n remains constant in the power transmission (F_n changes with change in position of the point of contact).

2. Only one pair of teeth takes the entire load (Load is often shared by more than on pairs).

3. Loads are static i.e. gears run at a low speed (There are dynamic loads in actual practice).

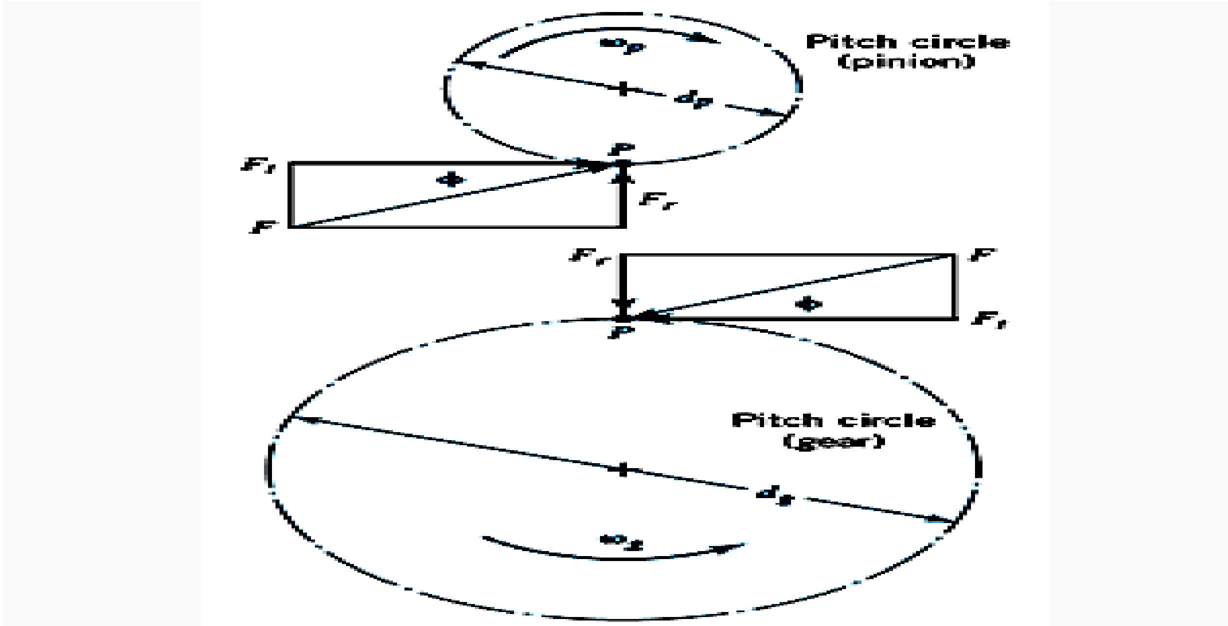


Figure 23.1 Normal Force Resolved into Tangential & Radial Components

23.2 Gear Material

As for any other component, selection of material depends upon performance requirements, material properties, manufacturing aspects & cost and availability. Gear material should have good strength, high endurance limit, good wear resistance, low coefficient of friction and good manufacturability. Gears are subjected to dynamic loads and very high bending and contact stresses.

Cast iron can be used for lighter stress conditions. It has good castability, machinability and wear properties. Also its vibration damping property makes the operation quiter. For light to medium duty gears steel castings and structural steels may be used. Hardened and tempered steel or case-hardened steels are used for heavy duty applications. For working conditions where special properties, like resistance to heat, corrosion or wear, are desired, alloy steels can be used bronzes, aluminium and zinc alloys are some other material that are used for manufacturing gears, due to their high strength and good sliding properties. Non-metallic materials like acetal, nylon and other plastics are also used for manufacturing gears. These are quieter, durable, cheaper and don’t require lubrication also but their bad carrying capacity is low. Gears can be manufactured with the help of casting, forging or machining.

23.3 Gear Tooth Failure

Gear tooth may fail in two ways – breakage of tooth due to overloading or fatigue or Surface damage of the tooth due to wear, pitting or scoring. These are discussed below:

23.3.1. Breakage of Tooth

Each tooth of gear is subjected to bending stress, which is maximum at the fillet of its base. Gear tooth may fail, if this stress reaches the yield strength of the material. Also as the load on the tooth is dynamic in nature, it leads to variable stresses and the tooth may fail due to fatigue.

23.3.2. Surface Damage of the Tooth

Surface of gear tooth is subjected to very high contact stresses at the point of contact between two mating gears, where the normal force acts. Surface of the tooth may get damaged due to these very high contact stresses. Gear-tooth surface damage is a very complex phenomenon as excessive loading and lubrication breakdown may lead to various combinations of abrasion, pitting, and scoring. These three basic types of surface deterioration are briefly discussed below:

Abrasion (Abrasive wear) is a type of wear caused by the presence of foreign particles on the surfaces of contact between two mating gears. These foreign particles may be present at the time of assembly or may enter along with the lubrication oil if it is not filtered properly.

Scoring is a form of adhesive wear, which is generally caused due to the failure of lubrication film between the two surfaces i.e. due to inadequate lubrication and occurs in gears rotating at very high speeds. Inadequate lubrication leads to increase in coefficient of sliding friction that combines with high sliding velocity and very high tooth loads to produce a high rate of localized heat generation. This high temperatures and pressure condition causes welding and tearing apart of the tooth surface, which is called scoring. It can be prevented by providing good surface finish and using an appropriate lubricant.

Pitting and Spalling are types of surface fatigue failures that are characterized by the separation of small bits of material from the surface of the tooth. Due to very high localized stresses, cracks are initiated on and under the contacting surfaces of the teeth, which start propagating, finally resulting in loss of material from the surface in the form of bits. If this phenomenon starts with surface cracks resulting in pits of relatively smaller size, it is called pitting. And if it originates with subsurface cracks resulting in thin flakes of surface material, it is called Spalling.

23.4 Beam Strength of Gear Tooth

Wilfred Lewis presented first recognized analysis of bending stress in gear tooth in 1892, which serves as the basis of bending stress analysis of gear teeth, even today. Lewis considered the gear tooth as a cantilever beam, subjected to bending moment due the tangential force acting on it, as shown in Figure 23.2.

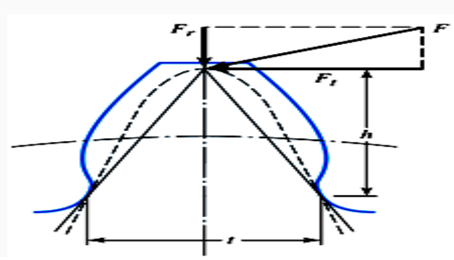


Figure 23.1 Gear Tooth as Cantilever

Lewis Analysis is based on following assumptions:

1. Only one pair of teeth takes the entire load (Load is often shared by more than on pairs).

2. The effect of radial component, Fr, is negligible (Fr induces compressive stress).
3. The load is uniformly distributed across the full face width. (Gear should be rigid and accurately machined for this).
4. Tooth sliding friction forces are negligible.
5. Effect of stress concentration is negligible.

Cross-section of the gear tooth, considered as a cantilever beam, varies from the free end to the fixed end. A parabola can be constructed within the tooth profile, as shown in Figure 23.1, giving the outline of a beam of uniform strength. Cross-section at XX is critical, where this parabola is tangent to the tooth profile. If b, t and h are width, thickness and height of the gear tooth, Bending Stress at section XX can be given by,

$$\sigma_b = \frac{My}{I} = \frac{F_t h(t/2)}{bt^3/12} = \frac{6F_t h}{bt^2}$$

Multiplying nominator and denominator by 'm' and rearranging,

$$F_t = mb\sigma_b\left(\frac{t^2}{6hm}\right) = mb\sigma_b Y$$

where Y = t^2 / 6hm = Lewis Form Factor

Above equation gives the relation between tangential force and bending stress in the tooth. increases with increase in . The maximum tangential load that can be transmitted by the tooth without bending failure is called its Beam Strength . Therefore in the above relation, if is replaced by allowable bending stress, of the tooth, corresponding value of is the beam strength.

$$\therefore\ S_b = mb[\sigma_b]\,Y$$

Value of for different systems can be taken as given below:

$$Y = \pi\left[0.124 - \frac{0.684}{Z}\right]\ for\ 14.5^\circ\ involute\ teeth$$

$$= \pi\left[0.154 - \frac{0.912}{Z}\right]\ for\ 20^\circ\ full\ depth\ teeth$$

$$= \pi\left[0.175 - \frac{0.950}{Z}\right]\ for\ 20^\circ\ stub\ teeth$$

For the gears to be safe in bending, σ_b should be less than $[\sigma_b]$ or F_t should be less than S_b . If gear and pinion are made of same material, pinion is the weaker member and design is based on its strength. But if those are made of different materials, it is evident from the Lewis Equation that the gear or pinion having lesser value of product [σb] X Y is weaker, as m and b are same for pinion and gear. Also as the gears are subjected to variable stresses, design is based on the endurance limit and allowable bending stress can be taken as, [σb] = S_e/f_{os}

LESSON 24 DESIGN OF SPUR GEARS-II

24.1 Effective Load on the Gear Tooth

As discussed above, is calculated for rated torque and rated speed, and therefore is average value of the tangential force on the tooth. But in actual conditions, torque developed by the power source and also the torque requirements of the driven machinery vary. To account for these variations in service conditions, F_t is multiplied with Service Factor (C_s). Also, F_t was calculated assuming that gears are rotating at a very slow speed, which is generally not the case. Gears rotate at appreciable speeds resulting in dynamic forces, which may occur due to inaccuracies in tooth profile, misaligned bearings, inertia and elasticity of the members. To account for the effect of this dynamic force, Velocity Factor (C_v) is used. Effective Tangential Force acting on the tooth is then given by, $F_{eff} = F_t \, . \, C_s / C_v$

Values of and can be taken from the table 24.1 and table 24.2 respectively.

Table 24.1 Service Factor,

Power Source	Driven Machinery		
	Uniform	Moderate Shock	Heavy Shock
Uniform	1.00	1.25	1.75
Light Shock	1.25	1.50	2.00
Medium Shock	1.50	1.75	2.25

Table 24.2 Barth's Formulae for Velocity Factor,

Range of Velocity	Commercial Name	Value of C_v
$v \le 8$ m/s	Ordinary Cut Gear	$\frac{3.05}{3.05 + v}$
$v \le 13$ m/s	Carefully Cut Gear	$\frac{4.58}{4.58 + v}$
$v \le 20$ m/s	Accurately Cut Gear	$\frac{6.1}{6.1 + v}$
$v \ge 20$ m/s	Precision Cut Gear	$\frac{5.56}{5.56 + \sqrt{v}}$

Therefore for the gear tooth to be safe,F_{eff} should be less than S_b.

