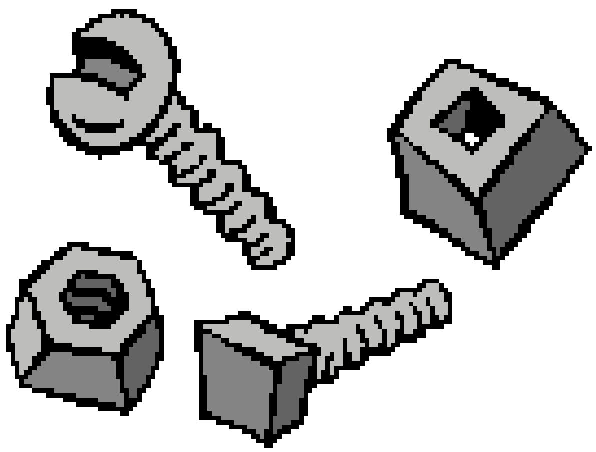
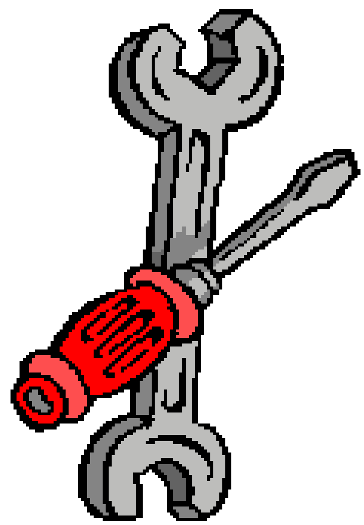
PRINCIPLES OF DAIRY MACHINE DESIGN





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Principles of Dairy Machine Design

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Module 1 Statics and Dynamics

Lesson-1

Basic Concepts in Statics and Dynamics, Force Systems, Equilibrium Conditions

1.1 INTRODUCTION:

Statics and dynamics are important terms in designing of any machine parts or equipments. The science of dynamics is based on the natural laws governing the motion of a particle.

1.1.1 Mechanics: It is a branch of science which deals with study of forces and their effects on structure, machines etc.

The branch of mechanics is further divided in to Engineering mechanics, mechanics of solids and mechanics of fluids.(fig 1.1)

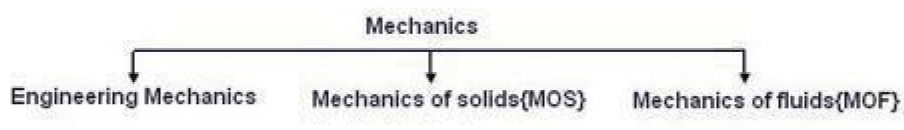

Fig1.1 Mechanics

1.1.2 Engineering mechanics: It deals with mechanics of rigid body and study of external forces and their effects on rigid body.

1.1.3 Mechanics of solids: Study of internal resisting forces developed in a deformable body under the action of external forces.

1.1.4 Mechanics of fluids: It deals with mechanics of compressive forces on the fluid & in compressive forces on fluid particles. Mechanics is an extensive field of operation which can be subdivided in various ways. A subdivision addressed in the given description of mechanics is based on the perspective of rest and movement.

1.1.5 Statics: Branch of a mechanics which deals with the study of forces on a body which is at rest.

1.1.6 Dynamics: It deals with study of motion and forces on a body which is in motion. Subcomponent of dynamics is kinematics.

1.1.7 Kinematics: The study of forces and the displacement of bodies, without addressing the cause of movement.

1.1.8 Newton's Laws: The basic laws for the displacement of a particle were first formulated by Newton. Newton's three laws are as follows.

1.1.9 First law (or) law of inertia: First Law states that when a body is in motion (or) is at rest unless until it is forced to change that state by forces imposed on it. The property with which a particle resists a change in its state of rest (or) movement is called its inertia. Newton's first law is therefore also known as the law of inertia.

1.1.10 Second law of motion: It states that the rate of change in momentum of a particle is proportional to the force applied to it, and takes place in the direction of that force. Second law is defined by the following formula.

$F = K.M.a$

Where, F= force in N

M= mass in kg

a = Acceleration in m/s^2

k= Proportionality constant

1.1.11 Third law of motion: It states that for every action there is an equal and opposite reaction. If a body A exerts a force on particle (or) body B, it will exert an equal and opposite force on body A.

1.2 FORCE SYSTEMS:

1.2.1 Force: It is defined as agent which produces (or) tends to produce destroys (or) tends to destroy motion. It may be noted that the force may have either of the two functions i.e. produces (or) tends to produce motion.

1.2.2 Resultant force: If a number of forces P, Q, R etc are acting simultaneously on a particle. It is possible to find out a single force, which would replace them, i.e. which would produce the same effect as produced by all the given forces. This force is called the resultant force and the forces P, Q, R etc are called component forces.

1.3 ANALYTICAL METHOD TO FIND OUT THE RESULTANT FORCE:

The resultant forces, of a given system of forces, may be found out analytically by methods as discussed below.

1.3.1 By parallelogram law of forces: It states that if two forces, acting simultaneously on a particle are represented in magnitude and direction by the two adjacent sides of a parallelogram; their resultant may be **Fig.1.2 Parallelogram law of forces** represented in magnitude and direction by the diagonal of the parallelogram which passes through their point of intersection:

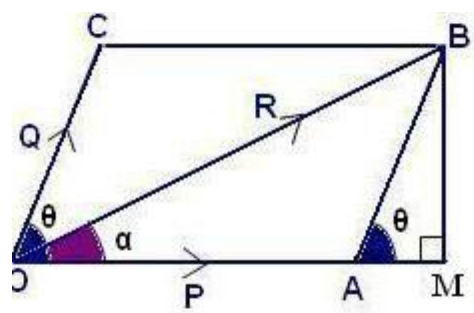

Fig.1.2 Parallelogram law of forces

Let the two forces P and Q, acting at 'O', be represented by the straight line OA and OC in magnitude and direction If parallelogram OABC be completed, with OA and OC as adjacent sides, the resultant 'R' of the forces OA and OC may be represented by the diagonal OB.

Let 'θ' be the angle between the forces P and Q. Extend the OA and draw perpendicular BM from B on this line. Let these lines meet at the point 'M' as shown in fig. no 1.2 from the geometry of the figure, we known that

$AB = Q$

$\therefore \angle BAM = \theta$

$BM = Q \sin\theta$ and $AM = Q \cos\theta$

In right angle triangle OBM, we know that

$$\therefore \quad OB = \sqrt{(OM^2 + BM^2)}$$

$$= \sqrt{((OA + AM)^2 + BM^2)}$$

$$= \sqrt{([(P + Q\cos\theta)^2 + (Q\sin\theta)^2]}$$

$$= \sqrt{([(P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta)}$$

$$= \sqrt{(P^2 + Q^2(\sin^2\theta + \cos^2\theta) + 2PQ\cos\theta)}$$

$$= \sqrt{(P^2 + Q^2 + 2PQ\cos\theta)}$$

Let the resultant OB makes an angle 'α' with the OA as shown in fig. 1.2

$$\tan\alpha = \frac{BM}{OM} = \frac{BM}{OM + AM}$$

$$= \frac{Q\sin\theta}{P + Q\cos\theta}$$

1.4 METHODS FOR THE DETERMINATION OF RESULTANT FORCE:

The magnitude of resultant force may be found out by the following methods

1. Analytical method

2. Graphical method

1.4.1 Analytical method: It is also known as the method of resolution of forces. The magnitude and direction of the resultant force of a given system of forces may be found out by the analytical method as discussed below.

Resolve all the forces vertically and find the algebraic sum of all the vertical components {i.e. $\sum V$.}

Resolve all the forces Horizontally and find the algebraic sum of all the horizontal components {i.e. ∑H.}

The resultant R of the given forces will be given by the each

$R=\sqrt{[\sum V^2+\sum H^2]}$

The resultant forces will be inclined at an angle θ, with Horizontal such that.

$\tan\theta = \sum V / \sum H$

1.4.2Graphical method for the resultant forces: The resultant forces of a given system of forces may be found out graphically by the follows methods as discussed below.

Triangle Law of Forces: It states that "if two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order, their resultant may be ,represented in magnitude and direction by the third side of the triangle, taken in opposite order".

The algebraic sum of the resolved parts of a number of forces, in a given direction is equal to the resolved part of their resultant in the same direction.

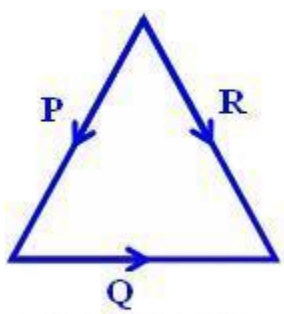

Fig.1.3 Triangle Law of Forces

Polygon Law of Forces: It states that "if more than two forces acting simultaneously on a particle, be represented in magnitude and direction by the sides of a polygon, taken in order, their resultant may be ,represented in magnitude and direction by the side closing of the polygon, taken in opposite order".

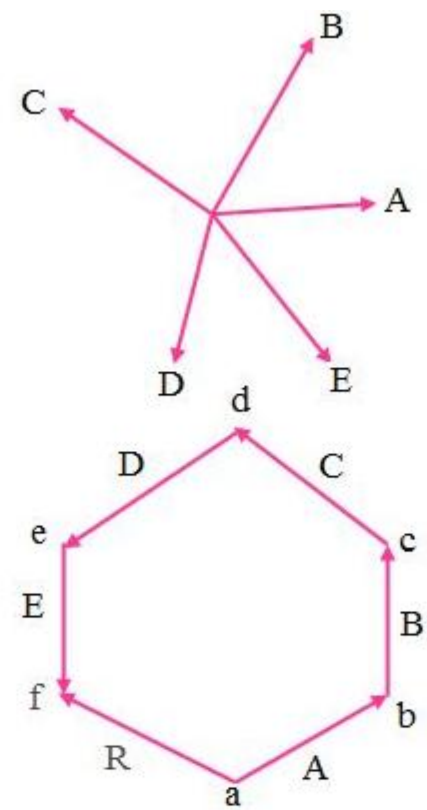

Fig.1.4 Polygon Law of forces

1. A, B, C, D, E are five forces.2. R is the resultant forces i.e. the closing side of the polygon taken in opposite order.

1.5 METHOD OF VECTORS TO FIND OUT THE RESULTANT FORCE:

This method also used to find out the resultant of given forces. First of all draw the space diagram for the given forces. Now select some suitable points and go on adding the forces vertically. Then the closing side taken in opposite direction, will represent the resultant.

1.5.1 Equilibrium of forces: If the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium. Such a set of forces, whose resultant is zero are called equilibrium forces.

1.5.2 Coplanar forces: The forces, whose lines of action lie on the same plane, are known as coplanar forces.

1.5.3 Concurrent forces: The forces, which meet at one point, are known as concurrent forces.

1.5.4 Lames theorem: It states that three coplanar forces acting on a point be in equilibrium, then ratio of force to the sine of the angle between the other two, forces are always equal to the ratio of other force to the sine of the angle between the remaining forces.(Fig.1.5)

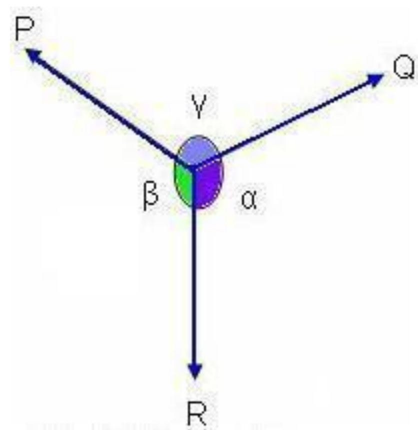

Fig.1.5 Lames Theorem

Mathematically,

$$P/\text{Sin}\alpha = \theta/\text{Sin}\beta = R/\text{Sin}\gamma$$

Where P, Q, R are three forces and α,β,γ are the angles.

1.6 GRAPHICAL METHOD OF STUDYING THE EQUILIBRIUM OF FORCES: The equilibrium of such forces may also be studied, graphically, by drawing the vector diagram.

1.6.1 Converse of the law of triangle of forces: If three forces acting at a point be represented in magnitude and direction by the three sides of a triangle taken in order, the forces shall be in equilibrium.

1.6.2 Converse of the law of polygon of forces: If any number of forces acting at a point be represented in magnitude and direction by the sides of a closed polygon, taken in order, the forces shall be in equilibrium.

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Lesson-2

Friction, Law of Friction

2.1 INTRODUCTION

Friction is a part of our everyday life. Nearly every movement we make involves friction, and we have instinctively learned to take advantage of friction, or the lack of friction, since our childhood. Simple devices that rely on friction are everywhere around us.

2.2 FRICTION:

The opposing force, which acts in the opposite direction of the movement of the body or particle, is called the force of friction or simply friction.

2.2.1 Static friction: It is the friction, experienced by a body, when at rest (Fig 2.1)

Example

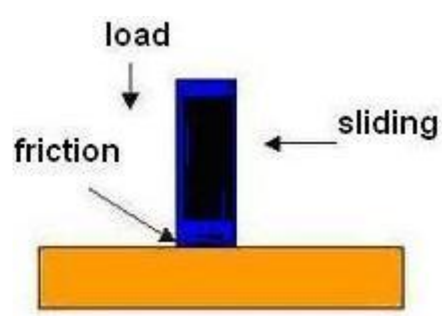

Fig.2.1 Static friction

2.2.2 Dynamic friction:

1. It is the friction, experienced by a body, when in motion (Fig2.2).

2. Dynamic friction is also called kinetic friction.

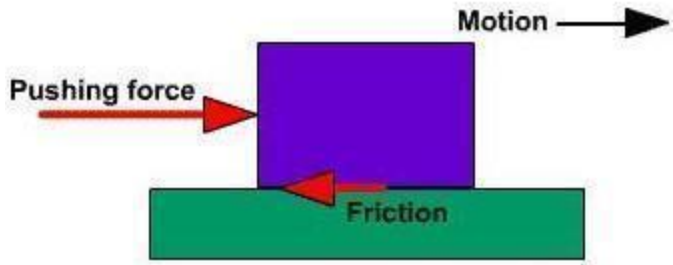

Fig.2. 2 Dynamic friction

2.3 LAW OF FRICTION:

2.3.1 Law of static friction: The forces of friction always act in a direction, opposite to that in which the body tends to move, if the forces of friction would have been absent.

The magnitude of the force of friction is exactly equal to the force, which tends the body to move.

The force of friction is independent of the area of contact between the two surface. The force of friction depends upon the roughness of the surfaces. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces.

Mathematically,

F/R=Constant

2.3. 2 Law of dynamic friction

- The force of friction always acts in a direction opposite to that in which the body is moving.
- The magnitude of kinetic limiting friction (F) bears a constant ratio to the normal reaction (R) between the two surfaces, known as coefficient of friction (μ).

2.3.2.1 Angle of Friction: : Consider a component of weight 'W' resting on an inclined plane, as shown in Fig. (2.3).

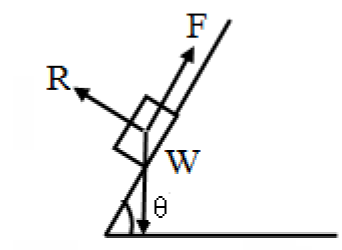

Fig. 2.3. Angle of friction

Let the angle of inclination (θ) be gradually increased, till the component just starts sliding down the plane. This angle of inclined plane, at which a body just begins to slide down the plane, is called the angle of friction. The body is in equilibrium under (1) Weight (W) of the body, acting vertically down wards.(2) Frictional force (F) acting up wards along the plane. (3) Normal reaction (R) acting at right angle to the plane.

2.4 COEFFICIENT OF FRICTION:

It is the ratio of the limiting friction (F) to the normal reaction, (R) between the two bodies, and is generally denoted by μ. Such that

$\mu = \tan\theta = F/R$

$F = \mu R$

μ = co-efficient of friction.

θ = angle of friction.

F=frictional force

R=Normal reaction between the two bodies.

P= Force

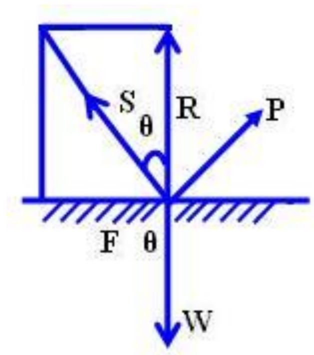

Fig.2.4Coefficient of friction

2.4.1 Limiting force of friction:

Let a component rests on a horizontal plane be acted upon by a gradually increasing force P parallel to the plane and passing through the centre of gravity of the body.

Initially, when P is small, the body does not move note it is on the point of motion. The body is said to be in non-limiting equilibrium. The magnitude of the force of friction is equal to P.

As P increases, a state is reached when the component is on the point of motion. The component is said to be in limiting equilibrium and force of friction developed is called the limiting force of friction.

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Lesson: 3
Solving Numerical

3.1. PROBLEM

Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But if they act at 60^0, their resultant i $\sqrt{13}$ N

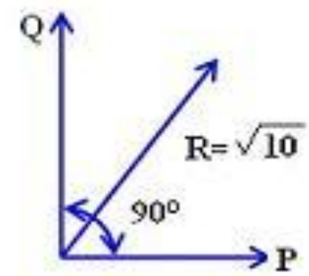
Sol. Let the two forces be P & Q

i. When acting at right angles.

Given resultant force,

$R = \sqrt{10}$ N

When the angle between the two forces is 90^0,



We, know that the resultant force

$$\therefore \quad R = \sqrt{P^2 + Q^2}$$

$$\sqrt{10} = \sqrt{P^2 + Q^2}$$

Squaring on both sides

$$P^2 + Q^2 = 10 \quad \text{——} \quad (1)$$

ii. When acting at 60^0

Given, resultant force,

$$R = \sqrt{13} \text{ N}$$

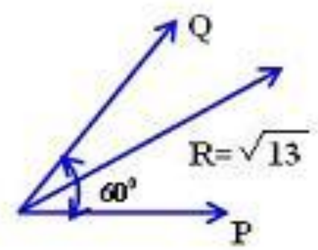


Fig.3.2

Using the relation,

$$R = \sqrt{(P^2 + Q^2 + 2PQ \cos \theta)}$$

$$\sqrt{13} = \sqrt{P^2 + Q^2 + 2PQ \cos 60^0}$$

Squaring on both sides

13 =

$13 = 10 + 2PQ \times 0.5$ $\quad$ $PQ = 3$, Therefore $2PQ = 6$ —— (2)

(OR)

Adding equations (1) and (2), we get

$(P+Q)^2 = P^2 + Q^2 + 2PQ = 10+6 = 16$

$P+Q = 4 \rightarrow (3)$

Similarly,

$(P-Q)^2 = P^2 + Q^2 - 2PQ = 10 - 6 = 4$

$P-Q = 2 \rightarrow (4)$

(OR) Solving the equations 3 & 4

P = 3 N

Q = 1N

3.2. PROBLEM

Two forces act at an angle of 120^0. The bigger force is of 40 N and the resultant is perpendicular to the smaller one. Find the smaller force.

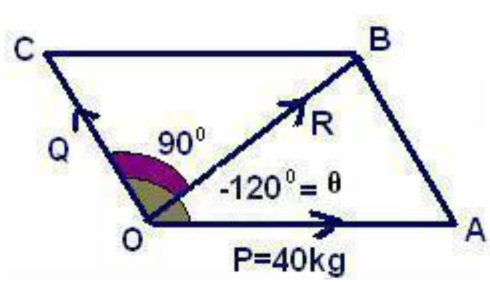


Fig.3.3

Sol. Bigger force P = 40 N

Angle between the resultant and the

Smaller force = 90^0

Angle between the two forces θ = 120^0

Angle between the bigger force and the resultant,

A=120-90=30^0

Let Q = Smaller force

Using the relation

$$\tan \propto = \frac{Q \sin \theta}{P + Q \cos \theta} = \frac{\Sigma \text{ Resolved components of all forces on Y axis}}{\Sigma \text{ Resolved components of all forces on X axis}}$$

$$\tan 30^0 = \frac{Q \sin 120^0}{40 + Q \cos 120^0} = \frac{P.\sin 0^0 + Q.\sin \theta}{P.\cos 0^0 + Q.\cos \theta}$$

$$\tan 30^0 = \frac{Q \sin 120^0}{40 + Q \cos 120^0}$$

$$\frac{1}{\sqrt{3}} = \frac{Q.\frac{\sqrt{3}}{2}}{40 - \frac{Q}{2}}$$

(θ = angle between two forces P&Q)

(angle between P and R)

$$40 - \frac{Q}{2} = Q.\frac{\sqrt{3}}{2}.\sqrt{3} = \frac{3}{2}.Q$$

$$2Q = 40$$

$$Q = 20 \text{ N}$$

3.3. PROBLEM

A body of mass100 N is suspended by two strings of 4 m & 3m lengths attached at the same horizontal level of 5 m a part. Find the tensions in strings.

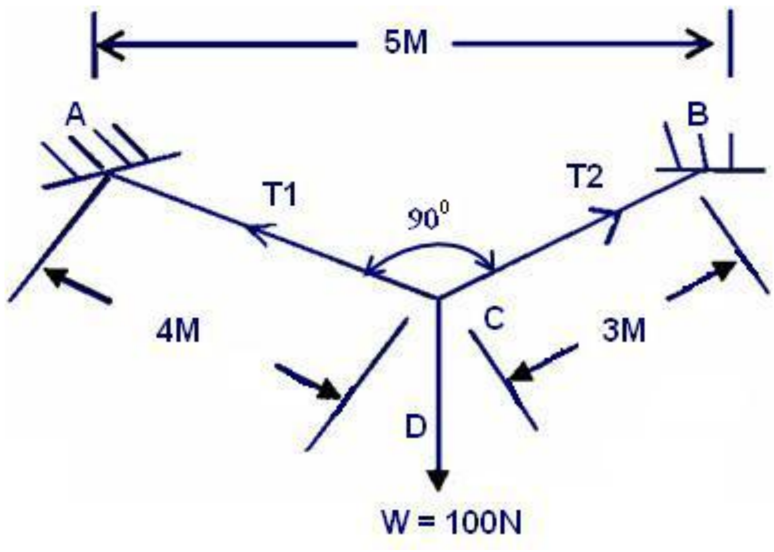


Fig.3.4

Sol since ? ABC

$$5^2 = 4^2 + 3^2$$

$\angle ACB = 90^0$

$\sin A = \frac{3}{5} = \angle A = 0.6 \longrightarrow = 36^0\ 52'$

$\angle B = 90 - 36^0 52' = 53^0 8'$

Let T_1 = Tension in AC

T_2 = Tension in BC

From geometry of fig 3.4 $\angle \theta = \angle B$ & $\angle \alpha = A$

$\angle BCD = 180^0 - \beta = 180^0 - 53^0\ 8'$

$\angle ACB = 90^0$

$\angle ACD = 180^0 - \alpha = 180^0 - 36^0\ 52'$

Apply Lames theorem at joint 'C'

$T_1 / \sin 36^0\ 52 = T_2 / \sin 53^0\ 8' = 100 / \sin 90$? $T_1 = 60$ N, $T_2 = 80$ N.

3.4. PROBLEM

A component of weight of 50 N is hauled along a rough horizontal plane, by a pull of 18 N acting at an angle of 14^0 with the horizontal. Find co-efficient of friction.

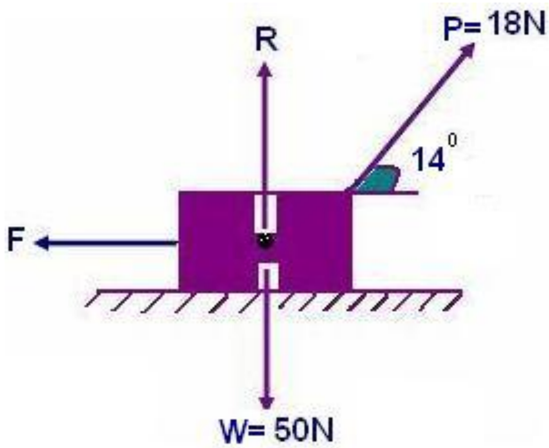


Fig.3.5

Sol Weight W = 50 N

Inclination of force

$\theta = 14^0$

Force P = 18 N

Let R = Normal reaction

μ = co-efficient of friction

F = Force of friction

Resolving the forces at right angles to the plane

$R = 50 - 18 \sin 14^0 = 45.65$ N → (1)

Now resolving the force along the plane

F = 18 Cos 14^0 = 17. 46 N → (2)

We, know that

F = μ R

$$\mu = F/R = \frac{17.46}{45.65} = 0.38$$

3.5. PROBLEM

A force of 40 N pulls a component of weight 60 N up an inclined of the plane, to the horizontal, is 30^0, calculate the co-efficient of friction.

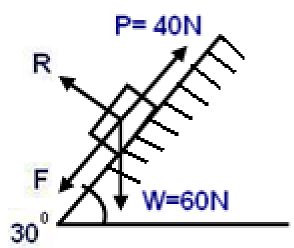


Fig.3.6

Sol Given weight, W = 60 N

Force P = 40 N

Inclination θ = 30^0

Let R = Normal reaction

μ = co-efficient of friction

F = Force of friction

Resolving forces along the inclined plane

$F = 40\text{-}60 \text{ Sin } 30^0 = 10 \text{ N} \rightarrow (1)$

Resolving force at right angles to the plane

$R = 60 \text{ Cos } 30^0 = 51.26 \text{ N} \rightarrow (2)$

Using the relation

$$F = \mu R$$

$$\mu \quad \frac{F}{R} = \frac{10}{51.96} = 0.193$$

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Lesson-4

Second moment of Inertia, Parallel Axis Theorem

4.1 INTRODUCTION:

In classical mechanics, moment of inertia, also called mass moment of inertia, rotational inertia, polar moment of inertia of mass, or the angular mass is a measure of an object's resistance to changes to its rotation. It is the inertia of a rotating body with respect to its rotation. The moment of inertia plays much the same role in rotational dynamics as mass does in linear dynamics, describing the relationship between angular momentum and angular velocity, torque and angular acceleration, and several other quantities. The symbol ***I*** and sometimes ***J*** are usually used to refer to the moment of inertia or polar moment of inertia.

