Theory of Machines

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Theory of Machines

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Module 1. Introduction to Theory of Machine

Lesson 1. Introduction of Theory of Machine

1.1 INTRODUCTION TO THEORY OF MACHINE

Simply speaking, "A machine is a device which received energy in some available form and utilizes it to do some particular type of work" or "A machine may be regarded as an agent for transmitting or modifying energy".

1.1.1 THE MACHINE

A machine is a combination of components which can transmit power in a controlled manner and which is capable of performing useful work. A machine consists of a number of kinematically related links.

A machine is a combination of resistant bodies (links or elements) with successfully constrained relative motions, which is used for transmitting other forms of energy into mechanical energy or transmitting and modifying available energy to do some particular kind of work.

INPUT	MACHINE	OUTPUT
Mechanical Electrical Hydraulic Chemical or Nuclear	**Kinematic Arrangement of Links** - Rigid - Rigid-Hydraulic - Rigid-Pneumatic	Mechanical Electrical Hydraulic or Thermal
Force Velocity Energy/Time Power	A machine has moving parts, which must be constrained and controlled.	Force Velocity Energy/Time Power

Machine Arrangement

Every machine will be found to consist of a system of parts (links or elements) connected together in such a manner that, if one be made to move, they all receive a motion, the relation of which to that of the first depends upon the nature of connections (i.e. joints).

The links may be rigid, rigid-hydraulic, or rigid-pneumatic. The power input may be mechanical, electrical, hydraulic, chemical, or nuclear. The power output may be mechanical, electrical hydraulic or thermal.

Examples of machines:

Heat engine- Receives heat energy and transformers it into mechanical energy.

Electric motor- Changes electric energy into mechanical energy.

A pump- Input electric power and output hydraulic power.

The majority of machines receives mechanical energy, and modify it so that the energy can be used for doing some specific task, for which it is designed, common examples of such machines being hoist, lathe, screw jack, etc.

Note:-It should be noted that machine must be capable of doing useful work. A series of kinematically related links put into motion with no output link, and which simply converts input energy to friction heat, is not a machine, unless the original purpose was only to generate heat.

1.1.2 CLASSIFICATION OF MACHINES

1. Machines for generating mechanical energy

   - Converts other forms of energy into mechanical work

   Examples: Steam engines, Steam turbines, I. C. engines, gas turbines, water turbines etc

2. Machines for transmitting mechanical energy into other form of energy

   - Known as converting machines

   Examples: Electric generators, air or hydraulic pumps, etc.

3. Machines for utilizing mechanical energy in the performance of useful work.

Examples: Lathe, and other machine tools, etc.

The transmission and modification of energy within the machine require the inclusion of a number of parts (links or elements), which are so selected that they will produce the desired motion and carry with safety the forces to which they are subjected so that the machine can perform its task successfully.

The study of relative motion between the various parts of a machine, and the forces which act on them, is covered under they field of "**Theory of machines**", or "**The Theory of Machines** may be defined as that branch of engineering science which deals with the study of relative motion between various elements of a machine and the forces which act on them.

1.2 DIFFERENCE BETWEEN MACHINE AND MECHANISM

In kinematics, a mechanism is a mean of transmitting, controlling, or constraining relative movement. The central theme for mechanisms is rigid bodies connected together by joints. It can also be defined as a combination of resistant bodies that are shaped and connected in such a way that they move with definite relative motion with respect to each other.

A machine is a combination of rigid or resistant bodies, formed and connected in such a way that they move with definite relative motions with each other and transmit force also. A machine has two functions: transmitting definite relative motion and transmitting force. The term mechanism is applied to the combination of geometrical bodies which constitute a machine or part of a machine.

MACHINE	MECHANISM
• It is a combination of links having relative motion and has the capability of modifying available energy in a suitable form. • A machine can consist of one or more than one mechanism. e.g. lathe machine has several mechanisms • Its objective is to transmit mechanical energy. • It is a combination of links which transmit and transform its motion only.	• A mechanism is a single system to transmit motion. • Its objective is to transmit motion only.

Example: A simple example of machine and mechanism is IC engine and slider crank mechanism. A slider crank mechanism converts rotary motion of crank into sliding motion of slider. Where as, in the IC engine the same mechanism is used to convert available mechanical energy at the piston into the required torque at the crank shaft.

1.3 KINEMATIC ANALYSIS OF MECHANSIM

Various mechanisms have its own set of outputs when they are put in motion. The analysis of the mechanism is done by calculating the position, velocity and acceleration at various points on the mechanisms. For the analysis of velocity & acceleration at any point on the mechanism we don't need to calculate forces & stresses acting in the parts of the mechanism. In other means, in analysis of motion of a particular mechanism we don't need to consider the cross section area or strength of the parts in that mechanism. Also, it does not matter whether the parts are made of cast iron or wood or anything else to study it motion analysis.

1.3.1 GRAPHICAL AND ANALYTICAL METHOD: Analysis of the Mechanism can be done by two types of methods, generally known as graphical and analytical methods. Each method has its own advantages and disadvantages. The graphical method is easy to follow and gives the visual image of the working of mechanism which can be applied in some simple problems. But for more complex problems analytical methods are more suitable. It is up to us by which method we want to solve the problem in hand. With the advent of high speed computing, analytical methods has very useful tool for solving complex problems. In this course will concentrate our study to graphical methods due its ease and simplicity.

1.4 SYNTHESIS OF DESIGN

In the design of a mechanism, we will consider stress analysis & others design parameters like bending, fatigue etc. to find the dimensions of the parts. The synthesis of a mechanism can be done by following two approaches. In the first approach the dimensions of the parts in a mechanism is found by considering load, stress & bending etc. in the different parts of the mechanism. In the second approach, the dimensions of the parts are assumed first and then the analysis is done to check its strength. The second method of synthesis is preferred by most of the engineers.

1.5 KINEMATICS OF MECHANISM

It involves the study of the relative motions of various parts of a mechanism without considering the forces producing the motion in the parts. It is the study from the geometric point of view by which we can know the displacement, velocity and acceleration at the various points on the parts of a mechanism.

1.6 DYNAMICS OF MECHANISM

It involves the calculations of the forces impressed upon various parts of a mechanism. The forces impressed on a mechanism can be divided into static & kinetics. In static, the study of forces is done when all the parts of the mechanisms are in equilibrium. Where in kinetics the study of inertia forces are done which may occur due to the combination of mass and motion of the parts.

Module 2. Planar Mechanism

Lesson 2. Planer Mechanism

2.1 LINK

A link is defined as a single part which can be a resistant body or a combination of resistant bodies having inflexible connections and having a relative motion with respect to other parts of the machine. A link is also known as kinematic link or element. Links should not be confused with the parts of the mechanism. Different parts of the mechanism can be considered as single link if there is no relative motion between them.

Example: The frame of any machine is considered as single link as there is no relative motion between the various parts of the frame. As shown in slider crank mechanism shown below, the frame is considered as one link (link 1) as there is no relative motion in frame itself. The crank here is link 2 & connecting rod is again single link (link 3). The slider or piston is link 4 as there is no relative motion it. In this way, many complex mechanisms can be describe by simple configuration diagram by considering the definition of a link.

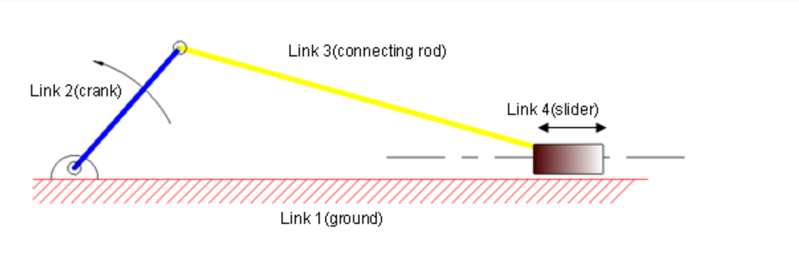


Fig: 2.1 Slider crank mechanism

2.1.1 TYPES OF LINKS: Links can be classified into **Binary, Ternary, Quaternary etc.** depending upon its ends on which revolute or turning pairs can be placed.

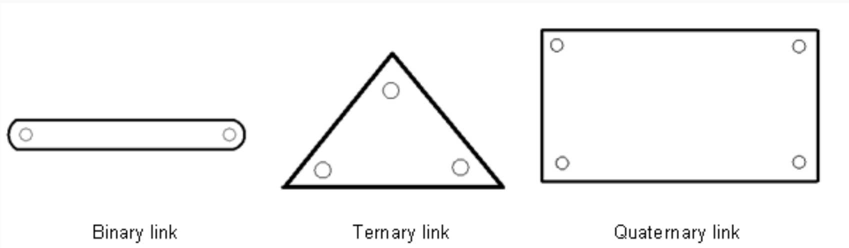


Fig.2.2 Types of links

The links can also be classified into **Rigid, Flexible , Fluid** according to its nature such as

Rigid link is the link which do not deform while transmitting the motion

Flexible link is the link which deform while transmitting the motion but does not affect its function of transmitting motion such as belts, chains etc.

Fluid link is the link which uses the fluid pressure to transmit the motion such as hydraulics jack, brakes and lifts.

2.2 RIGID BODY

A rigid body is a body in which the distance between the two points on the body remains constant or it does not deform under the action of applied force. In actual practice no body is perfectly rigid but we assume it to be rigid to simplify our analysis.

2.3 RESISITANT BODY

A Resistant body is a body which is not a rigid body but acts like a rigid body whiles its functioning in the machine. In actual practice, no body is the rigid body as there is always some kind of deformation while transmitting motion or force. So, the body should be resistant one to transmit motion or force.

Examples: The cycle chain is the resistant body as it acts like rigid body while transmitting motion to the rear wheel of the cycle, Belt in belt and pulley arrangement.

2.4 KINEMATIC PAIR OR PAIR

A kinematic pair is a connection between rigid bodies, which permits relative motion between them. When the links are supposed to be rigid in kinematics, then, there cannot be any change in the relative positions of any two chosen points on the selected link. In other words, the relative position of any two points does not change and it is treated as one link. Due to this rigidness, many complex shaped links can be replaced with simple schematic diagrams for the kinematic and synthesis analysis of mechanism.

2.4.1 CLASSIFICATION OF PAIRS

Kinematic pairs can be classified according to

a) Type of contact between elements

b) Type of relative motion

c) Nature of constraint or Type of closure

2.4.1 a) Type of contact between elements

Lower Pairs : A pair of links having surface or area contact between the members is known as a lower pair. The surfaces in contact of the two links are similar.

Examples: Nut turning on a screw, shaft rotating in a bearing, universal joint, etc.

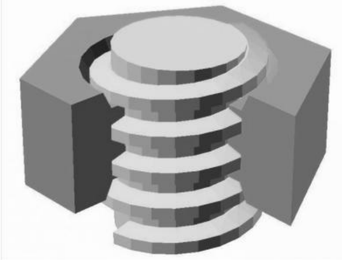


Fig. 2.3 Nut and screw (lower pair)

Higher Pair : When a pair has a point or line joint contact between the links, it is known as a higher pair. The contact surfaces of the two links are dissimilar.

Examples: Wheel rolling on a surface, cam and follower pair, tooth gears, ball and roller bearings, etc.

Fig. 2.4 ball and roller bearing (higher pair)

2.4.1 b) Type of relative motion

Sliding Pair : When two pairs have sliding motion relative to each other.

Examples: piston and cylinder, rectangular rod in rectangular hole.

Turning Pair : When one element revolves around another element it forms a turning pair.

Examples: shaft in bearing, rotating crank at crank pin.

Screw Pair : This is also known as helical pair. In this type of pair two mating elements have threads on it or its relative motion takes place along a helical curve.

Examples: Nut and screw pair as shown in figure 2.4, Screw jack

Rolling Pair : When one element is free to roll over the other one.

Examples: Ball and rolling as shown in figure 2.5, motion of wheel on flat surface

Spherical pair : When one element move relative to the other along a spherical surface.

Examples: Ball and socket joint

Explanation

In slider crank mechanism (Fig.2.6), crank (link 2) rotates relative to ground (link 1) and form a turning pair. Similarly, crank (link 2), connecting rod (link 3) and connecting rod (link 3), slider (link 4) also form turning pairs. Slider (link 4) reciprocates relative to ground (link 1) and form a sliding pair.

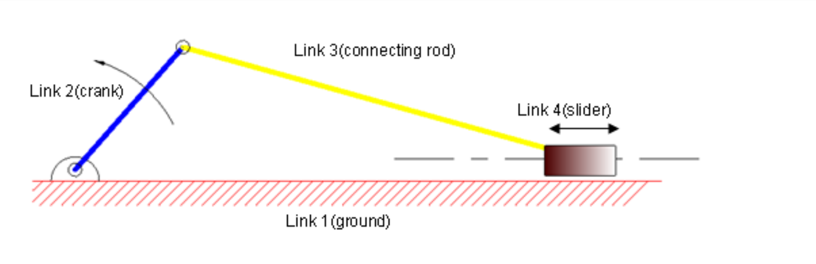


Fig.2.5 Slider crank mechanism

It should be noted here that the slider crank mechanism showed here is useful only in kinematic analysis and synthesis of the mechanism as actual physical appearance will be different and more complex than showed here. For designing the machine component the different approach will be followed.

2.4.1 c) Nature of Constraint or Type of closure

Closed pair : One element is completely surrounded by the other.

Examples: Nut and screw pair

Open Pair : When there is some external mean has been applied to prevent them from separation.

Examples: cam and follower pair

Lesson 3. Degree of Freedom

3.1 DEGREE OF FREEDOM

An object in space has six degrees of freedom.

Translatory motion along X, Y, and Z axis (3 D.O.F.)