Above calculation of F_{eff} gives approximate estimation of the effective tangential force acting on the gear tooth and can be used in the initial stages of design. For more accurate calculation, Buckingham's equation is used, which takes into account inertia of connected masses, elasticity of gear material, tooth profile inaccuracies etc. All these important aspects are neglected in previous method as the velocity factor depends only on the pitch line velocity of gears. According to Buckingham, effective tangential load on the gear tooth is given by,

$$F_{eff} = C_s F_t + F_d$$

where, F_d is the additional load due to dynamic conditions, called as 'Incremental Dynamic Load' and is given by,

$$F_d = \frac{21v(Ceb + F_t)}{21v + \sqrt{(Ceb + F_t)}}$$

where, v = pitch line velocity (m/s)

e = sum of errors between two meshing teeth (mm)

b = face width of the tooth (mm)

C = Deformation Factor (N/mm^2), which is given by,

$$C = \frac{k}{\left(\frac{1}{E_p} + \frac{1}{E_g}\right)}$$

where, and are Modulii of Elasticity of pinion and gear respectively and is a constant that depends upon the form of the tooth. Value of can be taken as given below:

$$\begin{aligned} k &= 0.107 \text{ for } 14.5^0 \text{full depth teeth} \\ &= 0.111 \text{ for } 20^0 \text{ full depth teeth} \\ &= 0.115 \text{ for } 20^0 \text{ stub teeth} \end{aligned}$$

Error e depends upon method of manufacturing and quality of gear. There are twelve grades from Grade 1 to Grade 12, in decreasing order of precision. Standard tables giving tolerances for these grades are available and error is expected to be equal to those.

24.2 Wear Strength of Gear Tooth

Wear Strength is the maximum value of tangential load that the tooth can transmit without surface damage. It can be calculated using the Buckingham's Equation as given below:

$$S_w = dbQK$$

where,

$$Q = \frac{2d_g}{d_g + d_p} = \frac{2Z_g}{Z_g + Z_p}$$

$$K = \frac{s_{es}^2 \sin\varphi}{1.4}\left[\frac{1}{E_p} + \frac{1}{E_g}\right]$$

Where is Surface Endurance Limit, which is related to Brinell Hardness Number as:

$$S_{es} = 2.75\,(BHN) - 70$$

24.3 Design Procedure for Spur Gear

1.Select suitable material and design stress.

$$[\sigma_b] = S_e/fos$$

2.Assume number of teeth on pinion (Zp), considering minimum teeth required to avoid interference.
3.Calculate and identify weaker of pinion and gear.
4.Select suitable value of service factor Cs,
5.Assume / calculate values of and Cv.

K is the ratio of face width to module and is known as face width ratio i.e. k = b / m . Value of lies between 9.5 and 12.5 and is generally assumed to be 10.

For Cv, if the centre distance between the gears is known, pitch line velocity (v) can be calculated and , corresponding to that can be calculated using Barth's formula given in table 24.2.

Therefore, if centre distance is unknown, assume and for known centre distance (CD) following steps can be followed:

$$d_p + d_g = 2CD \text{ and } \frac{d_g}{d_p} = VR \text{ (Velocity Ratio)}$$

$$d_p\left(1 + \frac{d_g}{d_p}\right) = 2CD$$

$$d_p(1 + VR) = 2CD$$

$$d_p = \frac{2CD}{1 + VR} \text{ and } d_g = \frac{2CD \times VR}{1 + VR}$$

Thus pitch circle diameter of pinion or gear can be calculated for known centre distance and desired velocity ratio and can then be calculated.

6. Calculation of Module

For the gear tooth to be safe in bending, $F_{eff} \leq S_b$ and to calculate limiting value of module,

$$F_{eff} = S_b$$

$$F_t \frac{C_s}{C_v} = mb[\sigma_b]Y$$

$$\frac{2T}{d}\frac{C_s}{C_v} = m^2 \frac{b}{m}[\sigma_b]Y$$

$$\frac{2T}{mZ}\frac{C_s}{C_v} = m^2 k[\sigma_b]Y$$

$$m = \sqrt[3]{\frac{2TC_s}{[\sigma_b]kYZC_v}}$$

Value of module can be estimated using this relation and closest standard value of module is selected.

7. If centre distance is unknown, calculate d = mZ, calculate $v\ (= \pi dN/60)\ \&\ C_v$ and recalculate . And for known centre distance, calculate Z = d / m and recalculate Y and m .

Repeat steps 6 and 7 until two consecutive values of module are found to be same.

8. Check the design for bending using Buckingham's equation. Calculate F_d & F_{eff} and compare it with S_b. If design is no safe higher values of standard module can be checked.
9. Check the design for wear strength using Buckingham's equation. F_{eff} should be ≤ Sw. If not, increase in surface hardness may be recommended.

MODULE 8.

LESSON 25 INTRODUCTION TO POWER SCREWS

25.1 Introduction

Power screw is a mechanical device used to convert rotary motion into linear motion and to transmit power. Unlike the threaded fasteners which are used to clamp the machine members, power screws are used to transmit power. Common applications of power screws are:

1. Screw jacks: to lift weight
2. Lead screw of lathe: for axial movement of tool and its precise positioning.
3. Tensile testing machine: to exert large force.
4. Vice: to clamp the work piece

Power screw comprises of two main components: screw and nut, and can operate in following three ways:

1. Screw rotates in bearings and nut moves axially
2. Screw rotates and also moves axially while nut is kept fixed
3. Nut rotates and screw moves in axial direction

Power screws have large load carrying capacity, are compact, provide large mechanical advantage, provide very accurate and precise linear motion, have smooth and noiseless operation, are reliable and have lesser cost. Disadvantages are that power screws have poor efficiency and high rate of wear.

25.2 Materials

Screw is subjected to torque, axial compressive load and bending moment also, sometimes. Screws are generally made of C30 or C40 steel. As the failure of power screws may lead to serious accident, higher factor of safety of 3 to 5 is taken. Threads may fail due to shear, which can be avoided by using nut of sufficient height. Wear is another possible mode of thread failure as the threads of nut and bolt rub against each other. Nuts are made of softer material than screws so that if at all the failure takes place, nut fails and not the screw, which is the costlier member and is also difficult to replace. Plastic, bronze or copper alloys are used for manufacturing nuts. Plastic is used for low load applications and has good friction and wear properties. Bronze and copper alloys are used for high load applications.

25.3 Thread Forms for Power Screws

Thread forms used in threaded fasteners are not suitable for power screws. The purpose of threads used in fasteners is to provide high friction to minimize the chances of loosening. But in power screws, low friction between nut and bolt is desired as those are to be used for power transmission. Types of threads commonly used for power screws are:

i) Square Threads ii) Acme or Trapezoidal Threads iii) Buttress Thread

These thread forms are shown in figure 25.1.

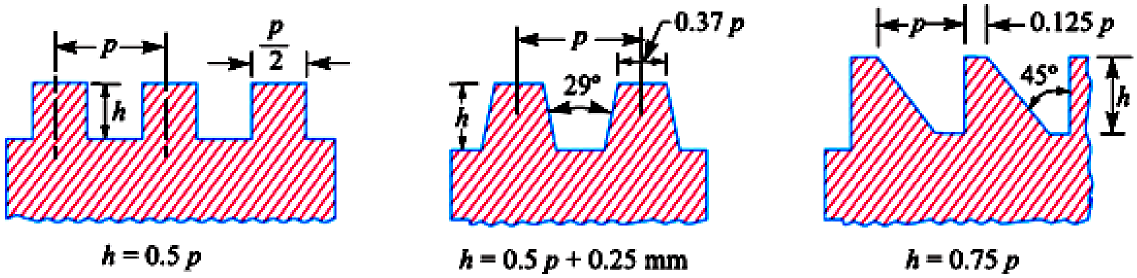


Figure 25.1 Thread Forms for Power Screws

25.3.1 Square Threads

Square thread can be used to transmit power in either direction. Square threads have maximum efficiency and there is no radial or bursting pressure on the nut, increasing the life of the nut and making its motion uniform. But it is difficult to manufacture square threads. These are difficult to cut with taps and dies and are usually cut on a lathe with a single point cutting tool, making it expensive. Also, it is not possible to compensate for wear in square threads as split nut cannot be used with it. Therefore, nut or screw has to be replaced, when worn out. The square threads are used in screw jacks, presses and clamping devices.

25.3.2 Acme or Trapezoidal Threads

Acme thread is used for power transmission. It has higher load carrying capacity in comparison to square threads, because of larger root thickness. Acme threads are manufactured on a milling machine using a multi-point cutting tool and are therefore economical to cut. Due to the slope provided on its sides, efficiency of acme threads is less than the square threads and the nut is subjected to radial or bursting pressure. Wear can be compensated in this case by using split nut, which is a nut cut into two halves along its diameter. These two halves are tightened together after certain intervals to take care of the wear taken place.

25.3.3 Buttress Thread

Buttress thread is designed to take large loads in one direction. This is the strongest of the thread forms due to greater root thickness. Its efficiency is comparable with the square threads, is easier to cut and is compatible with the split nut also. Buttress thread finds its application in light jack screws and vices.

25.4 Torque Required to Raise & Lower the Load

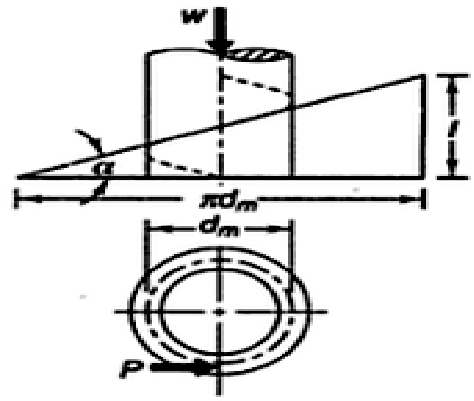


Figure 25.2 Development of Thread

Power screws are used to convert the applied torque into useful axial force, e.g. screw jack converts torque into axial force which is used to lift load. It is important to know the torque required to raise or lower a given load.

Let p = Pitch of the screw

d_m = Mean diameter of the screw

α = Helix angle

l = Lead of the screw

P = Effort applied at the circumference of the screw to lift the load

W = Load to be lifted

μ = Coefficient of friction,

between the screw and nut

A screw thread can be considered as an inclined plane wrapped around a cylinder to form helix and the relative motion between the nut and the screw against the external load is analogous to the movement of a weight on an inclined plane. For simplicity of the analysis, this is assumed to be a point load, though the actual load is distributed on the thread surface. Figure 25.2 shows the right angled triangle formed by un-wrapping a single thread. Thread forms the hypotenuse of the triangle and can be considered as an inclined plane. Length of the base of the triangle is p d_m and its height is equal to lead, l. This gives the following relation between helix angle, mean diameter and lead of the screw:

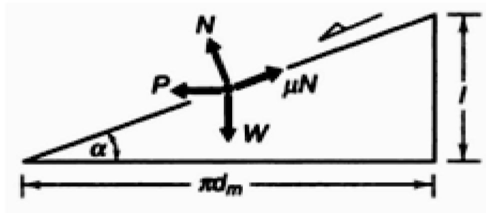


Figure 25.3 Force Diagram for Raising Load

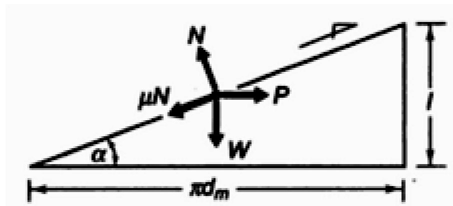


Figure 25.3 Force Diagram for Raising Load

Figure 25.3 and figure 25.4 show the forces acting at a point on the inclined plane, while raising and lowering the load respectively. Forces acting at the point are: P – Applied Effort, W – Load to be raised and lowered, N – Normal Reaction and mN – Frictional Force.