4.2 MOMENT OF INERTIA:

The measure of an object's resistance to changes to its rotation is the inertia of a rotating body with respect to its rotation.

The moment of inertia is that property of a body which makes it reluctant to speed up or slow down in a rational manner. In fact moment of inertia means second moment of mass.

4.3 LAW OF MOMENT:

It states that, if a number of coplanar forces acting at a point are in equilibrium, the sum of the clockwise moments must be equal to the sum of the anticlockwise moments about any point.

It is an important law in the field of statics and is used for finding out the reaction of forces in frames etc.

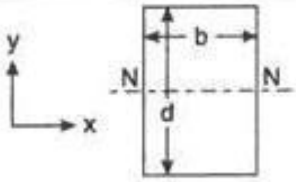
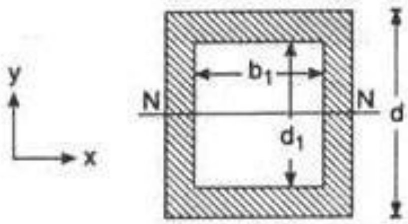
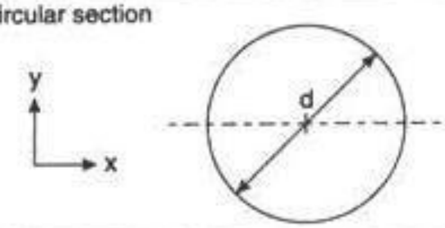
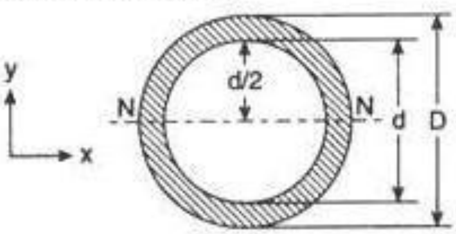
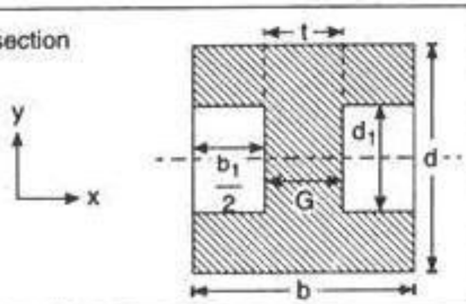
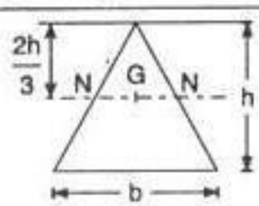
Type of section	Moment of Inertia	y_{max}	Section modulas (Z)
Rectangle or paralleogram 	$I_{xx} = \frac{bd^3}{12}$ $I_{yy} = \frac{db^3}{12}$	$\frac{d}{2}$ $\frac{b}{2}$	$Z_{xx} = \frac{bd^2}{6}$ $Z_{yy} = \frac{db^2}{6}$
Hollow rectangular section 	$I_{xx} = \frac{bd^3}{12} - \frac{b_1 d_1^3}{12}$ $I_{yy} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12}$	$\frac{d}{2}$ $\frac{b}{2}$	$Z_{xx} = \frac{1}{6d}(bd^3 - b_1 d_1^3)$ $Z_{yy} = \frac{1}{6b}(db^3 - d_1 b_1^3)$
Circular section 	$I_{xx} = \frac{p}{64} d^4$ $I_{yy} = \frac{p}{64} d^4$	$\frac{d}{2}$ $\frac{d}{2}$	$Z_{xx} = \frac{p}{32} d^3$ $Z_{yy} = \frac{p}{32} d^3$
Hollow curcular section 	$I_{xx} = I_{yy} = I$ $I_{yy} = \frac{p}{64}(D^4 - d^4)$	$\frac{D}{2}$	$Z_{xx} = Z_{yy} = Z$ $Z = \frac{p}{32D}(D^4 - d^4)$
I-section 	$I_{xx} = \frac{bd^3}{12} - \frac{b_1 d_1^3}{12}$ $I_{yy} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12}$ or $I_{xx} = \frac{1}{12}(bd^3 - (b - t)\, d_1^3)$	$\frac{d}{2}$ $\frac{b}{2}$	$Z_{xx} = \frac{1}{6d}(bd^3 - b_1 d_1^3)$ $Z_{yy} = \frac{1}{6b}(db^3 - d_1 b_1^3)$
Triangle 	$I_G = \frac{bh^3}{36}$	$\frac{2}{3}h$	$Z_G = \frac{bh^2}{24}$

Fig.4.1 Moment of inertia of composite areas

4.4 PARALLEL AXIS THEOREM:

4.4.1 Statement: If the moment of inertia of a plane area about an axis through its center of gravity be denoted by I_g, the moment of inertia of the area about axis AB, parallel to the first, and at a distance h from the center of the gravity is given by

$I_{AB}=I_G+Ah^2$

I_{AB}=M.I about AB

I_G= M.I about c.g

A=Area of the section.

h=distance between C.G of the section and the axis AB

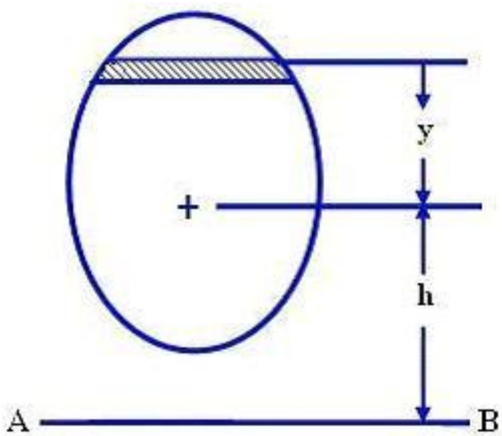


Fig.4.2 Parallel axis theorem

Proof: Consider a strip of an area δa at a distance 'y' from the c.g of a section as shown in fig. 4.2.

Then $I_G = \sum \delta a y^2$ (therefore $I = mr^2$)

We also know that

$I_{AB}=\sum \delta a \, (h+y)^2$

$=\sum \delta a\,(h^2+y^2+2hy)$

$=\sum h^2\,\delta a + \sum \delta a y^2 + 2h\sum \delta a y$

$=Ah^2 + I_G + 0$

$\sum \delta a y$ is the algebraic sum of the moments of all the areas of strips about the axis through c.g and is equal to 'Ay' where 'y' is the distance between the c.g of the section and axis through the c.g which is zero

Hence $I_{AB}=I_G+Ah^2$

4.5. MOMENT OF INERTIA OF A RECTANGULAR SECTION

Consider a rectangular section ABCD as shown in fig. whose moment of inertia is required to be found out.

Let b= Width of the section and

d= Depth of the section

Now consider a strip PQ of thickness dy parallel to X-X axis and at a distance y from it as shown in the figure 4.3

Area of the strip

$\delta a= b.dy$

We know that moment of inertia of the strip about X-X axis,

$= \text{Area}\,\delta y^2 = (b.dy)\,y^2$

Now moment of inertia of the whole section may be found out by integrating the above equation for the whole length of the lamina i.e. from –d/2 to +d/2,

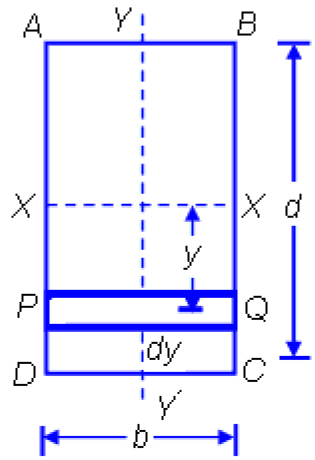


Fig.4.3. Rectangular section

$$I_{xy} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} b.y^2.dy - b\int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2.dy$$

$$= b\left[\frac{(y)^3}{3}\right]_{-\left(\frac{d}{2}\right)}^{\frac{d}{2}} = b\left[\frac{(d/2)^3}{3} - \frac{(-\frac{d}{2})^3}{3}\right] - \frac{bd^3}{12}$$

$$\text{Similarly, } I_{yy} - \frac{db^3}{12}$$

4.6. MOMENT OF INNERTIA OF A CIRCULAR SECTION

Find the moment of inertia of a rectangular section 30 mm wide and 40 mm about X-X axis and Y-Y axis.

Solution: Given: Width of the section (b) = 30 mm and depth of the section (d) = 40 mm. We know that moment of inertia of the section an axis passing through its centre of and parallel to X-X axis.

$$I_{xx} = \frac{bd^3}{12} = \frac{30 \, x \, (40)^3}{12} = 160 \, x \, 10^3 \, mm^4 \qquad Ans.$$

Similarly $I_{yy} = \frac{db^3}{12} = \frac{40 \, x \, (30)^3}{12} = 90 \, x \, 10^3 \, mm^4 \qquad Ans.$

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Lesson-5

Solving Numerical

5.1 PROBLEM

Find the moment of inertia of a hollow rectangular section about its Centre of gravity, if the external dimensions are breadth 6 cm, depth 8 cm and internal dimension are breadth 3 cm and depth 4 cm respectively.

Sol Outer breadth b = 6cm

Inner breadth b_1 = 3cm

Outer depth d = 8cm

Inner depth d_1 = 4cm

Using the relation

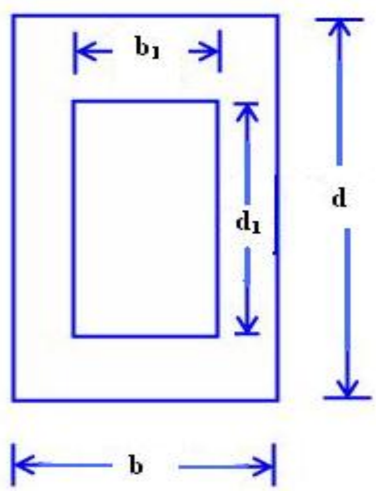

Fig.5.1

$$I_{xx} = \frac{bd^3}{12} - \frac{b_1\ d_1{}^3}{12}$$

$$= \frac{6 \times 8^3}{12} - \frac{3 \times 4^3}{12}$$

$$= \frac{6 \times 8 \times 8 \times 8}{12} - \frac{3 \times 4 \times 4 \times 4}{12}$$

$= 256-16 = 240\ cm^4$

Similarly

Similarly,

$$I_{yy} = \frac{db^3}{12} - \frac{d_1 b_1{}^3}{12}$$

$$= \frac{8 \times 6^3}{12} - \frac{4 \times 3^3}{12}$$

$$= \frac{8 \times 6 \times 6 \times 6}{12} - \frac{4 \times 3 \times 3 \times 3}{12}$$

$$= 144 - 9 \quad = 135\ cm^4$$

5.2 PROBLEM

Find the M.I of rectangular section shown in fig. about faces

AB & BC

b = 4cm

d = 6cm

M.I about AB

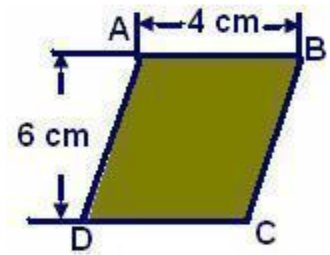

Fig.5.2

M.I of the section about an axis through its C.G and parallel to x – x axis

$$I_G = \frac{bd^3}{12} = \frac{4x6^3}{12} = \frac{4 \times 6x\ 6x\ 6}{12} = 72\ cm^4$$

Distance of c.g. and the face AB h = 3cm

∴ M.I about face AB

$$I_{AB} = I_G + Ah^2 = 72 + (4 \times 6) \times 3^2$$
$$= 288\ cm^4$$

M.I about BC

Similarly, M.I. of the section about an axis through its c.g. & Parallel to y – y axis.

$I_{GY} = db^3 / 12 = 6 \times 4^3 / 12 = 6 \times 4 \times 4 \times 4 / 12 = 32\ cm^4$

Distance of c.g. & the face BC, h = 2 cm

∴ M. I about the face BC $I_{BC} = I_G + Ah^2 = 32 + (4 \times 6) \times 2^2 = 128\ cm^4$

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Module 2: Dynamics

Lesson-6

Dynamics

6.1 INTRODUCTION:

In the field of physics, the study of the causes of motion and changes in motion is dynamics. In other words the study of forces and why objects are in motion. Dynamics includes the study of the effect of torques on rotating components. These are in contrast to Kinematics, the branch of classical mechanics that describes the motion of objects without consideration of the causes leading to the motion.

6.2 RIGID BODY:

A body is said to be rigid, if the relative positions of any two particles in it do not change under the action of the forces. In Fig. 6.1 (*a*) points *A* and *B* are the original position in a body.

After application of a system of forces $F1$, $F2$, $F3$, the body takes the position as shown in Fig. (a,b)A^1 and B^1 are the new positions of A and B. If the body is treated as rigid, the relative position of A^1B^1 and AB are the same *i.e.*

$A^1B^1 = AB$.

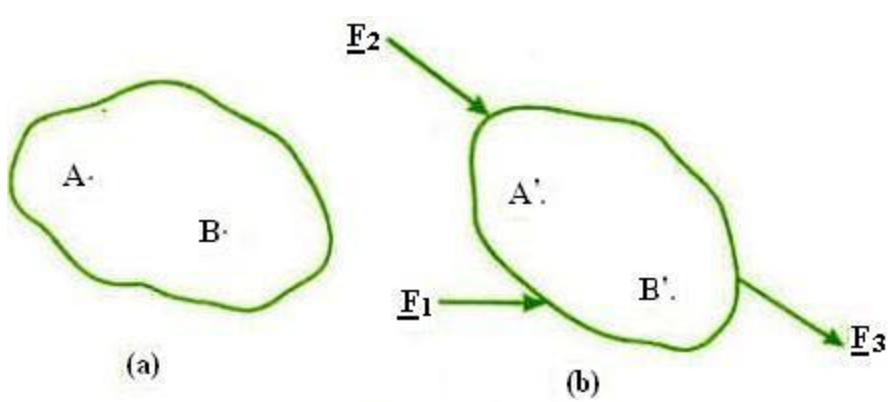

Fig.6.1 Rigid body

6.3 TRANSLATION:

Orientation of the body never changes. Every line segment on the body remains parallel to its original direction.

1. The movements of the points in a rigid body are parallel.
2. Every point has same velocity and acceleration
3. Motion of one point completely describes the motion of the entire rigid body. We generally choose the motion of the center of mass to denote the motion of the rigid body.

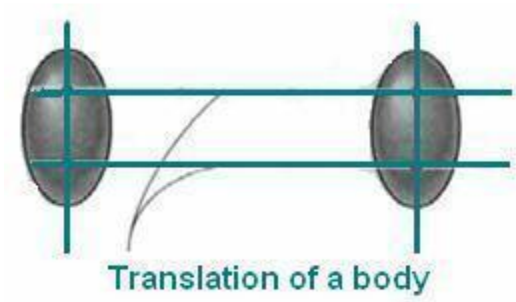

Fig.6.2 Translation

6.4 PURE ROTATION:

The rigid body rotates about a fixed axis

1. All points are rotating about same fixed axis.
2. No two points have same linear velocity or linear acceleration although all of They have same angular velocity and angular acceleration.

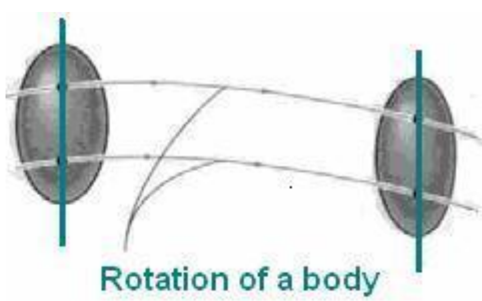

Fig. 6.3 Pure rotation

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Lesson-7

Work and Mechanics of Materials

7.1 INTRODUCTION:

Mechanical work is the amount of energy transferred by a force acting through a distance. Like energy, it is a scalar quantity, with SI units of joules. The term work was first coined in 1826 by the French mathematician Gaspard-Gustave Coriolis.

7.2 WORK:

When- ever a force acts on a body and the body under goes a displacement, the work is said to be done.

As shown is figure the body on which a force 'P' is acting on a body moves through a distance's',

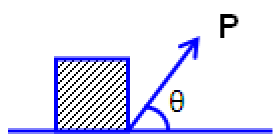

Fig.7.1 Work

The work done by the force 'P'

W = Force x distance

= P x S.

If the force 'P' inclined at an angle 'θ' to the body as shown in fig

Work done x

$$W = P \cos\theta \times S$$

Force P = Resolved component of the force in the direction of motion

$$= P \cos\theta$$

7.3 MECHANICS OF MATERIAL:

It is a branch of engineering mechanics which deals with the behavior of elastic bodies (or) deformable bodies. Mechanics of materials mainly deals with the

1. Stresses
2. Strains

7.3.1 Strength: Strength is defined as the maximum internal resistance force (stress) that a material can withstand without any failure.

7.3.2 Stiffness: It is an ability of a material i.e. the resistance against the deformation.

7.3.3 Homogeneous: It is the uniform property of the material at all

7.3.4 Isotropic: It is the property of the material at any given points the properties of given material are same in all direction.

7.3.5 Orthotropic: At a point in 3 mutually perpendicular direction properties are different. An orthotropic material has two or three mutually orthogonal two fold axes of rotational symmetry so that its mechanical properties are, in general, different along each axis. Orthotropic material are thus anisotropic; their properties depend on direction in which they are measured.

7.3.6 Allotropic (or) Anisotropic: At any given point the properties are different in different directions. Any material whose physical properties depend upon direction relative to some defined axes.

Assumptions: Usually, for all engineering calculation, unless otherwise specified, following assumptions are made.

Material is assumed to be homogeneous and isotropic.

Material is assumed to be solid and continuous.

No internal forces prior to loading (self weight = 0).

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Lesson :8
Solving Numerical

8.1 PROBLEM:

1. A trolley weighing 2000 N moves on a level track for a distance of 2 km. The resistance of the track is 5 N per 1000 N weight of trolley find the work done.

Sol: Given,

Resistance force,

$$P = \frac{5}{1000} X\ 2000 = 10\ N$$

Distance, S = 2km = 2000 m

Using the relation

W.D = P X S

= 10 X 2000 = 20,000 N-m

8.2 PROBLEM:

A man is drawing, water from a well, with a light bucket which leaks uniformly. The bucket when full weight 20 N and when it arrives at the top, it weight 10 N Find the work done, if depth of the well is 30mt.

Sol: Initial weight P_1 = 20 N

Final weight P_2 = 10 N

$$\therefore \text{Average weight } P = \frac{P_1 + P_2}{2} = \frac{20+10}{2} = 15\ N$$

Depth of the well, S = 30 m

Work done = P X S

= 15 x 30 = 450 N-m

8.3 PROBLEM:

Calculate the work done in pulling a block weighing 200 N for length of 10m up a smooth inclined plane which makes 30^0 with the horizontal.

Sol: Weight of the block = 200 N

Inclination of block = 30^0

$$= 200 \text{ Sin } 30 = 200\frac{1}{2} = 100 \text{ N}$$

Distance S = 10 m

Using the relation

Work done = P X S

= 100 x 10 = 1000 N-m

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Module 3: Stress

Lesson-9

Stress Analysis

9.1 INTRODUCTION:

Important factor in determining the size of component is the load carried. Stress on member should not be so high that the member may fail in service.

9.2 STRESS:

Whenever any external force is applied on a body, it causes deformation. Any deformation of a body resulting from the action of external forces causes internal forces within the material. These internal forces per unit area are called stress. Depending upon the plane in which it acts, the stress may be Normal, Tangential or Oblique

9.2.1 Tensile stress: When a section is subjected to axial pull P acting normally across the section, the resistance set up is called Tensile and the corresponding strain is called Tensile Strain.

Expressed as kg/cm^2 or N/m^2

P = Load or Force acting on the body, kg or N

A = Cross sectional area of the body cm^2 or m^2

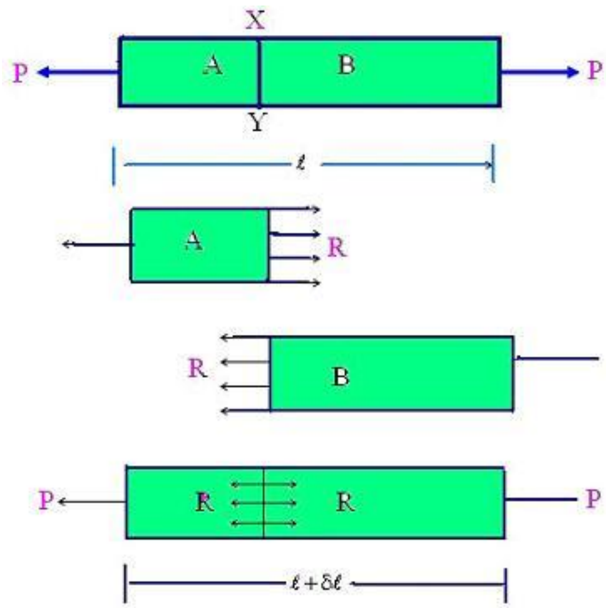
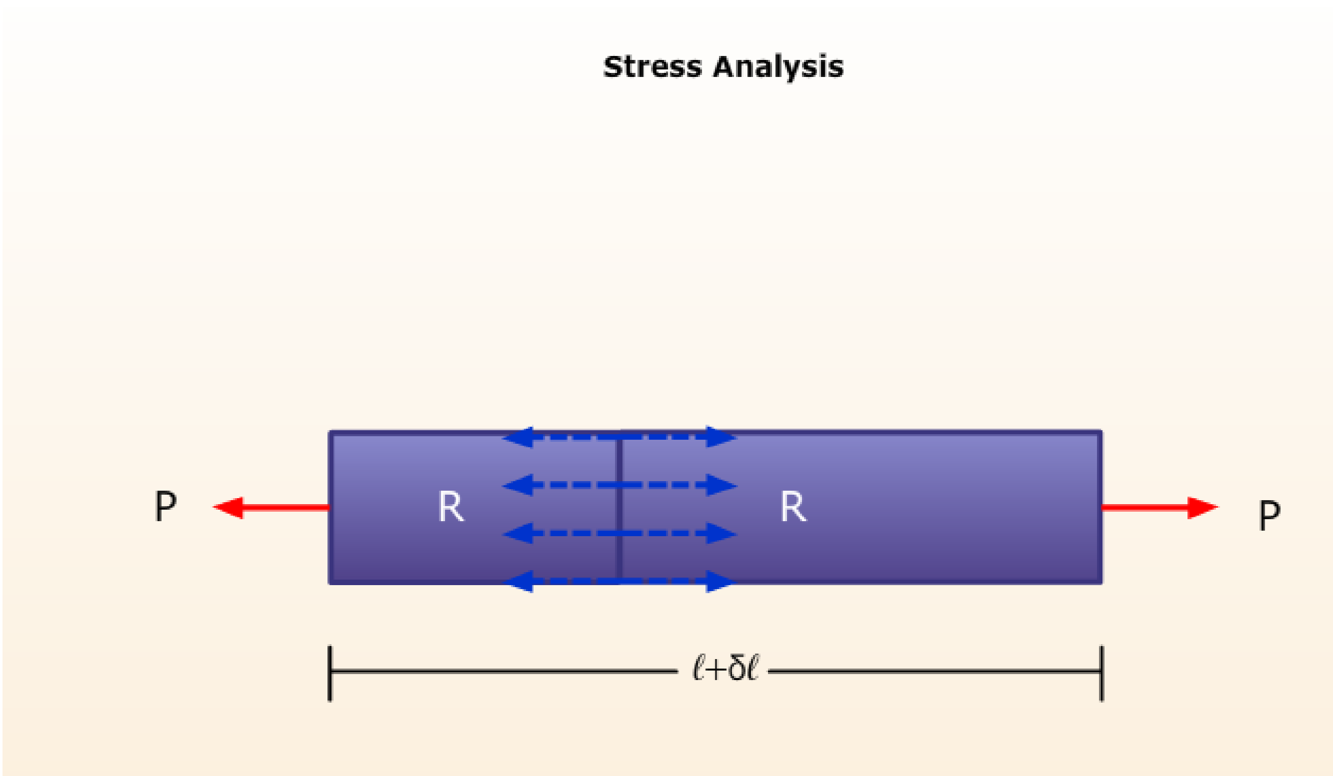


Fig.9.1 Stress



For Detailed Pictorial View of Stress Analysis

9.2.2 Compressive stress: When the section is subjected to axial pushes P acting normally across the section, the resistance set up is compressive stress and corresponding strain is compressive strain. Hence the original length will shorten.