Rotary motion about X, Y, and Z axis (3 D.O.F)

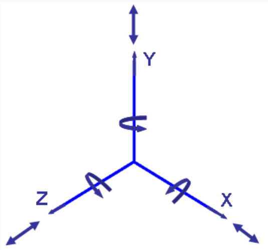


Fig.2.6 Degree of freedom

The rigid body has 6 DOF in space but due to formation of linkage one or more DOF is lost due to the presence of constraint on the body. The total number constraints cannot be zero as the body has to be fixed at some place to make the linkage possible. Thus the degree of freedom is given by

DOF= 6- (Numbers of Restraints)

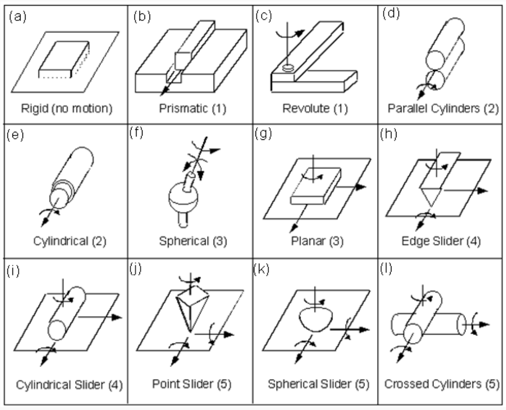


Fig.2.7 Pairs having varying degree of freedom

Table 2.1

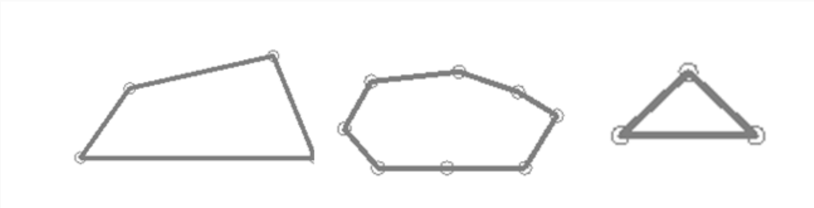
S. No.	Geometrical Shapes involved	Restraints on		Degree of freedom	Total restraints
		Translatory motion	Rotary motion		
(a)	Rigid	0	0	0	6
(b)	Prismatic	2	3	1	5
(c)	Revolute	3	2	1	5
(d)	Parallel cylinders	2	2	2	4
(e)	Cylindrical	2	2	2	4
(f)	Spherical	3	0	3	3
(g)	Planer	1	2	3	3
(h)	Edge slider	1	1	4	2
(i)	Cylindrical slider	1	1	4	2
(j)	Point slider	1	0	5	1
(k)	Spherical slider	1	0	5	1
(l)	Crossed cylinder	1	0	5	1

Table 2.2

Figure	Explanation for DOF
2.9 a	(0) As there is no motion hence DOF is zero
2.9 b	(1) As movement is possible only in Z direction.
2.9 c	(1) As it can revolve around Y axis
2.9 d	(2) As one element can move in Z axis & also revolve around Z axis
2.9 e	(2) As element inside can revolve around Z axis and also move in Z axis
2.9 f	(3) As element can revolve around X,Y&Z axis
2.9 g	(3) As element can revolve around Y axis & can move in Z & X axis
2.9 h	(4) As element can revolve around Z & Y axis & can move in Y axis
2.9 i	(4) As element can revolve around Z & Y axis & can move in Z & X axis
2.9 j	(5) As an element can revolve around X,Y&Z axis & can move in X & Z axis
2.9 k	(5) As element can revolve around X,Y & Z axis & can move in X & Z axis
2.9 l	(5) As element can revolve around X,Y & Z axis & can move in X & Z axis

3.2. KINEMATIC CHAIN

a) Kinematic chain: A kinematic chain is an assembly of links which are interconnected through joints or pairs, in which the relative motions between the links is possible and the motion of each link relative to the other is definite.



a. kinematic chain **b. non kinematic chain** **c. redundant chain**

Fig.2.8 kinematic chains

b) Non-kinematic chain: In case the motion of a link results in indefinite motions of others links, it is a non-kinematic chain. The reason for this indefinite motion lies in the fact that if we give motion to any of the link in the chain then the other links can take indefinite position.

c) Redundant chain: There is no motion possible in the redundant chain. It can be observed from the figure 2.9 c that this chain is locked due to its geometry.

3.3 DEGREE OF FREEDOM IN A MECHANISM

Degrees of freedom of a mechanism in space can be explained as follows:

Let

N = total number of links in a mechanism

F = degrees of freedom

J_1 = number of pairs having one degree of freedom

J_2 = number of pairs having two degree of freedom and so on.

When one of the links is fixed in a mechanism

Then, the number of the movable links are = N - 1

Degrees of freedom of (N- 1) movable links = 6(N-1)

(Because each movable link has six degree of freedom) Each pair having one degree of freedom imposes 5 restraints on the mechanism reducing its degrees of freedom by $5J_1$ this is because of the fact that the restraint on any of the link is common to the mechanism as well. Other pairs having 2, 3, 4 and 5 degrees of freedom reduce the degree of freedom of the mechanism by putting constraints on the mechanism as well.

Then, the DOF can be given by

$$F = 6(N-1) - 5J_1 - 4J_2 - 3J_3 - 2J_4 - 1J_5$$

Most of the mechanism we generally study are two dimensional in nature, such as slider-crank mechanism in which translatory motion is possible along two axes(one restraint) and rotary motion about only one axis(two restraints).Thus there are three general restraints in a two dimensional mechanism. This can be shown with the help of figure 2.10 that a link has three degree of freedom in two dimensions.

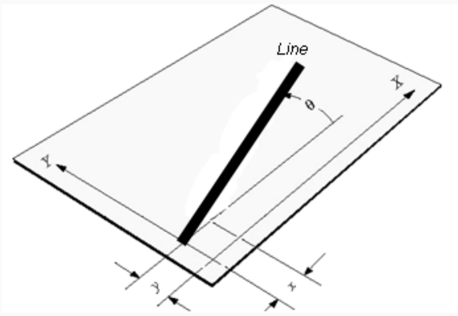


Fig.2.9 a line in a plane has three DOF: x, y, θ

Therefore, for plane mechanism, the following relation can be used for degrees of freedom,

$$F = 3 (N-1) - 2J_1 - 1J_2$$

This equation is known as Gruebler's criterion for degrees of freedom of plane mechanism. It should be noted here that gruebler's criterion does not take care of geometry of the mechanism so it can give wrong prediction. So, inspection should be done in certain cases to find the degrees of freedom.

Example: 2.1 Find the degree of freedom of the mechanism given below.

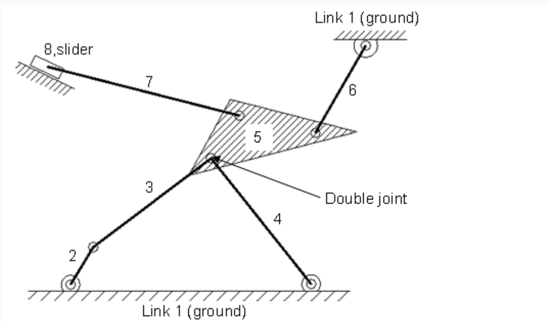


Fig.2.10

Solution:

Number of links=8

Numbers pairs having one degrees of freedom=10 by counting

How to calculate pairs

Pair 1 Link 1 (ground) and link 2 constitute a single turning pair

Pair 2 Link 2 and link 3 constitute a single turning pair

Pair 3 Link 3 and link 5 constitute a single turning pair

Pair 4 Link 4 and link 5 constitute a single turning pair

Pair 5 Link 5 and link 6 constitute a single turning pair

Pair 6 Link 6 and ground (link 1) constitute a turning pair

Pair 7 Link 5 and link 7 constitute a turning pair

Pair 8 Link 7 and link 8 constitute a turning pair

Pair 9 Link 8 and ground (link 1) constitute a sliding pair

Pair 10 Link 4 and ground (link 1) constitute a turning pair

As all the pair calculated have one degree of freedom so there is only term J_1 is used as it denotes the pair having single degree of freedom.

$J_1 = 10$ (as all pairs have one degree of freedom)

$F = 3\ (N-1) - 2J_1 - 1J_2$

DOF=3(8-1)-2×10=1

The degree of freedom is one for this mechanism.

Example 2.2 Find the degree of freedom of the mechanism given below.

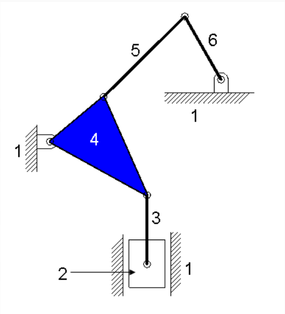


Fig. 2.11

Solution:

Number of links = 6

Number of Pairs = 7

J_1 = 7 (six turning pairs and one sliding pair)

DOF=3(6-1)-2×7=1

The degree of freedom is one.

Example 2.3: Find the mobility or degree of freedom of the following mechanism.

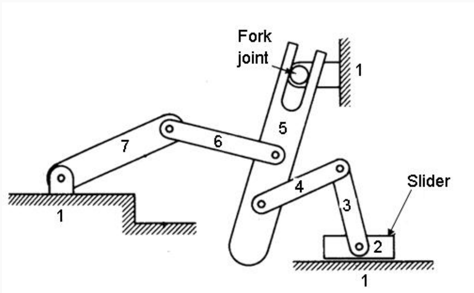


Fig. 2.12

Solution:

Number of links = 7

Number of Pairs = 8

J_1 = 7 (six turning pairs and one sliding pair)

J_2 = 1 (Fork joint is two DOF joint)

DOF=3(7-1)-2×7-1×1=3

The degree of freedom is one.

Lesson 4.

4.1 FOUR BAR MECHANISM

The four bar linkage, as shown in figure 2.13 below, is a basic mechanism which is quite common. Further, the vast majority of planar one degree-of-freedom (DOF) mechanisms have "equivalent" four bar mechanisms. The four bar has two rotating links (2 and 4)) which have fixed pivots. One of the levers would be an input rotation, while the other would be the output rotation. The two levers have their fixed pivots with the ground link (1) and are connected by the coupler link (3).

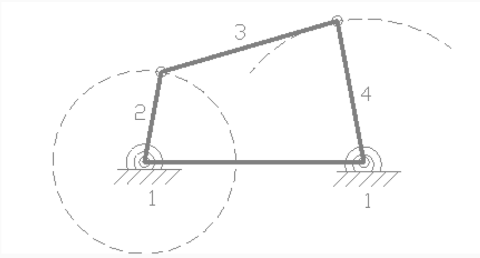


Fig.2.13 four bar mechanism

Crank (2) **-** a ground pivoted link which is continuously rotatable.

Rocker (4) **-** a ground pivoted link that is only capable of oscillating between two limit positions and cannot rotate continuously.

Coupler (3) - a link opposite to the fixed link.

1. If the length of any link is greater than the sum of lengths of other three links then it cannot act as four bar linkage.

2. If the sum of the lengths of the largest and the shortest links is less than the sum of the lengths of the other two links, then the linkage is known as a class-1 four bar linkage

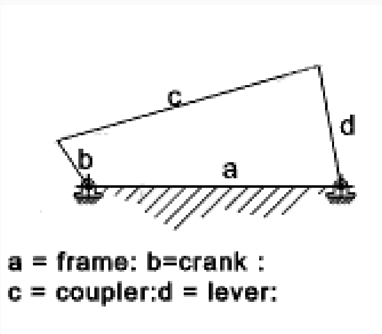


Fig.2.14 crank rocker

In fig 2.14 the links adjacent to the shortest link b is fixed. The mechanism such obtained is known as crank-lever or crank rocker mechanism.

If the shortest link b is fixed, the mechanism obtained is crank-crank or double crank mechanism.

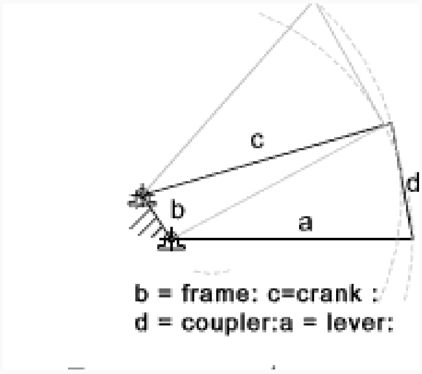


Fig.2.15 double crank

If the link opposite to the shortest link is fixed then the mechanism is know as double-rocker or double lever mechanism.

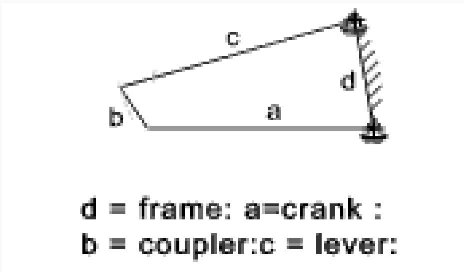


Fig. 2.16 double rocker

4.2 INVERSIONS OF SLIDER CRANK MECHANISM

Different mechanisms are obtained when we fixed different links of a Kinematic chain and the phenomenon is known as inversion of mechanism. A slider crank mechanism has the following inversions.

First Inversion

This inversion is obtained when link 1 is fixed (as shown in fig 2.17) and links 2 and 4 are made the crank and the slider respectively.

Application: This mechanism is commonly used in I.C. engines, steam engines and reciprocating compressor mechanism.