Both the cases are discussed below:

Considering equilibrium in horizontal and vertical direction,

Raising the Load	Lowering the Load
$P = \mu N \cos\alpha + N \sin\alpha$	$P = \mu N \cos\alpha - N \sin\alpha$
$W = N \cos\alpha - \mu N \sin\alpha$	$W = N \cos\alpha + \mu N \sin\alpha$

Dividing,

Raising the Load	Lowering the Load
$P = \dfrac{W(\mu \cos\alpha + \sin\alpha)}{(\cos\alpha - \mu \sin\alpha)}$	$P = \dfrac{W(\mu \cos\alpha - \sin\alpha)}{(\cos\alpha + \mu \sin\alpha)}$
$P = \dfrac{W(\mu + \tan\alpha)}{(1 - \mu \tan\alpha)}$	$P = \dfrac{W(\mu - \tan\alpha)}{(1 + \mu \tan\alpha)}$

Substituting, $\mu = \tan\varphi$, where φ is the pressure angle

Raising the Load	Lowering the Load
$P = \dfrac{W(\tan\varphi + \tan\alpha)}{(1 - \tan\varphi \tan\alpha)}$	$P = \dfrac{W(\tan\varphi - \tan\alpha)}{(1 + \tan\varphi \tan\alpha)}$
$P = W \tan(\varphi + \alpha)$	$P = W \tan(\varphi - \alpha)$

The torque required to raise/lower the load is given by,

Raising the Load	Lowering the Load
$T_r = \dfrac{Pd_m}{2} = \dfrac{Wd_m}{2} \tan(\varphi + \alpha)$	$T_l = \dfrac{Pd_m}{2} = \dfrac{Wd_m}{2} \tan(\varphi - \alpha)$

Figure 25.5 shows load acting on the surface of a trapezoidal thread. In this case the load acting normal to the surface of the thread is W.secq. As frictional force is a function of the normal load, it increases by a factor 'secq'. Effort and torque required to lift or lower the load in case of trapezoidal threads can be obtained by replacing 'm' with 'm secq'. For example:

Effort required for raising the load,

$$P = \frac{W(\mu \sec\theta + \tan\alpha)}{(1 - \mu \sec\theta \tan\alpha)}$$

Effort required for lowering the load,

$$P = \frac{W(\mu \sec\theta - \tan\alpha)}{(1 + \mu \sec\theta \tan\alpha)}$$

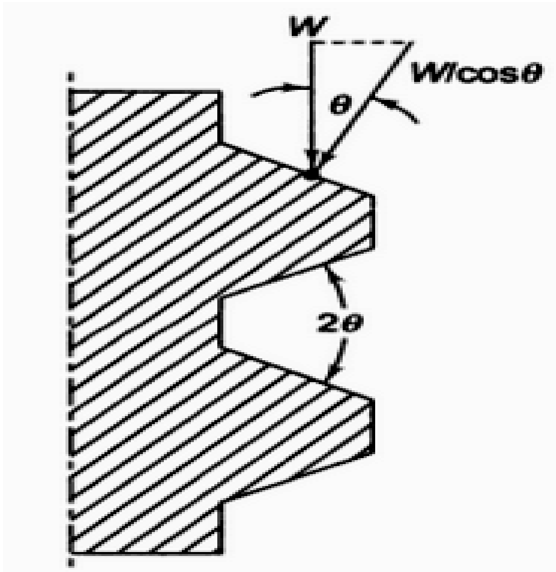


Figure 25.5 Force Diagram for Trapezoidal Thread

25.5 Condition for Self-locking

The torque required to lower the load is given by,

$$T_l = \frac{Wd_m}{2} \tan(\varphi - \alpha)$$

From the equation, it is clear that if , the torque required to lower the load is negative i.e. in such condition load will descend without requiring any external torque. Such condition is known as overhauling of the screw. But this an undesired condition for applications like screw jacks as it may lead to accident. On the other hand, if φ > a, a positive torque is required to lower the load and the load cannot descend on its own without application of external torque. Such screws are called self-locking screws. So the condition for the screw to be self-locking is:

$$\varphi > \alpha \quad \text{or} \quad \tan\varphi > \tan\alpha \quad \text{or} \quad \mu > \frac{l}{\pi d_m} \quad \text{or} \quad \mu\pi d_m > l$$

25.6 Efficiency of a Square Screw Thread

Referring to figure 25.3, suppose the load W moves from the lowest point to the highest point along the inclined plane. Then the output and input work is given by,

$$\text{Work output} = W.l$$

$$\text{Work input} = P.\pi d_m$$

The efficiency of the screw is given by,

$$\eta = \frac{\text{work output}}{\text{work input}} = \frac{Wl}{P\pi d_m} = \frac{W}{P}\tan\alpha = \frac{\tan\alpha}{\tan(\varphi + \alpha)}$$

LESSON 26 DESIGN OF POWER SCREWS

26.1 Materials

Screw is subjected to torque, axial compressive load and bending moment also, sometimes. Screws are generally made of C30 or C40 steel. As the failure of power screws may lead to serious accident, higher factor of safety of 3 to 5 is taken. Threads may fail due to shear, which can be avoided by using nut of sufficient height. Wear is another possible mode of thread failure as the threads of nut and bolt rub against each other. Nuts are made of softer material than screws so that if at all the failure takes place, nut fails and not the screw, which is the costlier member and is also difficult to replace. Plastic, bronze or copper alloys are used for manufacturing nuts. Plastic is used for low load applications and has good friction and wear properties. Bronze and copper alloys are used for high load applications.

26.1 Design of Screw and Nut

26.2.1 Compressive & Torsional Shear Stress in Screw Body

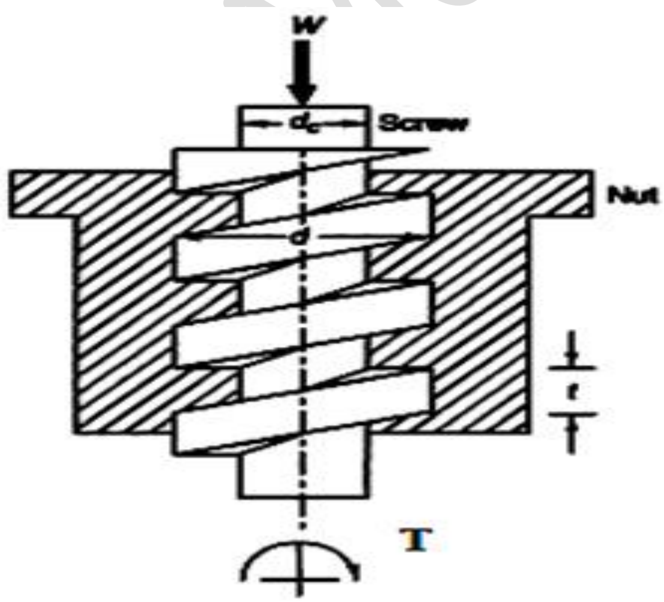


Figure 26.1 Screw and Nut

Screw body is subjected to axial compressive load, W and torque, T as shown in figure 26.1.

Direct Compressive Stress, $\sigma_c = \dfrac{W}{\frac{\pi}{4} d_c^2}$

Torsional Shear Stress, $\tau = \frac{T\,r}{J}$

where T = Applied Torque

J = Polar moment of inertia $= \pi d_c{}^4/32$

r = $d_c/2$

The principal shear stress is given by,

$$\tau_{max.} = \sqrt{\left(\frac{\sigma_c}{2}\right)^2 + (\tau)^2}\ should\ be\ \le [\tau]$$

26.2.2 Shear Stress in Threads

The threads of the screw and nut are subjected to transverse shear forces due to the load W. Shear stress is maximum near the root diameter for the threads of the screw.

Maximum transverse shear stress in the threads of screw is given by,

$$\tau_s = \frac{W/n}{\pi d_c b} = \frac{W}{\pi d_c n b}\ \text{should be}\ \le [\tau_s]$$

where, b – thread thickness at the core diameter

n - number of threads in engagement

In the threads of the nut, transverse shear stresses is maximum at the nominal diameter and is given by,

$$\tau_n = \frac{W/n}{\pi d b} = \frac{W}{\pi d n b}\ \text{should be}\ \le [\tau_n]$$

26.2.3 Bearing Pressure Between Surfaces of Screw and Nut

The bearing pressure between the contacting surfaces of the screw and nut is given by,

$$P_{bearing} = \frac{\text{load}}{\text{projected area}} = \frac{W/n}{\frac{\pi}{4}(d^2 - d_c^2)} = \frac{4W}{\pi(d^2 - d_c^2)n}\ \text{should be}\ \le [P_{bearing}]$$

26.3 Design of Screw Jack

Screw Jack is a device, in which screw mechanism is used to raise or lower the load. Manually operated, portable type screw jack is the simplest and most commonly used. Its construction is shown in figure 26.2. In this, nut is fixed to the frame and remains stationary. When screw is rotated with the help of the handle, it moves axially. A cup is provided at the top to support the load. Cup remains stationary as the screw rotates and they rub against each. Applied torque has to overcome this friction also, which is known as collar friction. To avoid screw to completely turning out of the nut, washer is fixed on the lower end of the screw.

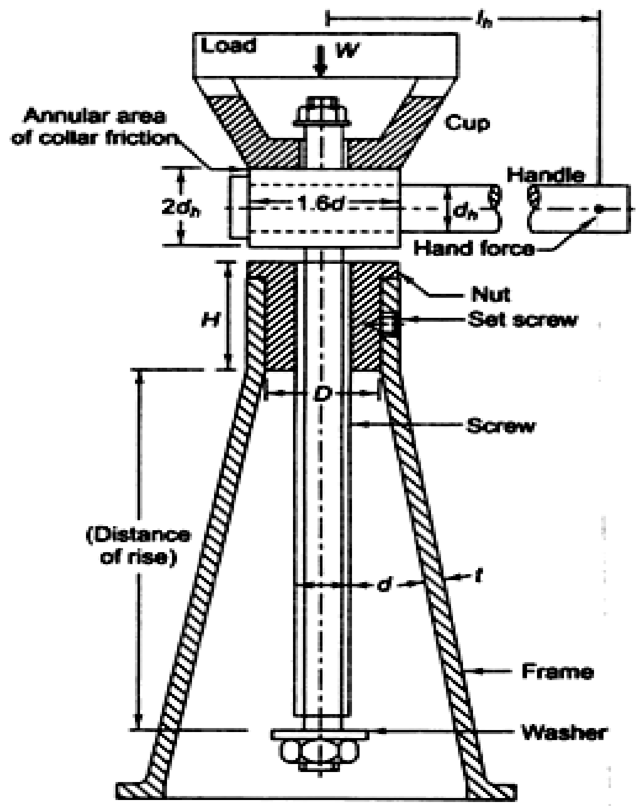


Figure 26.2 Screw Jack

In order to design a screw jack for a load W, the following procedure may be adopted:

1. Calculate core diameter of the screw, considering only the compressive stress,

$$\sigma_c = \frac{W}{\frac{\pi}{4} d_c^2} \leq [\sigma_c]$$

Value of core diameter should be standardized and corresponding values of mean diameter (d_m) and outer diameter (d) should be taken.