Fig.9.2 Compressive stress

9.3 STRAIN:

When a straight bar is subjected to a tensile load, the length of the bar increases. The extent of elongation of the bar is called Strain. The elongation per unit length of the bar is called unit strain. Thus, any element in a material subjected to stress is said to be strained. The strain (e) is the deformation produced by stress. The various types of strains are explained below

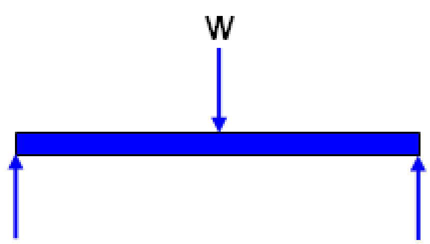


Fig.9.3 Strain

9.3.1 Tensile Strain:

A piece of material, with uniform cross-section, subjected to a uniform axial tensile stress, will increase its length from and the increment of length is the actual deformation of the material . The fractional deformation or the tensile strain is given by

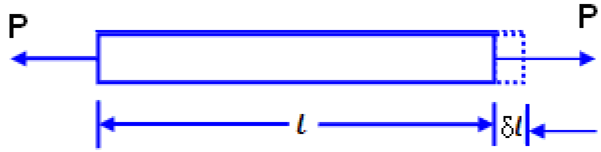


Fig.9.4 Tensile strain

9.3.2 Compressive Strain:

Under compressive forces, a similar piece of material would be reduced in length from

1to (1– δl)

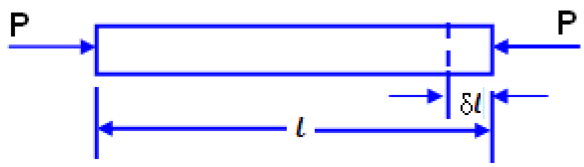


Fig.9.5 Compressive strain

The fractional deformation again gives the strain e_c et

Where $e_c = \frac{\delta l}{l}$

9.3.3 Shear Strain:

In case of a shearing load, a shear strain will be produced this is measured by the angle through which the body distorts.

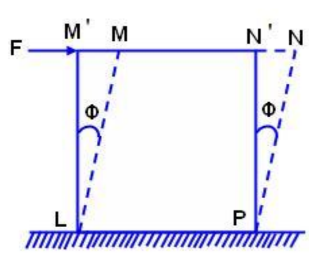


Fig.9.6 Shear strain

A rectangular block LMNP fixed at one face and subjected to force F. After application of force, it distorts through an angle Φ and occupies new position LM'N'P. The shear strain (es) is given by

$$e_s = \frac{NN'}{NP} = \tan\Phi$$

= Φ (radians) …… since Φ is very small.

The above result has been obtained by assuming NN' equal to arc (as NN' is small) drawn with centre P and radius PN.

9.3.4 Volumetric Strain:

It is defined as the ratio between change is volume and original volume of the body, and is denoted by e_v

$$\therefore \quad e_v = \frac{\text{change in volume}}{\text{original volume}} = \frac{\delta V}{V}$$

The strains which disappear with the removal of load are termed as elastic strains and the body which regains its original position on the removal of force is called an elastic body. The body is said to be plastic if the strains exist even after the removal of external force. There is always a limiting due to load up to which the strain totally disappears on the removal of load- the stress corresponding to this load is called elastic limit.

9.4 ELASTICITY:

It is the property of a material, which enables it to regain its original shape and dimensions when the load is removed.

A material is said to be perfectly elastic if the deformation due to external loading entirely disappears on removal of the load. For every material, a limiting value of the load for a given resisting section is found, up to and within which, the resulting strain entirely disappears on removal of the load. The value of the intensity of stress corresponding to this limiting load is known as the Elastic limit of the material. Within this limit, the material behaves like a perfect spring, regaining its original dimensions on removal of the load. Beyond the elastic limit, the material gets into the plastic stage.

Robert Hooke discovered experimentally that within elastic limit, stress varies directly as strain.

i.e $\quad \text{Stress} \propto \text{Strain or } \frac{\text{Stress}}{\text{Strain}} = \text{a constant}$

This constant is termed as Modulus of Elasticity.

(i) Young's modulus: It is the ratio between tensile stress and tensile strain or compressive stress and compressive strain. It is denoted by E. It is the same as modulus of elasticity.

$$\text{or} \quad E = \frac{\sigma}{e}\left(= \frac{\sigma_t}{e_t} \text{ or } \frac{\sigma_c}{e_c}\right)$$

(ii) Modulus of rigidity: It is defined as the ratio of shear stress τ to shear strain, e_s and is denoted by C,N or G. It is also called shear modulus of elasticity.

$$\text{or} \quad \frac{\tau}{e_s} = C, N \text{ or } G$$

(iii) Bulk or volume modulus of elasticity: It may be defined as the ratio of normal stress (on each face of a solid cube) to volumetric strain and is donated by the letter K.

$$\text{or} \quad \frac{\sigma_n}{e_v} = K$$

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Lesson-10

Hooke's Law

10.1 INTRODUCTION:

In mechanics Hooke's law of elasticity is an approximation which states that the extension of a spring is in direct proportion with the load applied to it. Many materials obey this law as long as the load does not exceed the material's elastic limit. Materials for which Hooke's law is a useful approximation are known as linear-elastic or "Hookean" materials.

10.2 HOOK'S LAW:

It states that, 'when a material is loaded, within its elastic limit, the stress is proportional to the strain'. Mathematically,

Stress / Strain = E,

which is a constant known as Young's Modulus or elasticity

It may be noted that Hooke's law equally holds good for tension as well as compression.

In case of shear stress (τ) and shear strain (γ), the relation is $\tau = G \cdot \gamma$

Where G is known as shear modulus of elasticity or modulus of rigidity.

10.3 STRESS-STRAIN DIAGRAM:

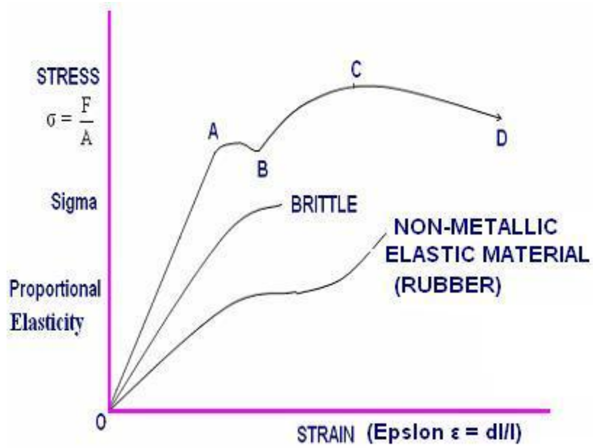

Fig.10.1 Stress and strain

When stresses for a specimen are plotted along ordinates and strains along the abscissa, we get stress strain diagram. The shape of the diagram remains the same for tensile as well as compressive stresses.

10.3.1 Ultimate stress:

$$\text{Ultimate stress} = \frac{\text{Max.Load}}{\text{Original Area of Cross section}}$$

10.3.2 Factor of safety:

$$F.S = \frac{\text{Ultimate stress}}{\text{Working stress}}$$

(for Brittle material)

10.3.3 Factor of safety:

$$F.S = \frac{\text{Yield stress}}{\text{Working stress}_*}$$

(for Ductile material)
***Working Stress:** Stress developed in the material during working.

Lesson-11

Poisson's Ratio

11.1 INTRODUCTION:

When a material is compressed in one direction, it usually tends to expand in the other two directions perpendicular to the direction of compression. This phenomenon is called the Poisson effect. Poisson's ratio m (mu) is a measure of the Poisson effect. On the molecular level, Poisson's effect is caused by slight movements between molecules and the stretching of molecular bonds within the material lattice to accommodate the stress. When the bonds elongate in the direction of load, they shorten in the other directions. This behaviour multiplied millions of times throughout the material lattice is what drives the phenomenon.

11.2 POISSON'S RATIO :

Whenever a bar is subjected to tensile load, its length will increase but its lateral dimension will decrease. Thus changes in longitudinal and lateral dimensions are of opposite nature.

The transverse dimension 'b' changes to δb and the deformation has the opposite signs. The ratio of transverse strain to the longitudinal strain is called Poisson's ratio and is designated as μ

$$\mu = (\delta b/b) \,/\, (\delta L \,/\, L)$$

The Poisson's ratio is dimensionless parameter.

For most of engineering materials its value is in between 0 to 0.5.

For most of the metals its value is in the range of 0.2 to 0.3.

The Modulus of Elasticity (E), the Modulus of Rigidity (G) and the Poisson's ratio, m are related by the relation,

$$E = 2\,G\,(1+\mu)$$

When same stress is applied on all the six faces of a cube then the ratio of normal stress to the volumetric strain (change of volume to original volume) is known as bulk modulus of elasticity, denoted by K. The relation between E and K is

$$E = 3K\,(1-2\mu)$$

Relation between E , K, G and μ

$$E = 9\,KG / (3K+G)$$

$$\mu = (3K-2G) / (K+2G) = (3K-2G) / 2(G+3K) = (3K-2G) / (2G+6K)$$

11.3 SHEAR STRESS:

When the section is subjected to two equal and opposite forces P acting tangentially across the resisting section, the resistance set up is called Shear Stress.

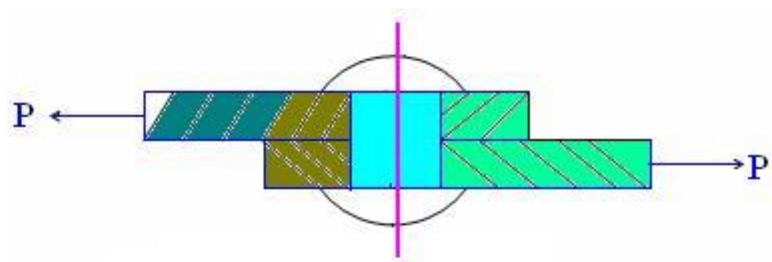

Fig.11.1 Shear stress

11.4 VOLUMETRIC STRAIN OF RECTANGULAR BAR:

V = l.b.t

$$\delta V = b.t.\,\delta l - t.l.\,\delta b - l.b.\,\delta t\,;$$

$$\frac{\delta V}{V} = \frac{\delta l}{l} - \frac{\delta b}{b} - \frac{\delta t}{t} = e - \frac{1e}{m} - \frac{1e}{m} = e(1 - \frac{2}{m})$$

11.5 VOLUMETRIC STRAIN OF ROD:

$$\delta v = \frac{\pi}{4} d^2 .\ \delta l - 2\frac{\pi}{4}\ d.\ l.\ \delta d$$

$$\frac{\delta V}{V} = \frac{\delta l}{V} - \frac{2\,\delta d}{d} = e - \frac{2.e}{m} = e\,(1 - \frac{2}{m})$$

11.6 BARS OF VARYING SECTIONS:

Total change in length will be equal to an algebra is sum of deformation in all (individual) sections.

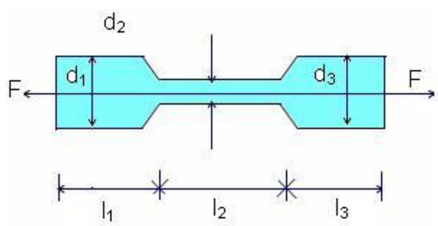

Fig.11.2 Bars of varying section

$$p_1 = \frac{F}{a_1}\ ;\ p_2 = \frac{F}{a_2}\ ;\ p_3 = \frac{F}{a_3}$$

Where, p, p_2, & p_3 are stresses developed in individual sections 1,2,&3 respectively

$$e_1 = \frac{P_1}{E}\ ;\ e_2 = \frac{P_2}{E}\ ;\ e_3 = \frac{P_3}{E}$$

$dl_1 = e_1 l_1$; $dl_2 = e_2\ l_2$; $dl_3 = e_3\ l_3$

Total length changed = $\delta l = \delta l_1 + \delta l_2 + \delta l_3$

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Lesson-12

Solving Numerical

12.1 PROBLEM:

A rod of 100m long and of 2mx2m cross section is under pull of 1000N force. If the modulus of elasticity of the material is $2x10^6$ N/m^2, determines the elongation of the rod.

Sol) Given Length of the rod, l=100m

Area of the rod, A=2x2=4m^2

Load P=1000N

Modulus of elasticity E=$2x10^6 N/m^2$

Let ∂ l=elongation of the rod.

Using the relation

$$E = \frac{\partial}{\epsilon}$$

$$= \frac{P}{A} / \frac{\partial l}{l}$$

$$= \frac{Pl}{A \partial l}$$

$$\delta l = \frac{Pl}{AE}$$

$$= \frac{1000X100}{4X2X10^{6}}$$

$$= \frac{1}{4X2X10}$$

$$=0.0125mt$$

12.2 PROBLEM:

A brace bar contain ABCD points 1 mt apart from each other of 10m² Area of cross section is subjected to axial force is as shown in fig. Take E=0.8x10⁶ N/m²

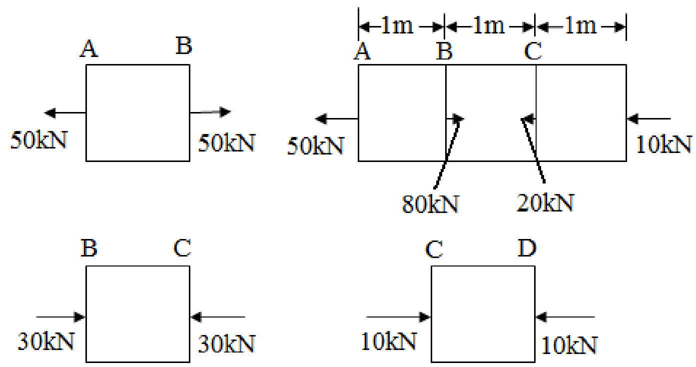


Fig.12.1 Brass bar

Given, Area $A = 10m^2$

$E = 0.8x10^6\ N/m^2$

Let δl=Total elongation of the bar.

Using the relations

$$\delta l = \frac{P_1 l_1 + P_2 l_2 + P_3 l_3}{AE} \qquad \{l_1 = l_2 = l_3 = l\}$$

$$= \frac{[P_1 + P_2 + P_3]}{AE}$$

$$\delta l = \frac{(50-30-10)}{10x0.8x10^6}$$

$$\delta l = \frac{10}{10x0.8x10^6}$$

$= 0.00000125$ m.

$= 125x10^{-8}$ m. {Increase in length}.

12.3 PROBLEM:

A compound bar is as shown in fig. Calculate the total strain of the composite bar?

Take E=200GPa

$=200 \times 10^3 N/mm^2$

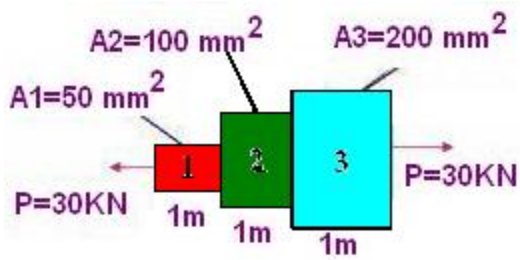


Fig.12.2 Compound bar

Total elongation $\delta\ell = \delta\ell_1 + \delta\ell_2 + \delta\ell_3$ or $\partial \ell = \partial \ell_1 + \partial \ell_2 + \partial \ell_3$

$$= \frac{P_1 l1}{A1E} + \frac{P_2 l2}{A2E} + \frac{P_3 l3}{A3E}$$

Given, that $P_1 = P_2 = P_3 = P = 30$ kN

In the present case, $l_1 = l_2 = l_3 = 1$ m

$$= \frac{Pl}{E}\left[\frac{1}{A1} + \frac{1}{A2} + \frac{1}{A3}\right]$$

$$= \frac{30 x 10^3 x 1000}{200 x 10^3}\left[\frac{1}{50} + \frac{1}{100} + \frac{1}{200}\right]$$

=5.25 mm {increase in length}

****** ☺ ******

Lesson-13

Torsion

13.1 INTRODUCTION:

In solid mechanics, torsion is the twisting of an object due to an applied torque. In circular sections, the resultant shearing stress is perpendicular to the radius. Torsion and bending produce normal and tangential stresses in the same plane simultaneously. The stresses produced by normal and tangential stresses in the same plane simultaneously. The stresses produced by torsion and bending are termed as compound stresses.

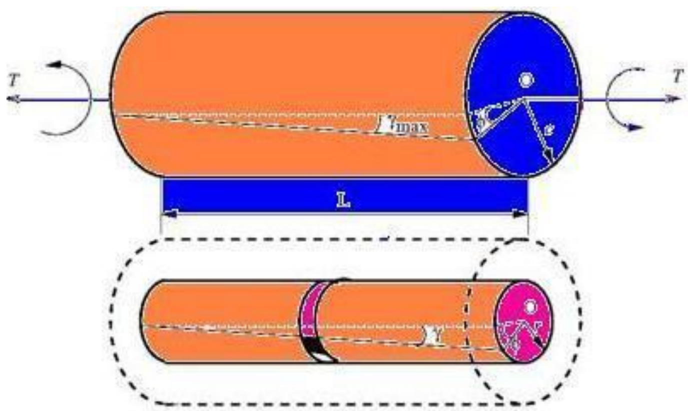
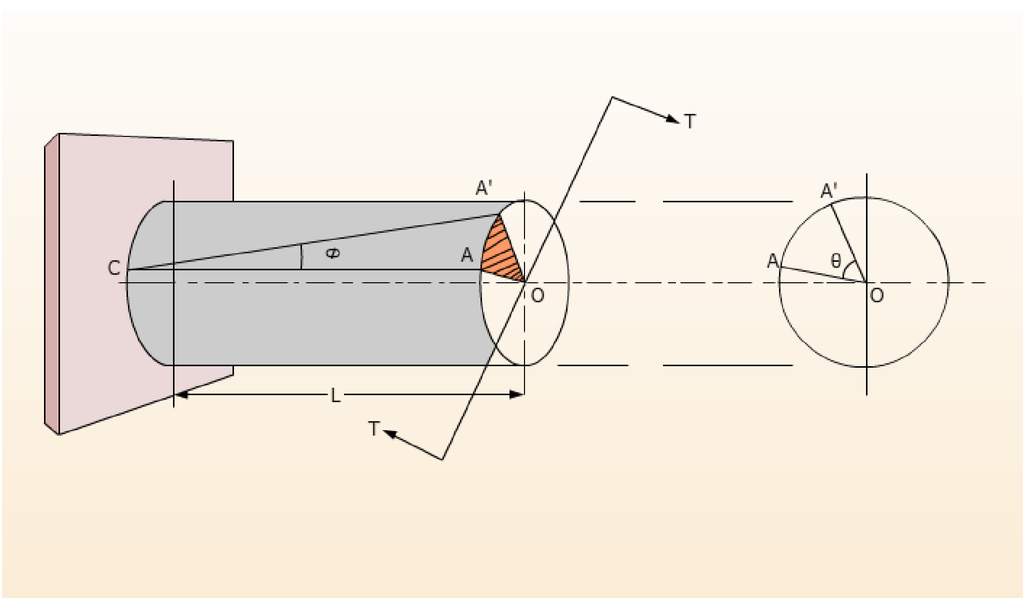


Fig.13.1 Torsion



Pictorial View of Torsion

13.2 TORSION:

When a member is twisted by a couple, the stress produced is pure shear. However, its intensity on any fiber depends on the distance of the fibre from the center line of the twisted member. Thus in a circular member the shear stress is proportional to the distance from the center line. Thus when a member is subjected to external torsional moment

T = Average Torque, M.m.

J = Polar moment of inertia , m^4

f_s or S_s= maximum shear stress, N/m^2

R= Radius = Distance of outer most fiber from the central line, m

G = Shear modulus , N/m^2

θ = maximum angular twist , radian

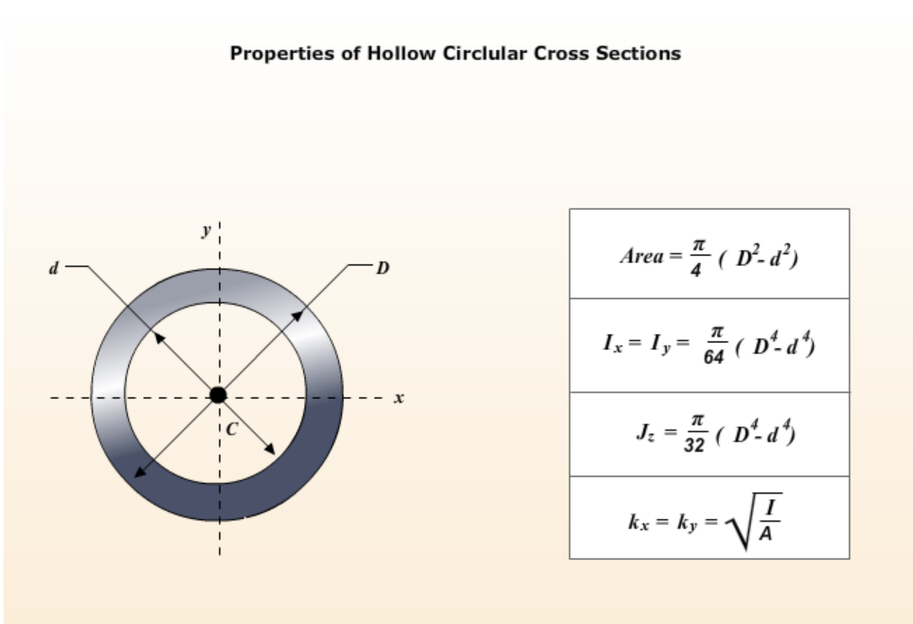
L = Length of shaft under twisting , m

Where τ_s-f_s or S_s is the maximum shear stress developed in the member and Z_p is the polar section modulus. In case of a solid or hollow circular section, Z_p = J / c where J is the polar moment of inertia and c is the distance from the axis to the most remote fibre where the stress is τ_s.

In case of solid round bar,

$Z_p = \pi D^3/16$ Hence, $\tau_s = 16\ T / (\pi D^3)$ [some refer S_s as f_s]

In case of hollow cylindrical bar,



Pictorial View of Torsion Properties

D= Outer Diameter

d = Inner diameter

13.3 TORSIONAL STRESSES AND STRAINS :

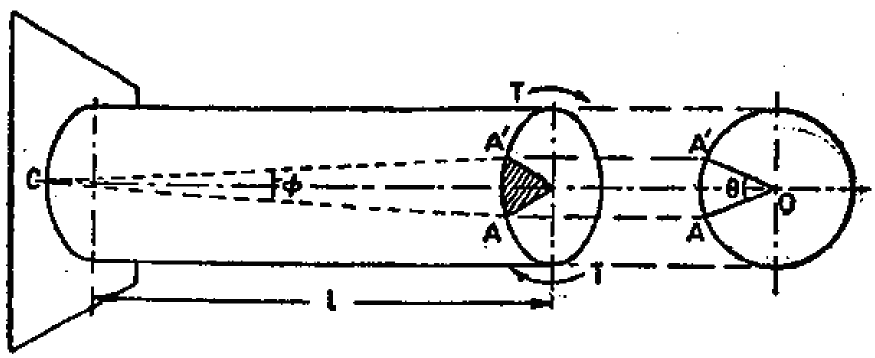


Fig 13.2.Torsional stresses

Consider a shaft fixed at one end, and subjected to a torque at the other as shown above.

Let T= Torque in kg-cm

l= Length of the shaft, and

R= Radius of the shaft.

As a result of this torque, every cross-section of the shaft will be subjected to shear stresses. Let the line CA on the surface of the shaft be deformed to CA^1 and OA to OA^1.

Let $\angle ACA^1 = \Phi$ in degrees

$\angle AOA^1 = \theta$ in radians

fs = Shear stress induced at the outer most surface, and

C = Modules of Rigidity, also known as torsional rigidity of the shaft material.

We know that shear strain

= Deformation per unit length.

$(AA^1)/ I = \tan \Phi$

$= \Phi$ (Φ being very small, $\tan\Phi = \Phi$)

We also know that the length of the arc $AA^1 = R\theta$

$\Phi = (AA^1 / I) = (R\theta / I)$ ------------(1)

Moreover deformation

= (Shear stress / Modulus of rigidity)

$\Phi = (fs / C)$ -------------(2)

Now from equation (1) and (2) we find that

$(fs / C) = (R\theta / I)$

$(fs / R) = (C\theta / I)$

if q be the intensity of shear stress, any layer at a distance r be the center of the shaft, then

$(q / r) = (fs / R) = (C\theta / I)$

13.4 STRENGTH OF THE SOLID SHAFT :

It means the maximum torque or power of the shaft can transmit from one to another. Now consider a solid circular shaft subjected to some torque.

Let R= Radius of the shaft, and

fs= Maximum shear stress developed in the

outer most layer of the shaft material

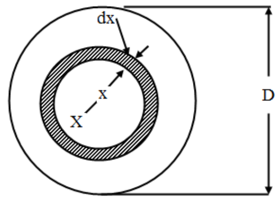


Fig 13.3. Strength of the solid shaft

Now consider an elementary ring of thickness dx at a distance from the center. We know that the area of this ring,

$da = 2\pi x \cdot dx$ ________________(1)

Shear stress at this section,

$fx = fs \times \frac{x}{R}$

Turning force = stressX Area = fx.da

$$= fs \times \frac{x}{R} \times da \qquad fx = fs\,\frac{x}{R}$$

$$= fs \times \frac{x}{R} \times 2\pi x \cdot dx \qquad da = 2\pi x \cdot dx$$

$$= \frac{2\pi fs}{R}\, x^2\, dx$$

We know that turning moment of this element,

DT= Turning force X Distance of the element from the axis of the shaft

$$dT = \frac{2\pi fs}{R}\, x^2\, dx \times x = \frac{2\pi fs}{R}\, x^3\, dx \qquad \text{\_\_\_\_\_\_\_\_(ii)}$$

The total torque, which the shaft can transmit, may be found out by integrating the above equation between r and R.

$$T = \int_0^R \frac{2\pi fs}{R}\, x^3\, dx = \frac{2\pi fs}{R} \int_0^R x^3\, dx$$

$$= \frac{2\pi fs}{R} \left[\frac{x^4}{4}\right]_0^R = \frac{2\pi fs}{R} \left(\frac{R^4 - r^4}{4}\right)$$

$$T = \frac{\pi}{16}\, fs \left(\frac{D^4}{D}\right)$$

$$T = \frac{\pi}{16}\, fs\, D^3$$

Where D is the external diameter of the shaft and is equal to 2R

Example 1: Find the torque which a shaft of 25 cm diameter can safely transmit, if the shear is not to exceed 460 kg/cm^2

Solution:

Given. Dia. Of shaft,

D=25 cm

Max. Shear stress, fs = 460 kg/cm^2

Let T= Torque transmitted by the shaft.