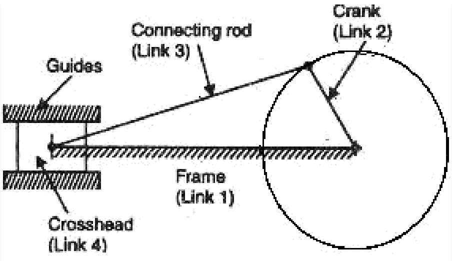


Fig.2.17 reciprocating engine

Second Inversion

By fixing link 2 of a slider mechanism gives second inversion. Rotary engine mechanism or gnome engine is the application of second inversion. It is a rotary cylinder V – type internal combustion engine used as an aero engine. The rotary engine has generally seven cylinders in one plane. The crank (link 2) is fixed and all the connecting rods from the pistons are connected to this link. In this mechanism when the pistons reciprocate in the cylinders, the whole assembly of cylinders, pistons and connecting rods rotate about the axis O, where the entire mechanical power developed, is obtained in the form of rotation of the crank shaft.

Application: Rotary engine mechanism or gnome engine

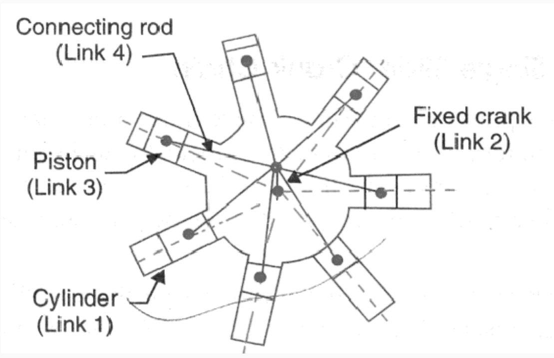


Fig.2.18 rotary engine

Third Inversion

By fixing the link 3(connecting rod) of the slider crank mechanism we can obtain third inversion (as shown in fig.2.19). It is used in hoisting engine mechanism and also in toys. In hoisting purposes, its main advantages lie in its compactness of construction as it allows simple method of supplying steam to the cylinder.

Application: It is used in hoisting engine mechanism and also in toys

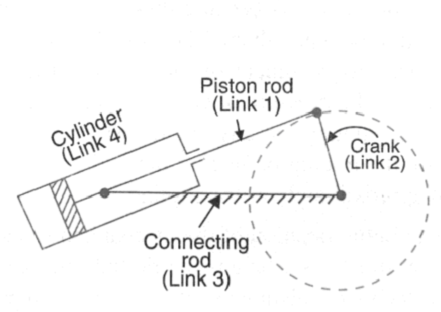


Fig.2.19 oscillating cylinder engine

Fourth Inversion

By fixing the link 4 of the slider crank mechanism we can obtain the fourth inversion of slider crank. Fixing the slider means that the slider should be fixed in position and also should be fixed in respect to rotation. In this case, the cylinder will have to be slotted to give passage to piston pin of connecting rod as the cylinder slides over the piston. Due to this difficulty, the shapes of the cylinder and piston are exchanged as shown in figure below.

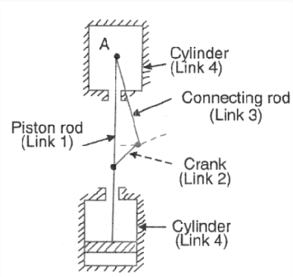


Fig.2.20 pendulum pump

Application: hand pump

Module 3. Velocity and Acceleration Analysis

Lesson 5.

5.1 INTRODUCTION

Different machine elements have different velocities & acceleration at different moments of time. So, we need to know their velocities & acceleration for their proper study and use of these mechanisms in various applications. We use configuration diagram of a machine which is represented by skeleton or a line diagram for their velocity and accelerations analysis.

There are two ways to analyze velocity and acceleration of a machine element, first is analytical and the second one is graphical. Analytical methods are complex as compare to graphical methods but with the use advance software's it's become easy to solve complex equations of analytical methods. In Analytical methods, once the mechanism is modeled and coded in computer the parameters are easily manipulated to create new design. Graphical methods on the other hand are easy to apply and are also accurate to an acceptable degree but we cannot easily change the design parameters in this method.

5.2 MOTION OF A LINK

Consider a rigid link OA of length r having uniform angular velocity ω rad/s. OA turns through a small angle δθ in small interval of time δt. Then A will travel along arc AA'.

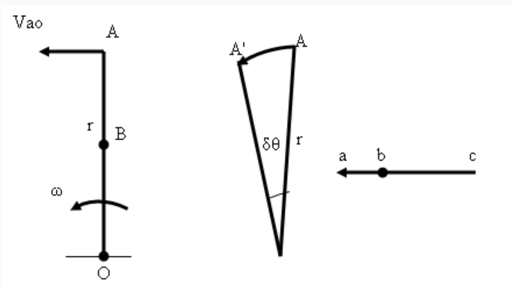


Fig. 3.1 motion of a link

Velocity of A relative to O = Arc AA'/ δt

$$V_{ao} = r\frac{\delta \theta}{\delta t}$$

In the limits, when δt tends to zero

$$V_{ao} = r\frac{d\theta}{dt}$$

$$\text{V}_{\text{ao}} = \text{ r } \times \omega \text{ }\left(\text{OA } \times \omega \right)$$

The direction of V_{ao} is along the displacement of A. Also, as δt approaches zero, AA' will be perpendicular to OA. Thus, velocity of A is ωr and perpendicular to OA. It is so because A can neither approach nor recede from O and thus, the only possible motion of A relative to O is in a direction perpendicular to OA.

Now, consider appoint B o the link OA

The velocity of point B = ω OB which is perpendicular to OB

If ob represents the velocity of B, it can be found that

$$\frac{\text{ob}}{\text{oa}} = \frac{\omega\ \text{OB}}{\omega\ \text{OA}} = \frac{\text{OB}}{\text{OA}}$$

The important conclusion here is that b divides the velocity vector in the same ratio as B divides the link. The velocity vector V_{ao} gives the velocity of A at a particular instant of time at other instant of time when OA takes other position the velocity vector will change its direction accordingly.

5.3 FOUR BAR MECHANISM ANALYSIS BY RELATIVE VELOCITY METHOD

Four bar mechanism is given below in the diagram having angular velocity ω rad/s in the clockwise direction

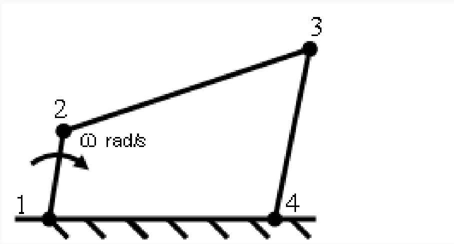


Fig. 3.2 four bar mechanism

Velocity of the point 3 relative to 1= velocity of point 3 relative to velocity of point 2+ velocity of point 2 relative to velocity of point 1.

$$V_{31} = V_{32} + V_{21}$$

The velocity of point 3 relative to 1 will be equal to velocity of point 3 relative to 4 as point 1 & point 4 both are fixed point, so we can write

$$V_{34} = V_{32} + V_{21}$$

Here

$V_{21} = \omega \times 12$, which is perpendicular to 12

Similarly,

V_{32} is unknown in magnitude but it will be perpendicular to link 23

V_{34} is unknown in magnitude but it will be perpendicular to link 43

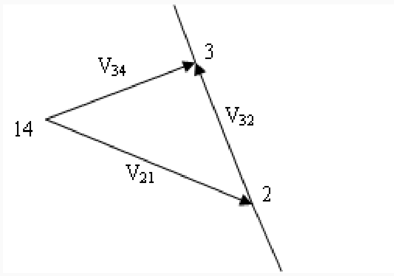


Fig. 3.3

Angular velocity of links

a. Velocity of 3 relative to 2, V_{32} link 3 moves in anti-clockwise about 2.

$$\text{V}_{\text{32}} \text{ = } \omega_{\text{32}} \times \text{23}$$

$$\omega_{\text{32}} = \frac{\text{V}_{\text{32}}}{\text{23}}$$

b. Velocity of link 34

Angular velocity of link 3 relative to 4 is in clockwise direction & is given by

$$\omega_{\text{34}} \text{ = } \frac{\text{V}_{\text{34}}}{\text{43}}$$

5.4 VELOCITY OF RUBBING AT PINS

Two links forms a turning pairs at points 1, 2, 3&4 as shown in figure 3.4. It can be seen here that a pin forms an integral part of a link and this pin fits into the hole in the other link which permits the relative motion between them. When two links are connected then the surface at the hole of one link rubs against the surface of pin of the other link. The rubbing velocity between the two links depends on the angular velocity of one link relative to the angular velocity of the other.

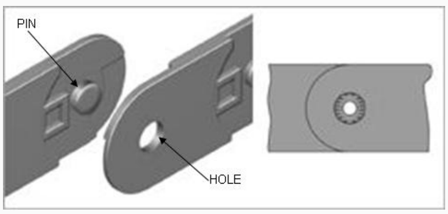


Fig.3.4 two links joined by pins

This can be explained at the points 1, 2, 3 & 4 in figure 3.2 of the four bar mechanism.

At pin 1

The pin at 1 connects links 12 and 41, the velocity of rubbing will depend only on links 12 as 41 is the fixed link.

Let r_1 = radius of the pin at point 1

Then the velocity of rubbing = $r_1 = \omega$

At pin 2

$\omega_{21} = \omega_{12} = \omega$ clockwise

$\omega_{23} = \omega_{32} = \frac{V_{23}}{23}$ counter clockwise

The direction of the two angular velocities of the links 12 and 23 are in opposite directions, the angular velocity of one link relative to the other is the sum of the two velocities.

Let r_2 = radius of the pin at point 2

Then the velocity of rubbing = $r_2 = \left(\omega_{12} + \omega_{23} \right)$

At pin 3

$\omega_{23} = \omega_{32}$ counter clockwise

$\omega_{41} = \omega_{34}$ clockwise

Let r_3 = radius of the pin at point 3

Then the velocity of rubbing = $r_3 = \left(\omega_{23} + \omega_{43} \right)$

At pin 4

Let r_4 = radius of the pin at point 4

Then the velocity of rubbing = $r_4 = \omega_{34}$

Lesson 6.

6.1 SLIDER CRANK MECHANISM ANALYSIS BY RELATIVE VELOCITY METHOD

Angular velocity of link OA is given ω and velocity of point A can be given by, $V_a = \omega \times OA = V_{AO}$ in the clockwise direction about O. V_a is perpendicular to OA. So we know the direction and magnitude of V_a. The velocity of slider B is along OB. The configuration and velocity diagram for this problem is given below.

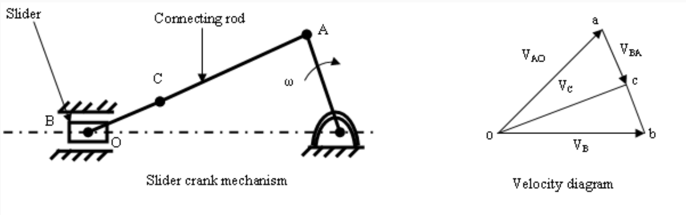


Fig. 3.5

Step 1. Take any point **o** and draw vector oa such that **oa**= ω × OA=V_{AO}, perpendicular to OA in some suitable scale as shown in the velocity diagram.

Step 2. The velocity of point B with respect to A (V_{BA}) is perpendicular to AB. From **a** draw a vector **ab** perpendicular to the line AB.

Step 3. The velocity of slider B is along the line of stroke OB, from **o** draw a line parallel to OB which will intersect vector **ab** at **b**. The vector **ob** represent the velocity of slider B (V_B).

The velocity of any point C on the connecting rod can be determined by the help of relation:

$$\frac{\mathbf{ac}}{\mathbf{ab}} = \frac{\text{AC}}{\text{AB}}\text{ or }\mathbf{ac} = \mathbf{ab} \times \frac{\text{AC}}{\text{AB}}$$

The point **c** can be located on the velocity diagram.
Join **o** with **c.** vector **oc** represents the absolute velocity of point C with respect to O.

Example 3.1 In the mechanism as shown below, the crank O_2A rotates at 600 r.p.m. in the anticlockwise direction. The length of link O_2A= 12 cm and of link O_1B= 60 cm. Find

a. Angular velocity of link O_1A.

b. Velocity of slider at B.

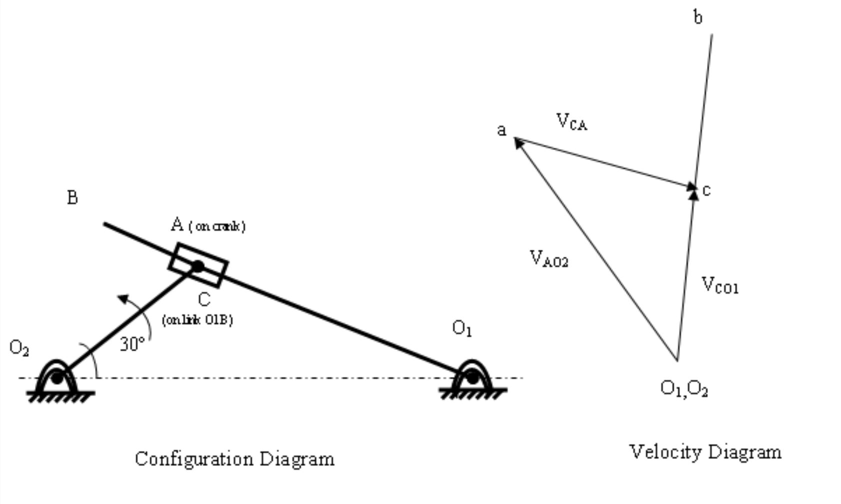


Fig. 3.6

Solution:

Given: N=600 r.p.m

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60} = 20\pi \text{ rad/sec}$$

The velocity of A with respect to O_2

$$V_{AO2} = \omega \times O_2A = 20\pi \times \frac{12}{100} = 7.53 \text{ m/s}$$

In the configuration diagram let us take a point C on the link O_1B. The velocity diagram can be drawn as explain below.

Step 1. O_1 and O_2 are fixed points on the configuration diagram so they are taken as single point **(o_1,o_2)** on the velocity diagram. The velocity of point A with respect to O_2, V_{AO2} = 7.53 m/s is drawn as vector **o_2a** in some suitable scale perpendicular to O_2A.