2. Calculate the effort at the handle and the torque required to lift load, W.

$$P = W\tan(\varphi + \alpha) \text{ and } T_r = \frac{Pd_m}{2}$$

3. Calculate the direct compressive stress and torsional shear stress and find the principal shear stresses.

$$\sigma_c = \frac{W}{\frac{\pi}{4}d_c^2} \qquad \text{and} \qquad \tau = \frac{T\,r}{J}$$

$$\text{Then,} \qquad \tau_{max.} = \sqrt{\left(\frac{\sigma_c}{2}\right)^2 + (\tau)^2} \quad \text{should be} \ \leq [\tau]$$

4. Find the height of nut (H), considering the bearing pressure on the nut.

$$P_{bearing} = \frac{\text{load}}{\text{projected area}} = \frac{W/n}{\frac{\pi}{4}(d^2 - d_c^2)} = \frac{4W}{\pi(d^2 - d_c^2)n} \text{ should be}$$

$$\leq \ [P_{bearing}]$$

Calculate the number of threads, n required for given allowable bearing pressure, by using equality sign in above relation.

Then, height of nut, H = number of threads X pitch = n p

5. Check the shear stress in the threads of screw and nut.

$$\tau_s = \frac{W/n}{\pi d_c b} = \frac{W}{\pi d_c n b} \text{ should be } \leq [\tau_s] \quad \text{and} \quad \tau_n = \frac{W/n}{\pi d b}$$

$$= \frac{W}{\pi d n b} \text{ should be } \leq [\tau_n]$$

Note that allowable value of shear stress is different for screw and nut as different materials are used for them.

6. Find outer diameter (D) of the nut by considering its tearing failure.

$$\sigma_{tearing} = \frac{W}{\frac{\pi}{4}(D^2 - d^2)} \text{ should be} \leq \left[\sigma_{tearing}\right]$$

7. Find outer diameter of the nut collar (D_1) considering crushing failure of the nut.

$$\sigma_{crushing} = \frac{W}{\frac{\pi}{4}(D_1^2 - D^2)} \text{ should be} \leq [\sigma_{tearing}]$$

8. Find thickness of the nut collar (t) considering shear failure of the collar.

$$\tau = \frac{W}{\pi D t} \text{ should be } \leq [\tau]$$

9. Assume outer and inner diameter of cup, coming in contact with the head as:

Outer Diameter, Do = 1.6 d

Inner Diameter, Di = 0.8 d

10. Calculate the torque required to overcome collar friction,

$$T_{collar} = \mu' W \frac{(D_o + D_i)}{4}$$

11. Total input torque required to lift load, W,

$$T = T_r + T_{collar}$$

This is the total torque to be applied with the help of hand lever. Assuming that a force of 300 N (approximately) can be applied by hand. Required length of the handle is then given by, $l_h = 300/T$

12. Calculate diameter of the handle (d_h) considering its bending failure.

$$\sigma_b = My/I \text{ should be } \leq [\sigma]$$

where, Applied bending moment, $M = P\,l_h$

Moment of inertia, I = $\pi d_h^4/32$

Distance of the farthest fiber from the neutral axis, $y = d_h/2$

13. Take height of the head = 2 d_h

14. Check the screw for buckling.

Effective or unsupported length of the screw is given by,

$$L = \text{Distance of Rise} + \text{Height of Nut}/2$$

Critical Load is given by,

$$W_{critical} = AS_y\left[1 - \frac{S_y}{4n\pi^2E}\left(\frac{L}{k}\right)^2\right]$$

Where, S_y = Yield Strength of Screw Material

E = Modulus of Elasticity of Screw Material

n = End Fixity Coefficient = 0.25 (for one end fixed and load end free)

k = Radius of gyration = 0.25 d_c

A = Cross-sectional Area of Screw

This critical load must be higher than the load for which the screw is designed.

15. Draw the sketch of the screw jack showing all its dimensions.

16. Find efficiency of the screw jack.

MODULE 9.

LESSON 27 INTRODUCTION TO BEARINGS

27.1 Introduction

Every mechanical system involves relative motion between different machine elements. Relative motion leads to loss of power due to friction and deterioration of contacting surfaces due to wear. Bearings are the machine elements that permit relative motion between two components and transmission of load from one to the other, with minimum friction. For example, there is relative motion between a transmission shaft and the housing, in which it is supported. Bearings are provided at the support points of the shaft and they help in reducing power losses due to friction between the shaft and the housing and transmit the loads from the shaft to the housing.

27.2 Classification of Bearings

27.2.1 Depending upon Direction of Load

Radial Bearings

Bearings used to support the load that acts perpendicular to the axis of shaft are called radial bearings. Refer figure 27.1a.

Thrust Bearings

Bearings used to support the load that acts parallel to the axis of shaft are called thrust bearings. Refer figure 27.1b.

27.2.2 Depending upon Nature of Contact

Sliding Contact Bearings

In case of sliding contact bearings, sliding takes place between the moving and fixed elements along the contact surfaces. To reduce friction and wear, sliding surfaces are separated by a lubricating oil film. Sliding contact bearings, shown in figure 27.2a, are also known as plain bearings, journal bearings and sleeve bearings. Applications: engine crankshaft bearings, centrifugal pumps, turbines, large size electric motors, concrete mixers, rope conveyors etc.

Rolling Contact Bearings

In case of rolling contact bearings, rolling elements (balls or rollers) are introduced between the surfaces having relative motion. These bearing thus have rolling friction instead of sliding friction. Rolling contact bearings, shown in figure 27.2b, are also known as antifriction bearings. Applications: automobile axles, gear boxes, machine tool spindles, small size electric motors, crane hooks etc.

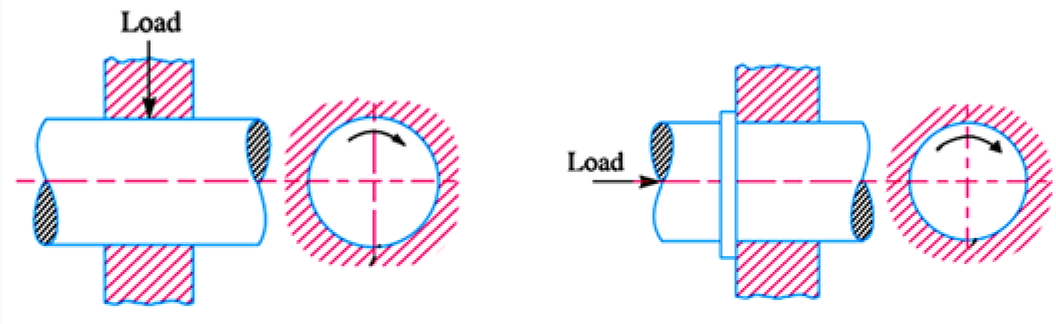


Figure 27.1 a. Radial Bearing b. Thrust Bearing

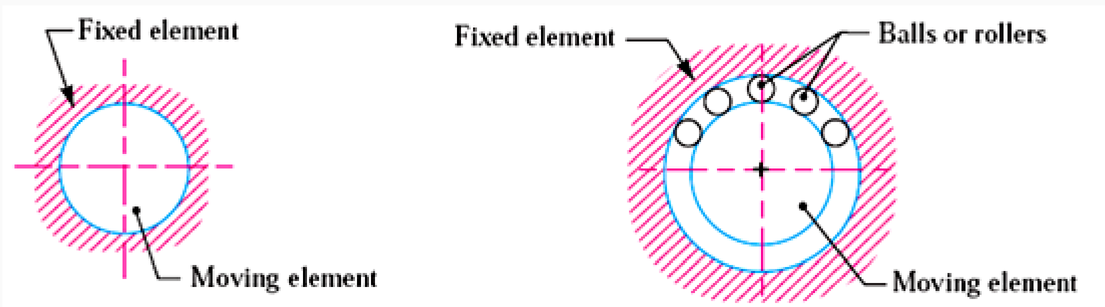


Figure 27.2 a. Sliding Contact Bearing b. Rolling Contact Bearing

27.3 Comparison of Sliding and Rolling Contact Bearings

In sliding contact bearings, starting friction is very high due to metal to metal contact between the two surfaces. Friction reduces gradually as the relative speed increases and the lubricating oil film gets established between the two surfaces having relative motion. But for the condition of pure rolling, friction is zero. That is why rolling contact bearings are also known as antifriction bearings. Although, in actual practice, because of the deformation of contacting surfaces, the type of contact changes from point/ line to surface contact, leading to a positive value of friction.

Because of lesser friction in the rolling contact bearings, the starting torque is very less as compared to operating torque. Due to this reason, these are used in driving units. Maintenance is easy and lubricant consumption is less. Also due to standardization, these are easy to replace. But are sensitive to shock and impact and have limited maximum speed and service life.

On the other hand, sliding contact bearings, due to large lubrication area and load absorbing capacity, are insensitive to impacts and shocks. These can operate at very high speed and have infinite service life. Sliding contact bearings have simple construction and are easy to mount and dismount. But the starting torque and lubricant consumption is very high.

27.4 Rolling Contact Bearings

In rolling contact bearings, the elements of the bearing have a rolling contact. These have following four main parts:

1. Outer Race
2. Inner Race
3. Balls/Rollers
4. Retainers

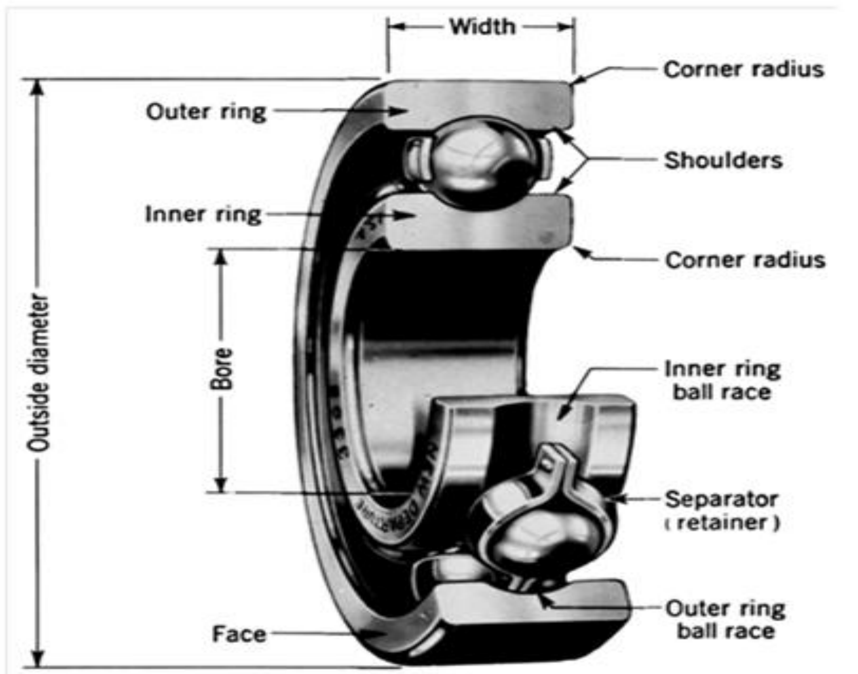


Figure 27.3 Rolling Contact Bearing

Inner race is mounted on the shaft and rotates along with it. The outer race, fitted in the housing, remains stationary. The two races are held concentric and rolling elements (balls/rollers) are kept between the two. Rolling elements are equally spaced along the circumference and are held separated from each other with the help of retainers. Figure 27.3 shows a typical rolling contact bearing.