Using the relation,

$$T = \frac{\pi}{16} fs\, D^3 \text{ with the usual notations.}$$

$$= \frac{\pi}{16} X\, 460\, X\, 25^3 = 1411300 \text{ kg-cm}$$

$$= 14113 \text{ kg-m}$$

Example 2.: A solid steel shaft is to transmit a torque of 10000 kg-m. If the shearing stress is not to exceed 450 kg/cm^2, find the minimum diameter of the shaft.

Solution:

Given Torque, T= 10,000 kg-m = 1000000 kg-cm

Max, shear stress,

fs= 450 kg/cm^2

Let D= Diameter of the shaft.

Using the relation,

Or

$$T = \frac{\pi}{16} fs\, D^3 \text{ with the usual notations.}$$

$$1000000 = \frac{\pi}{16} X\, 450\, D^2$$

$$D^3 = \frac{1000000 X\, 16}{\pi\, X\, 450} = 11320$$

$$D = 22.45 \text{ cm}$$

13.5 STRENGTH OF THE HALLOW SHAFT:

It means the maximum torque or power a hallow shaft can transmit from on pulley to another. Now consider a hallow circular shaft subjected to some torque

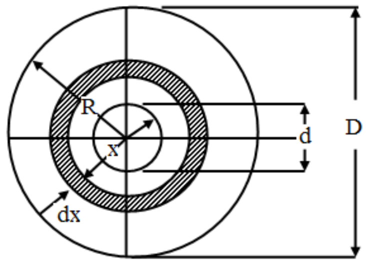


Fig 13.4. Hallow shaft

Let R= Outer radius of the shaft,

r = Inner radius of the shaft, and,

fs= Maximum shear stress developed in the outer most layer of the shaft material

Now consider an elementary ring of thickness dx at a distance x from the center as shown in fig above. We know that the area of this ring, diff $a = \pi x^2$ w.r.t x

$$\frac{da}{dx} = 2\pi x$$

$$da = 2\pi x \cdot dx \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

Shear stress at this section,

$$fx = fs \times \frac{x}{R}$$

Turning force = stress X Area = fx.da

$$= fs \times \frac{x}{R} \times da$$

$$= fs \times \frac{x}{R} \times 2\pi x \cdot dx$$

$$= \frac{2\pi fs}{R} x^2 dx$$

$$fx = fs \frac{x}{R}$$

$$da = 2\pi x \cdot dx$$

We know that turning moment of this element,

DT= Turning force X Distance of the element from the axis of the shaft

$$dT = \frac{2\pi fs}{R} x^2 dx \times x = \frac{2\pi fs}{R} x^3 dx \qquad \text{\_\_\_\_\_\_\_\_(ii)}$$

Now the total torque, which the shaft can transmit, may be found out by integrating the above equation between r and R

$$T = \int_r^R \frac{2\pi fs}{R} x^3 dx = \frac{2\pi fs}{R} \int_r^R x^3 dx$$

$$= \frac{2\pi fs}{R} \left[\frac{x^4}{4}\right]_r^R = \frac{2\pi fs}{R} \left(\frac{R^4 - r^4}{4}\right)$$

$$T = \frac{\pi}{16} fs \left(\frac{D^4 - d^4}{D}\right)$$

Where D is the external diameter of the shaft and is equal to 2R and 2d is the internal diameter of the shaft and is equal to 2r

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Lesson-14

Power Transmission

14.1 INTRODUCTION:

Power transmission is the movement of energy from its place of generation to a location where it is applied to performing useful work. Power transmission is normally accomplished by belts, ropes, chains, gears, couplings and friction clutches. They are subjected to twisting and bending moments.

Power is defined formally as units of energy per unit time. In SI units:

$$\text{Watt} = \frac{\text{joule}}{\text{second}} = \frac{\text{Newton x meter}}{\text{second}}$$

14.2 MODES OF POWER TRANSMISSION:

14.2.1 Belts:

A belt is a loop of flexible material used to link two or more rotating shafts mechanically. Belts may be used as a source of motion, to transmit power, or to track relative movement. Belts are looped over pulleys. In a two pulley system, the belt can either drive the pulleys in the same direction, or the belt may be crossed, so that the direction of the shafts is opposite. As a source of motion, a conveyor belt is one application where the belt is adapted to continually carry a load between two points.

14.2.2 Ropes:

Ropes were used as belts for the transmission of power before belts became common. Because of the low coefficient of friction between the rope and the pulleys, multiple loops were usually used, either as a single rope passing several times around the pulleys, or multiple ropes on the same pulleys. The advantage of multiple ropes is that if one fails, the others will continue to transmit power; however, it is difficult to get equal tension. A single rope will distribute the tension evenly, but the tension will also have to take the rope leaving the last groove on one pulley, pass it back over the other loops of the rope, and place it on the first groove of the other pulley, to keep the rope from moving off the end of the pulleys.

14.2.3 Chains:

A chain drive can be used in a variety of machines such as bicycles and motorcycles. In addition to these machines, there are also many other vehicles which also have requirements for the chain drive. It is a basic way of transmitting mechanical power from one place to another. As we can see, one of its main uses is to convey power to the wheels of a vehicle.

14.2.4 Gears:

A gear is a rotating machine part having cut teeth, or cogs, which mesh with another toothed part in order to transmit torque. Two or more gears working in tandem are called a transmission and can produce a mechanical advantage through a gear ratio and thus may be considered a simple machine. Geared devices can change the speed, magnitude, and direction of a power source. The most common situation is for a gear to mesh with another gear; however a gear can also mesh a non-rotating toothed part, called a rack, thereby producing translation motion instead of rotation.

14.2.5 Couplings:

A coupling is a device used to connect two shafts together at their ends for the purpose of transmitting power. Couplings do not normally allow disconnection of shafts during operation.

14.3. SHAFTS:

A shaft is a rotating machine element which is used to transmit power from power from one place to another. The power is delivered to the shafts by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power one shaft to another, the various members such as pulleys, gears etc. are mounted on it. These members along with the forces exerted upon them causes the shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines.

14.3.1. Types of shafts: The following two types of shafts are important from the subject point of view:

1. **Transmission shafts:** These shafts transmit power between the source and the machine absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmitting shafts. Since these shafts carry machine parts such as pulley, gears etc., therefore they are subjected to bending in addition to twisting.
2. **Machine Shafts:** These shafts from an integral part of the machine itself. The crank should be examples of machine shaft.

14.3.2. Stresses in shafts: The following stresses are induced in the shafts:

1. Shear stress due to the transmission of torque(i.e. due to torsional load)
2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements gears, pulleys est.'s well as due to the weight of the shafts itself.
3. Stresses due to combined torsional and bending loads.

14.3.3. Maximum permissible working Stresses for Transmission Shafts: According to American Society of Mechanical Engineers(ASME) code for the design of transmission shafts, the maximum permissible working stresses in tension or compression may be such as

a. 112 MPa for shafts without allowances for key ways.

a. 84 MPa for shafts with allowance for key ways.

For shafts purchased under definite physical specification, the permissible tensile stress (σ_t) be taken as 60 per cent of the elastic limit in tension (σ_{el}), but not more than 36 per cent of the ultimate tensile strength (σ_u). In other words, the permissible tensile stress,

σ_t=0.6 σ_{el} or 0.36 σ_u , whichever is less.

The maximum permissible shear stress may be taken as

a. 56 MPa for shafts without allowances for key ways.

b. 42 MPa for shafts with allowance for key ways.

For shafts purchased under definite physical specification, the permissible shear stress (τ) be taken as 30 per cent of the elastic limit in tension (σ_{el}), but not more than 18 per cent of the ultimate tensile strength (σ_u). In other words, the permissible tensile stress,

$\tau = 0.3\ \sigma_{el}$ or $0.18\ \sigma_u$, whichever is less.

14.3.4. Design shafts: The shafts may be designed on the basis of

1. Strength
2. Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered:

a. Shafts subjected to twisting moment or torque only,
b. Shafts subjected to bending moment only,
c. Shafts subjected to combined twisting and bending moments, and
d. Shafts subjected to axial loads in addition to combined torsional and bending loads.

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Lesson-15

Solved Examples

15.1 PROBLEM:

A solid shaft subjected to a torque of 1500N-m. Find the necessary diameter of the shaft, if the allowable shear stress is 600N/cm^2. The allowable twist is 1^0 for every 20 diameter length of the shaft. Take G=8*10^5 N/cm^2

Sol) Given Torque

T=1500 N-m =1500*10^2 N-cm

Allowable shear stress

τ_s=600N/cm^2

Angle of Twist θ=1^0=π/180 radians

Length of shaft, l=20D

G=8*10^5 N/cm^2

D=diameter of the shaft

First we shall find out the diameter of the shaft for its strength and stiffness

i. For strength

Using the relation
$T=(\pi/16)\,\tau_s D^3$
$1500*10^2= (\pi/16)*600*D^3$
On solving
D=10.8cm.

ii. For stiffness

Using the relation

$T/J = \tau_s/R = G\,\theta/L$

Where T=Torque

J=Polar moment of inertia in $cm^4 = (\pi/32)*D^4$

R=Radius of curvature in cm

G=Rigidity modulus in N/cm^2

L=Length of shaft in cm

Θ=Angular Twist

$$\frac{1500 \times 10^2}{\pi/32 * D4} = \frac{8 \times 10^5 * (\pi/180)}{20D}$$

On solving

D=13cm

Therefore we shall provide diameter of 13 cm i.e. greater of two values

For the same problem we can also find out the power P by using the power transmission relation that is

$P = 2\pi NT/60$

Take N= 1200rpm

$$P = \frac{2\pi * 1200 * 1500}{60}$$

P=188.4 kW

15.2.PROBLEM: Find the torque which a shaft of 25 cm diameter can safely transmit, if the shear is not to exceed 460 kg/cm^2

Solution:

Given. Dia. of shaft,

D = 25cm

Max. Shear stress, fs = 460 kg/cm^2

Let T = Torque transmitted by the shaft

Using the relation,

T= (π/16) x fs x D^3 with the usual notations

T= (π/16) x 460 x 25^3 = 1411300 kg/cm

14113 kg/m

15.3PROBLEM: A solid steel shaft is to transmit a torque of 10000 kg-m. If the shearing stress is not to exceed 450 kg/cm^2, find the minimum diameter of the shaft.

Solution:

Given Torque, T= 10,000 kg-m = 1000000 kg-cm

Max, shear stress,

fs= 450 kg/cm^2

Let D= Diameter of the shaft.

Using the relation,

Or

T= (π/16) x fs x D^3 with the usual notations

1000000 = (π/16) x 450 x D^3

$D^3 = (1000000 \times 16) / (\pi \times 450)$

$= 11320$ cm

$D = 22.45$ cm

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Module 4: Beams And Bending Moments

Lesson-16, 17

Shear Force & Bending Moment

16.1 INTRODUCTION:

Shear force and bending moment diagrams are analytical tools used in conjunction with structural analysis in structural design, determining the value of shear force and bending moment at a given point of an element. Using these diagrams the type and size of a member of a given material can be determined.

16.2 SHEAR FORCE:

Shear force at a point or at a given section is defined as the algebraic sum of vertical forces (Including the reactions) either to the left or right to the sections.

Shear force is different at different sections sections. To know the variation of shear force we draw shear force diagrams.

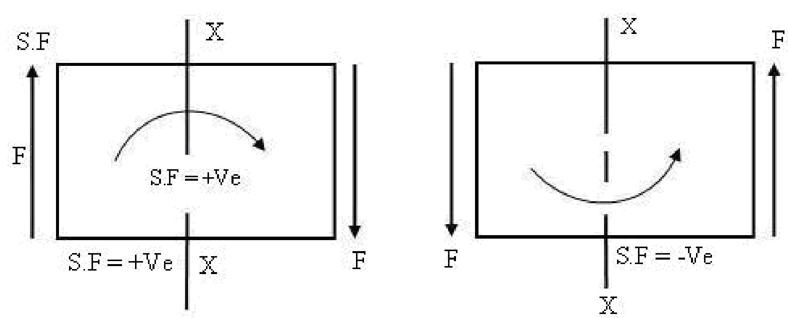


Fig.16.1 Shear force

Shear force may be considered to be +Ve if it is acting in the upward direction of the left hand side of the section (or) when it is acting in the downward direction on the right hand side (or) section & vice-versa.

16.3 BENDING MOMENT:

Bending moment at a section of beam is the algebraic sum of moment of forces to the right or left of the section. It Acts in a plane as axis (in axis). Bending moments either to the left (or) right of the section. Sign convention for shear force & bending moment.

Bending moment is said to be +ve when it is acting in the C.W (clock wise) as L.H.S (Left hand side) of the section (or) when it is acting in the A.C.W direction on the R.H.S as the section & vice versa.

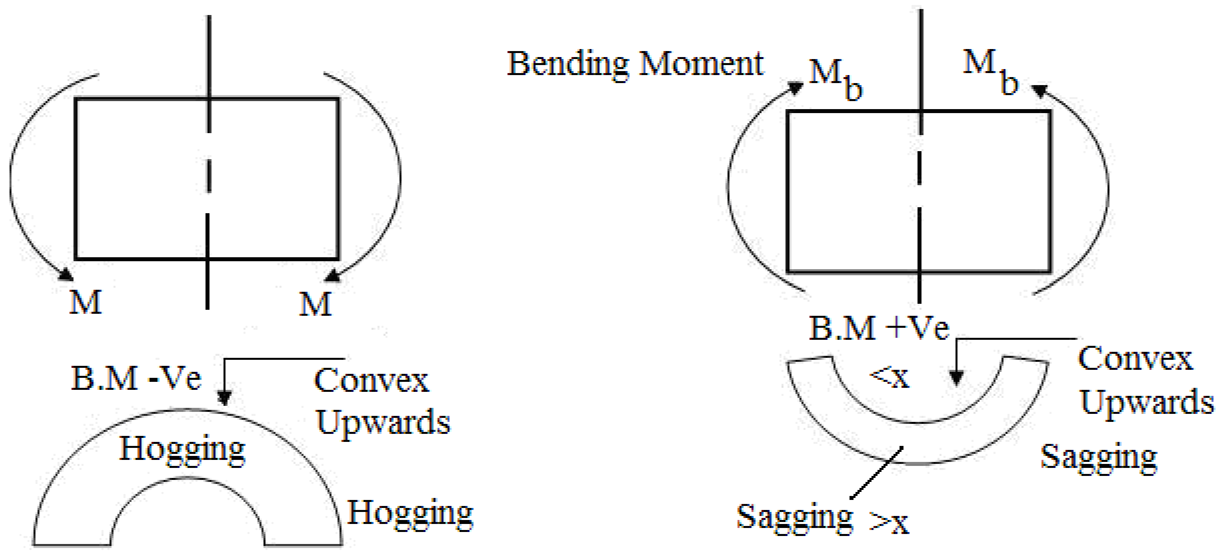


Fig.16.2 Bending moment

16.4 SHEAR FORCE DIAGRAM AND BENDING MOMENT FOR CANTILEVER BEAMS

16.4.1: Cantilever Beam with Point Load at the Free End

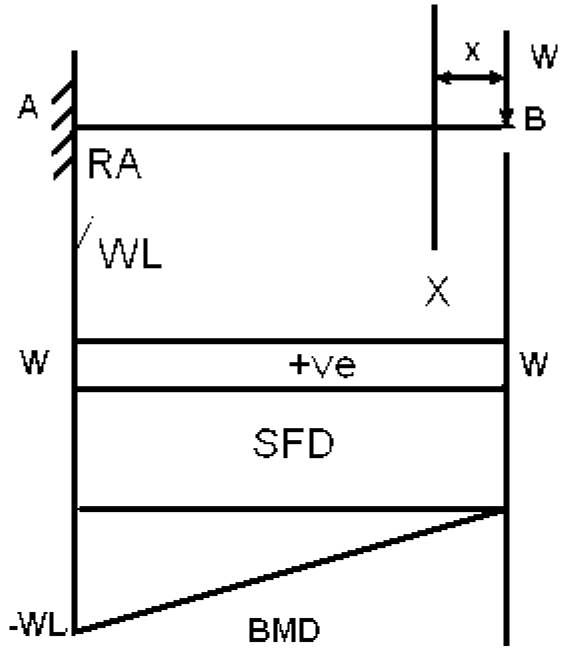


Fig.16.3 Cantilever beam

In cantilever beam we start from free end let us considered X-X section at a distance of x from free end.

Reaction RA is given by the height S.F.D.

SHEAR FORCE at X-X,

$S.F_{X-X}=W$

Moment at X-X,

$M_{X-X}=-W_X$

At B,

X=0,

$(S.F)_B=W$

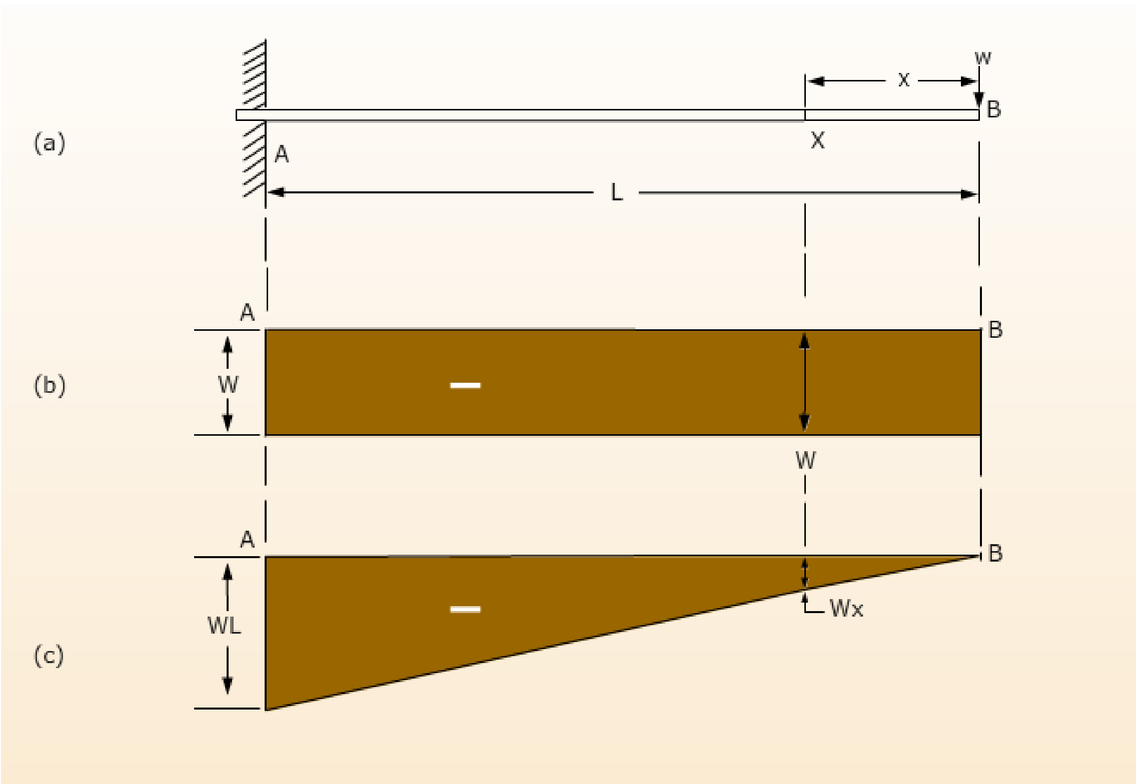
$M_B=0$

At A,

X=L,

$(S.F)_A=W$

$M_A=-WL$



16.4.2: Cantilever Beam Is Subjected To the Moment At The Free End

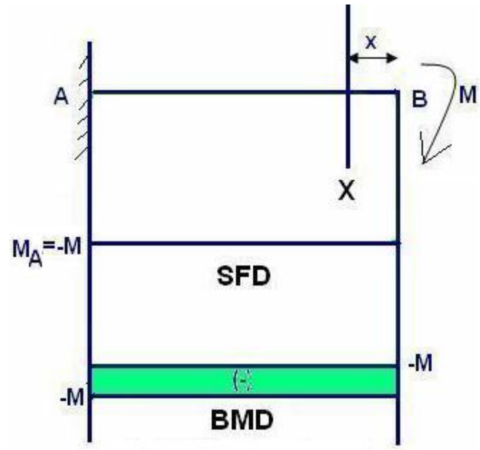


Fig.16.4 Cantilever beam

M_x=-M

M_B=-M

$(S.F)_{X-X}$=0

At B,

M_B=-M

SF_B=0, SF_A=0

Whenever there is point load acting there is a sudden change in S.F.D

Similarly whenever a concentrated moment is acting there is a sudden change is S.F.D

In presence of point load there is a sudden change in S.F.D at that corresponding point.

In presence of concentrated moment there is a sudden change in B.M.D at that corresponding point.

16.4.3: Cantilever Beam Subjected To The Moment At The Middle Point of The Beam

M_X=-M

M_A=-M

M_C=-M

M_B=-M

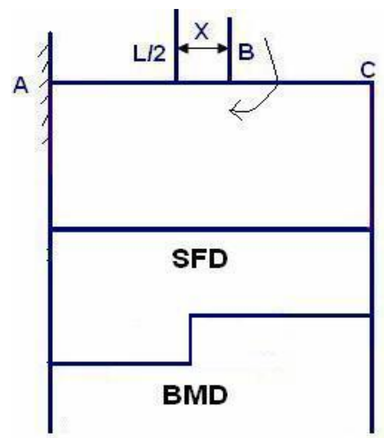


Fig.16.5 Middle point

16.4.4: Cantilever Beam Subjected To Uniform Distributed Load

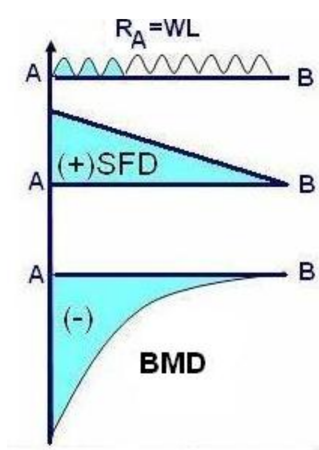


Fig.16.6 Uniform distribution load

At section X-X

$SF_{X-X}=WX$

$M_{X-X}=-WX^2/2$

At A, X=L

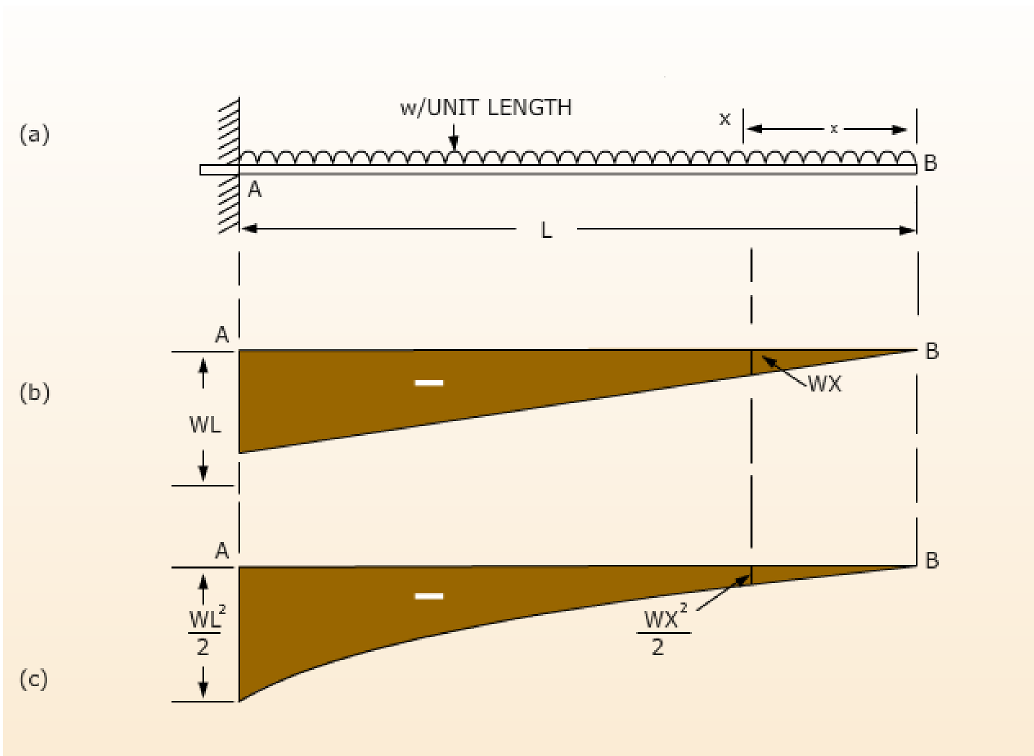
$SF_A=WL$

$M_A=-WL^2/2$

At B

$SF_B=0$

$M_B=0$



Pictorial View of Uniform Distribution Load

16.4.5: Cantilever Beam Subjected To Uniform Varying Load

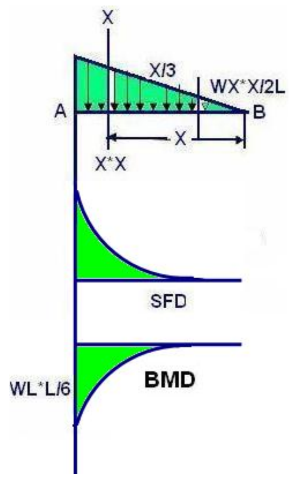


Fig.16.7 Varying load

B.M variation is always more than first order of S.F variation

Always cantilever beams we get first order type parabola and simply supported beam we get second order type parabola.