Step 2. The velocity of point C with respect to A, V_{CA} along the path of slider, O_1B. From **a** draw a vector **ac** representing V_{CA} along O_1A. This will contain point **c**.

Step 3. The velocity of point C with respect to O_1 is V_{CO1} and it will be perpendicular to O_1A.Then from **o_1** draw a vector o_1c representing V_{CO1} this will intersect **ac** at point **c**.

Step 4. Locate the point **b** corresponding to point B such that

$$\frac{\mathbf{o_1 b}}{\mathbf{o_1 c}} = \frac{O_1B}{O_1C}$$

o_1b= 5.55 m/s by measurement

o_1b=V_{BO1}

Angular velocity of link O_1A

$$\omega_{O1A}=\omega_{O1B}=\frac{V_{BO1}}{O_1B}=\frac{5.55}{0.60}$$

= 9.25 rad/sec

(anticlockwise direction)

6.2 INSTANTANEOUS CENTRE OF ROTATION

A rigid body undergoing in plane motion, there always exist a point in the plane of motion at which the velocity is zero at that particular instant. This point is called instantaneous centre of rotation or I-centre. It may or may not lie on the body. If the location of this point is determined then we can simplify the velocity analysis. To locate the I-centre, we use the fact that the velocity of a point on a body is always perpendicular to the position vector from I-centre to that point. If the velocity at two points A and B are known, I-centre will lie at the intersection of the perpendiculars to the velocity vectors through A and B.

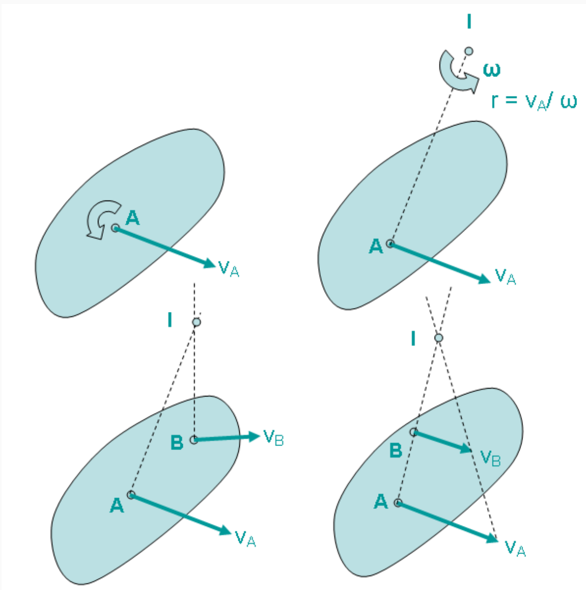


Fig. 3.7 I-centre of a body

If the velocity vectors at A and B are perpendicular to the line AB, I-centre will lie at the intersection of the line AB with the line joining the extremities of the velocity vectors at A and B.If the velocity vectors are equal & parallel, I-centre will lie at infinity and the angular velocity is zero (pure translation).

6.3 NUMBER OF I-CENTRE

The number of I-centre 'N' in a mechanism is given by

$$\text{N} = \frac{n\,(n - 1)}{2}$$

Where n = No. of links

The number of I-centre in a four bar mechanism will be $\frac{4 \times (4 - 1)}{2} = 6$.

6.4 HOW TO LOCATE I-CENTRE

There are two methods to locate the I-centre

1. by using rules to locate I-centre by inspection

2. by applying Aronhold-Kennedy Theorem

Rules to locate I-centre

- The centre of the pivot is the I-centre for two links in the pivoted joint.

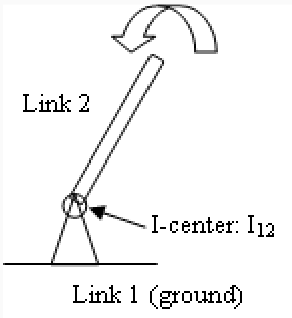


Fig. 3.8

- The I-centre lies at infinity perpendicular to the path of motion of the slider.

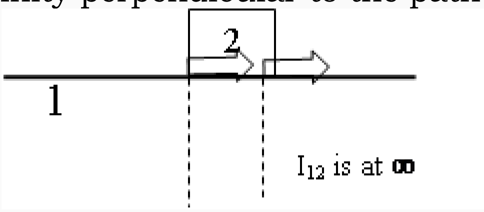


Fig. 3.9

- In pure rolling or in case of no slip between the two links. The I-centre will lie at the point contact between the two links.

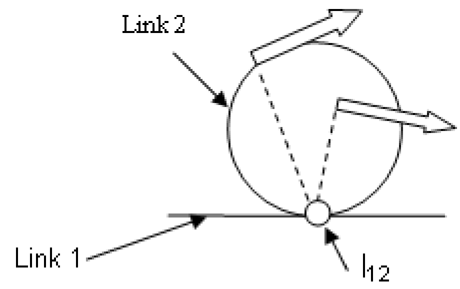


Fig. 3.10

Lesson 7.

7.1 ARONHOLD-KENNEDY THEOREM

It states that "if three bodies, having relative motion with respect to each other will have three I-centres all of which lies on the same line". It is very useful in the mechanism where there are three links and will have three I-centres, if two them are known then third will lie on the line joining the two I-centres.

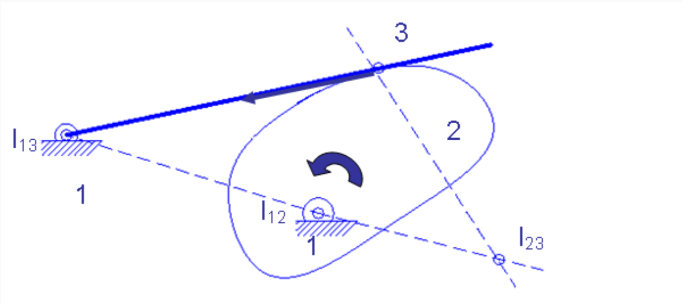


Fig. 3.11 Aronhold –Kennedy's theorem

as shown in the above figure 3.11, the I-centre I_{13} and I_{12} are located by observation , then the third I-centre I_{23} will lie on the line extended from I_{13} & I_{12} and cutting the line perpendicular to the velocity .

7.2 METHOD FOR LOCATING I-CENTRE

Step 1. The number of I-centre in a four bar mechanism is

$$n = \frac{n(n - 1)}{2}$$

Where n = No. of links

The number of I-centre in a four bar mechanism will be

$$\frac{4 \times \left(4 - 1\right)}{2} = 6$$

So there are 6 I-centre in a four bar mechanism

Step 2. These I-centre's is shown below

Links	1	2	3	4
I-centre's	I_{12}	I_{23}	I_{34}	-
	I_{13}	I_{24}	-	-
	I_{14}	-	-	-

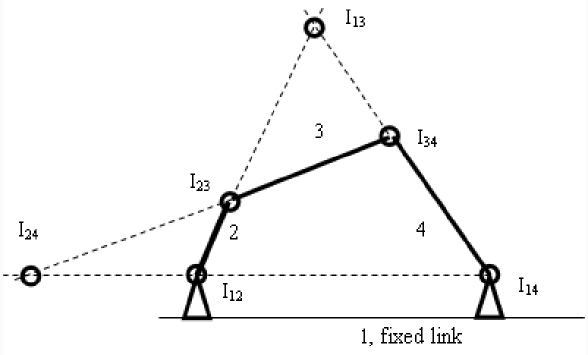


Fig. 3.12

Step 3. I_{12} and I_{14} are found to be fixed I-centre because they will change their position during the rotation of crank or at any position of crank. The I_{23} and I_{34} are permanent types of I-centre as they will change during crank movement but will remain at the joint of link 2 & 3.

Step 4. I_{13} and I_{24} are secondary I-centre and are located by circle diagram given below.

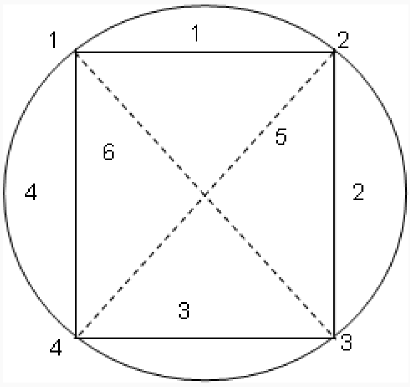


Fig. 3.13 circle diagram

Take four points 1, 2, 3 and 4 on the circle equal to the number of links in the mechanism. Join these point by solid lines 1-2,2-3,3-4 and 4-1.These lines will be specify the I-centre's I_{12},I_{23},I_{34} and I_{14}.Two remaining I-centre's I_{24} and I_{13} are located by joining points 1 to 3 and 2 to 4 showed by dotted lines.

7.3 TYPES OF I-CENTRE

There are basically three types of I-centre **Fixed, Permanent, Neither fixed nor permanent**

The fixed I-centre means its location will remain same during the relative motion between the links because one of the links involve is fixed to the ground as I-centres I_{12} & I_{14} in the figure 3.12. The permanent I-centres are those whose location will change depending on the position of links but they will remain at permanent joining points of the two links as I-centres I_{23} & I_{34} in figure 3.12. The remaining two I-centres I_{24} & I_{13} are neither fixed nor permanent types of I-centres as it will change continuously depending on the location of other I-centres.

Example: 3.2 Slider-crank mechanism has a crank length of 125 mm which is rotating at 600 r.p.m. (clockwise direction). Find the velocity of slider and angular velocity of connecting rod if the length of connecting rod is 500 mm and crank makes an angle of 45^0 with the inner dead centre.

Solution:

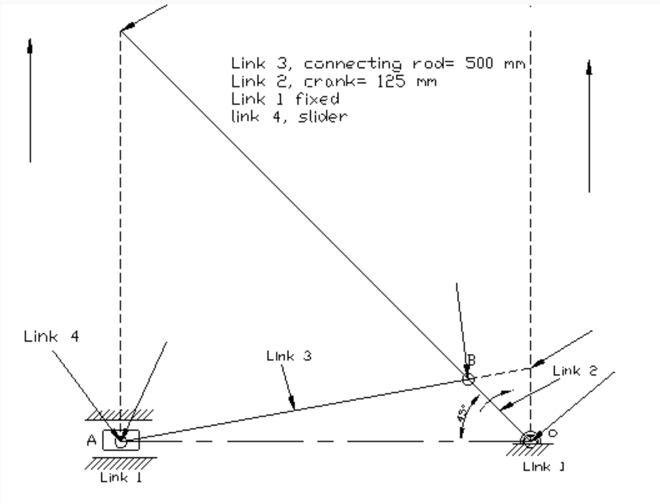


Fig.3.14 slider crank mechanism

Find the number and location of all the I-centers by following the method as describe in above article. Then, Locate the fixed and permanent I-centre I_{12},I_{34} and I_{23} by inspection. As slider (link 4) moves in a straight line so I-centre I_{14} will lie at infinity. Locate remaining two I-centre's I_{24} and I_{13} by using Kennedy theorem.

By measuring from the above diagram we can have

$I_{13}A$ = 581 mm

$I_{13}B$ = 690 mm

Velocity of slider A

$$\omega \times \text{OB} = \frac{2\pi \times 600}{60} = 62.90 \text{ rad/s}$$

$$V_B = \omega_{OB} \times OB = 62.90 \times 0.125 = 7.86 \text{ m/s}$$

$$\frac{V_A}{I_{13}A} = \frac{V_B}{I_{13}B}, V_A = 0.581 \times 7.86/0.69 = 6.61 \text{ m/sec}$$

Angular velocity of connecting rod

$$\omega_{AB} = \frac{V_A}{I_{13}A} = \frac{6.61}{0.581} = 11.37 \text{ rad/s}$$

Example 3.3: Find the linear velocities of the points B, C & D by using I-centre method and also find the angular velocities of the links AB, BC and CD for the diagram shown below.

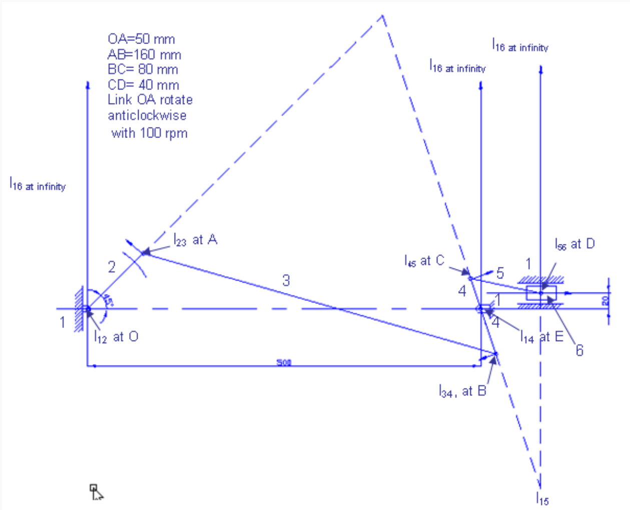


Fig: 3.15

Solution:

Step 1. Write down all the I-center's in the table

I_{12}	I_{23}	I_{34}	I_{45}	I_{56}
I_{13}	I_{24}	I_{35}	I_{46}	
I_{14}	I_{25}	I_{36}		
I_{15}	I_{26}			
I_{16}				

Step 2. Locate all the fixed and permanent I-centre by inspection

$I_{12}, I_{23}, I_{34}, I_{45}, I_{56}, I_{16}, I_{14}$

Step 3. Locate I-centre I_{15} by using Kennedy theorem as it will be used in finding the slider velocity

Step 4. Locate I_{13} by Kennedy theorem as it is common I-center of points A and B.