Various types of rolling contact bearings are shown in figure 27.4 and are discussed in the following articles.

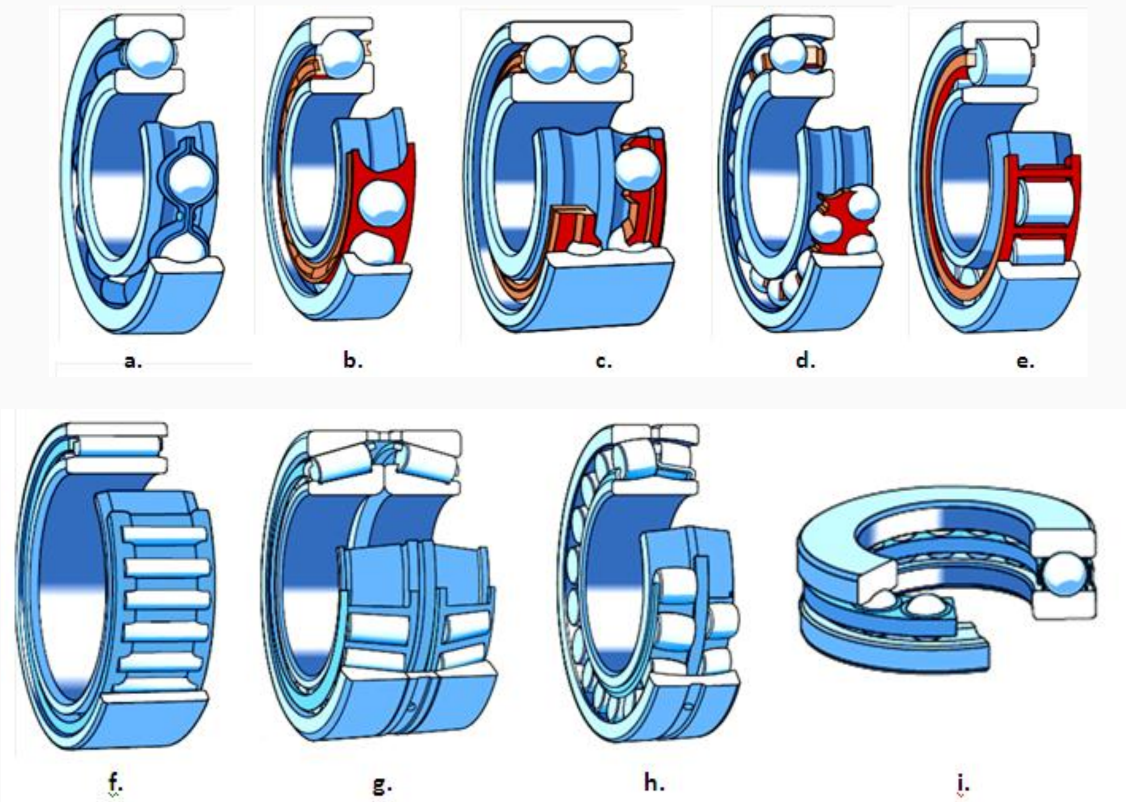
27.4.1 Single Row Deep Groove Ball Bearing

It is the most common type of rolling contact bearing. It can withstand both axial and radial loads. Under radial loads, contact between balls and races is along a vertical line. Rings get displaced slightly under the axial load and the balls roll in contact with side walls of the races. These bearings are used for supporting shafts in the gearbox.

27.4.2 Single Row Angular Contact Bearing

In this case, races are provided with shoulders (a higher and a lower); in such a way that line through contact points of ball makes an acute angle with the bearing shaft axis. Due to this

angular contact and elliptical contact area, angular contact bearings are suitable for heavy axial loads. Also because of larger number of rolling elements, it can withstand higher radial loads. But it can take up axial loads only in one direction (towards the higher shoulder) and are therefore used in pairs (placed in opposite direction) if axial load is to be supported in both directions. It finds applications in supporting wheel hubs, shafts of differential gear and steering gears like rack and pinion.



a. Single Row Deep Groove Ball Bearing
b. Single Row Angular Contact Bearing
c. Double Row Angular Contact Ball Bearing
d. Self Aligning Ball Bearings
e. Cylindrical Roller Bearings
f. Needle Roller Bearings.
g. Taper Roller Bearings
h. Spherical Roller Bearings
i. Single Row Thrust Ball Bearing

Figure 27.4 Types of Rolling Contact Bearing

27.4.3 Double Row Angular Contact Ball Bearing

It is compounded unit of two single row angular contact bearings that can support axial load in opposite directions. These bearings can thus withstand high axial loads in both directions, in addition to the radial load. These bearings are used to support shafts with worm gears, angled spur gears, bevel gears etc.

27.4.4 Self Aligning Ball Bearings

These bearings are provided with a spherical outer race, which allows the deflection of inner race and shaft with respect to the outer race. This helps in compensating deflection or misalignment of the shaft. Radial load capacity of a self aligning ball bearing is lesser than a corresponding deep groove ball bearing. These bearings are used in applications where chances of shaft bending, mounting inaccuracies, misalignment etc. are there e.g. transmissions, agricultural machinery, convening machinery etc.

27.4.5 Cylindrical Roller Bearings

Cylindrical rollers have a line contact with the races due to which cylindrical roller bearing can support larger radial load in comparison to a ball bearing of corresponding size. But their axial load carrying capacity is very less and accurate alignment is also required. Races are provided with flanges to guide the rollers. These bearings are used in electric motors, gearboxes, rail car axles etc.

27.4.6 Needle Roller Bearings

These are special type of cylindrical roller bearings in which the rolling elements are of relatively smaller diameter (1. 5mm to 4.5mm) with length to diameter ratio of 3 to 8. These can take high radial loads and can withstand fluctuating loads also. But these are not suitable for axial loading and are also very sensitive to shaft misalignment. These bearings are used for supporting connecting rods, swivel arms, rocker shafts, spindles etc.

27.4.7 Taper Roller Bearings

In taper roller bearings, taper rollers are used, which have a shape of frustum of a cone. These bearings can take both radial and thrust loads. Rollers and races are so shaped and designed that the production lines drawn from cone and races converge at one point on the axis of the bearing. This helps in attaining pure rolling without slippage along the entire length of the roller. These are also used in pairs to support axial thrust in both directions. Double row taper roller bearings are used to support higher radial and axial thrust in compact space. Taper roller bearings are used for shafts having worm and bevel gears, machine tool spindles, cable pulleys and wheel hubs.

27.4.8 Spherical Roller Bearings

Spherical roller bearings have two rows of spherical rollers that run on a common cylindrical surface of outer race. Inner race retains two rows of rollers and outer spherical race helps in accommodating misalignment between the two races. Due to this advantage of compensation of misalignment and angular deflection of shafts, these also fall in the category of self-aligning bearings. These bearings can withstand high axial as well as radial loads. These are used for cable pulleys, propelling shafts, heavy wheels, crankshafts etc.

27.4.9 Single Row Thrust Ball Bearing

Single row thrust ball bearings have two grooved annular disc plates with balls are retained between them. These are designed for taking only axial loads in one direction with one stationary race way and other attached to rotating member. Due to sliding of balls, friction is higher in these bearings. To decrease friction and reduce contact area, diameter of grooves is kept larger than that of balls. But this restricts the use of thrust ball bearings to lower speeds as at higher speeds the centrifugal force pushes the balls outwards. Moderate misalignment only can be tolerated. These bearings are used for injection pump governor linkage steering boxes and other applications for supporting thrust loads.

LESSON 28 SELECTION OF ANTIFRICTION BEARINGS

28.1 Rated Life of a Bearing

Life of a bearing is the number of revolutions or number of hours at constant speed that the bearing runs, before the first evidence of fatigue crack in the balls or races. But test data shows large variation in the lives of identical bearings, operating under similar conditions. Because of this reason, life of bearings is expressed in terms of statistical average life of a group of bearings.

Rated life of a group of apparently identical bearings is defined as the number of revolutions that 90 % of bearings will complete or exceed before the first evidence of fatigue crack. It is also known as minimum life and L_{10} life.

Reliability is the ratio of number of bearings that complete L million revolutions to the total number of bearings tested. Therefore, by definition, rated life (L_{10}) corresponds to 90% reliability of the bearing. Depending upon the type of application, bearings with reliability greater than 90% or less than 90% may also be required. Relation between bearing life and reliability is as follows:

$$\frac{L}{L_{10}} = \left[\frac{\log_e\left(\frac{1}{R_x}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/b}$$

where, L_{10} = rated life of the bearing (life corresponding to 90% reliability)

L = life of the bearing corresponding to x % reliability

R_{90} = reliability of 90% (0.9)

R_x = reliability of x %

Using this relation, for given rated life of bearing, life of bearing corresponding to a reliability of x % (L) can be calculated.

28.2 Static Load Carrying Capacity

Static load carrying capacity (C_0) of a bearing is defined as the static load corresponding to a total permanent deformation of balls and races, at the most heavily stressed point of contact, equal to 0.0001 of the ball diameter.

The bearing is subjected to some static load, when the shaft is stationary. This leads to plastic deformation in the balls and races. This deformation increases with increase in the static load.

It has been established that a total permanent deformation of 0.0001 of ball diameter, at the most heavily stressed point of contact, can be tolerated without affecting operational properties of the bearing.

Different formulae have been developed for calculation of static load carrying capacity. However for selection of bearings, use of these formulae is not necessary; as the values of static load carrying capacity are directly given in manufacturer's catalogue.

28.3 Dynamic Load Carrying Capacity

Dynamic load carrying capacity (C) is defined as the constant stationary radial load (in case of radial bearings) or constant axial load (in case of thrust bearings), which a group of apparently identical bearings, with stationary outer ring can endure for a rated life of one million revolutions with only 10 % failures.

Various relations have been developed for calculating dynamic load carrying capacity also, but again its value for different bearings is available in manufacturer's catalogue. Table 28.1 gives basic dimensions and load capacities of different types of bearings.

28.4 Equivalent Bearing Load

In many applications, bearings are subjected to both radial and axial loads and are sometimes required to operate with stationary inner race and rotating outer race.

The equivalent dynamic load of a bearing is defined as the constant radial load in radial bearings or trust load in trust bearings, which if applied to the bearing, would give same life as attained by the bearing under actual conditions of loading. It is given by,

$$P = XVF_r + YF_a$$

where, V = race rotation factor = 1 if inner race rotates

= 1.2 if outer race rotates

X = radial load factor

Y = thrust load factor

Values of X and Y are available in manufacturer's catalogue. Table 28.2 gives values of X and Y for deep groove ball bearings.