At X-X

$(SF)_X = WX^2/2L$

$M_{X-X} = -WX^3/6L$

At B, X=0

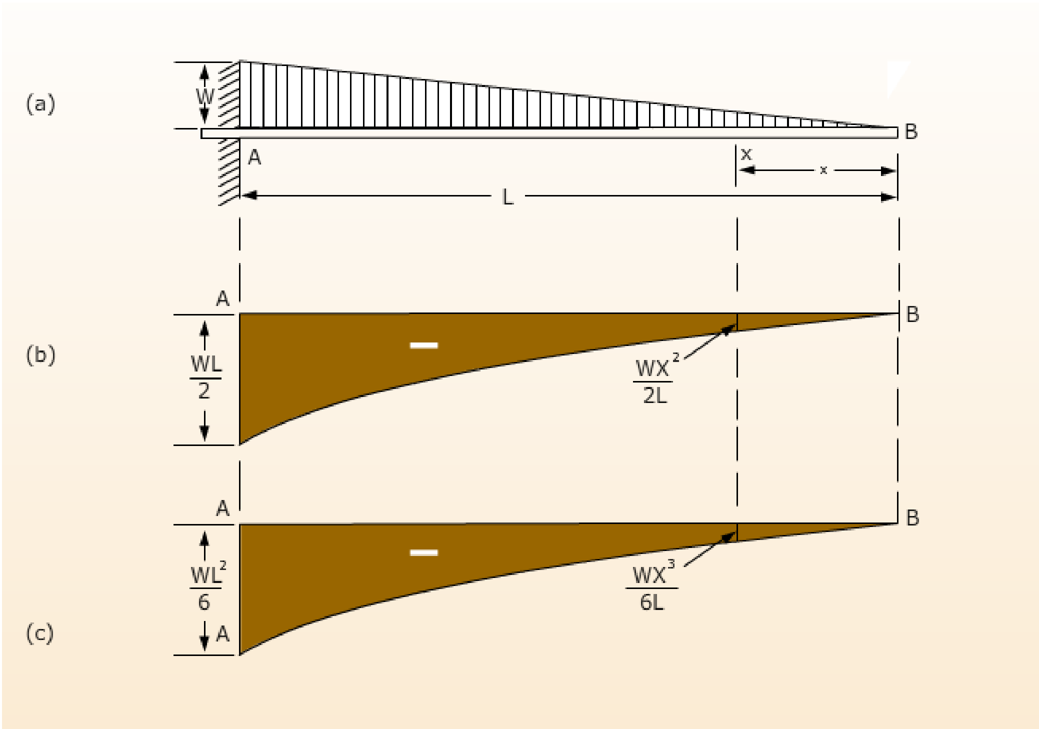
$(SF)_B = 0$

$M_B = 0$

At A, x=L

$SF_A = WL/2$

$M_A = -WL^2/6$



Pictorial View of varying load

16.5 S.F.D and B.M.D for SIMPLY SUPPORTED BEAMS

16.5.1: Simply Supported Beam Subjected To Point Load at Mid Section

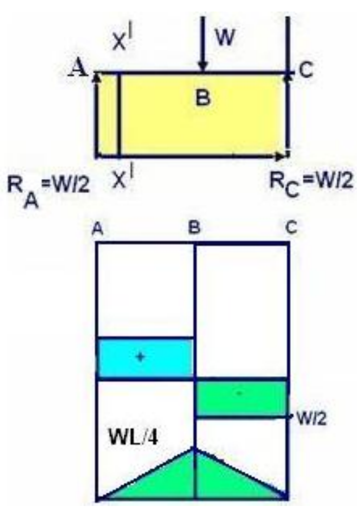


Fig.16.8 S.F.D and B.M.D

R_A=W/2, R_C=W/2 AB: (x: 0 to L/2)

$(SF)_{X-X}$=+W/2

$(BM)_{X-X}$=(W/2)X

SF_A=W/2 BM_A=0

SF_B=W/2, M_B=WL/4

BC (X: L/2-L)

$(SF)_{X-X}$=W/2-W=-W/2

$(BM)_{X-X}$=W/2(X)-W(X-L/2)

SF_B=-W/2

M_B=WL/4

SF_C=-W/2

M_C=WL/2-WL/2=0

16.5.2: Simply Supported Beam Carrying Uniform Distributed Load

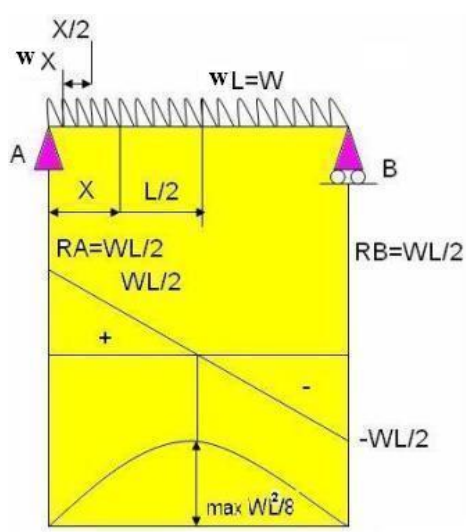


Fig.16.9 Simply supported beam carrying UDL

AB (X=0 to L)

$SF_{X-X} = \omega L/2 - \omega X$

$M_{X-X} = \omega X L/2 - \omega X^2/2$

$SF_A = \omega L/2$

$M_A = 0$

$SF_B = \omega L/2 - \omega L$

$= - \omega L/2$

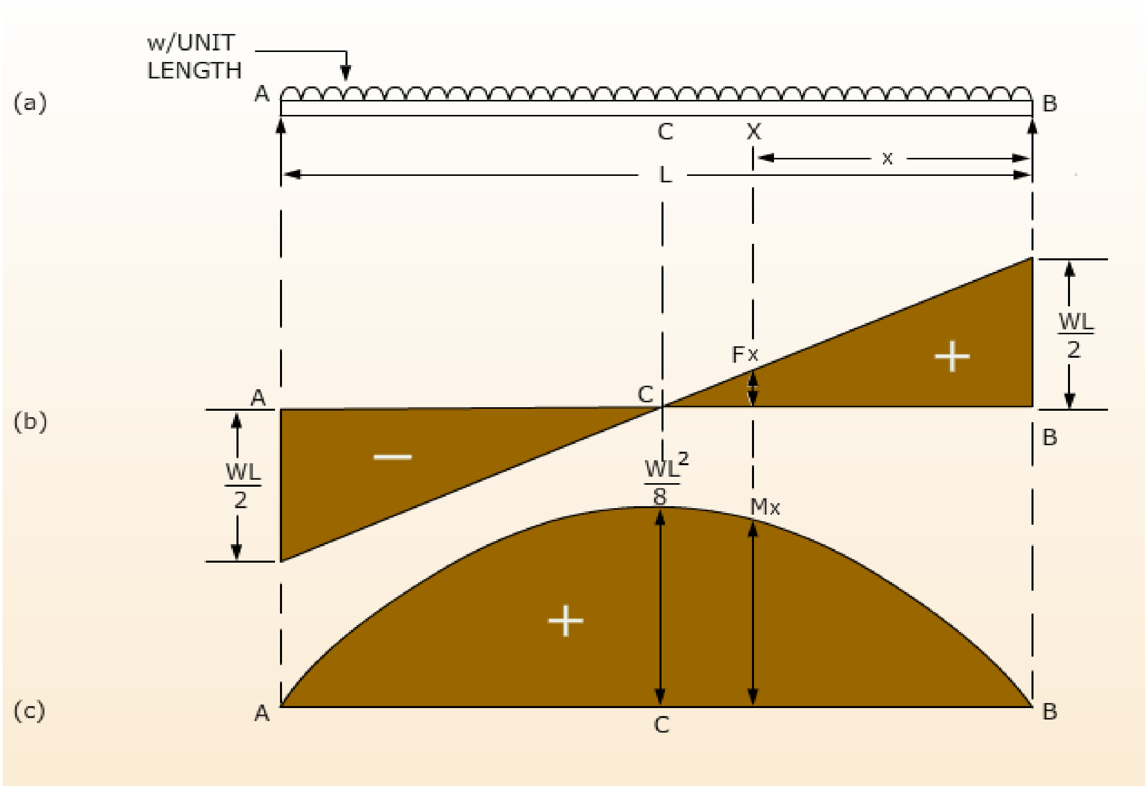
$M_B = \omega L^2/2 - \omega L^2/2 = 0$

Mmax=L

$SF_{X-X} = \omega L/2 - \omega X$

X=L/2=0

$M_{X-X} = \omega L/2 * X - \omega X^2/2 = \omega L^2/4 - \omega L^2/2 = - \omega L^2/8$



Pictorial View of Simply supported beam carrying UDL

W – Point load

ω – Uniformly distributed load

16.5. 3: Simply supported beam with point load at a distance 'a' from point A.

Taking moments of forces at C of the beam the beam about A.

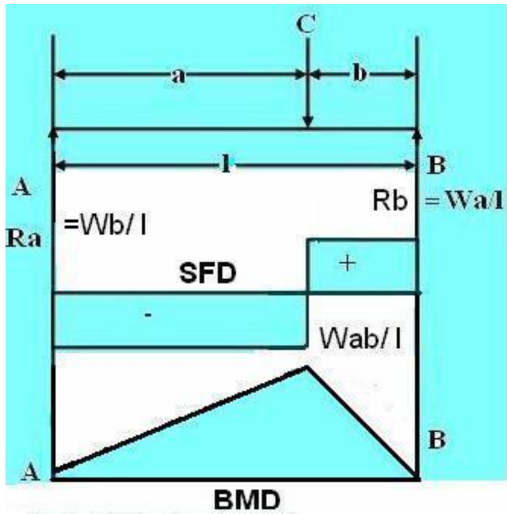


Fig.16.10 Simply supported beam with point load

R_b .l = Wa.

R_b = Wa/l

R_a = W-Wa/l

= W(l-a)/l

R_a = Wb/l { a+b = l}

For any section between A&C

Shear force

$S.F_x = R_a$

= Wb/l.

For any section b/w C&B

$S.F_x = +V_b = +$ (Wa/l).

At any section between A&C distance x from

At any section b/w A&C distance x from

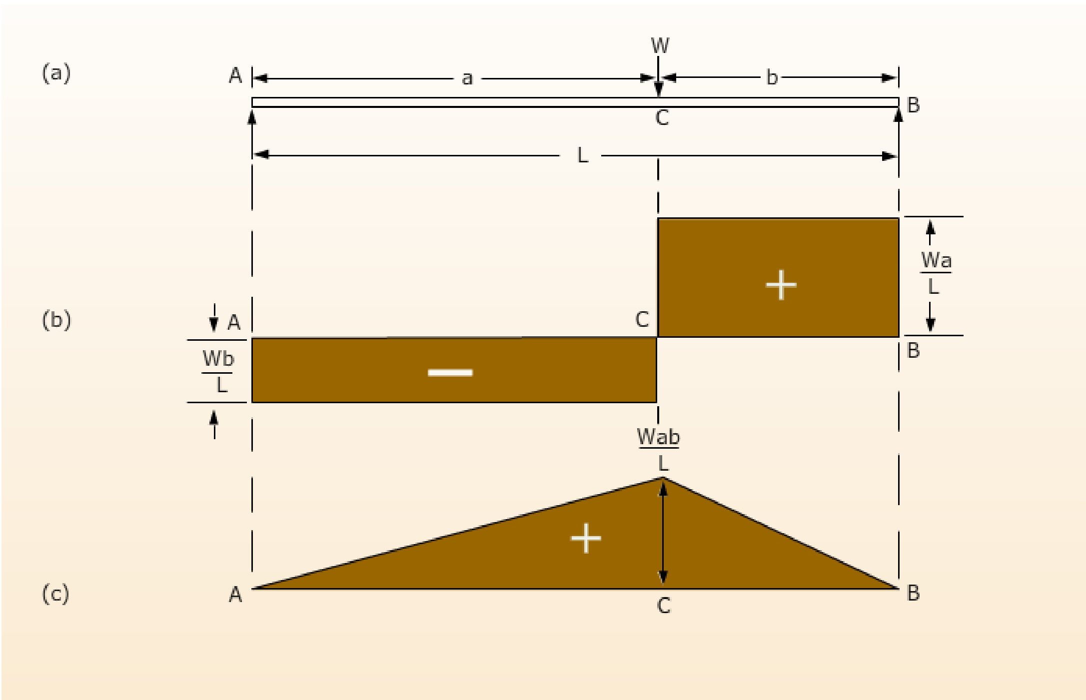
$B.M_x = +(Wb\ /1)\ .x$

{B.M = Bending moment}

$X = 0 \rightarrow M_x = 0$

$X = a \rightarrow M_x = (Wab/l)$

Hence bending moment increases uniformly from zero at the end A to Wab/l at C and B.M will decreases uniformly from Wab/l at C to zero at the end B.



Pictorial View of Simply supported beam with point load at A distance

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Lesson-18

Pure Bending

18.1 INTRODUCTION:

Pure bending refers to flexure of a beam under a constant bending moment. Therefore, pure bending occurs only in regions of a beam where the shear force is zero. In contrast, non uniform bending refers to flexure in the presence of shear forces, which means that the bending moment changes as we move along the axis of the beam. An example of pure bending would be a beam with two couples, one on each end acting in opposite directions.

18.2 ASSUMPTION IN THE THEORY OF SIMPLE BENDING:

The following assumption is made in the theory of simple bending:

1. The material of the beam is perfectly homogeneous (i.e., of the same kind throughout) and isotropic (i.e., of equal elastic properties in all directions).
2. The beam materials are stressed within its elastic limit and, thus, obey Hooks law.
3. The transverse sections, which were plane before bending, remain plane after bending also.
4. Each layer of the beam is free to expand or contract, independently, of the layer, above or below it.
5. The value of E(Young modulus of elasticity) is the same in tension and compression

18.3 THEORY OF SIMPLE BENDING:

Consider a small length dx of a simply supported beam subjected to a bending moment M. Now consider two section AB and CD, which are normal to the axis of the beam RS. Due to the action of the bending moment, the beam as a whole will bend. Since we are considering a small length of dx of the beam, therefore, the curvature of the beam, in this length, is taken to be circular. A little consideration will show, that all the layer of the beam, which were originally of the same length doing not remain of the same length any more. The top layer of the beam AC has suffered compression, and reduced to A^1C^1. As we proceed towards the lower layer of the beam, we find that the layers have, no doubt, suffered compression, but to a lesser degree; until we come across the layer RS, which has suffered no change in its length, through bent into R^1S^1. If we further proceed towards the lower layers are stretched; the amount of extension increases as we proceed lower, until we come across the lowermost layer BD which has been stretched to B^1D^1.

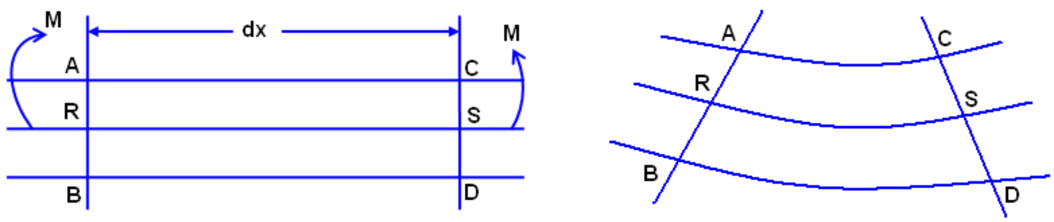
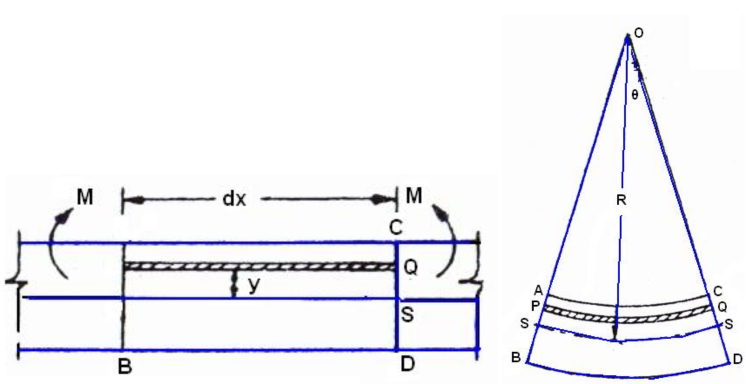


Fig 18.1 simple bending

Now we see that the layers above RS have been compressed and that bellow have stretched. The amount, by which a layer is compressed or stretched, depends upon the position of the layer with reference to RS. This layer RS, which is neither compressed nor stretched, is known as neutral plane or neutral layer. The theory of bending is called theory of simple bending.

18.4 BENDING STRESS:



(A) Before Bending B) After Bending

Fig 18.2. Bending stress

Consider a small length dx of a beam subjected to a bending moment. As a result of this moment, let this small length of beam bend into an arc of a circle with O a center.

Let M= Moment acting at the beam,

θ = angle subtended at the center by the arc, and

R= Radius of curvature of the beam

Now consider a layer PQ at a distance y from RS the neutral axis of the beam. Let this layer be compressed to P^1Q^1 after bending.

Decrease in length of this layer

$\delta l = PQ - P^1Q^1$

Strain $e = (\delta l / \text{Original length}) = (PQ - P^1Q^1) / (PQ)$

Now from the geometry of the curved beam, we find that the two section OP^1Q^1 and OR^1S^1 are similar

$(P^1Q^1) / (R^1S^1) = (R - y) / R$

Or

$1 - (P^1Q^1) / (R^1S^1) = 1 - (R - y) / R$

Or

$(R^1S^1) - (P^1Q^1) / (PQ) = (y / R)$

$(PQ) - (P^1Q^1) / (PQ) = (y / R)$ ($PQ = R^1S^1 = RS$ = Neutral axis)

$e = (y / R)$ ($e =(PQ) - (P^1Q^1) / (PQ)$)

It is thus obvious, that the stain of a layer is proportional to its distance from the neutral axis.

We also know that the stress,

f= Strain X Elasticity = e.E

= (y / R) x E = y x (E / R) (e = y / R)

Since E and R are constants in this expression, therefore the stress at any point is directly proportional to y, i.e., the distance of the point from the neutral axis.

The above expression may also be written as:

f / y = E / R ----------(1)

18.5 POSITION IF NEUTRAL AXIS:

The line of intersection of the neutral layer, with any normal cross-section of a beam, is known as neutral axis of the section. We have that on one side of the neutral axis there are compressive stresses, whereas on the other there are tensile stresses. At the neutral axis, there is no stress of any kind.

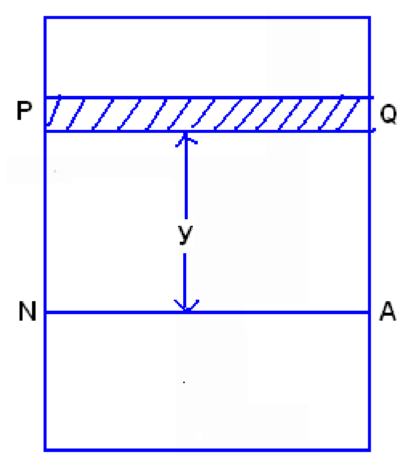


Fig.18.3 Position of neutral axis

18.6 MOMENT OF RESISTANCE:

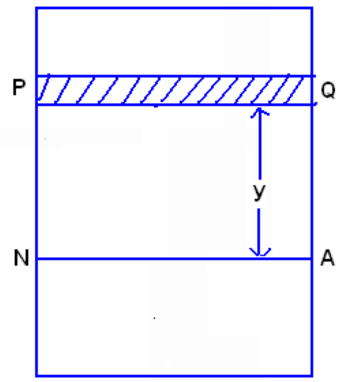


Fig.18.4 Moment of resistance

We have already known that on one side of the neutral axis there are compressive stresses, and on the other there are tensile stresses. These stresses from a couple, whose moment must be equal to the external moment M. The moment of this couple, which resist the external bending moment, is known as moment of resistance. Consider a section of the beam.

Let NA be the neutral axis of the section. Now consider a small layer PQ of the beam section at a distance y from the neutral axis.

Let

δa = Area of this layer

$f = y \times \frac{E}{R}$

Total stress in this layer

$y \times \frac{E}{R} \times \delta a$

Moment of this section about the neutral axis

$= y \times \frac{E}{R} \times \delta a \times y = \frac{E}{R} \times y^2 \times \delta a$

The algebraic sum of all such moments about the neutral axis must be equal to M. Therefore

$M = \Sigma \frac{E}{R} \times y^2 \times \delta a = \frac{E}{R} \times \Sigma y^2 \times \delta a$

The expression ? y^2 * δa represent the moment of inertia of the area of the section about the neutral axis. Therefore

$M = \frac{E}{R} \times I$ (where I = Moment of inertia)

Or

$$\frac{M}{I} = \frac{E}{R}$$

We have already

$$\frac{f}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

18.7 SECTION MODULUS:

We have

M / I=б/y=E / R

M / I= б / y

б or f_s =M / I*y

$б_{max}$ =M /I /y_{max} where Z = I/y_{max}

= M / Z Z=section modulus.

18.8. EXAMPLES OF SECTION MODULUS FOR DIFFERENT SECTIONS:

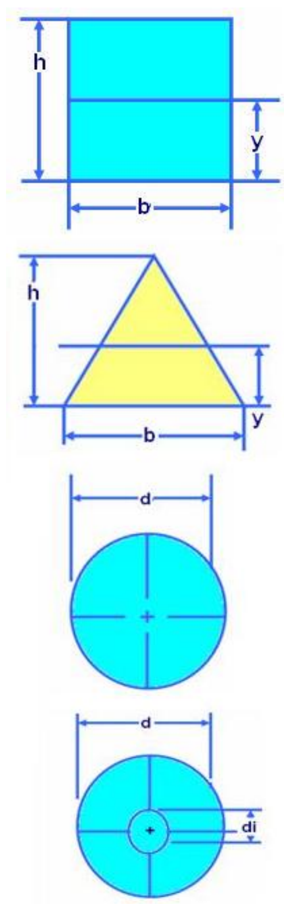


Fig.18.5

I = Moment of inertia, m^4 , m^4

Z = Section modulus, m^3 , m^3

y = Centroidal distance, m

I = b * h^3 / 12 = 1/12 $b.h^3$

Z = b * h^2 / 6 = 1/6 $b.h^2$

y = h / 2

I = b * h^3 / 36 = $b.h^3/36$

Z = b * h^2 / 12 = $b.h^2/12$

y = h / 3

I = $\prod d^4$ / 64

$$I = \frac{\pi}{64}(d^4 - d^4_i)$$

$$Z = \frac{\pi}{32}\ (d^4 - d^4_i/d)$$

y = d / 2

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Lesson-19

Solved Examples Based On Shear Force and Bending Moment Diagrams

19.1 PROBLEM:

A cantilever beam is subjected to various loads as shown in figure. Draw the shear force diagram and bending moment diagram for the beam.

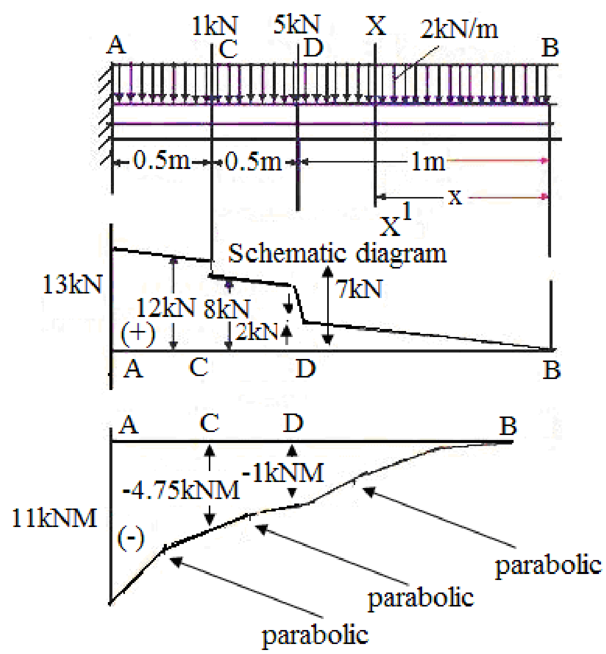

Fig.19.1 Shear force and bending moment

Solution: Consider a section (X – X') at a distance x from section B. shear force. between B and D;

Shear force Fx = + wx

At x = 0, Fb = 0 ...(1)

x = 1 m; Fd just right = 2 × 1 = 2 kN

S.F. between D and C; F_x = + wx + 5

At x = 1 m; F_D just left = (2 × 1) + 5 = 7 kN

At x = 1.5 m; F_c just right = (2 × 1.5) + 5 = 8 kN

S.F. between C and A

Fx = + wx + 5 + 4

At x = 1.5 m; F_c just left = 2 × 1.5 + 5 + 4 = 12 kN

At x = 2 m; F_A = 2 × 2 + 5 + 4 = 13 kN

Bending moment between C and A;

$M_x = -(wx).x/2$

$Mx = wx^2/2 - 5(x - 1) - 4(x - 1.5)$

At x = 1.5 m; $Mc = -2 \times (1.5)^2 / 2 - 5(1.5 - 1) - 4(1.5 - 1.5)$

Mc = –4.75 kN- m

x = 2.0 m; $Ma = -2 \times (2)^2 / 2 - 5(2.0 - 1) - 4(2.0 - 1.5)$

Ma = - 11 kN-m

(The sign of bending moment is taken to be negative because the load creates hogging).