Step 5. find $V_A = \omega \times OA$

Step 6. considering I-center I_{13} because point A and point B rotate about this I-center and have zero velocity at this point. So use

eq. $\frac{V_A}{I_{13}A} = \frac{V_B}{I_{13}B}$ and find V_B

Step 7. similarly, considering I-center I_{14}, use eq. $\frac{V_B}{I_{14}B} = \frac{V_C}{I_{14}C}$ and find V_C

Step 8. similarly, considering I-center I_{15}, use eq. $\frac{V_C}{I_{15}C} = \frac{V_D}{I_{15}D}$ and find V_D

Angular velocity of links AB can be found by using eq. ${\omega}_{AB} = \frac{V_A}{AI_{13}}$

Similarly find velocities for links BC and CD

Lesson 8.

8.1 ACCELERATION ANALYSIS

The rate of change of velocity with respect to time is known as acceleration. It has magnitude as well as direction. So, it is a vector quantity. Consider two points A and B on a rigid link. Point B moves relative with respect to A with an angular velocity of ω rad/s and angular acceleration of α rad/s^2.V_{BA} is the velocity of point B relative to A is perpendicular to the line joining A and B (as shown in figure 3.16a).V_{BA} is equal to ω × AB. Acceleration has two components radial or centripetal and tangential.

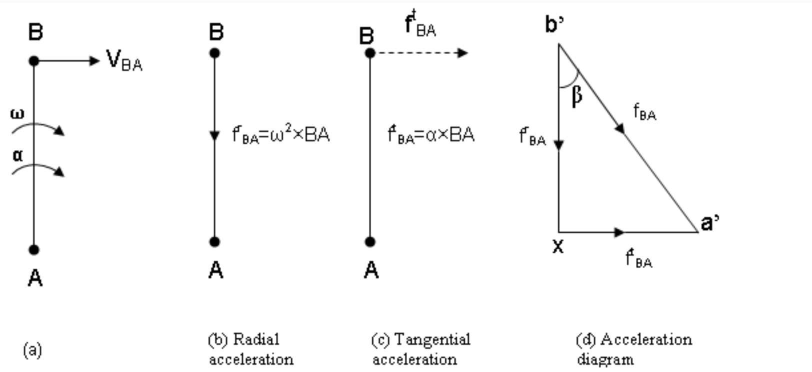


Fig. 3.16 acceleration components

8.1.1 RADIAL OR CENTRIPETAL COMPONENT OF ACCELERATION

The magnitude of radial component is given by f^r_{BA}= ω^2×BA where f^r_{BA}= Radial component of acceleration of point B respect to point A (as given in fig: 3.16b). Its direction is given from B to A. When the angular velocity of a rotating link or a particle moving in a circular path is not constant, but is subjected to an angular acceleration (with consequent peripheral or linear acceleration), the resultant acceleration may be found by adding the vectors representing the centripetal and peripheral (or tangential) accelerations. Since the centripetal acceleration is always radial and perpendicular to the instantaneous direction of the motion and the peripheral acceleration is perpendicular to the radius, these two accelerations are always mutually perpendicular and the resultant acceleration is easily found by applying the wall known relationship which exist between the sides of a right angled triangle, that is by extracting the square root of the sum of the squares of the respective accelerations.

8.1.2 TANGENTIAL COMPONENT OF ACCELERATION

It can be defined as the rate of change velocity V_{BA} in the tangential direction. Its magnitude will be equal to α×BA. It is represented by $f^t_{BA;}$ the tangential component of acceleration of B with respect to A. f^t_{BA} is perpendicular to BA and parallel to V_{BA}.

The total acceleration of point B with respect to A is the vector sum of their components of radial and tangential acceleration. This can be written as

$${{\text{f}}_{{\text{BA}}}}={\text{}}{{\text{f}}^{\text{r}}}_{{\text{BA}}}+{\text{}}{{\text{f}}^{\text{t}}}_{{\text{BA}}}={\text{}}{\omega ^{\text{2}}} \times {\text{BA}} + {\text{ }}\alpha\times {\text{BA}} = {\text{ }}\left({{\omega ^{\text{2}}} + {\text{ }}\alpha } \right){\text{}} \times {\text{ BA}}$$

Here f_{BA} is the total acceleration of B with respect to A

8.1.3 How to draw acceleration diagram

Step 1. Draw **b'x** = f^r_{BA} in some suitable scale parallel to AB and its direction will be from B to A.

Step 2. From point **x** draw **xa'** = f^t_{BA} = α×BA which is perpendicular to AB. α is known in this case.

Step 3. Join **a'** with **b'** which shows f_{BA}, which represents the total acceleration of B with respect to A. The acceleration of B relative to A is inclined at an angle of β with AB.

$${\text{Tan }}\beta= \frac{\alpha }{{{\omega ^{\text{2}}}}}$$

When α = 0, AB rotates at the uniform angular velocity, f^t_{BA} = 0 and thus f^r_{BA} represents the total acceleration.

When ω = 0, A has a linear motion, f^r_{BA} = 0 and thus the tangential acceleration is the total acceleration.

Example: 3.3 A motor-car passes round a curve of 30.5 m radius and at a given instant has a speed of 82 km/h (or 8.89 m/s). The car is accelerating at the rate of 16 km/h in 3 sec. find the resultant acceleration.

Sol:

Centripetal acceleration, $f^r = r\omega^2 = v^2/r$

$$f^r = (32000/3600)^2 / 30.5 = 2.59 \text{ m/s}^2$$

Tangential acceleration, $f^t = (16000/3600)/3 = 1.48 \text{ m/s}^2$

Resultant acceleration, $f = \sqrt{(2.59^2 + 1.48^2)} = \sqrt{8.8985} = 2.983 \text{ m/s}^2$

Example 3.4 The crank of a slider crank mechanism rotates a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine: the angular velocity and angular acceleration of the connecting rod when the crank makes the angle of 45^0 with form the inner dead centre.

Solution.

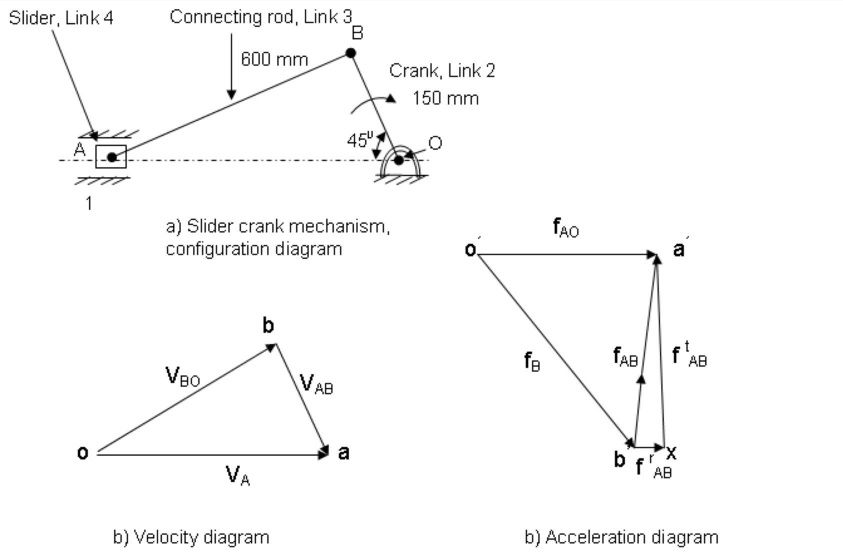


Fig. 3.17

The crank OB = 150 mm = 0.15 m

N_{BO}=300 r.p.m.

$$\omega_{OB} = \frac{2\pi\times 300}{60} = 31.4 \text{ rad/s}$$

V_{BO}= ω_{OB} × OB= 31.4 × 0.15= 4.71 m/s

Velocity diagram is drawn by following these steps

Step 1. V_{BO}, the velocity of point B with respect to O is known and it is perpendicular to OB. Take any point **o** and draw vector **ob** which will represent V_{BO}.

Step 2. V_{AB} is perpendicular to AB, from **b** draw a vector **ba** perpendicular to AB representing V_{AB}.

Step 3. V_A ,the velocity of slider is along path OA. From **o** draw vector **oa** which will intersect vector **ba** at point **a.**

By measurement **oa**=V_A=4 m/s

ba=V_{AB}=3.34 m/s

The radial acceleration of B with respect to O,f^r_{BO} is given as

$$f^r_{BO} = f^r_B = \frac{V^2_{BO}}{OB} = \frac{(4.71)^2}{0.15} = 147.8 \text{ m/s}^2$$

$f^t_{BO}=0$ as crank rotates with the constant speed

Radial acceleration of A with respect to B is given as

$$f^r_{AB}=\frac{V^2_{AB}}{AB}=\frac{(3.34)^2}{0.60} = 18.5\ \text{m}/\text{s}^2$$

The acceleration diagram is drawn by following these steps

Step 1. Draw vector **o'b'** = f^r_{BO} parallel to BO and 147.8 m/s^2 in magnitude.

Step 2. The radial acceleration of A with respect to B, f^r_{AB} is known in magnitude i.e. 18.5 m/s^2 and parallel to AB. from **b'** draw vector **b'x**=f^r_{AB}. from **x** draw **xa'** perpendicular to **b'x** representing tangential component of acceleration of A with respect to B.

Step 3. The acceleration of slider A with respect to O, f_{AO} is along the line of stroke of OA. From **o'** draw vector **o'a'** representing f_{AO}. **o'a'** intersects **xa'** at point **a'.** Now join **o'** to **a'**. Thus **o'a'** is $f_{AO}=f_A$

By measurement f_A=**o'a'**= 109 m/s^2

Angular velocity of the connecting rod is given by

$$\omega_{AB}=\frac{V_{AB}}{AB}=\frac{3.34}{0.60} = 5.56\ \text{rad}/\text{s}$$

Angular acceleration of the connecting rod AB, α_{AB} is given by

$$\alpha_{AB}=\frac{f^t_{AB}}{AB}=\frac{107.5}{0.60} = 179.1\ \text{rad}/\text{s}^2$$

(f^t_{AB}=**xa'**=107.5 by measurement)

Lesson 09

15.1 INTRODUCTION

Power can be transmitted in number of ways i.e by belts, rops, chains and couplings but gears form very important part of power transmission. With the help of gears definite velocity ratio is achieved and they found applications in many machines such as transmission of automobiles, machine tools, rolling mills, and clocks. A gear is a toothed circular part which mesh with another toothed part to provide specific output. The output may be in the form of speed or torque. The purpose of projections or teeth is to reduce slipping. Two or more gears working together is called a gear train. Intermeshing gears always turn in counter directions.

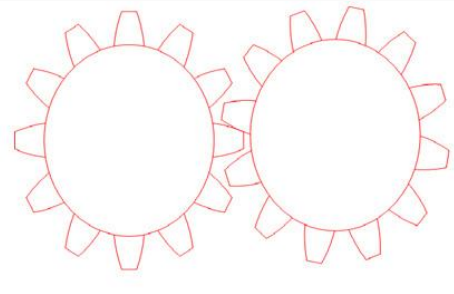


Fig.4.1 Meshing of two gears

Gears of different sizes can be combined together to design different mechanisms depending upon particular requirement. The gears are generally designed to prevent failure against static and dynamic loads. Gears can be made from cast iron, steel, bronze, phenolic resin, nylon and Teflon etc. The major advantages of gear drives include transmission of exact velocity ratio, high efficiency and compact layout.

15.2 HISTORY OF GEARS

Gears are as old as any other machinery, the mankind is using. The early Greeks and Romans made considerable use of gears. The Antikythera mechanism is an example of an ancient geared device, that was designed to calculate astronomical positions (built between 150 and 100 BC). The first known geared mill was built about 27 B.C. In the fourth century, BC Aristotle wrote about wheels using friction between smooth surfaces to transmit motion.

Early man used wooden gears to grind wheat and hammer metals. During the beginning of the Christian era gears were used in many machines such as clocks, waterwheels and windmills. Philon of Byzantium, Archimedes, Dionysius of Alexandria and Leonardo da Vinci have made use of gears in various machines.

15.3 GEAR CLASSIFICATION

The gears may be classified into three types as discussed below:

1. According to relative position of shafts: The shafts between which motion has to be transmitted may be

1. Parallel
2. Intersecting
3. Non-intersecting and Non-parallel

2. DEPENDING UPON THE PERIPHERAL VELOCITY

a) When the velocity of gears is less than 3m/s, the gears are termed as Low Velocity Gears.

b) When the velocity of gears is between 3m/s to 15 m/s, the gears are termed as Medium Velocity Gears.

c) When the velocity of gears is greater than 15m/s, they are termed as high velocity gears.

3. According to contact of Gears**:**

It may be classified as

i) External Gearing

ii) Internal Gearing

In External Gearing the gears mesh externally while in internal gearing the gears mesh internally with each other.

15..3.1 Parallel Shafts

The following are the main types of gears connecting parallel shafts.

1. Spur Gear

The teeth of spur gears are straight and run parallel to the axis of the shaft.

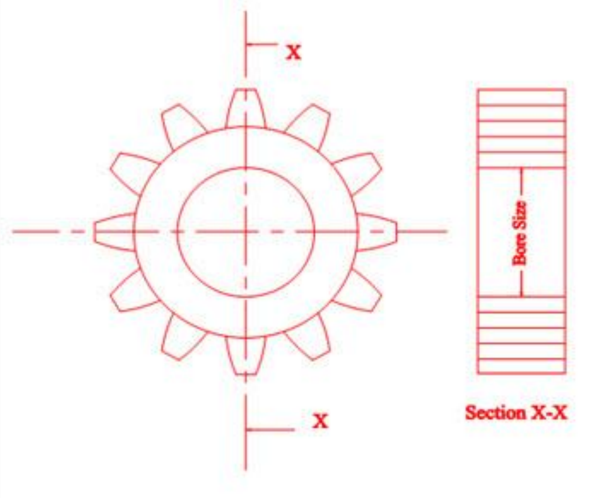


Fig.4.2 Spur gear

Spur gears may have external or internal contact depending on the type of layout. They are the most common type of gear and are quite simple to manufacture. Spur gears are mainly used in tractor transmissions, blenders, clothes dryers, and flour mills.