Table 28.1 Dimensions & Load Carrying Capacities of Ball Bearings

Bearing No.	Bore (mm)	Outside Diameter (mm)	Width (mm)	Load Carrying Capacity (kN)							
				Single Row Deep Groove		Single Row Angular Contact		Double Row Angular Contact		Self-aligning	
				C_0	C	C_0	C	C_0	C	C_0	C

6200 6300	10	30 35	9 11	2.24 3.60	4 6.3	----- -----	----- -----	4.55 -----	7.35 -----	1.8 -----	5.7 -----
6201 6301	12	32 37	10 12	3.00 4.30	5.4 7.65	----- -----	----- -----	5.6 -----	8.3 -----	2 3	5.85 9.15
6202 6302	15	35 42	11 13	3.55 5.20	6.1 8.8	3.75 -----	6.30 -----	5.6 9.3	8.3 14	2.16 3.35	6 9.3
6203 6303 6403	17	40 47 62	12 14 17	4.40 6.30 11.00	7.5 10.6 18	4.75 7.2 -----	7.8 11.6 -----	8.15 12.9 -----	11.6 19.3 -----	2.8 4.15 -----	7.65 11.2 -----
6204 6304 6404	20	47 52 72	14 14 19	6.55 7.65 15.60	10 12.5 24	6.55 8.3 -----	10.4 13.7 -----	11 14 -----	16 19.3 -----	3.9 5.5 ------	9.8 14 -----
6205 6305 6405	25	52 62 80	15 17 21	7.10 10.40 19.00	11 16.6 28	7.8 12.5 -----	11.6 19.3 -----	13.7 20 -----	17.3 26.5 -----	4.25 7.65 -----	9.8 19 -----
6206 6306 6406	30	62 72 90	16 19 23	10.00 14.60 23.20	15.3 22 33.5	11.2 17 -----	16 24.5 -----	20.4 27.5 -----	25 35.5 -----	5.6 10.2 -----	12 24.5 -----
6207 6307 6407	35	72 80 100	17 21 25	13.70 17.60 30.50	20 26 43	15.3 20.4 -----	21.2 28.5 -----	28 36 -----	34 45 -----	8 13.2 ------	17 30.5 -----
6208 6308 6408	40	80 90 110	18 23 27	16 22 37.5	22.8 32 50	19 25.5 -----	25 35.5 -----	32.5 45.5 -----	39 55 -----	9.15 16 -----	17.6 35.5 -----
6209 6309 6409	45	85 100 120	19 25 29	18.3 30 44	25.5 41.5 60	21.6 34 -----	28 45.5 -----	37.5 56 -----	41.5 67 -----	10.2 19.6 -----	18 42.5 -----
6210 6310 6410	50	90 110 130	20 27 31	21.2 35.5 50	27.5 48 68	23.6 40.5 -----	29 53 -----	43 73.5 -----	47.5 81.5 -----	10.8 24 -----	18 50 -----
6211 6311 6411	55	100 120 140	21 29 33	26 42.5 60	34 56 78	30 47.5 -----	36.5 62 -----	49 80 -----	53 88 -----	12.7 28.5 -----	20.8 58.5 -----
6212 6312 6412	60	110 130 150	22 31 35	32 48 67	40.5 64 85	36.5 55 -----	44 71 -----	63 96.5 -----	65.5 102 -----	16 33.5 -----	26.5 68 -----
6213 6313 6413	65	120 140 160	23 33 37	35.5 55 76.5	44 72 93	43 63 -----	50 80 -----	69.5 112 -----	69.5 118 -----	20.4 39 -----	34 75 -----

6214 6314 6414	70	125 150 180	24 35 42	39 63 102	48 81.5 112	47.5 73.5 -----	54 90 -----	71 129 -----	69.5 137 -----	21.6 45 -----	34.5 85 -----
6215 6315 6415	75	130 160 190	25 37 45	42.5 72 110	52 90 120	50 81.5 -----	56 98 -----	80 140 -----	76.5 143 -----	22.4 52 -----	34.5 95 -----
6216 6316 6416	80	140 170 200	26 39 48	45.5 80 120	57 96.5 127	57 91.5 -----	63 106 -----	96.5 160 ------	93 163 -----	25 58.5 -----	38 106 -----
6217 6317 6417	85	150 180 210	28 41 52	55 88 132	65.5 104 134	65.5 102 -----	71 114 -----	100 180 -----	106 180 -----	30 62 -----	45.5 110 -----
6218 6318 6418	90	160 190 225	30 43 54	63 98 146	75 112 146	76.5 114 -----	83 122 ------	127 ----- -----	118 ----- -----	36 69.5 -----	55 118 -----

Table 28.2 Values of Radial Load Factor (X) and Thrust Load Factor (Y) for Deep Groove Ball Bearings

$\frac{F_a}{C_0}$	$\frac{F_a}{F_r} \leq e$		$\frac{F_a}{F_r} \geq e$		e
	X	Y	X	Y	
0.0025	1	0	0.56	2.0	0.22
0.04				1.8	0.247
0.07				1.6	0.27
0.13				1.4	0.31
0.25				1.2	0.37
0.50				1.0	0.44

28.5 Load Life Relationship

Relationship between bearing life, dynamic load carrying capacity and equivalent dynamic load, is given by,

$$L_{10} = \left(\frac{C}{P}\right)^k$$

where, L_{10} = rated bearing life

C = dynamic load carrying capacity

P = equivalent dynamic load

k = load life exponent = 3 for ball bearings

= 10/3 for roller bearings

Also,	$$L_{10} = \left(\frac{C}{P}\right)^k$$

where, L_{10h} = rated bearing life in hours

N = speed of rotation in r.p.m.

28.6 Selection of Bearings

Following steps are generally followed in selection of antifriction bearings:

1. Determine radial and axial forces (F_r and F_a respectively) acting on the shaft.
2. Calculate the diameter of the shaft.
3. Select suitable type of the bearing from manufacturer's catalogue.

Following guidelines can be used for selecting ball bearings:

Type of Ball Bearing	F_a/F_r
Single Row Deep Groove	0.5 – 0.8
Double Row Deep Groove	0.8 – 1.5
Angular Contact	1.5 – 2.0
Self Aligned	0.2 – 0.5

Select the lowest series of the selected category, depending upon the shaft diameter. Note the value of static load carrying capacity, C_0. Refer table 28.1.

4. Select value of race rotation factor, V.

5. Determine values of radial and thrust load factors (X and Y) corresponding to the calculated values of F_a and F_r, and value of C_0 of selected bearing. Refer table 28.2 for deep groove ball bearings.

6. Calculate equivalent dynamic load of the bearing,

$$P = XVF_r + YF_a$$

7. Decide expected life of the bearing in millions of revolutions (L).

8. Calculate the required dynamic load carrying capacity for expected life, using load life relationship.

$$C_{req.} = P(L)^{1/k}$$

9.Dynamic load carrying capacity of the selected bearing should be greater than the required value calculated above, i.e.

$$C \text{ should be} \geq C_{req.}$$

If , choose higher series of the bearings from the catalogue and repeat the procedure from Step 3.

MODULE 10.

LESSON 29 DESIGN OF LEVERS

29.1 Introduction

Lever is a simple mechanical device, in the form of a straight or curved link or a rigid rod, pivoted about the fulcrum. It works on the principle of moments and is used to get mechanical advantage and sometimes to facilitate the application of force in a desired direction. Examples of levers are: straight tommy bar used to operate screw jack, bell crank lever, rocker arm, lever of lever loaded safety valve etc. Figure 29.1 shows the construction of a simple lever. P is the applied effort required to overcome load, W.

Ratio of load to effort is called Mechanical Advantage and ratio of effort arm length to load arm length is called leverage.

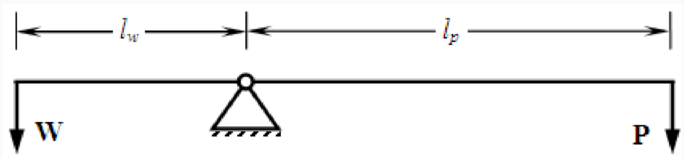


Figure 29.1 Lever

$$\text{Mechanical Advantage} = \frac{\text{Load}}{\text{Effort}} = \frac{W}{P}$$

$$\text{Leverage} = \frac{\text{Effort Arm Length}}{\text{Load Arm Length}} = \frac{l_p}{l_w}$$

29.2 Classes of Levers

Depending upon the position of load point, effort point and fulcrum, levers are classified into following classes:

Class I Levers	Lever having the fulcrum located between the load point and effort point is called Class I lever. Examples are rocker arm, bell crank lever etc. Mechanical advantage of such levers is greater than one as effort arm is larger than the load arm.
Class II Levers	Lever having load point located between the fulcrum and effort point is called Class II lever. Lever used in safety valve is an example of lever of this class. The effort arm is larger than the load arm; therefore the mechanical advantage is more than one.
• Class III Levers	• Lever having effort point located between the fulcrum and load point is called Class III lever. The effort arm, in this case, is smaller than the load arm; therefore the mechanical advantage is less than one. Due to this, the use of such type of

	levers is not recommended. However a pair of tongs, the treadle of a sewing machine etc. are examples of this type of lever.

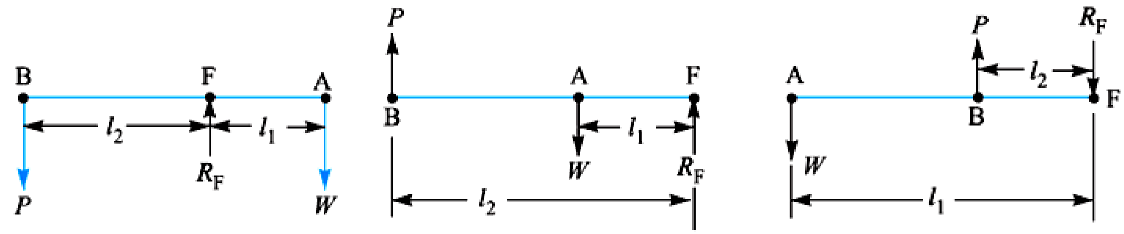


Figure 29.2 Class I, Class II and Class III Lever

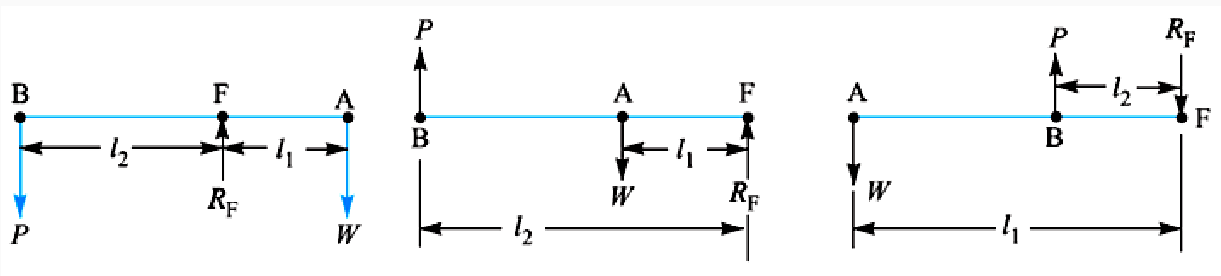


Figure 29.2 shows levers of different classes.

29.3 Design of Lever

Design of lever involves determination of various dimensions of the lever for a specified load or output force required. For a specified load or output force desired, effort required can be calculated using principle of moments. Due to these forces, arms of the lever are subjected to bending and are designed based on that. Reaction force acting on the fulcrum can be calculated. Fulcrum of the lever is a pin joint and is designed based on bending and bearing considerations. Design procedure is discussed below.