The shape of bending moment diagram is parabolic in shape from B to D, D to C, and, also C to A.

19.2 PROBLEM:

A cantilever beam carries a uniform distributed load of 60 kN/m as shown in figure. Draw the shear force and bending moment diagrams for the beam.

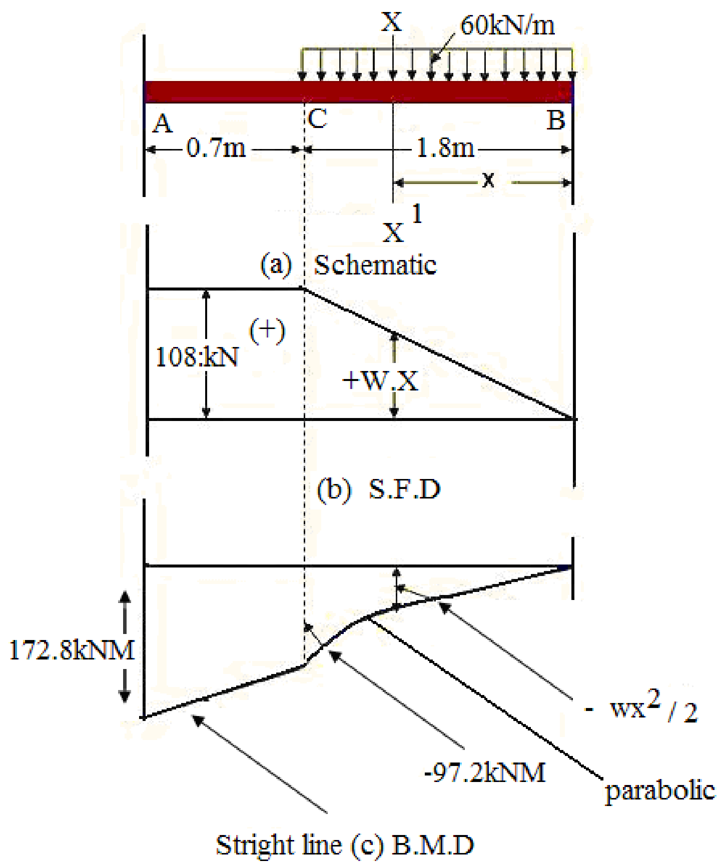

Fig. 19.2 Simply supported beam carrying UDL upto some extent

Solution: Consider a section (X – X') at a distance x from end B.

Shear force = Total unbalanced vertical force on either side of the section.

S.F. between B and C $F_x = + w.x$ ...(1)

A x = 0, $F_B = w.0 = 0$

x= 1.8 m; $F_C = 60 \times 1.8 = 108$ kN

Since, there is no load between points A and C; for this region Fx remains constant. Therefore, from C to A;

$F_x = + 108$ kN

(The sign is taken to be positive because the resultant force is in downward direction on right hand side of the section).

SFD will be triangular from B to C and a rectangle from C to A.

Now,

Bending moment between B and C $M_x = -(wx).x/2$

$= -wx^2/2$

At $x = 0$; $M_B = -w.0/2 = 0$

$x = 1.8$ m; $M_C = -60/2.(1.8)2 = -97.2$ kN m

For region C to A; $M_x = -w\,(1.8)(x - 1.8/2)$

$= -60 \times 1.8\,(x - 0.9)$

$= -108\,(x - 0.9)$

At $x = 1.8$ m; $M_C = -108\,(1.8 - 0.9)$

$= -97.2$ kN m

$x = 2.5$ m; $M_A = -108\,(2.5 - 0.9)$

$= -172.8$ kN m

(The sign of bending moment is taken to be negative because the load creates hogging).

BMD is parabolic in nature from B to C and straight line from C to A.

19.3 PROBLEM:

A simply supported beam overhanging on one side is subjected to a uniform distributed load of 1 kN/m. Sketch the shear force and bending moment diagrams and find the position of point of contra-flexure.

Solution: Consider a section (X – X’) at a distance x from end C of the beam.

To draw the shear force diagram and bending moment diagram we need R_A and R_B.

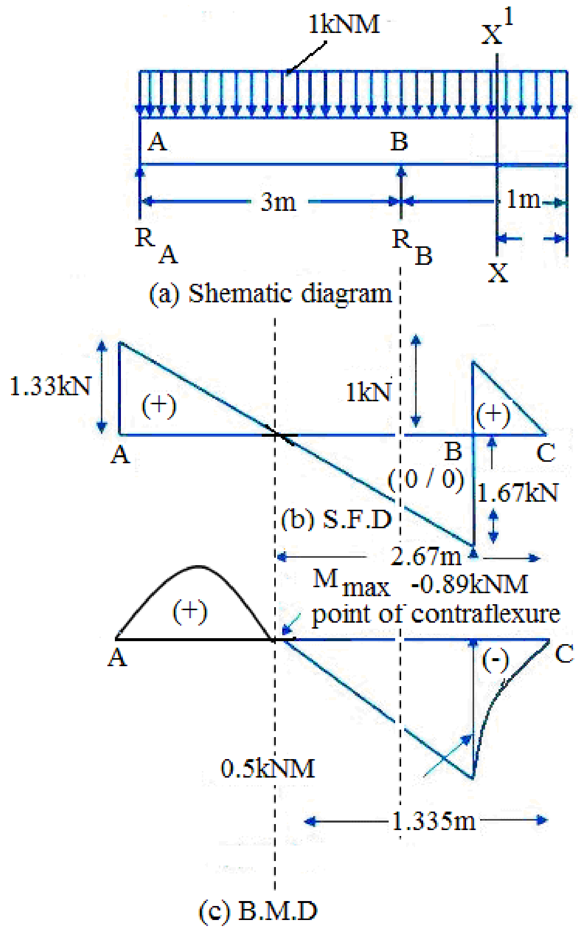

Fig.19.3 simply supported beam carrying -UDL

By taking moment of all the forces about point A.

$R_B \times 3 - w/2 \times (4)^2 = 0$

$R_B = 1 \times (4)^2 / 2 \times 3 = 8/3$ kN

For static equilibrium;

$R_A + RB - 1 \times 4 = 0$

$R_A = 4 - 8/3 = 4/3$ kN

To draw shear force diagram we need shear force at all salient points:

Taking a section between C and B, SF at a distance x from end C. we have,

$Fx = + \omega .x$ kN

At x = 0; FC = 0

x = 1 m; FB just right = 1 × 1 = + 1 kN

Now, taking section between B and A, at a distance x from end C, the SF is:

$Fx = \omega\ x - 8/3 = \omega.x\text{-}8/3$

When, x = 1 m; $F_B = 1 \cdot 1 - 8/3 = \omega.x\text{-}8/3 = -5/3$ kN $= -1.67$ kN

At x = 4 m; F_A = 4 – 8/3 = + 4/3 kN = + 1.33 kN

The shear force becomes zero;

$Fx = \omega \cdot x - 8/3 = 0$

═ x = 2.67 m

(The sign is taken positive taken when the resultant force is in downward direction the RHS of the section).

To draw bending moment diagram we need bending moment at all salient points. Taking section between C and B, bending moment at a distance x from end C, we have

Mx = – wx2 / 2 = –1.x2/2 kN m $= \omega \cdot x^2/2 = -1 \cdot x^2/2$

When x = 0, M_C = 0

At x = 1 m. MB = –1 × (1)2 / 2 = –0.5 kN m

Taking section between B and A, at a distance x from C, the bending moment is:

Mx = –x2/2 + 8/3 (x – 1)

At x = 1 m, M_B = – 0.5 kN m

x = 4 m; $M_A = -(4)^2/2 - 8/3\,(4 - 1) = 0$

The maximum bending moment occurs at a point where

$dM_x / dx = 0$

$= d/dx\,[-x^2 / 2 + 8/3\,x - 8/3] = 0$

$= -1/2 \times 2x + 8/3 = 0$

= x = 8/3 m from end C.

$= M_{max} = -1/2\,(8/3)^2 + 8/3\,(8/3 - 1) = 0.89$ kN m

The point of contraflexure occurs at a point, where

$M_x = 0$

$= -x^2/2 + 8/3\,(x - 1) = 0$

$= x^2 = 16/3\,(x - 1)$

$= x^2 - 16/3\,x + 16/3 = 0$

x = 1.335 m or 4 m

4. A simply supported beam is subjected to a combination of loads as shown in figure. Sketch the shear force and bending moment diagrams and find the position and magnitude of maximum bending moment.

Solution: To draw the shear force diagram and bending moment diagram we need R_A and R_B.

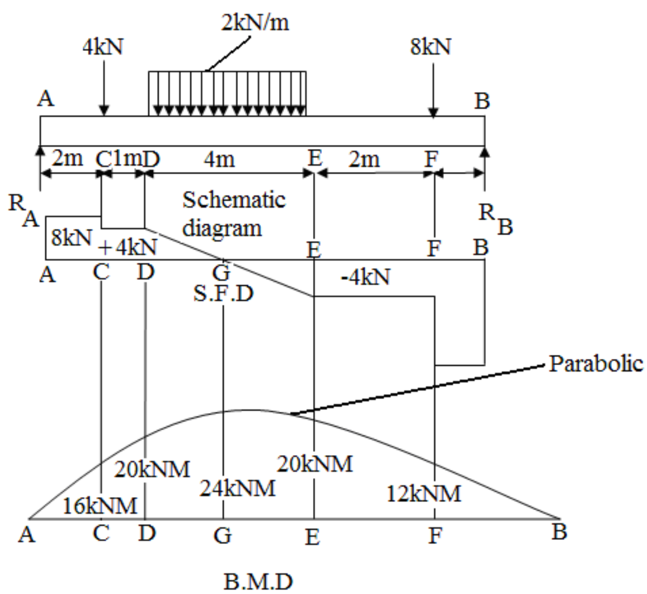

Fig.19.4 Shear force and bending moment

By taking moment of all the forces about point A.

We get $R_B \times 10 - 8 \times 9 - 2 \times 4 \times 5 - 4 \times 2 = 0$

$R_B = 12$ kN

From condition of static equilibrium $\Sigma Fy = 0$

$R_A + R_B - 4 - 8 - 8 = 0$

$R_A = 20 - 12 = 8$ kN

To draw shear force diagram we need shear force at all salient points:

For AC; $F_A = + R_A = 8$ kN

For CD, $F_C = + 8 - 4 = 4$ kN

$F_D = 4$ kN

For DE, $Fx = 8 - 4 - 2\ (x - 3) = 10 - 2x$

At x = 3 m; F_D = 10 – 6 = 4 kN

At x = 7 m; F_E = 10 – 2 × 7 = –4 kN

The position for zero SF can be obtained by 10 – 2x = 0

x = 5 m

For EF; Fx = 8 – 4 – 8 = – 4 kN

For FB; Fx = 8 – 4 – 8 – 8 = – 12 kN

To draw BMD, we need BM at all salient points.

For region AC, Mx = + 8x

At x = 0; M_A = 0

x = 2; M_C = 8 × 2 = 16 kN m

For region CD; Mx = + 8x – 4 (x – 2)

At x = 2 m; M_C = 8 × 2 – 4 (2 – 2) = 16 kN m

At x = 3 m; M_D = 8 × 3 – 4 (3 – 2) = 20 kN m

For region DE,

Mx = + 8x – 4 (x – 2) – 2(x – 3)2 / 2 = 10x – x2 – 1

At x = 3 m; M_D = 8 × 3 – 4 (3 – 2) – 2(32 – 3)2 / 2 = 20 kN m

At x = 7 m; M_E = 10 × 7 – (7)2 – 1 = 20 kN m

At x = 5 m; M_G = 10 × 5 – (5)2 – 1 = 24 kN m

For region EF,

Mx = 8x – 4 (x – 2) – 2 × 4 (x – 5) = 48 – 4x

At x = 9 m, M_F = 120 – 12 × 9 = 12 KN m

At x = 10 m; M_B = 120 – 12 × 10 = 0

The shear force diagram and bending moment diagram can now be drawn by using the various values of shear force and bending moment. For bending moment diagram the bending moment is proportional to x, so it depends, linearly on x and the lines drawn are straight lines.

The maximum bending moment exists at the point where the shear force is zero, and also dM/dx = 0 in the region of DE

d/dx (10 x – x2 – 1) = 0

10 – 2x = 0

X = 5 m

Mmax = 10 × 5 – (5)2 – 1 = 24 kN m

Thus, the maximum bending is 24 kN m at a distance of 5 m from end A.

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Lesson- 20

Flexural Stress

20.1 INTRODUCTION

Flexural strength, also known as modulus of rupture, bend strength, or fracture strength a mechanical parameter for brittle material, is defined as a material's ability to resist deformation under load. The transverse bending test is most frequently employed, in which a rod specimen having either a circular or rectangular cross-section is bent until fracture using a three point flexural test technique. The flexural strength represents the highest stress experienced within the material at its moment of rupture. It is measured in terms of stress, here given the symbol σ.

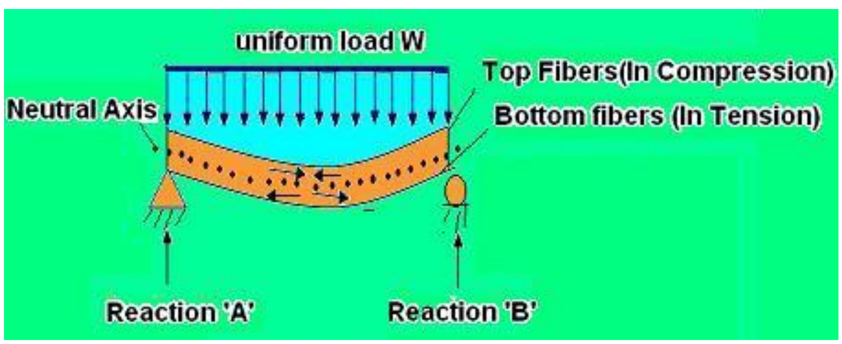

Fig.20.1 Beam diagram

20.2 FLEXURAL STRESS

When a member is being loaded similar to that in figure 20.1 flexure stress (or bending stress) will result. Bending stress is a more specific type of normal stress. When a beam experiences load like that shown in figure 20.1 on the top fibers of the beam undergo a normal compressive stress. The stress at the horizontal plane of the neutral axis is zero. The bottom fibers of the beam undergo a normal tensile stress. It can be concluded therefore that the value of the bending stress will vary linearly with distance from the neutral axis.

Calculating the maximum bending stress is crucial for determining the adequacy of beams, rafters, joists, etc.

$\sigma_b = MY/I$

σ_b=Bending stress

M=Calculated Bending Moment, M.m

Y=Vertical distance away from neutral axis,

I=Moment of inertia about the neutral axis, m^4

20.3 SHEAR STRESSES IN BEAMS RELATIONS BETWEEN CENTRE, TORSIONAL AND FLEXURAL LOADS:

The shear stress in any section at a distance 'y' from neutral axis is given by

$\sigma_{avg} = F / b\ d$

$J_{max} = FAy / Ib$

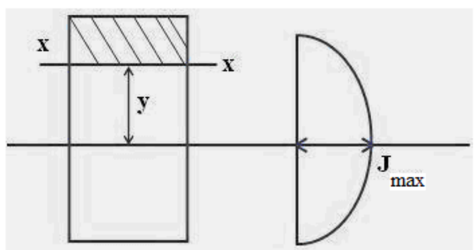

Fig.20.2 Shear stress distribution in rectangular section

Where

σ_{avg} = Complimentary (or) Horizontal shear

stress at a layer x – x.

F = Shear Force at a section, M

A = Area above 👉below the section x – x, m^2

y = Distance of centroid of area 'A' from neutral axis ,m

I = Moment of Inertia about the neutral axis, m^4

b = Width of section x – x , m

Average shear stress is given by

$\sigma_{Avg} = F / A$

Where F = Shear force

A = Cross sectional Area

For Rectangular cross section:

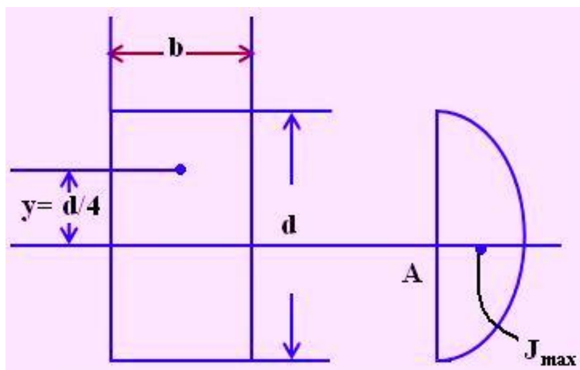

Fig.20.3 Shear stress distribution in rectangular section

$\sigma_{avg} = F/ b.d$ $\sigma_{max} = FAy /I * b$

$l = 1/12\ b.d^3$

$$= \frac{F.(b.d/2)\ (d/4)}{1/12\ bd^3\ .b}$$

$= 3/2\ F/b.d$

$\sigma_{max} = 3/2\ \sigma_{avg}$

For square cross section:

Same as rectangular cross section.

$\sigma_{max} = 3/2\ \sigma_{avg}$

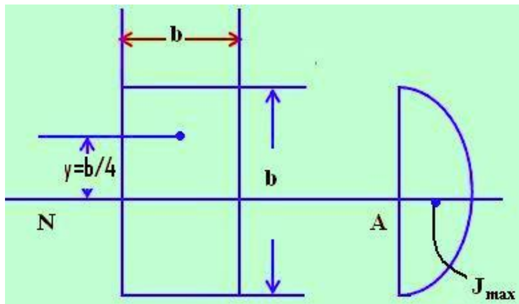

Fig.20.4 Shear stress distribution in square section

For Circular cross section:

$Y = 4R/3\Lambda$

$\sigma_{avg} = F/\ (\Lambda\ /4)D^2$

$$J_{max} = \frac{F.\ 1/2\ \ \pi\ /4\ D^2\ \ (4R/3\ \pi\ )}{(\pi\ /64).\ D^4.D}$$

$\sigma_{max} = 4/3\ \sigma_{avg}.$

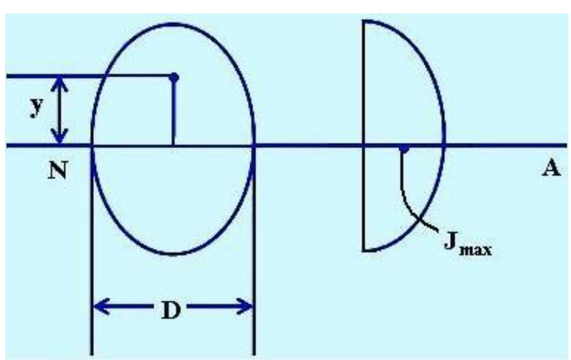

Fig.20.5 Shear stress distribution in circular section

Note:

A cross section having more area at N.A is strong in resisting shear. {except circle}.

If bending and shear are acting together in a beam, the main design criterion is the bending only. Therefore a rectangular section is commonly used as a beam.

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Lesson-21

Solving Numerical

21.1 PROBLEM:

A beam made of cast iron having a section of 50 mm external diameter and 25 mm internal diameter is supported at two points 4 m apart. The beam carries a concentrated load of 100 N at its centre. Find the maximum bending stress induced in the beam.

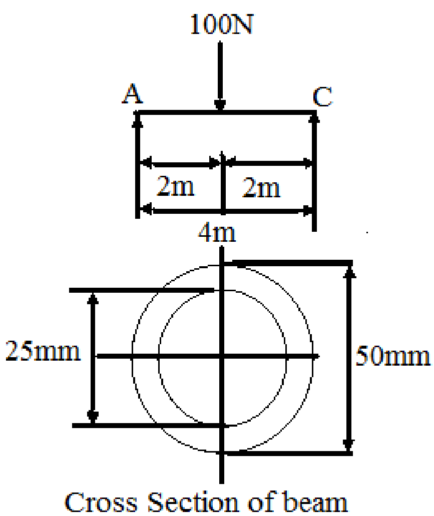

Fig.21.1 Bending stress

Solution: Given that

Outer diameter of cross-section D_0 = 50 mm

Inner diameter of cross-section D_i = 25 mm

Length of spam 'L' = 4 m

Load applied W = 100 N

For a simply supported beam with point load at center

For AC $M_x = R_A.X = W/2\ x$

When x = 0, MA = 0

At x = L/2 M_C = W/2.L/2 = WL/4

By symmetry M_B =

Maximum bending moment occurs at center
and its value is WL/4

$\Rightarrow M = WL/4 = 100 \times 4 / 4 = 100$ Nm

From bending equation σ/y = M/I, where s is bending or flexure stress

M is bending moment , I is moment of inertia
y is the distance of point from neutral axis.

$\sigma = M.y/I = M(D_0/2) / \pi/64 [D_0^4 - D_1^4]$

$\sigma = (100 \times 100) \times (50/2) / \pi/64 ((50^4 - (25)^4))$

$\sigma = 8.692$ N/mm^2

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Module 5: Properties Of Material, Failures And Factor Of Safety

Lesson-22

Design: Procedures, Specification, Strength, Design Factors, Factor of Safety Selection of Factor of Safety

22.1 INTRODUCTION:

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one.

From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality.

In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, and Engineering Drawing.

22.2 CLASSIFICATIONS OF MACHINE DESIGN:

The machine design may be classified as follows:

22.2.1 Adaptive design: In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be alternation or modification in the existing designs of the product.

22.2.2 Development design: This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

22.2.3 New design: This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design. The designs, depending upon the methods used, may be classified as follows:

- **Rational design:** This type of design depends upon mathematical formulae of principle of mechanics.
- **Empirical design:** This type of design depends upon empirical formulae based on the practice and past experience.
- **Industrial design:** This type of design depends upon the production aspects to manufacture any machine component in the industry.
- **Optimum design:** It is the best design for the given objective function under the specified constraints. It may be achieved by minimizing the undesirable effects.
- **System design:** It is the design of any complex mechanical system like a motor car.
- **Element design:** It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.
- **Computer aided design:** This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimization of a design.

22.3 DESIGN PROCEDURE AND SPECIFICATIONS:

The general procedure for design is widely available in the literature .The following procedure is representative of those found in the literature.

- Identification of need
- Problem statement or definition of goal
- Research
- Development of specifications
- Generation of ideas
- Creation of concepts based on the ideas
- Analysis of alternative concepts

- Prototype and laboratory testing
- Selection and specification of best concept
- Production
- Marketing
- Maintenance and repairs

22.4 GENERAL PROCEDURE IN MACHINE DESIGN:

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows:

- Recognition of need. First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.
- Synthesis (Mechanisms). Select the possible mechanism or group of mechanisms which will give the desired motion.
- Analysis of forces. Find the forces acting on each member of the machine and the energy transmitted by each member.
- Material selection. Select the material best suited for each member of the machine.
- Design of elements (Size and Stresses). Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform more than the permissible limit.
- Modification. Modify the size of the member to agree with the past experience and judgement to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.
- Detailed drawing. Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.
- Production. The component, as per the drawing, is to be manufactured in the workshop.

22.5 FACTOR OF SAFETY:

It is defined, in general, as the ratio of the maximum stress to the working stress. Mathematically,

Factor of safety =Maximum stress/Working or design stress

In case of ductile materials e.g. mild steel, where the yield point is clearly defined, the factor of Safety is based upon the yield point stress. In such cases,

Factor of safety =Yield point stress/Working or design stress

In case of brittle materials e.g. cast iron, the yield point is not well defined as for ductile materials.

Therefore, the factor of safety for brittle materials is based on ultimate stress.

Factor of safety =Ultimate stress/Working or design stress.

22.6 SELECTION OF FACTOR OF SAFETY:

The selection of a proper factor of safety to be used in designing any machine component depends upon a number of considerations, such as the material, mode of manufacture, type of stress, general service conditions and shape of the parts. Before selecting a proper factor of safety, a design engineer should consider the following points:

- The reliability of the properties of the material and change of these properties during service.
- The reliability of test results and accuracy of application of these results to actual machine parts.
- The reliability of applied load.
- The certainty as to exact mode of failure.
- The extent of simplifying assumptions.
- The extent of localized stresses.

- The extent of initial stresses set up during manufacture.
- The extent of loss of life if failure occurs.
- The extent of loss of property if failure occurs.

Each of the above factors must be carefully considered and evaluated. The high factor of safety results in unnecessary risk of failure. The values of factor are safety based on ultimate strength for different materials and type of load.

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Lesson-23

Material And Properties: Static Strength, Ductility, Hardness, Fatigue, Designing For Fatigue Conditions.

23.1 INTRODUCTION

The knowledge of materials and their properties is of great significance for a design engineer. The machine elements should be made of such a material which has properties suitable for the conditions of operation. We will discuss the commonly used engineering materials and their properties in Machine Design.

23.2 CLASSIFICATION OF ENGINEERING MATERIALS

The engineering materials are mainly classified as:

1. Metals and their alloys, such as iron, steel, copper, aluminum, etc.
2. Non-metals, such as glass, rubber, plastic, etc.