Lesson 10.

2. Helical Gear

Just like spur gears, helical gears are also used connect parallel shafts to transmit power. In helical gears teeth are cut at an angle which results in quiet and smooth running operation by providing more contact area at the time of teeth engagement.

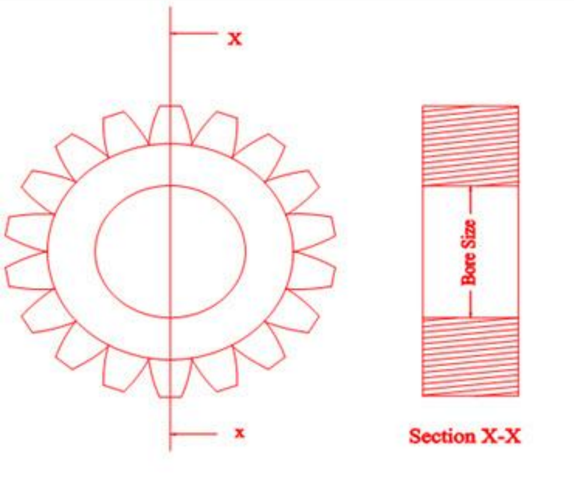


Fig.4.3 Helical gear

The applications of helical gears are in automobile transmission, machine tools, and compressors.

3. Herringbone gears:

Since Herringbone gears look like two helical gears joined together, so they may be referred as "double helical gears. Herringbone gears generally find applications in heavy duty vehicles.

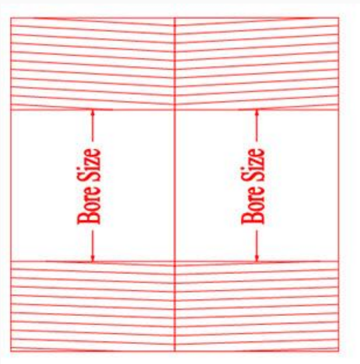


Fig.4.4 Herringbone gears (or double-helical gears)

4. Rack and pinion

A rack is a straight gear that meshes with smaller gear (pinion) to convert rotary power and motion in a linear movement or vice-versa.

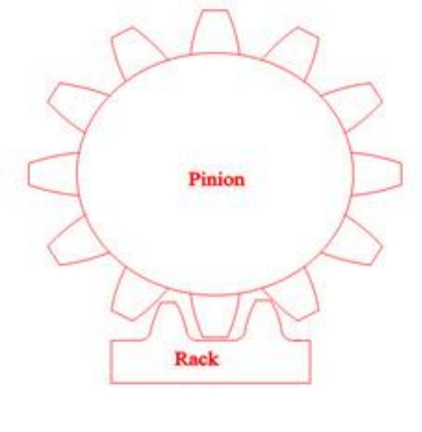


Fig.4.5 Rack and pinion

INTERSECTING SHAFTS

The following are the main types of gears connecting intersecting shafts.

1. Bevel Gear

Bevel gears find applications which generally require power transmission at right angles. The teeth on such gears may be straight, spiral or hypoid. The bevel gear has many applications such as differential of automobiles, printing machines, and processing plants.

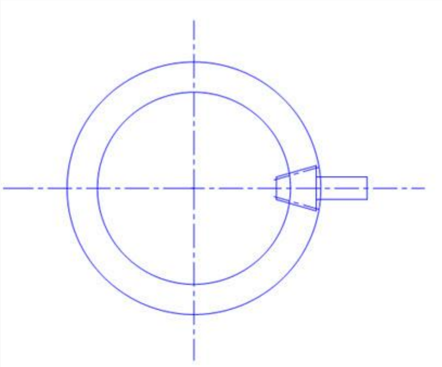


Fig.4.6 Bevel gears

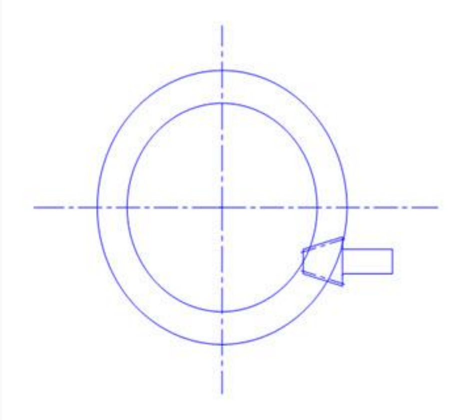


Fig.4.7 Hypoid gear

Lesson 11.

NON-INTERSECTING AND NON-PARALLEL (SKEW SHAFTS)

The following are the main types of gears connecting non intersecting and non-parallel shafts.

1. WORM AND WORM GEAR

The major function of worm gear is that of speed reduction. With the help of worm gears speed reductions greater than 300:1 can be achieved. Such types of gears are generally used in automobile steering mechanism.

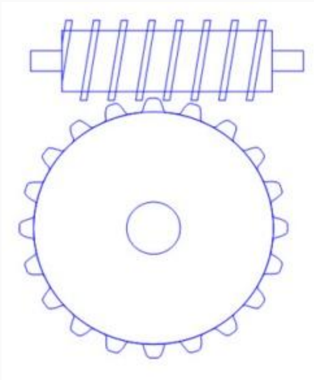


Fig. 4.8 Worm and worm gear

GEAR TERMINOLOGY

The Bureau of Indian standards (BIS) in their codes IS: 2458(1965) and IS: 2467(1965) has defined various parameters of gears as such:

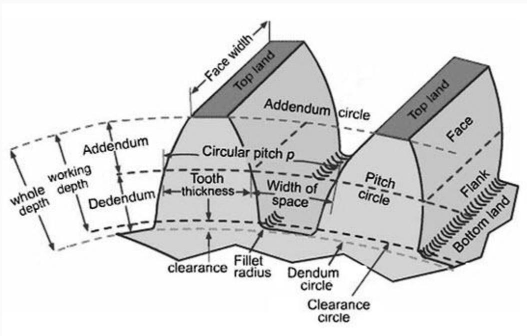


Fig 4.9 GEAR TERMINOLOGY

- **Pitch circle**: It is theoretical circle which divides the gear into two imaginary parts addendum and dedendum
- **Pitch circle diameter**: it is also called PCD or bolt circle diameter. Pitch circle diameter as the name suggests is the diameter of pitch circle. Circular

parts are assembled with other parts by passing bolts through holes drilled at PCD.

- **Addendum**: The vertical height along the circumference from the pitch circle to the gear top is called addendum.
- **Addendum circle**: it is an imaginary circle passing through top of gear.
- **Dedendum**: The vertical height along the circumference from the pitch circle to the bottom of gear is called **Dedendum**.
- Dedendum circle: it is an imaginary circle passing through bottom of gear.
- **Clearance**: The difference between the dedendum of one gear and the addendum of the mating gear.
- **Face and Flank of a tooth**: Axially the pitch circle divides the tooth in to two parts. The portion axially above the pitch circle is called face and below pitch circle is called flank.
- **Tooth thickness**: The thickness of the tooth along the pitch circle is called tooth thickness
- **Tooth space**: It is the distance between adjoining teeth of a gear.
- **Fillet:** A curvature called fillet is provided to connect tooth to the root circle
- **Pitch point**: Where two gears mesh their point of contact is called pitch point.
- **Backlash**: When gears mesh, there is clearance between the tooth of two mating gears. This clearance is referred to as backlash
- **Pressure angle**: The angle between the common normal at the point of tooth contact and the common tangent to the pitch circles. It refers to the angle through which forces are transmitted between meshing gears.
- **Diametral pitch:** The diametral pitch is the number of teeth divided by the pitch diameter.

$$\text{Diametral Pitch} = \frac{\text{Number of teeth}}{\text{PCD}}$$

- **Module** is defined as Pitch diameter divided by number of teeth.

$$\text{Module} = \frac{\text{PCD}}{\text{Number of Teeth}}$$

Lesson 12.

18.1 According to fundamental law of gearing, if the gears have meshed properly, the line of action should be straight and pass through the Pitch Point of the gears. Two tooth profiles that satisfy the above condition and used extensively in gear manufacturing process are involute and cycloidal profiles. An involute curve can be imagined as it is traced by a point on a stretched string during its unwinding from a cylinder. Whereas when two generating circles roll on the pitch circle they trace the cycloidal tooth profile. Involute profiles have constant pressure angle, are easy and cheap to manufacture but cycloidal profile tooth have variable pressure angle, more precise to manufacture so are relatively costlier than involute profiles. Pressure angle is necessary for quiet operation of gears. In cycloidal gears, the pressure angle is maximum at the start and end of engagement and is zero at pitch point. So the running of cycloidal gears is little noisy. But at the same time cycloidal gears are more robust due to wider flanks. Exact center distance has to be maintained in cycloidal gears while in the involute gears the center distance of mounting shafts can be varied by adding correction factor.

18.2 GEAR TRAINS

When two or more gears mesh it is called a **gear train.** Gears mesh with each other to transmit rotational motion from one shaft to another. There is a **driver** gear and a **driven** gear and the **gear ratio** produced by the train depends on the number of teeth on each gear. As already described the meshing gears rotate in opposite directions and in order to have the same direction of rotation of the driver gear and the driven gear an idler gear is added between the driver and driven gears.

18.3 VELOCITY RATIO

Velocity ratio of gear train is defined as the ratio of the speed of the driver gear to the speed of the driven gear and the ratio of speed of two meshing gears varies inversely as the number of teeth.

$$\text{Speed ratio} = \frac{\text{Speed of driver gear}}{\text{Speed of driven gear}}$$

$$= \frac{\text{No.of teeth on driver gear}}{\text{No.Of teeth on driven gear}}$$

Or

$$\text{Speed ratio} = \frac{1}{\text{Train Value}}$$

Lesson 13.

19.1 TYPES OF GEAR TRAINS

1. Simple gear train
2. Compound gear train
3. Planetary gear train

19.1.1 SIMPLE GEAR TRAIN

As the name suggests this is the simplest type of gear train in which one gear is mounted on single shaft. Such gear trains are generally used where there is no constraint of space and large centre distance can be maintained between the gear mounting shafts. The gear ratio produced by the train depends on the number of teeth on each gear. Idler gears have no effect on the speed ratio or train value of gear train. For different gear ratios, different combinations of gears are required. It may be worth mentioning here that meshing gears must have same module.

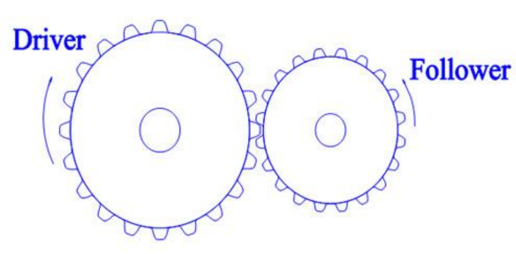


Fig 4.10 Simple Gear Train

19.1.2 COMPOUND GEAR TRAIN

In Compound gear train more than one gear rotate on single shaft. Such type of gear trains are used where the designer wants a compact layout and when large changes in speed or power output are needed.

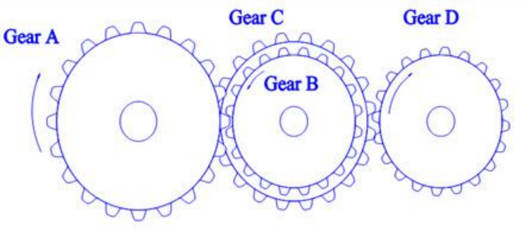


Fig 4.11 Compound Gear Trains

Speed ratio of compund gear train is given by

$$\text{Speed ratio} = \frac{\text{Speed of first driver}}{\text{Speed of lastdriver}}$$

$$= \frac{\text{Product of number of teeth on driven}}{\text{Product of number of teeth on driver}}$$

19.1.3 Epicyclic Gear Train

Epicyclic means to move upon and around in circular manner. Consider the diagrammatic representation of simple epicyclic gear train in the fig 4.12. In this gear train gear B meshes with gear A. Considering the arm AB to rotate about axis of gear A, then the gear B is forced to rotate upon and around gear A.

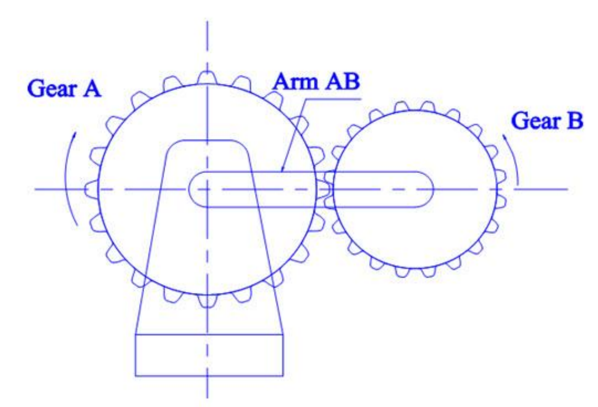


Fig 4.12 Epicyclic Gear Train

Compound epicylic gear trains are also referred to as **Planetary gear trains**. In such gear trains one or more gears orbit about the central axis of the train.