29.3.1 Determination of Forces

If the load and effort are parallel to each other, as shown in figure 29.2, reaction on the fulcrum is the algebraic sum of these two forces. But if the load and effort are inclined to each other at an angle q, as shown in figure 29.3, reaction (R) at the fulcrum can be determined as:

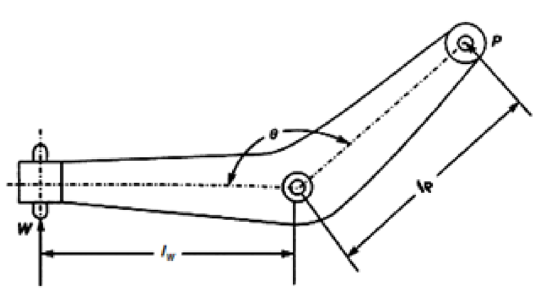


Figure 29.3 Angled Lever

$$R = \sqrt{W^2P^2 - 2WP\cos\theta}$$

29.3.2 Design of Lever Arms

Arms are subjected to bending moment and their section is estimated from bending stress consideration. Figure 29.4 shows lever with fulcrum located between the load and the effort point. Bending moment is zero at the point of application of forces and is maximum at the fulcrum. Maximum Bending Moment is given by,

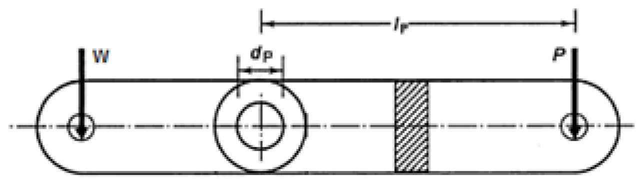
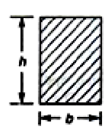
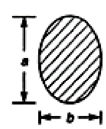
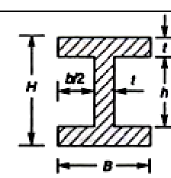


Figure 29.3 Lever Arms

Maximum Bending Stress is then given by, $M = Pl_p$

Most commonly used sections for lever arms are: rectangular, elliptical and I-section. Values of moment of inertia, I and distance of farthest fibre from neutral axis, y for these sections are given in table 29.1.

Table 29.1 Common Sections used for Lever Arms

Section	Rectangular	Elliptical	I-section
Shape			
I	$\frac{bh^3}{12}$	$\frac{\pi a^3 b}{64}$	$\frac{BH^3 - bh^3}{12}$
y	h/2	a/2	H/2
Typical Proportions	$h = (2 \text{ to } 5)b$	$a = (2 \text{ to } 3)b$	$H = (4 \text{ to } 6)t$ $B = (3 \text{ to } 4)t$

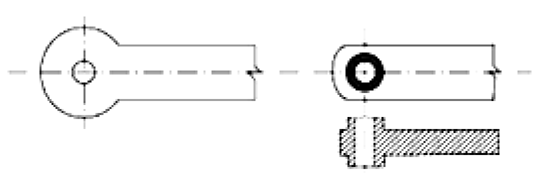


Figure 29.4 Section of Lever Arm at Fulcrum

Therefore using suitable values of I and y for selected section, its dimensions can be finalised so that the bending stress remains within the allowable limits. Often the arms are made with cross-section reducing from central portion to the point of application of load. This is done to save material using uniform strength condition. Critical section of the lever (section of maximum bending moment) becomes weak due to hole made for pin. To compensate for the reduced strength, width of that section is increased or boss is provided as shown in figure 29.4.

29.3.3

Design of Fulcrum

Fulcrum of lever is a pin joint as shown in figure 29.5. Pin is designed based on bearing and bending considerations as discussed below.

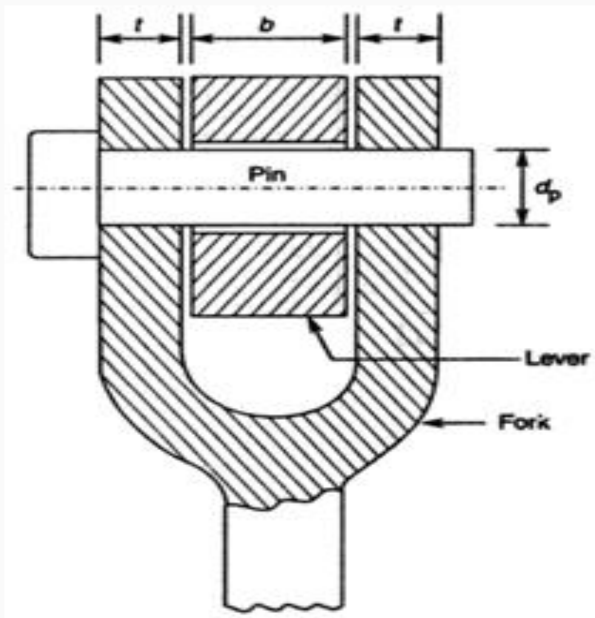


Figure 29.5 Lever Fulcrum

Bearing Failure

The permissible bearing pressure ($[P_{bearing}]$) depends upon relative velocity, frequency of relative motion and the lubrication condition between the pin and the bush. The usual range of allowable bearing pressure for brass/bronze bush and steel pin is 10-25 N/mm². Lower values are used for high relative velocity, frequent motion and intermittent lubrication conditions. If d_p and l_p are diameter and length of the pin respectively, bearing pressure is given by,

$$P_{bearing} = \frac{\text{load}}{\text{projected area}} = \frac{R}{d_p\, l_p} \text{ should be } \leq [P_{bearing}]$$

Shear Failure

Pin is subjected to double shear and maximum shear stress is given by, $\tau = \frac{R}{2\left(\frac{\pi}{4}d_p^2\right)} = \frac{2R}{\pi d_p^2}$ should be $\leq [\tau]$

Bending Failure

As discussed in the design of pin for knuckle joint, when the pin is loose in the eye, which is a desired condition here for relative motion, pin is subjected to bending moment. It is assumed that: Load acting on the pin is uniformly distributed in the eye and uniformly varying in the two parts of the fork. Maximum Bending Moment (at centre) is given by,

$$M = \frac{R}{2}\left(\frac{b}{2} + \frac{a}{3}\right) - \frac{R}{2}\left(\frac{b}{4}\right) = \frac{R}{2}\left(\frac{b}{4} + \frac{a}{3}\right)$$

Maximum Bending Stress in the pin, $\sigma_b = \frac{My}{I}$ should be $\leq [\sigma]$

where, $I = \frac{\pi d_p^4}{64}$ and $y = \frac{d_p}{2}$

29.4 Lever Material & Factor of Safety

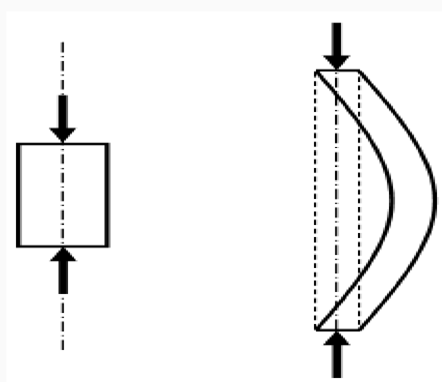
Levers are generally forged or cast. It is difficult to forge curved levers with complicated cross-sections and have to be cast. As the levers are subjected to tensile stress due to bending, cast iron is not recommended to be used as material for levers. Aluminium alloys are generally used for levers. For severe loading and corrosive conditions, alloy steels are used. Suitable heat treatment processes are also often employed to improve wear and shock resistance of lever. Factor of safety of 2 to 3 on yield strength is generally used. For severe loading conditions or fatigue loading higher factor of safety is also taken.

LESSON 30 DESIGN OF COLUMNS

30.1 Introduction

A slender machine component, having considerable length in comparison to its cross-sectional dimensions, subjected to a compressive load is known as column, strut, pillar or stanchion. Examples of columns are push rods of valve mechanisms, piston rods in hydraulic / pneumatic cylinders, connecting rods, power screws etc.

If a machine member with comparable length and cross-sectional dimensions is subjected to axial compressive load as shown in figure 30.1a, compressive stresses are developed and it deforms according to Hook's law. Failure occurs when the compressive stress reaches the yield strength. But when a machine member, having considerably larger length than cross-sectional dimensions, is subjected to compressive load as shown in figure 30.1b, it may fail due to buckling. Buckling is sudden large lateral deflection, which occurs when the compressive load reaches certain limit called critical load ($W_{critical}$). It is different from lateral deflection of a beam. Lateral deflection of beam increases gradually with increase in lateral load. On the other hand, in case of buckling, there is no load in the lateral direction and there is no lateral deflection also before the axial compressive load reaches the critical load. Once the load reaches critical load, sudden lateral deflection takes place resulting in collapse of the column. Thus buckling leads to sudden and total failure that takes place without any warning.



(a) (b)

Figure 30.1 Machine Members subjected to Compressive Load

30.2 Slenderness Ratio

Slenderness ratio is an important parameter that affects the critical load. It is defined as the ratio of length of the column to least radius of gyration of the cross-section about its axis.

$$\text{Slenderness Ratio} = \frac{l}{k}$$

where, l = length of the column

k = radius of gyration of the cross-section about its axis = $\sqrt{I/A}$

where, A = Area of Cross-section

I = Least Moment of Inertia of the Cross-section

Buckling does not take place if the slenderness ratio is less than 30 and such members are designed on the basis of compressive stresses only. But if the slenderness ratio is more than 30, buckling takes place at critical load and critical load becomes the design criteria.

30.3 Critical Load Prediction

Various scientists have developed different relations for the prediction of critical load, out of which only two basic relations are discussed here. For the column too be safe in buckling, maximum load to which the column is subjected, must be less than the critical load.

30.3.1 Euler's Equation

According to Euler, buckling takes place when the compressive load reaches following critical value.

$$W_{critical} = \frac{n\pi^2 EI}{l^2} = \frac{n\pi^2 EA}{(l/k)^2}$$

where, E = Modulus of Elasticity of Column Material

n = End Fixity

= 4 (both ends fixed)

= 2 (one end fixed and other end hinged)

= 1 (both ends hinged)

= 0.25 (one end fixed and other end free)

30.3.2 Johnson's Equation

According to Johnson, critical load is given by,

$$W_{critical} = AS_y\left[1 - \frac{S_y}{4n\pi^2 E}\left(\frac{l}{k}\right)^2\right]$$

where, S_y = Yield Strength of Screw Material

LESSON 31 DESIGN OF THIN CYLINDRICAL & SPHERICAL SHELLS

31.1 Pressure Vessels

Pressure vessels are the vessels used to store or supply fluids under pressure. Stored fluid may undergo a change of state inside the pressure vessel e.g. in steam boiler or may undergo some chemical reaction. In nuclear / thermal power plants, chemical industries and various other industries, pressure vessels are used for storage and supply of different fluids, like water, gas, steam, air etc. Pressure vessels are generally made of steel plates by bending them to desired shapes and joining the ends by welding. Pressure vessels have to be designed very carefully as their failure may cause dangerous accident due to explosion or leakage of fluid.

31.2 Classification of Pressure Vessels

31.2.1 According to Dimensions

Thin Shell Pressure Vessels

Pressure vessels with inner diameter to wall thickness ratio greater than 20 are called thin shell pressure vessels. These are used in boilers, tanks, pipes etc.

Thick Shell Pressure Vessels

Pressure vessels with inner diameter to wall thickness ratio less than 20 are called thick shell pressure vessels. These are used in high pressure cylinders, tanks and gun barrels etc.

31.2.2 According to Shape

Depending upon the geometric shape, the pressure vessels can be classified into Cylindrical, Spherical or Conical pressure vessels.