The metals may be further classified as:

(a) Ferrous metals and (*b*) Non-ferrous metals.

The ferrous metals are those which have the iron as their main constituent, such as cast iron, wrought iron and steel.

The non-ferrous metals are those which have a metal other than iron as their main constituent, such as copper, aluminum, brass, tin, zinc, etc.

23.3 PHYSICAL PROPERTIES OF METALS: The physical properties of the metals include luster, colour, size and shape, density, electric and thermal conductivity, melting point, etc.

23.4 MECHANICAL PROPERTIES OF METALS: The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness. We shall now discuss these properties as follows:

23.4.1 Static strength. It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a material to an externally applied force is called stress.

23.4.2 Stiffness. It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

23.4.3 Elasticity. It is the property of a material to regain its original size and shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines.

23.4.4 Plasticity. It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

23.4.5 Ductility. It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms-percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice are mild steel, copper, aluminum, nickel, zinc, tin and lead.

23.4.6 Brittleness. It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads snap off without giving any sensible elongation. Cast iron is a brittle material.

23.4.7 Malleability. It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminum.

23.4.8 Toughness. It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed up to the point of fracture. This property is desirable in parts subjected to shock and impact loads.

23.4.9 Machinability. It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.

23.4.10 Resilience. It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume with inelastic limit. This property is essential for spring materials.

23.4.11 Creep. When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow a permanent deformation called **creep.** This property is considered in designing internal combustion engines, boilers and turbines.

23.4.12 Fatigue. When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as fatigue. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.

23.4.13 Hardness. It is a very important property of the metals and has a wide variety of meanings .It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal. The hardness is usually expressed in numbers which are dependent on the method of making the test. The hardness of a metal may be determined by the following tests:

1. Brinell hardness test
2. Rockwell hardness test
3. Vickers hardness
4. Shore scleroscope.

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Lesson-24

Theories Of Failure, Stresses In Elementary Machine Parts

24.1 INTRODUCTION

It has already been discussed in the previous chapter that strength of machine members is based upon the mechanical properties of the materials used. Since these properties are usually determined from simple tension or compression tests, therefore, predicting failure in members subjected to uniaxial stress is both simple and straight-forward. But the problem of predicting the failure stresses for members subjected to bi-axial or tri-axial stresses is much more complicated. In fact, the problem is so complicated that a large number of different theories have been formulated. The principal theories of failure for a member subjected to bi-axial stress are as follows:

24.2 PRINCIPAL THEORIES OF FAILURES

- Maximum principal (or normal) stress theory (also known as Rankine's theory).
- Maximum distortion energy theory (also known as Hencky and Von Mises theory).
- Maximum shear stress theory (also known as Guest's or Tresca's theory).
- Maximum strain energy theory (also known as Haigh's theory).
- Maximum principal (or normal) strain theory (also known as Saint Venant theory).

Since ductile materials usually fail by yielding *i.e.* when permanent deformations occur in the material and brittle materials fail by fracture, therefore the limiting strength for these two classes of materials is normally measured by different mechanical properties. For ductile materials, the limiting strength is the stress at yield point as determined from simple tension test and it is, assumed to be equal in tension or compression. For brittle materials, the limiting strength is the ultimate stress in tension or compression.

24.2.1 Maximum principal or normal stress theory (Rankine's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal or normal stress in a bi-axial stress system reaches the limiting strength of the material in a simple tension test. Since the limiting strength for ductile materials is yield point stress and for brittle materials the limiting strength is ultimate stress, therefore according to the above theory, taking factor of safety (*F.S.*) into consideration, the maximum principal or normal stress ($\sigma t1$) in a bi-axial stress system is given by

$\sigma t1 = \sigma yt / F.S$ for ductile material

$= \sigma u / F.S$ for brittle material

Where, σyt = Yield point stress in tension as determined from simple tension test, and

σu = Ultimate stress.

24.2.2 Maximum distortion energy theory (Hencky and Von Mises Theory):

According to this theory, the failure or yielding occurs at a point in a member when the distortion strain energy (also called shear strain energy) per unit volume in a bi-axial stress system reaches the limiting distortion energy (*i.e.* distortion energy at yield point) per unit volume as determined from a simple tension test. Mathematically, the maximum distortion energy theory for yielding is expressed as

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = (\sigma_{yt} / \text{F.O.S})^2$$

Where,

σ_{yt} is yield stress

F.O.S. = Factor of safety.

This theory is mostly used for ductile materials in place of maximum strain energy theory.

24.2.3 Maximum shear stress theory (Guest's Or Teresa's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum shear stress in a bi-axial stress system reaches a value equal to the shear stress at yield point in a simple tension test. Mathematically,

$$\tau_{max} = \tau_{yt} / F.O.S.$$

Where

τ_{max} = Maximum shear stress in a bi-axial stress system,

τ_{yt} = Shear stress at yield point as determined from simple tension test,

F.O.S. = Factor of safety.

24.2.4 Maximum strain energy theory (Haigh's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the strain energy per unit volume in a bi-axial stress system reaches the limiting strain energy per unit volume as determined from simple tension test. We know that strain energy per unit volume in a bi-axial stress system,

$$U_1=1/2E[\sigma_{t1}^2 + \sigma_{t2}^2 – ((2\sigma_{t1}* \sigma_{t2})/M)]$$

$$U_2=1/2E[\sigma_{yt}/F.O.S]^2$$

According to the above theory $U_1=U_2$

$$1/2E[\sigma_{t1}^2 + \sigma_{t2}^2 – ((2\sigma_{t1}* \sigma_{t2})/M)] =1/2E[\sigma_{yt}/F.O.S]^2$$

$$\text{Or } [\sigma_{t1}^2 + \sigma_{t2}^2 – ((2\sigma_{t1}* \sigma_{t2})/M)]= [\sigma_{yt}/F.O.S]^2$$

This theory may be used for ductile materials.

24.2.5 Maximum principal strain theory (Saint Venant's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal (or normal) strain in a bi-axial stress system reaches the limiting value of strain as determined from a simple tensile test. The maximum principal (or normal) strain in a bi-axial stress system is given by

$$\epsilon_{max} = (\sigma_{t1}/E) - (\sigma_{t2}/m*E)$$

According to the above theory,

$\epsilon_{max} = (\sigma_{t1}/E) - (\sigma_{t2}/m.E) = \epsilon = (\sigma_{yt}/E*F.O.S)$…………………………

Where,

σ_{t1} and σ_{t2} = Maximum and minimum principal stresses in a bi-axial stress system,

ε = Strain at yield point as determined from simple tension test,

$1/m$ = Poisson's ratio,

E = Young's modulus, and

F.O.S. = Factor of safety.

From equation (*i*), we may write that

$$\sigma_{t1} - (\sigma_{t2}/m) = (\sigma_{yt}/F.O.S)$$

This theory is not used, in general, because it only gives reliable results in particular cases.

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Lesson-25

Solving Numerical

The load on a bolt consists of an axial pull of 20 KN together with a transverse

Shear force of 10 KN. Find the diameter of bolt required according to

- Maximum principal stress theory.
- Maximum distortion energy theory.
- Maximum shear stress theory.
- Maximum strain energy theory; and
- Maximum principal strain theory.

Take Permissible Tensile stress at elastics limit=100Mpa and Poisson's ratio=0.3

Nomenclature

Load=P

$б_{t1}$ = normal tensile stress in x direction

$б_{t2}$ = normal tensile stress in y direction

$б_1$, $б_2$= Principal stresses

ȶ =Shear stress

$б_{t(el)}$= Permissible Tensile stress at elastics limit

Sol) Given load=$P_{tensile}$ =20KN

P_{shear}=10KN

(stress(sigma symbol))$_{tensile(elongation)}$=100Mpa=100N/MM^2

$\mu = 1/m = 0.3$

Let d=diameter of the bolt in mm

Therefore Cross sectional area of the bolt

$A = (\Pi / 4) \times d^2 = 0.785 * d^2\ mm^2$

We know that axial tensile stress

$б_{t1} = (P_{t1} / A) = (20 / 0.785d^2) = (25.47 / d^2)\ kN / mm^2$

And transverse shear stress

Shear stress(τ) = $(P_s / A) = (10 / 0.785d^2) = (12.73 / d^2)\ kN/mm^2$

1). According to maximum principal stress theory.

We know that maximum principal stress,

$$\text{Normal stress (Sigma symbol) } б_{t1} = \frac{б_1 + б_2}{2} + \frac{1}{2}\left[\sqrt{(б_1 \quad б_2)^2}\right]$$

$$= б_{t1/2} + \frac{1}{2}\sqrt{б_1^2} + \sqrt{4τ^2}$$

$$= \frac{25.47}{2d^2} + \frac{1}{2}\sqrt{\left[\frac{25.47}{d^2}\right]^2} + \sqrt{4\left[\frac{12.73}{d^2}\right]^2}$$

$$= \frac{12.73}{d^2} + \frac{1}{2} * \frac{12.73}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{12.73}{d^2}\left\{1 + \frac{1}{2}\left[\sqrt{4+4}\right]\right\}$$

$= 30365 / d^2\ kN/mm^2$

According to maximum principal stress theory

$б_{t1} = б_{t(el)}$

$(30365 / d^2) = 100$

$d^2 = (30365 / 100) = 303.65$

$= 17.42mm$

2). According to maximum shear stress theory

We know that maximum shear stress.

$$\tau_{max} = 1/2\sqrt{(\sigma 1 - \sigma 2)^2} + \sqrt{4(\tau)^2}$$

$$= 1/2\sqrt{\sigma 1^2} + \sqrt{4(\tau)^2}$$

$$= 1/2 \sqrt{[\frac{25.47}{d^2}]^2} + \sqrt{4[\frac{12.73}{d^2}]^2}$$

$$= 1/2 * \frac{12.73}{d^2} * \sqrt{4+4} = \frac{\sqrt{8}}{2} * \frac{12.73}{d^2}$$

$$= \frac{18.000}{d^2}\ kN/mm^2 = \frac{18000}{d^2}\ N/mm^2$$

According to maximum shear stress theory

$$\tau_{max} = \frac{\sigma t(el)}{2} \Rightarrow \frac{18000}{d^2} = \frac{100}{2}$$

$$d^2 = \frac{18000}{50} = 360$$

$$d = 18.97\ mm$$

3.) According to maximum principal strain theory

We know that maximum principal stress

$$\sigma_{t1} = \frac{\sigma 1}{2} + \frac{1}{2}\sqrt{\sigma 1^2} + \sqrt{4\tau^2}$$

$$= \frac{25.47}{2d^2} - \frac{1}{2}\sqrt{[\frac{25.47}{d^2}]^2} + \sqrt{4[\frac{12.73}{d^2}]^2}$$

$$= \frac{12.73}{d^2} - \frac{1}{2} * \frac{12.73}{d^2}[\sqrt{4+4}]$$

$$= \frac{12.73}{d^2}[1-\sqrt{2}] = \frac{5.270}{d^2}\ kN/mm^2$$

$$= \frac{5270}{d^2}\ N/mm^2$$

We know that accordingly to maximum principal strain theory,

$$\frac{\sigma_T}{E} - \frac{\sigma_T}{ME} = \frac{\sigma_t(el)}{E}$$

Therefore

$$\frac{30365}{d^2} + \frac{5270 \times 0.3}{d^2} = 100$$

Or

$$\frac{30365}{d^2} + \frac{1581}{d^2} = 100$$

$$\frac{31946}{d^2} = 100$$

$$\frac{31946}{100} = d^2$$

Or d=17.87mm

4).According to maximum strain energy theory

We know that according to maximum strain energy theory

$$б_{t1}^2 + б_{t2}^2 - \frac{2б_{t1} \times б_{t2}}{M} = (б_{t(el)})^2$$

$$[\frac{30365}{d^2}]^2 + [\frac{-5270}{d^2}]^2 - 2 \times \frac{30365}{d^2} * \frac{-5270}{d^2} * 0.3 = (100)^2$$

$$\frac{922 \times 10^6}{d^4} + \frac{27.7 \times 10^6}{d^4} + \frac{96.0 \times 10^6}{d^4} = 10000$$

$$\frac{92200}{d^4} + \frac{2770}{d^4} + \frac{9600}{d^4} = 1$$

$104570 = d^4$

(Or) d=17.8mm

5). Maximum distortion energy theory

According to this theory

$$б_{t1}^2 + б_{t2}^2 - 2\, б_{t1} * б_{t2} = (б_{t(el)})^2$$

$$[\frac{30365}{d^2}]^2 + [\frac{-5270}{d^2}]^2 - 2 \times \frac{30365}{d^2} * \frac{-5270}{d^2} = (100)^2$$

$$\frac{922 \times 10^6}{d^4} + \frac{27.7 \times 10^6}{d^4} + \frac{320 \times 10^6}{d^4} = (10)^4$$

$$\frac{92200}{d^4} + \frac{2770}{d^4} + \frac{32000}{d^4} = 1$$

126970 = d4

d = 18.87mm

Module 6: Power Transmission

Lesson-26 and 27
Design of a Drive System, Design of Length and Thickness of Belt

26.1 INTRODUCTION:

Among flexible machine elements, perhaps V-belt drives have widest industrial application. These belts have trapezoidal cross section and do not have any joints. Therefore, these belts are manufactured only for certain standard lengths. To accommodate these belts the pulleys have V shaped grooves which make them relatively costlier. Multiple groove pulleys are available to accommodate number of belts, when large power transmission is required. V-belt drives are most recommended for shorter center distances. V belt can have transmission ratio up to 1:15 and belt slip is very small. As the belts are endless type, V-belt drives do not suffer from any joint failure and are quiet in operation. V-belts constitute fabric and cords of cotton, nylon etc. and impregnated with rubber.

26.2 NOMENCLATURE OF V-BELT :

A typical V-belt section is shown in Fig.26.1. The geometrical features of the belt section are indicated in the figure. The pitch line, which is also marked as N-A, is the neutral axis of the belt section. The design calculations for V-belt drives are based on the pitch line or the neutral axis. These belts are available in various sections depending upon power rating.

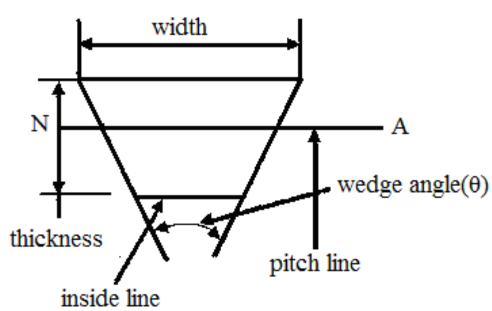

Fig.26.1 Nomenclature of V-belt

26.3 STANDARD V-BELT SECTIONS :

The standard V-belt sections are A, B, C, D and E. The table below contains design parameters for all the sections of V-belt. The kW rating given for a particular section indicates that, belt section selection depends solely on the power transmission required, irrespective of number of belts. If the required power transmission falls in the overlapping zone, then one has to justify the selection from the economic view point also.

Table 26.1: Standard V-belt configuration

Section	kW Range	Minimum pulley pitch diameter(mm)	Width (mm)	Thickness (mm)
A	0.4-4	125	13	8
B	1.5-15	200	17	11
C	10-70	300	22	14
D	35-150	500	32	19
E	70-260	630	38	23

As for example, a single belt of B section may be sufficient to transmit the power, instead of two belts of A section. This may increase the cost as well as weight of the pulley, as two-grooved pulley is required. In general, it is better to choose that section for which the required power transmission falls in the lower side of the given range.

Another restriction of choice of belt section arises from the view point of minimum pulley diameter. If a belt of higher thickness (higher section) is used with a relatively smaller pulley, then the bending stress on the belt will increase, thereby shortening the belt life.

V- Belt Equation

V-belts have additional friction grip due to the presence of wedge. Therefore, modification is needed in the equation for belt tension. The equation is modified as,

$$\frac{T_1 - mv^2}{T_2 - mv^2} = e^{\mu\alpha/(\sin\theta/2)}$$

Where, T_1= Tension in belt on tight side

T_2 =Tension in belt slack side

V=Linear velocity of the belt

m= Mass of belt per unit length θ

Where θ is the belt wedge angle

Selection of V- belt

The transmission ratio of V belt drive is chosen within a range of 1:15.

Depending on the power to be transmitted a convenient V-belt section is selected.

The belt speed of a V-belt drive should be around 20m/s to 25 m/s, but should not exceed 30 m/s.

From the speed ratio, and chosen belt speed, pulley diameters are to be selected from the standard sizes available.

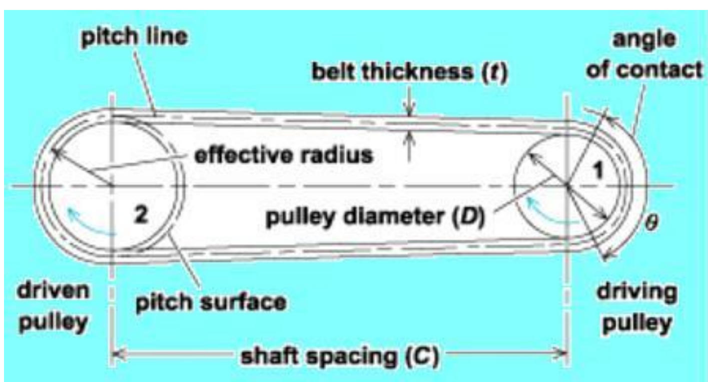

Fig.26.2 Standard V-belt section

Depending on available space the center distance is selected, however, as a guideline,

$$d_2 < C < 3(d_2 + d_1)$$

Where C is the center distance between pulleys, d_2 and d_1 are the diameter of larger and smaller pulleys respectively.

The belt pitch length can be calculated if C, d_2 and d_1 are known. Corresponding inside length then can be obtained from the given belt geometry. Nearest standard length, selected from the design table, is the required belt length above, the design power and modified power rating of a belt can be obtained. Therefore,

$$\text{Number of belts} = \frac{\text{Design power}}{\text{Modified power rating of one belt}}$$

26.4 CHAIN DRIVES :

We have seen in belt and rope drives that slipping may occur. In order to avoid slipping, steel chains are used. The chains are made up of rigid links which are hinged together in order to provide the necessary flexibility for swapping around the driving and driven wheels. The wheels have projecting teeth and fit into the corresponding recesses, in the links of the chain. The wheels and the chain are thus constrained to move together without slipping and ensures perfect velocity ratio. The toothed wheels are known as sprocket wheel or simply sprockets. The sprockets and the chain are thus constrained to move together without slipping and ensure perfect velocity ratio.

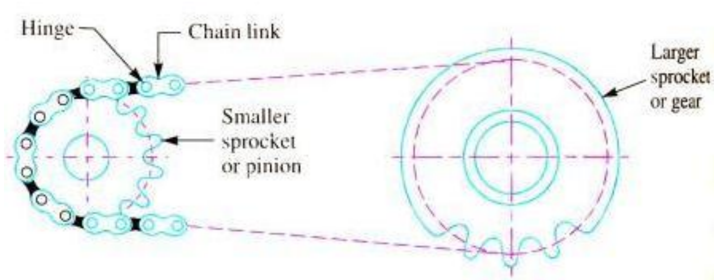

Fig.26.3. Chain Drives

The chains are mostly used to transmit motion and power from one shaft to another, when

Centre distance between their shafts is short such as in bicycles, motor cycles, agricultural machinery, conveyors, rolling mills, road rollers etc. The chains may also be used for long centre distance of up to 8metres. The chains are used for velocities up to 25 m / s and for power up to 110kW. In some cases higher power transmission is also possible.

26.4.1 Advantages and Disadvantages of Chain Drive over Belt or Rope Drive:

Following are the advantages and disadvantages of chain drive over belt or rope drive:

Advantages:

1. As no slip takes place during chain drive, hence perfect velocity ratio is obtained.
2. Since the chains are made of metal, therefore they occupy less space in width than a belt or rope drive.
3. It may be used for both long as well as short distances.
4. It gives high transmission efficiency (up to 98 percent).
5. It gives fewer loads on the shafts.
6. It has the ability to transmit motion to several shafts by one chain only.
7. It transmits more power than belts.
8. If permits high speed ratio of 8 to l0 in one step.
9. It can be operated under adverse temperature and atmospheric conditions.

Disadvantages

1. The production cost of chains is relatively high.
2. The chain drive needs accurate mounting and careful maintenance, particularly lubrication and slack adjustment.
3. The chain drive has velocity fluctuations especially when unduly stretched.

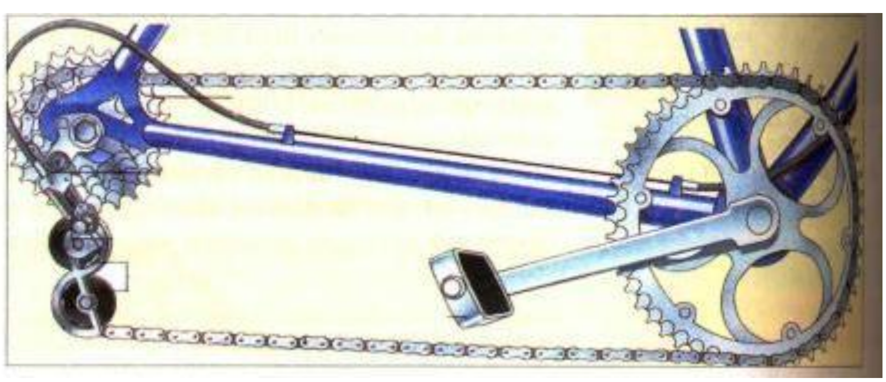

Fig.26.4. Rope Drives

26.4.2 The following terms are frequently used in chain drive:

I. **Pitch of chain:** It is the distance between the hinge centre of a link and the corresponding hinge center of the adjacent link; it is usually denoted by p.

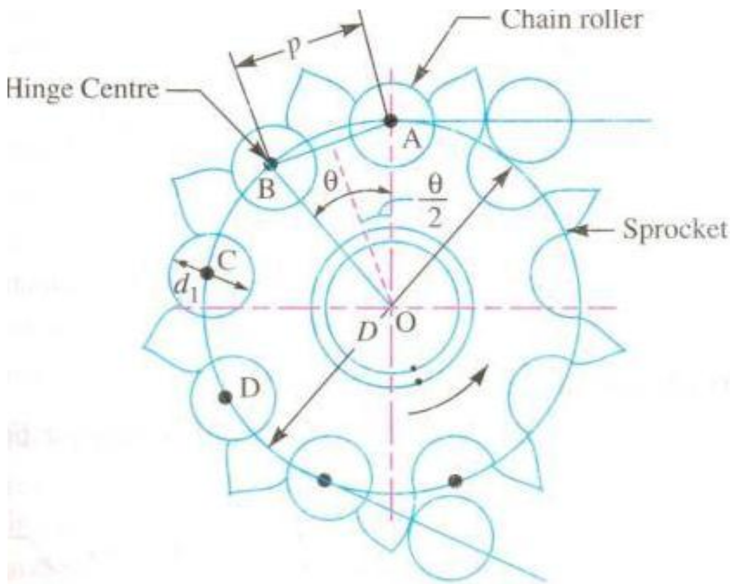

Fig.26.5. Pitch of circle

2. **Pitch circle diameter of chain sprocket:** It is the diameter of the circle on which the hinge centres of the chain lie, when the chain is wrapped round a sprocket. The points A, B, C, and D are the hinge centres of the chain and the circle drawn through these centres is called pitch circle and its diameter (D) is known as pitch circle diameter.

26.4.3. Relation between Pitch and Pitch Circle Diameter:

A chain wrapped round the sprocket since the links of the chain are rigid, pitch of the chain does not lie on the arc of the pitch circle. The pitch length becomes a chord. Consider one pitch length AB of the chain subtending an angle θ at the centre of sprocket

(Or pitch circle)

Let

D = Diameter of the pitch circle, and

T = Number of teeth on the sprocket.

We find that pitch of the chain,

$$P = AB = 2AO \sin\left(\frac{\theta}{2}\right) = 2 \times \left(\frac{D}{2}\right)\sin\left(\frac{\theta}{2}\right) = D \sin\left(\frac{\theta}{2}\right)$$

We know that

$$\theta = \frac{360^0}{T}$$

$$P = D \sin\left[\frac{360^0}{2T}\right] = D \sin\left[\frac{180^0}{T}\right]$$

$$D = P \operatorname{cosec}\left[\frac{180^0}{T}\right]$$

Or

The sprocket outside diameter (D_0), for satisfactory operation is given by

$D_0 = D + 0.8\ d_1$

Where d_1 = Diameter of the chain roller

Relation between Chain Speed and Angular Velocity:

Since the links of the chain are rigid, therefore they will have different positions on the sprocket in different instants. The relation between the chain speed (v) and angular velocity of the sprocket (ω) also varies with the angular position of the sprocket.

Theory of mechanics:

For the angular position of the sprocket

$v = \omega \times OA$

and for the angular position of the sprocket

$v = \omega \times OX = \omega \times OC \cos(\theta/2) = \omega \times OA \cos(\theta/2)$ ($OC = OA$)

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Lesson-28

Solving Numerical

28.1 PROBLEM:

Design a V belt drive for the following:

Drive: AC motor, operating speed is 1440 rpm and operates for over 10 hours. The equipment driven is a compressor, which runs at 900 rpm and the power transmission is 20kw

Solution:

Since it is a V belt drive, let us consider belt speed, v = 25 m/sec.