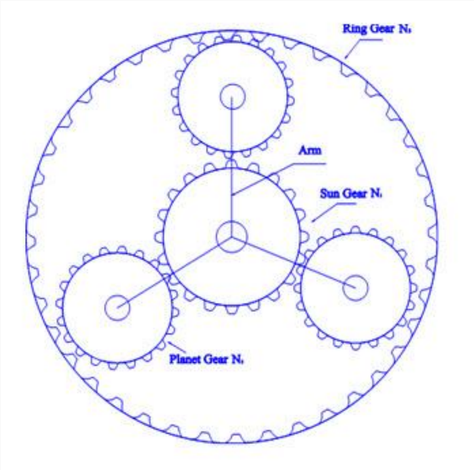


Fig 4.13 Planetary Gear Train

As shown in the figure 4.13 the sun gear, N_1 engages all three planet gears simultaneously. All three planet gears are attached to a plate (the planet carrier), and they engage the inside of the ring gear. The output shaft is attached to the ring gear, and the planet carrier is held stationary. Different gear ratios can be produced depending on which gear is used as the input, which gear is used as the output, and which one is held stationary. The major applications of Epicylic Gear train is in differential of automobiles, lathe machines etc.

Lesson 14.

Consider epicyclic train as shown in the figure 4.13. Let T_B, T_C and T_D be the number of teeth on sun gear N_1, Planet gears N_2 and internally tooth ring N_3.

First suppose that spider/arm is fixed. Sun gear N_1 is given +1 movement the planet gear C turns through – $\frac{T_B}{T_C}$ and internal gear D by – $\frac{(T_B)}{T_C}\frac{(T_C)}{T_D} = -\frac{T_B}{T_D}$ Secondly when spider A is fixed, sun wheel b rotates through +X revolution and planet C will rotate through –X $\frac{T_B}{T_C}$ and Internal Gear through –X $\frac{T_B}{T_D}$

Thirdly each element of an epi cyclic train is given +y revolution and finally the motion of each element is added up and entered in fourth row.

Table of Motion

Sr. no.	Motion of different elements of the system	Revolutions of different elements			
		Arm	Sun Gear	Planet Gear	Ring Gear
1	Spider is fixed, Sun gear rotates +1 revolution anticlockwise	0	+1	$-\frac{T_B}{T_C}$	$-\frac{(T_B)}{T_C}\frac{(T_C)}{T_D} = -\frac{T_B}{T_D}$
2	Spider is fixed, sun gear rotates +X revolutions	0	+X	$-X\frac{T_B}{T_C}$	$-X\frac{T_B}{T_D}$
3.	Add +y revolutions to every member of the system	+Y	+Y	+Y	+Y

4.	Motion of individual members of the system	+Y	X+Y	$Y - X\frac{T_B}{T_C}$	$Y - X\frac{T_B}{T_D}$

Q1. The compound gear train is as shown in the figure 4.14. The power shaft is connected to gear A having 30 teeth and rotates at 1000rpm. The spur gears B, C, D and E have 60, 20, 75 and 25 teeth respectively. Calculate the rpm of gear F if it has 90 teeth.

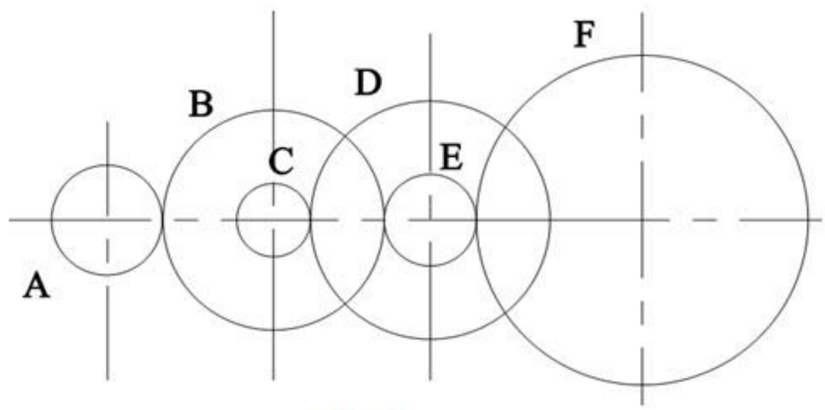


FIG 4.14

Sol:- let N_F = speed of gear F

$T_A = 30$

$T_B = 60$

$T_C = 20$

$T_D = 75$

$T_E = 25$

$T_F = 90$

$$\frac{N_A}{N_F} = \frac{T_B}{T_A}\frac{T_D}{T_C}\frac{T_F}{T_E}$$

$$\frac{1000}{NF} = \frac{60 \; X \; 75 \; X \; 90}{30 \; X \; 20 \; X \; 25}$$

$$N_F = \frac{1000 \; X \; 30 \; X \; 20 \; X \; 25}{60 \; X \; 75 \; X \; 90} = 37.03\text{rpm}$$

Lesson 15.

Q2:- An arm carries two gears A and B. The number of teeth on gears A is 30 and that on B is 50.Determine the speed of gear B if 1) arm rotates at 100 rpm in anticlockwise direction and gear A is fixed ii) if arm rotates at 100 rpm in anticlockwise direction and Gear A rotates at 200 rpm in the clockwise direction?

Sol:-

Given T_A = 30

T_B = 50

N_C = 100 rpm (anticlockwise)

The gear train in shown in the figure 5.15

By Tabular method.

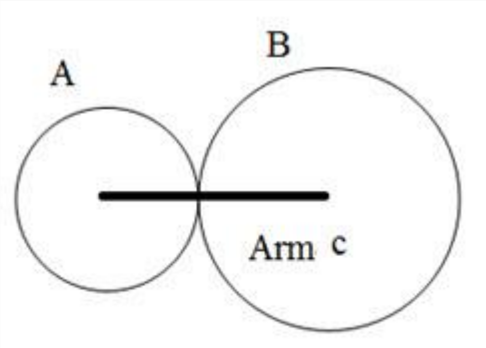


FIG. 4.15

Sr No	Motion of different elements of the system	Revolutions of different elements		
		Arm c	Gear A	Gear B
1.	Arm is fixed A rotates +1 Revolution	0	+1	$\frac{-T_A}{T_B}$
2.	Arm is fixed: A rotates + x revolution	0	+x	$\frac{-x\,T_A}{T_B}$
3.	Add +y revolutions to every member of the system	Y	+y	+y
4.	Motion of individual members of the system	+y	x+y	y-$\frac{x\,T_A}{T_B}$

Speed of gear B when gear A is fixed

Speed of arm is 100rpm anticlockwise

Y= +100rpm

Also x+y =0 therefore

X=-y=-100rpm

Speed of gear B=

$$N_B = y - X\frac{T_A}{T_B} = 100 + 100 \times \frac{30}{50} = 160\text{rpm}$$

Case-II

When the speed of gear B when gear A makes 200 rpm clockwise

X + y= -200

X= -200-y= -200-100= -300rpm

Speed of gear B

$$N_B = y - x \frac{T_A}{T_B}$$

$$= 100 + 300 \times \frac{30}{50}$$

$$= 100 + 180 = 280 \text{rpm}$$

Q3:- The pitch circle diameter of internally toothed ring as shown in the figure 4.16 is 200mm and module is 4mm. For five revolutions of sun wheel C, arm attached to planet gear makes only one revolution. Assuming Ring R stationary, design the gear train for number of teeths.

Sol:-

Given D_R = 200mm

m=4mm

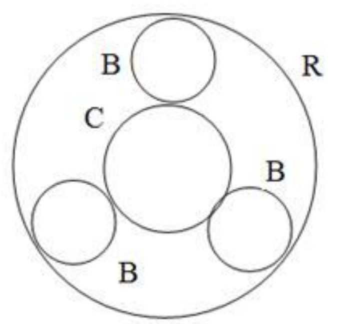


FIGURE 4.16

Sr. No	Motion of different elements of the system	Revolutions of different elements			
		Spider A	Sun wheel C	Planet wheel B	Internal gear R
1	Spider A fixed, sun gear wheel C rotates +1 revolution	0	+1	$\frac{-T_C}{T_B}$	$\frac{-T_C}{T_B} \times T_B/T_R = \frac{-T_C}{T_R}$
2.	Spider A fixed sunwheel C rotates +x revolution	0	+x	$\frac{-x\,T_C}{T_B}$	$\frac{-x\,T_C}{T_R}$
3.	Add +y revolutions to every member of the system	Y	Y	Y	+Y
4.	Motion of individual members of the system	+y	X+y	$y-\frac{x\,T_C}{T_B}$	$y-\frac{x\,T_C}{T_R}$

Sun gear C makes +5 revolutions, the spider A makes +1 revolution

Therefore y = +1

X y = +5

X = 5-y = 5-1=4

Since internally toothed ring R is stationery

$$y - x\frac{T_C}{T_R} = 0 \text{ or } 1-4 \times \frac{T_C}{T_R} = 0$$

$$\frac{T_C}{T_R} = \frac{1}{4} \text{ or } T_R = 4T_C$$

$$T_R = \frac{200}{4} = 50$$

$$T_C = \frac{50}{4} = 12.5 = 12.5$$

Also $d_C + 2\ d_B = d_R$

$12.5 + 2T_B = 50$

$2T_B = 50 - 12.5$

$T_B = 18.75$ Ans.

Lesson 16. BELT, ROPS AND CHAIN DRIVE

22.1 INTRODUCTION

A belt transmits power from one shaft to another by means of pulleys. Belts are one of the commonest and cheapest sources of power transmission. Power transmission through belts, rops and chains are achieved by specifically designed system. The pulleys may rotate in same or opposite directions depending upon the type of arrangement. The factors which play important role in designing belt drives are speed of shafts (driving and driven), shaft layout depending upon available space, type of application and power to be transmitted. The material of belt may be leather, cotton or fabric or rubber. Leather belts are made most widely used material for making belts. The belts are cleaned from time to time so that they remain dust free. The cotton belts are generally cheaper than leather belts and can be used in more hot and humid climate. They find applications is conveyors, hoist and machinery items. Rubber belts are quite flexible in nature and are therefore used in paper mills and saw industry.

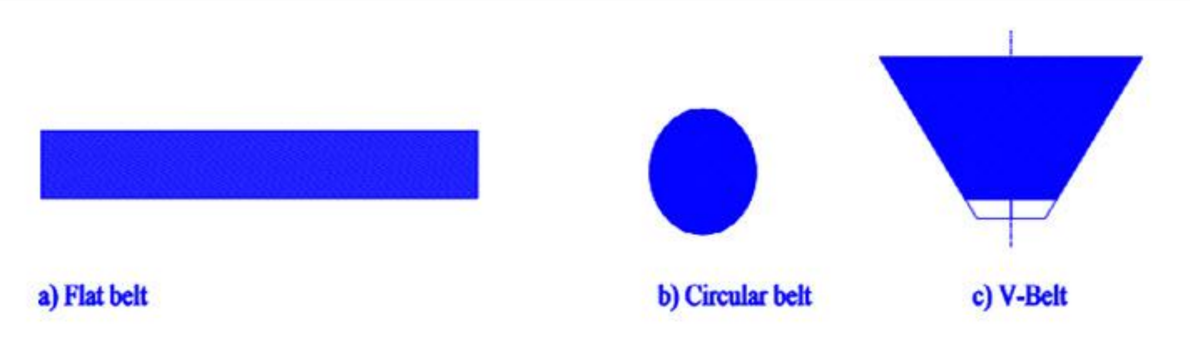


Fig 5.1 Types of belts

Belts may be flat, V- or circular cross-section. Belts find application in flour mills, power machines and electrical generators. Belt drives are simple to construct, require less maintenance and hence economical to use, are robust and are generally efficient source of power transmission. The major disadvantage of belt drives are slippage, and creep. Belts should be cleaned from time to time for best output.

Lesson 17.

23.1 TYPES OF FLAT BELT DRIVES

Flat belt drives may be of any of the following types:

1) Open belt drives

In this type of arrangement the shafts are parallel to each other and rotate in the same direction as shown in the fig 5.2. The lower side belt also known as tight belt side has more tension than the upper side belt known as slack side. The driver pulls the belt from one side and delivers it to other side known as slack side.

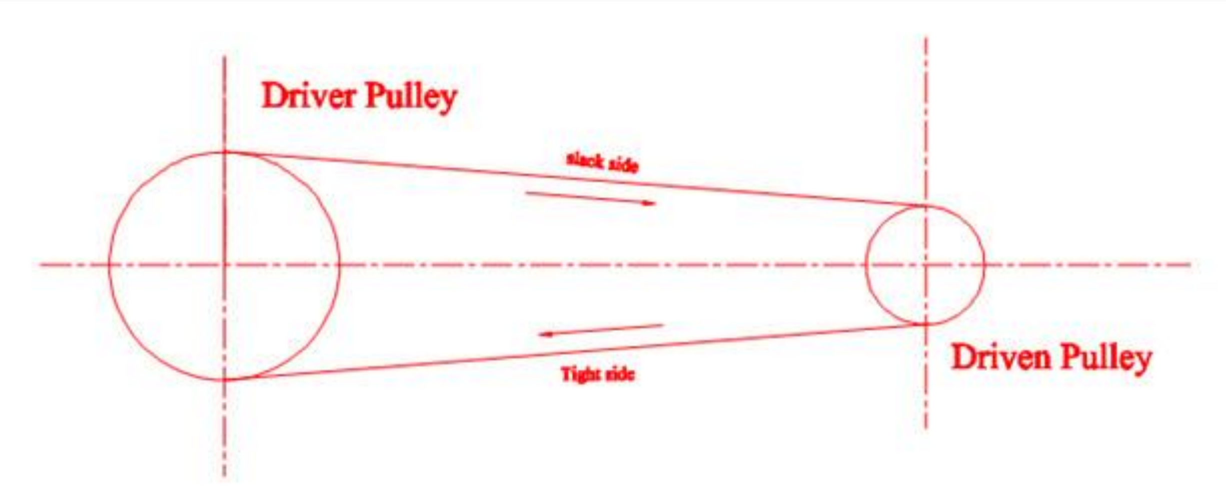


Fig 5.2 open belt drive

2) Crossed or twist belt drives

The arrangement of crossed belt drives is shown in the figure 5.3. Here the shafts are arranged in parallel. The belts rub against each other which cause wear and tear. There is driver pulley and driven pulley in the arrangement and they rotate in opposite direction.