31.2.3 According to End Construction

Open Ended Pressure Vessels

e.g. cylinder with a piston.

Due to internal fluid pressure, circumferential hoop stress is induced.

lose Ended Pressure Vessels

e.g. a tank.

Due to internal pressure, circumferential hoop stress and longitudinal stress is induced.

Design of pressure vessels with inner diameter to wall thickness ratio greater than 20 (i.e. thin shell) and having cylindrical and spherical shape, are discussed in the following articles.

31.3 Design of Thin Cylindrical Shells

31.3.1 Circumferential Hoop Stress

Due to internal pressure, cylindrical shell may fail along the longitudinal section as shown in figure 31.1. To avoid this kind of failure, the circumferential hoop stress should not exceed the yield strength of the material. Consider free body diagram of half portion of the cylinder as shown in figure 31.2.

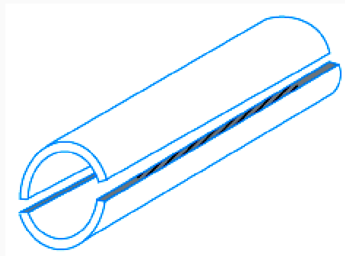


Figure 31.1 Failure of Thin Cylindrical Shell along the Longitudinal Axis

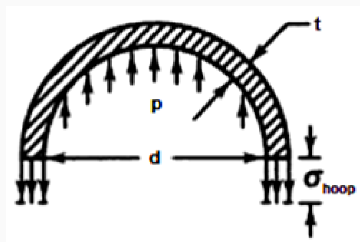


Figure 31.2 Hoop Stress in Thin Cylindrical Shell

Let p = Internal Pressure

d = Internal Diameter of Cylinder

t = Wall Thickness of Cylinder

l = Length of Cylinder

Considering the equilibrium of forces,

$$\text{Internal Pressure} \times \text{Projected Area} = \text{Hoop Stress} \times \text{Resisting Area}$$

$$p \times dl = \sigma_{hoop} \times 2tl \quad \text{or} \quad \sigma_{hoop} = \frac{pd}{2t} \text{should be} \leq [\sigma]$$

31.3.2 Longitudinal Stress

In addition to hoop stress, longitudinal stress is also induced in close ended pressure vessels. Thin cylindrical shells may fail along the transverse section (as figure 31.3), if this longitudinal stress reaches the yield strength of the material. Consider free body diagram of the upper half of the cylinder as shown in figure 31.4. Considering the equilibrium of forces,

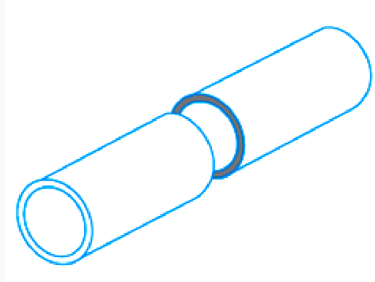


Figure 31.3 Failure of Thin Cylindrical Shell along the Transverse Section

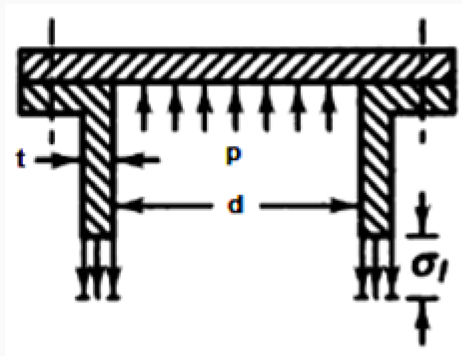


Figure 31.4 Longitudinal Stress in Thin Cylindrical Shell

31.4 Considering the equilibrium of forces,

$$\text{Internal Pressure} \times \text{Projected Area} = \text{Longitudinal Stress} \times \text{Resisting Area}$$

$$p \times \frac{\pi}{4} d^2 = \sigma_l \times \pi dt \quad \text{or} \quad \sigma_l = \frac{pd}{4t} \text{ should be} \leq [\sigma]$$

It can be observed from the expressions of transverse hoop stress and longitudinal stress that

$$\sigma_{hoop} = 2\,\sigma_l$$

Therefore hoop stress should be the design criteria for thin cylindrical shells subjected to internal pressure and desired minimum wall thickness can be estimated from the design equation based on allowable value of hoop stress.

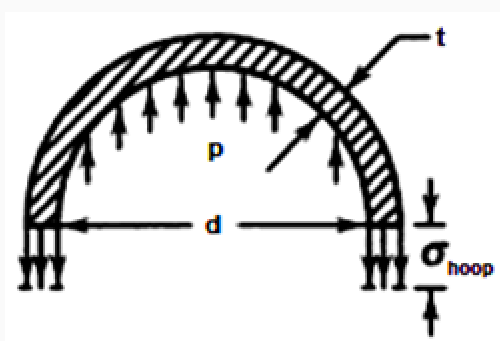


Figure 31.5 Hoop Stress in Thin Spherical Shell

31.5 Design of Thin Spherical Shells

Consider free body diagram of half portion of a sphere as shown in figure 31.5.

Let p = Internal Pressure

d = Internal Diameter of Sphere

t = Wall Thickness of Sphere

Considering the equilibrium of forces,

$$\text{Internal Pressure} \times \text{Projected Area} = \text{Hoop Stress} \times \text{Resisting Area}$$

$$p \times \frac{\pi}{4} d^2 = \sigma_{hoop} \times \pi dt \quad \text{or} \quad \sigma_{hoop} = \frac{pd}{4t} \text{ should be} \leq [\sigma]$$

Desired minimum wall thickness can be estimated from the design equation based on allowable value of hoop stress.

LESSON 32 DESIGN OF CURVED BEAMS

32.1 Introduction

Beam having its neutral axis curved in unloaded condition is known as curved beam. Neutral axis and centroidal axis of a curved beam do not coincide. Neutral axis of curved beams is shifted towards the centre of curvature. Also the bending stress in case of curved beam, varies hyperbolically with the distance from the neutral axis and variation is not linear as in case of straight beams. Figure 32.1 shows distribution of stress in curved beam.

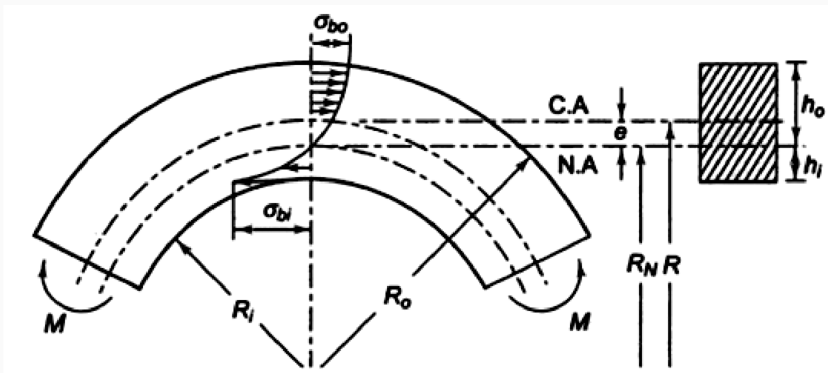


Figure 32.1 Distribution of Stress in a Curved Beam

In stress analysis of curved beam, following assumptions are made:

Let R_o = radius of outer fibre (mm)

R_1 = radius of inner fibre (mm)

R = radius of centroidal axis (mm)

R_N = radius of neutral axis (mm)

h_i = distance of inner fibre from neutral axis (mm)

h_o = distance of outer fibre from neutral axis (mm)

M = bending moment with respect to centroidal axis (N-mm)

A = area of the cross-section (mm2)

Eccentricity between centroidal and neutral axis is given by,

$$e = R - R_N$$

Bending stress in any fibre, at distance 'y' from the neutral axis, is given by,

$$\sigma_b = \frac{My}{Ae(R_N - y)}$$

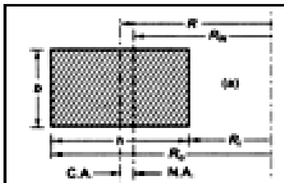
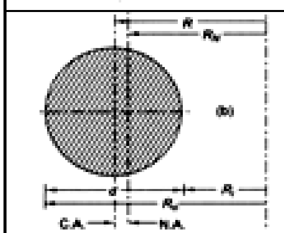
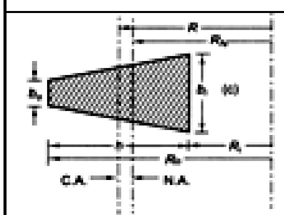
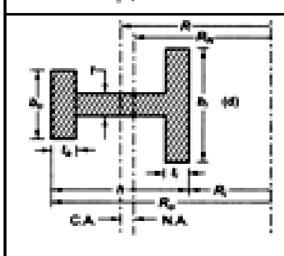
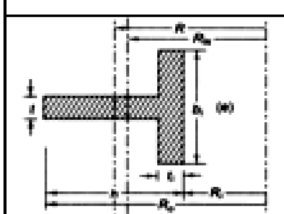
Equation indicates that stress has hyperbolic distribution with respect to y. Maximum stress occurs either at the inner fibre or at the outer fibre.

Stress at the inner fiber is given by, $\sigma_{b_i} = \dfrac{Mh_i}{AeR_i}$

Stress at the outer fiber is given by, $\sigma_{b_o} = \dfrac{Mh_o}{AeR_o}$

In symmetrical cross-sections (circular, rectangular etc.), maximum bending stress always occurs at the inner fibre. But for unsymmetrical sections, stress must be calculated for inner and outer fibre to find its maximum value. Value of e is generally very small but its precise calculation is very important to avoid error in stress determination. Expressions for calculation of R and R_N, for some standard cross-sections are given in table 32.1.

Table 32.1 R and R_N for some Standard Cross-sections

	$R_N = \dfrac{h}{\log_e\left(\frac{R_o}{R_i}\right)}$ $R = R_i + \dfrac{h}{2}$
	$R_N = \dfrac{(\sqrt{R_o} + \sqrt{R_i})^2}{4}$ $R = R_i + \dfrac{d}{2}$
	$R_N = \dfrac{\left(\frac{b_i + b_o}{2}\right)h}{\left(\frac{b_i R_o - b_o R_i}{h}\right)\log_e\left(\frac{R_o}{R_i}\right) - (b_i - b_o)}$ $R = R_i + \dfrac{h(b_i + 2b_o)}{3(b_i + b_o)}$
	R_N $= \dfrac{t_i(b_i - t) + t_o(b_o - t) + th}{b_i \log_e\left(\frac{R_i + t_i}{R_i}\right) + t \log_e\left(\frac{R_o - t_o}{R_i + t_i}\right) + b_o \log_e\left(\frac{R}{R_o - \cdots}\right)}$ $R = R_i + \dfrac{\frac{1}{2}th^2 + \frac{1}{2}t_i^2(b_i - t) + t_o(b_o - t)\left(h - \frac{t_o}{2}\right)}{t_i(b_i - t) + t_o(b_o - t) + th}$
	$R_N = \dfrac{t_i(b_i - t) + th}{(b_i - t)\log_e\left(\frac{R_i + t_i}{R_i}\right) + t \log_e\left(\frac{R_o}{R_i}\right)}$ $R = R_i + \dfrac{\frac{1}{2}th^2 + \frac{1}{2}t_i^2(b_i - t)}{t_i(b_i - t) + th}$