Design power

P_{des} = service factor(C_{sev}) * required power(P)

= 1.3 * 20kW = 26kW

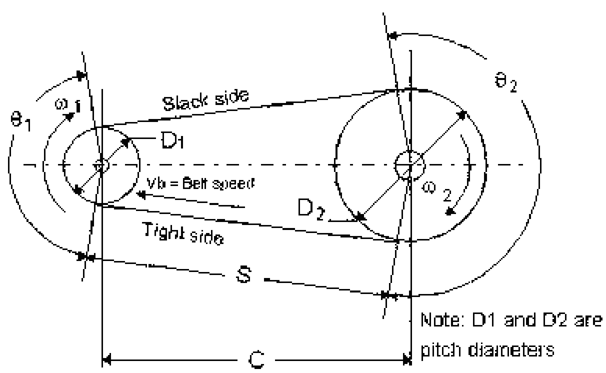

Fig. 28.1

The value 1.3 is selected from design data book for the given service condition. Now,

Hence, obvious choice for belt section is **C**

Now,

V= π*d_s*n / 60 X 100

$$25 = \frac{\pi \times d_s \times 1440}{60 \times 1000}$$
$$\therefore d_S = 331.6 \text{ mm}$$
$$\therefore d_L = 1.6 \times 331.6 = 530.6 \text{mm}$$

Where n=Rotational speed RPM

d_s = Diameter of small pulley

d_L = Diameter of large pulley

V = Velocity of belt

standard sizes are,

d_S=315 mm and d_L=530 mm

d_S=355 mm and d_L = 560 mm.

First combination gives the speed ratio to be 1.68

Second combination gives the speed ratio to be 1.58.

So, it is better to choose the second combination because it is very near to the given speed ratio. Therefore, selected pulley diameters are d_S=355 mm and d_L= 560 mm.

Center distance, C should be such that, $d_L < C < 3(d_L + d_S)$

Let us consider, C = 1500 mm, this value satisfies the above condition.

Considering an open belt drive, the belt length,

$$L_o = \frac{\pi}{2}(d_L + d_S) + 2C + \frac{1}{4C}(d_L - d_S)^2$$
$$= \frac{\pi}{2}(560 + 355) + 3000 + \frac{1}{6000}(560 - 355)^2 \approx 4444 \text{ mm}$$

Inside length of belt = 4444 – 56 = 4388 mm

The nearest value of belt length for C-section is 4394 mm (from design data book)

Therefore, the belt designation is **C: 4394/173**

Power rating (kW) of one C-section belt

Equivalent small pulley diameter is,

$d_{ES} = C_{SR}\, d_s$ = 355 X 1.12 = 398 mm

C_{SR} = 1.12 is obtained from the hand book

For the belt speed of 23 m/sec, the given power rating (KW) =12.1 KW

For the obtained belt length, the length correction factor C_{V1}=1.04

Determination of angle of wrap

$$\beta = \sin^{-1}\left(\frac{d_L - d_s}{2C}\right) = 3.92^0$$
$$\alpha_L = 180 + 2\beta = 187.84^0 = 3.28 \text{ rad}$$
$$\alpha_s = 180 - 2\beta = 172.16^0 = 3.00 \text{ rad}$$

For the angle of wrap of 3.00 radian (smaller pulley), the angle of wrap factor, C_{VW} is found to 0.98 for a C section belt.

Therefore, incorporating the correction factors,

$$\text{Modified power rating of a belt (kW)} = \text{Power rating of a belt (kW)} \times C_{vw} \times C_{vl} = 12.1 \times 0.98 \times 1.04 = 12.33 \text{ kW}$$

$$\text{Number of belts} = \frac{26}{12.33} = 2.1 \approx 2$$

2 numbers of C 4394\173 belts are required for the transmission of 20kW.

Lesson-29

Bearing: Journal and Anti-Friction Bearing. Selection of Ball, Tapered Roller and Thrust Bearing

29.1 INTRODUCTION

A bearing is a machine element which supports another moving machine element (known as journal). It permits a relative motion between the contact surfaces of the members, while carrying the load. A little consideration will show that due to the relative motion between the contact surfaces, a certain amount of power is wasted in overcoming frictional resistance and if the rubbing surfaces are in direct contact, there will be rapid wear. In order to reduce frictional resistance and wear and in some cases to carry away the heat generated, a layer of fluid known as lubricant may be provided. The lubricant used to separate the journal and bearing is usually a mineral oil refined from petroleum, but vegetable oils, silicon oils, greases etc., may also be used.

29.2 JOURNAL BEARING:

A solid bearing, as shown in Fig. is the simplest form of journal bearing. It is simply a block of cast iron with a hole for a shaft providing running fit. The lower portion of the block is extended to form a base plate or sole with two holes to receive bolts for fastening it to the frame. An oil hole is drilled at the top for lubrication. The main disadvantages of this bearing are

1. There is no provision for adjustment in case of wear, and
2. The shaft must be passed into the bearing axially,

i.e. endwise.

Since there is no provision for wear adjustment, therefore this type of bearing is used when the shaft speed is not very high and the shaft carries light loads only.

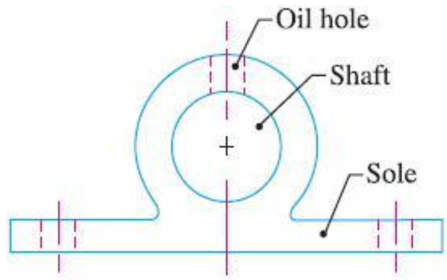

Fig.29.1 Journal bearing

29.3 ANTI FRICTION BEARING:

In rolling contact bearings, the contact between the bearing surfaces is rolling instead of sliding as in sliding contact bearings. The ordinary sliding bearing starts from rest with practically metal-to-metal contact and has a high coefficient of friction. It is an outstanding advantage of a rolling contact bearing over a sliding bearing that it has a low starting friction. Due to this low friction offered by rolling contact bearings, these are called antifriction bearings.

29.3.1 Advantages of anti friction bearing:

- Low cost of maintenance, as no lubrication is required while in service.
- Small overall dimensions.
- Reliability of service.
- Easy to mount and erect.
- Cleanliness.

29.3.2 Disadvantages of anti friction bearing:

- More noisy at very high speeds.
- Low resistance to shock loading.
- More initial cost.
- Design of bearing housing complicated.

29.4 BALL TAPERED ROLLER BEARING:

The ball and roller bearings consist of an inner race which is mounted on the shaft or journal and an outer race which is carried by the housing or casing. In between the inner and outer race, there are balls or rollers as shown in Fig.no:29.2. A number of balls or rollers are used and these are held at proper distances by retainers so that they do not touch each other. The retainers are thin strips and are usually in two parts which are assembled after the balls have been properly spaced. The ball bearings are used for light loads and the roller bearings are used for heavier loads.

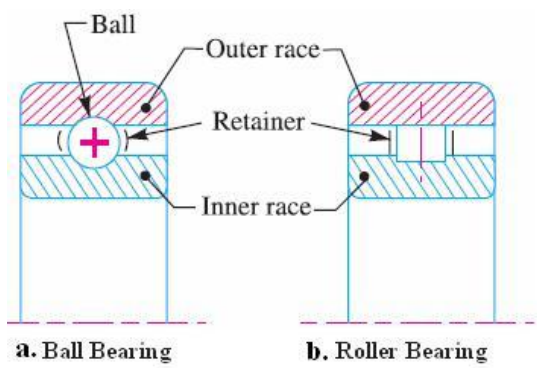

Fig.29.2 Ball tapered roller bearing

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Module 7: Springs

Lesson-30

Springs, Helical And Leaf Springs. Energy Stored In Springs.

30.1 INTRODUCTION:

When flexibility (or) deflection in a mechanical system is specifically desired some form of spring can be used. Otherwise, the elastic deformation of an engineering body is usually a disadvantage. Springs are employed to exert forces (or) torques in a mechanism (or) to absorb the energy of suddenly applied loads. Springs frequently operate with high values for the working stress and with loads that are continuously varying.

30.2 SPRING

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed.

30.2.1 Objectives of springs:

Following are the objectives of a spring when used as a machine member:

1. Cushioning, absorbing, or controlling of energy due to shock and vibration.

- Car springs or railway buffers.
- To control energy, springs-supports and vibration dampers.

2. Control of motion Maintaining contact between two elements (car and its follower)

In a cam and a follower arrangement, widely used in numerous applications, a spring maintains contact between the two elements. It primarily controls the motion.

Creation of the necessary pressure in a friction device (a brake or a clutch)

A person driving a car uses a brake or a clutch for controlling the car motion. A spring system keeps the brake in disengaged position until applied to stop the car. The clutch has also got a spring system (single springs or multiple springs) which engages and disengages the engine with the transmission system.

Restoration of a machine part to its normal position when the applied force is withdrawn (a governor or valve). A typical example is a governor for turbine speed control. A governor system uses a spring controlled valve to regulate flow of fluid through the turbine, thereby controlling the turbine speed.

3. Storing of energy

In clocks or starters:

> The clock has spiral type of spring which is wound to coil and then the stored energy helps gradual recoil of the spring when in operation. Nowadays we do not find much use of the winding clocks.

30.3 COMMONLY USED SPRING MATERIALS:

One of the important considerations in spring design is the choice of the spring material. Some of the common spring materials are given below.

Hard-drawn wire: This is cold drawn, cheapest spring steel. Normally used for low stress and static load. The material is not suitable at subzero temperatures or at temperatures above 1200^0C.

30.3.1 Oil-tempered wire: It is a cold drawn, quenched, tempered, and general purpose spring steel. However, it is not suitable for fatigue or sudden loads, at subzero temperatures and at temperatures above 1800^0C. When we go for highly stressed conditions then alloy steels are useful.

30.3.2 Chrome vanadium: This alloy spring steel is used for high stress conditions and at high temperature up to 2200^0C. It is good for fatigue resistance and long endurance for shock and impact loads.

30.3.3 Chrome silicon: This material can be used for highly stressed springs. It offers excellent service for long life, shock loading and for temperature up to 2500^{0}C.

30.3.4 Music wire: This spring material is most widely used for small springs. It is the toughest and has highest tensile strength and can withstand repeated loading at high stresses. However, it can not be used at subzero temperatures or at temperatures above 1200^{0}C.

Normally when we talk about springs we will find that the music wire is a common choice for springs.

30.3.5 Stainless steel: Widely used alloy spring materials.

30.3.6 Phosphor bronze / spring brass: It has good corrosion resistance and electrical conductivity. That's the reason it is commonly used for contacts in electrical switches. Spring brass can be used at subzero temperatures.

30.3.7 Spring manufacturing processes: If springs are of very small diameter and the wire diameter is also small then the springs are normally manufactured by a cold drawn process through a mangle. However, for very large springs having also large coil diameter and wire Diameter one has to go for manufacture by hot processes. First one has to heat the wire and then use a proper mangle to wind the coils.

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Lesson-31

Design and Selection of Springs

31.1 INTRODUCTION :

Two types of springs which are mainly used are, helical springs and leaf springs. We shall consider in this course the design aspects of two types of springs.

- Helical Spring
- Leaf Spring

31.2 HELICAL SPRING :

The figures below show the schematic representation of a helical spring acted upon by a tensile load F and compressive load F. The circles denote the cross section of the spring wire. The cut section, i.e. from the entire coil somewhere we make a cut, is indicated as a circle with shade.

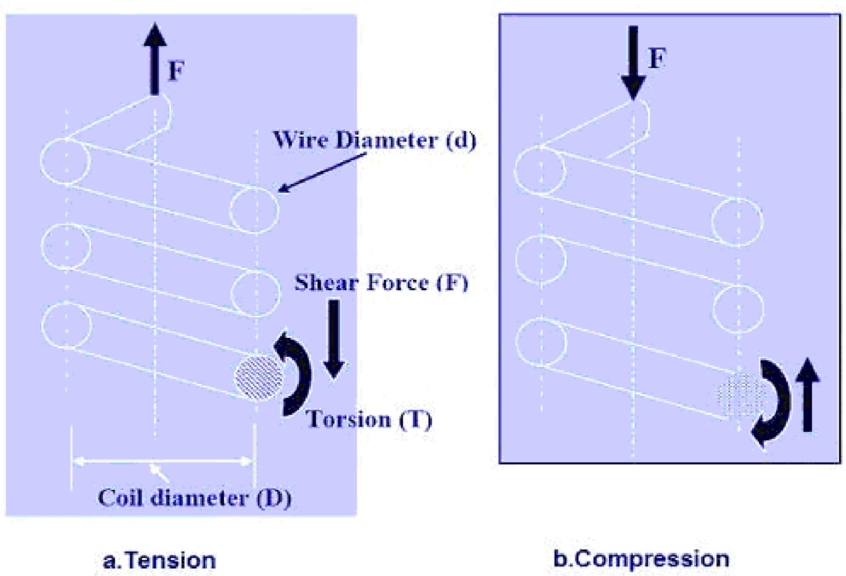

Fig.31.1 Helical spring

31.2.1 stresses in the helical spring wire:

From the free body diagram, we have to find out the direction of the internal torsion T and internal shear force F at the section due to the external load F acting at the centre of the coil.

The cut sections of the spring, subjected to tensile and compressive loads respectively, are shown separately in the Fig.no31.2. The broken arrows show the shear stresses (τ_T) arising due to the torsion T and solid arrows show the shear stresses (τ_F)due to the force F. It is observed that for both tensile load as well as compressive load on the spring, maximum shear stress ($\tau_T + \tau_F$) always occurs at the inner side of the spring. Hence, failure of the spring, in the form of crack, is always initiated from the inner radius of the spring.

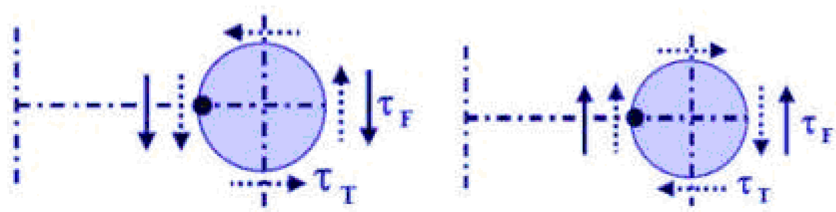

Fig.31.2 Stresses in the helical spring wire

The radius of the spring is given by D/2. Note that D is the mean diameter of the spring.

The torque T acting on the spring is

T=F*(D/2)

If d is the diameter of the coil wire and polar moment of inertia, $I_P = (\pi d^4 / 32)$ the shear stress in the spring wire due to torsion is

$$\tau_T = \frac{Tr}{I_p} = \frac{F \times \frac{D}{2} \times \frac{d}{2}}{\frac{\pi d^4}{32}} = \frac{8FD}{\pi d^3}$$

Average shear stress in the spring wire due to force F is

$$\tau_F = \frac{F}{\frac{\pi d^2}{4}} = \frac{4F}{\pi d^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

Therefore, maximum shear stress the spring wire is

Finally, $$\tau_T + \tau_F = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

or $$\tau_{max} = \frac{8FD}{\pi d^3}\left(1 + \frac{1}{\frac{2D}{d}}\right)$$

or $$\tau_{max} = \frac{8FD}{\pi d^3}\left(1 + \frac{1}{2C}\right)$$

Where, C=D/d, is called the spring index.

$$\tau_{max} = (K_s)\frac{8FD}{\pi d^3} \quad \text{where, } K_s = 1 + \frac{1}{2C} \quad \ldots\ldots 4$$

$$\tau_{max} = (K_w)\frac{8FD}{\pi d^3} \quad \text{Where } K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

The above equation gives maximum shear stress occurring in a spring. K_s & K_W the shear stress correction factor when the spring is in static loading and is in fatigue loading.

31.2.2 Deflection of helical spring

Where, **N** is the number of active turns and **G** is the shear modulus of elasticity. Now what is an active coil? The force F cannot just hang in space, it has some material contact with the spring. Normally the same spring wire will be given a shape of a hook to support the force F. The hook etc., although is a part of the spring, they do not contribute to the deflection of the spring. Apart from these coils, other coils which take part in imparting deflection to the spring are known as active coils.

31.3 LEAF SPRING

A Leaf spring is a simple form of spring commonly used in the suspension of vehicles.

Fig.31.3 Leaf spring

Characteristics:

Leaf spring, is also called as a semi-elliptical spring; as it takes the form of a slender arc shaped length of spring steel of rectangular cross section.

- The center of the arc provides the location for the axle, while the tie holes are provided at either end for attaching to the vehicle body.
- Leaves are stacked one upon the other to ensure rigidity and strength.
- It provides dampness and springing function.
- It can be attached directly to the frame at the both ends or attached directly to one end, usually at the front, with the other end attached through a shackle, a short swinging arm.
- The shackle takes up the tendency of the leaf spring to elongate when it gets compressed and by which the spring becomes softer.
- Thus depending upon the load bearing capacity of the vehicle the leaf spring is designed with graduated and un-graduated leaves.

31.3.2 Leaf springs-fabrication stages:

Because of the difference in the leaf length, different stress will be there at each leaf. To compensate the stress level, prestressing is to be done.Prestressing is achieved by bending the leaves to different radius of curvature before they are assembled with the center clip. The radius of curvature decreases with shorter leaves.

The extra initial gap found between the extra full length leaf and graduated length leaf is called as nip. Such prestressing achieved by a difference in the radius of curvature is known as nipping.

31.3.3 Nipping In leaf springs:

31.3.3.1 Applications:

Mainly in automobiles suspension systems.

31.3.3.2 Advantages:

It can carry lateral loads.

It provides braking torque.

It takes driving torque and withstands the shocks provided by the vehicles.

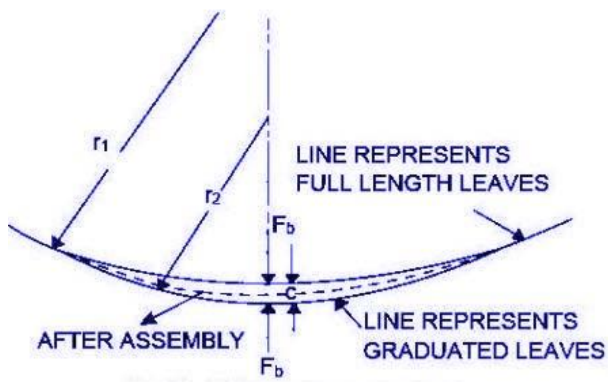

Fig.31.4 Nipping in leaf spring

31.3.4 Design of leaf spring

Consider a single plate fixed at one end and loaded at the other end as shown in Fig.31.5. This plate may be used as a flat spring.

Let t = Thickness of plate,

b = Width of plate, and

L = Length of plate or distance of the load W from the cantilever end. We know that the maximum bending moment at the cantilever end A, Flat Spring (Cantilever Type)

$M = W.L$

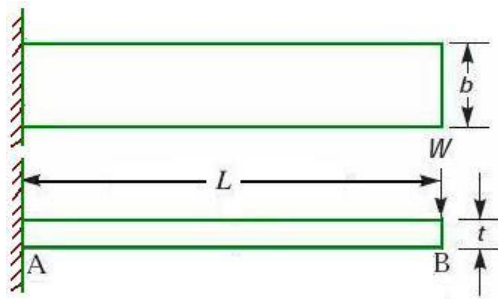

Fig.31.5 Design of leaf spring

and section modulus,

$$Z = \frac{I}{y} = \frac{b\,t^3/12}{t/2} = \frac{1}{6} \times b.t^2$$

Bending stress in such a spring, Fig.31.5 Design of leaf spring

$$\sigma = \frac{M}{Z} = \frac{W.L}{\frac{1}{6} \times b.t^2} = \frac{6\,W.L}{b.t^2}$$

We know that the maximum deflection for a cantilever with concentrated load at the free end is given by

$$\delta = \frac{W.L^3}{3E.I} = \frac{W.L^3}{3E \times b.t^3/12} = \frac{4\,W.L^3}{E.b.t^3}$$

$$= \frac{2\,\sigma.L^2}{\ }$$

$$\left(\because \sigma = \frac{6W.L}{b.t^2} \right)$$

If the spring is not of cantilever type but it is like simply supported beam, and load $2\,W$ act at the centre, as shown then

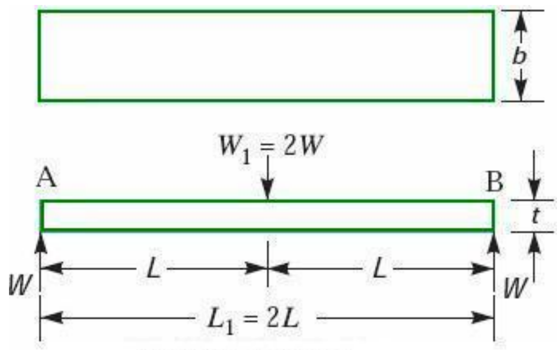

Fig.31.6 Flat spring

Maximum bending moment in the centre, $M = W.L$

Section Modulus = $Z = b.t^2/6$

Bending Stress = $\sigma = \frac{M}{Z} = \frac{W.L}{b.t^2/6}$

$= \frac{6\,W.L}{b.t^2}$

31.3.4.1 Length of Leaf Spring Leaves

The length of the leaf spring leaves may be obtained as discussed below:

Let $2L1$ = Length of span or overall length of the spring,

l = Width of band or distance between centres of *U*-bolts. It is the ineffective length of the spring,

nF = Number of full length leaves,

nG = Number of graduated leaves, and

n = Total number of leaves = nF + nG.

We have already discussed that the effective length of the spring,

$2L = 2L1 - l$ ...(When band is used)

=2L1-(2\3)L ... (When *U*-bolts are used)

It may be noted that when there is only one full length leaf (*i.e.* master leaf only), then the number of leaves to cut will be *n* and when there are two full length leaves (including one masters leaf)then the number of leaves to be cut will be (n-1). If a leaf spring has two full length leaves, then the length of leaves is obtained as follows:

$$\text{Length of smallest leaf} = \frac{\text{Effective length}}{n-1} + \text{Ineffective length}$$

$$\text{Length of next leaf} = \frac{\text{Effective length}}{n-1} \times 2 + \text{Ineffective length}$$

Similarly, length of $(n-1)$th leaf

$$= \frac{\text{Effective length}}{n-1} \times (n-1) + \text{Ineffective length}$$

The *n*th leaf will be the master leaf and it is of full length. Since the master leaf has eyes on both sides, therefore

$$\text{Length of master leaf} = 2L_1 - \pi(d+t) \times 2$$

where d = Inside diameter of eye, and

t = Thickness of master leaf.

The approximate relation between the radius of curvature (R) and the camber (y) of the spring is given by

$$R = \frac{(L_1)^2}{2y}$$

The exact relation is given by

$$y(2R + y) = (L_1)^2$$

where L_1 = Half span of the spring.

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Lesson-31

Solving Numerical

32.1 NUMERICAL:

A helical spring of wire diameter 6mm and spring index 6 is acted by an initial load of 800N. After compressing it further by 10mm the stress in the wire is 500MPa. Find the number of active coils. **G** = 84000MPa.

Solution:

D = Spring index (C) x d = 36mm

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$\tau_{max} = (K_w) \times (8FD / \Pi d^3)$

$K_w = (4C-1 / 4C-4) + (0.615/ C) = 1.2525$

Or

$500 = 1.2525 \times (8F \times 36/ \pi \times F)$

Therefore F = 940.86 N

$K = F / \delta$
$= (940.6 - 800) / 10$
$= 14$ N/mm
$K = (Gd^4) / (8D^4N)$
Or
$N = (Gd^4) / (K8D^3N)$
$= (84000 \times 6^4) / 14 \times 8 \times 3 \times 6^3$
= ~ 21tuns

32.1 PROBLEM:

Design a leaf spring for the following specifications: Total load = 140 kN ; Number of springs supporting the load = 4 ; Maximum number of leaves= 10; Span of the spring = 1000 mm ;

Permissible deflection = 80 mm.Take Young's modulus, E = 200 kN/mm^2 and allowable stress in spring material as 600 MPa.

Solution:

Given: Total load = 140 kN ; No. of springs = 4; n = 10 ; $2L$ = 1000 mm or

L = 500 mm; δ = 80 mm; E = 200 kN/mm^2 = 200 × 103 N/mm^2; σ = 600 MPa = 600 N/mm^2

We know that load on each spring,

$2W$ =Total load/No. of springs

=140/4

=35 kN

There fore,

W = 35 / 2 = 17.5 kN = 17500 N

Let t = Thickness of the leaves, and

b = Width of the leaves.

We know that bending stress (σ),

$$600 = \frac{6\,W.L}{n.b.t^2} - \frac{6 \times 17\,500 \times 500}{n.b.t^2} - \frac{52.5 \times 10^6}{n.b.t^2}$$

There fore $n.b.t^2 = 52.5 \times 106 / 600 = 87.5 \times 103$ ...*(i)*

And deflection of the spring (δ),

$$80 = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{6 \times 17\,500\,(500)^3}{n \times 200 \times 10^3 \times b \times t^3} = \frac{65.6 \times 10^6}{n.b.t^3}$$

Dividing equation ***(ii)*** by equation ***(i)***, we have

$$\frac{n.b.t^3}{n.b.t^2} = \frac{0.82 \times 10^6}{87.5 \times 10^3} \quad \text{or} \quad t = 9.37 \text{ say } 10 \text{ mm}$$

Now from equation ***(i)***, we have

$$b = \frac{87.5 \times 10^3}{n.t^2} = \frac{87.5 \times 10^3}{10\,(10)^2} = 87.5 \text{ mm}$$

and from equation ***(ii)***, we have

$$b = \frac{0.82 \times 10^6}{n.t^3} = \frac{0.82 \times 10^6}{10\,(10)^3} = 82 \text{ mm}$$

Taking larger of the two values, we have width of leaves,

$b = 87.5$ say 90 mm **Ans.**