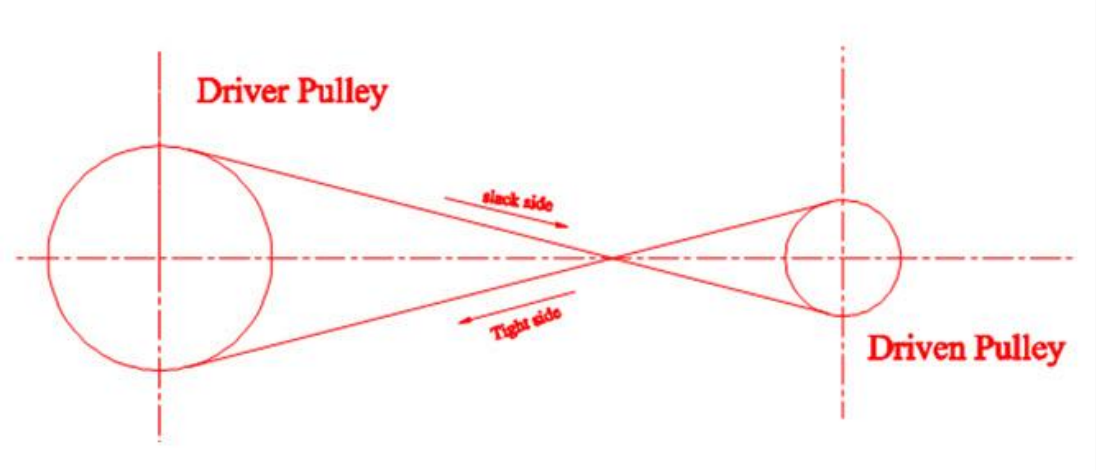


Fig 5.3 cross belt drive

3) Quarter turn belt drives

In this type of arrangement the shafts are arranged at right angles. The arrangement of the same is shown in the figure 5.4

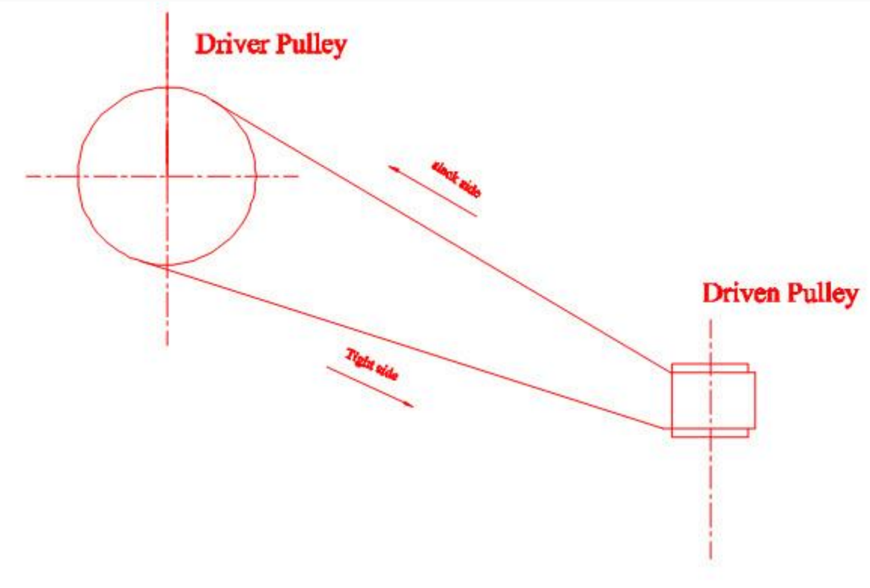


Fig 5.4 Quarter turn belt drive

4) Compound belt drives

Compound belt drives are used where power has to be transmitted through number of pulleys. The schematic diagram is as shown in the figure 5.5

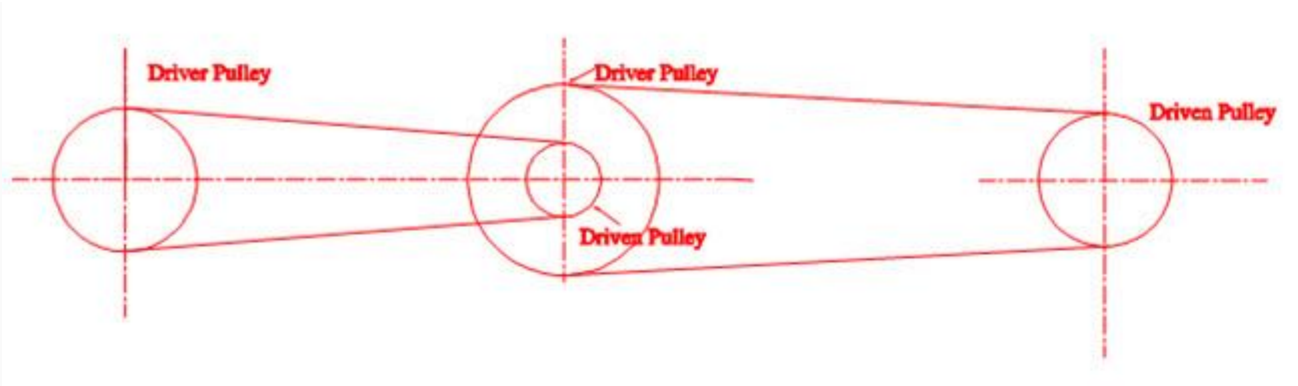


Fig 5.5 Compound belt drive

5) Stepped or cone belt drives:

Such belt drives are commonly used for changing the speed of driven shaft. The construction details for the same is shown in the figure 7.6

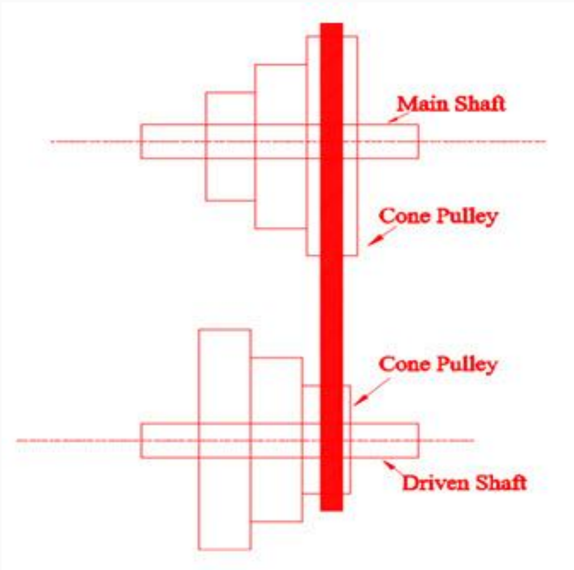


Fig 5.6 stepped

Lesson 18.

24.1 Consider a pulley 1 driving the pulley 2. The pulley 1 is called driver and pulley 2 is called driven. So velocity ratio may be defined as the ratio of speed of driven to that of driver or ratio of diameter of driver to that of driven.

Mathematically the equation is:

$$\frac{N_2}{N_1} = \frac{D_1}{D_2}$$

N_1 = Speed of driver pulley in rpm

N_2 = Speed of driven pulley in rpm

D_1 = Diameter of driver pulley in mm

D_2 = Diameter of driven pulley in mm

When the thickness of the belt (t) is considered, then velocity ratio is given by

$$\frac{N_2}{N_1} = \frac{\left(D_1 + t \right)}{\left(D_2 + t \right)}$$

24.2 SLIP OF BELT DRIVES

The forward motion of the driver without carrying the belt with it is called slip of the belt and is generally expressed in percentage. For slippage velocity ratio is given by

$$\frac{N_2}{N_1} = \frac{\left(D_1 + t \right)}{\left(D_2 + t \right)} \times \left(1 - \frac{S}{100} \right)$$

Where $S = S_1 + S_2$ (total percentage of slip)

S_1 = Percentage slip between driver and the belt and

S_2 = Percentage slip between belt and the follower

24.3 Length of open belt drive

Consider an open belt drive; both the pulleys rotate in the same direction as shown in the figure 5.7

Let r_1 and r_2 be radii of larger and smaller pulley

X= distance between the centres of two pulleys i.e. (C_1, C_2)

L= Total length of the belt

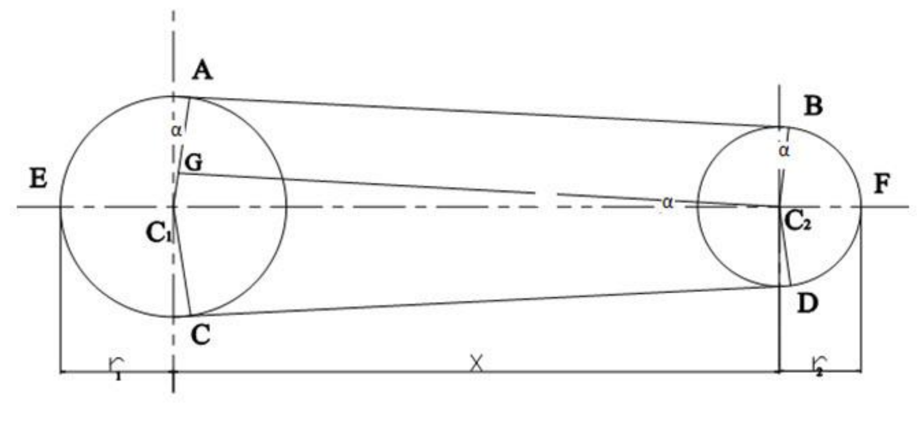


Fig. 5.7 Length of open belt drive

Let the belt leaves the larger pulley at A and C and smaller pulley at B and D. Through C_2 draw C_2G parallel to AB.

From the figure, C_2G will be perpendicular to C_1A.

Let the angle $GC_2C_1=\alpha$ radians

We know that length of belt is given by

L= Arc CEA+AB+Arc BFD+DC

=2(Arc AE+AB+Arc BF)

From the figure,

$$\sin\alpha = \frac{C_1G}{C_1C_2} = \frac{(C_1A - AG)}{C_1C_2} = \frac{(r_1 - r_2)}{x}$$

$$\text{Arc AE} = r_1\left(\frac{\Pi}{2} + \alpha\right)$$

$$\text{Similarly Arc BF} = r_2\left(\frac{\Pi}{2} - \alpha\right)$$

$AB= GC_2= [(C_1C_2)^2 – (C_1G)^2]^{1/2}= [x^2-(r_1-r_2)^2]^{1/2}$

Expanding by binomial theorem

$$AB = x\left[\left(1-\frac{1}{2}\frac{(r_1-r_2)}{x}\right]^{1/2} + \ldots\right] = x - \frac{(r_1-r_2)^2}{2x}$$

Substituting the values of arc AE from equation, arc BF and AB from equations we get

$$L\,2\left[r_1\left(\frac{\Pi}{2}+\alpha\right) + x - \frac{(r_1-r_2)^2}{2x} + r_2\left(\frac{\Pi}{2}-\alpha\right)\right]$$

$$= 2\left[r_1\left(\frac{\Pi}{2}+\alpha\right) + x - (r_1-r_2)^2/2x + r_2\left(\frac{\Pi}{2}-\alpha\right)\right]$$

$$= 2\left[r_1\frac{\Pi}{2} + r_1.\alpha + x - \frac{(r_1-r_2)^2}{2x} + r_2\frac{\Pi}{2} - r_2\alpha\right]$$

$$= \Pi\,(r_1+r_2) + 2\,\alpha\,(r_1-r_2) + 2x - \frac{(r_1-r_2)^2}{x}$$

Substituting the value of $\alpha = \dfrac{(r_1-r_2)}{x}$

$$L = \Pi\,(r_1+r_2) + 2\,(r_2-r_1)\,(r_1-r_2) + 2x - \frac{(r_1-r_2)^2}{x}$$

$$\frac{\Pi}{2}(d_1+d_2) + 2x + \frac{(d_1-d_2)^2}{4x}$$

24.4 Ratio between belt tensions

A driven pulley rotating in clockwise direction is shown in figure 5.8

Let T_1 = Tension on tight side of the belt

T_2 = Tension on slack side of the belt

θ = Angle of contact of belt with pulley in radians

μ = Coefficient of friction between the belt and pulley.

Considering a small portion of belt XY in contact with the pulley. X makes angle $\delta\theta$ at the centre of the pulley. Let T be the tension in the belt at X and $(T + \delta T)$ be the tensions in the belt at Y.

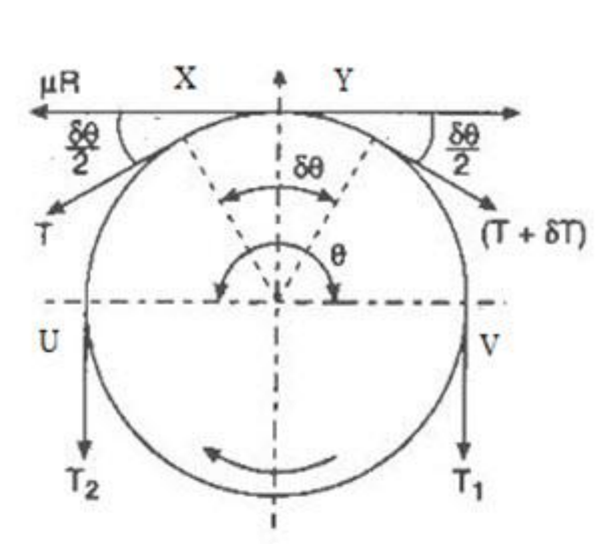


Fig 5.8 Ratio of driving tensions

The belt portion XY is in equilibrium under the action of following forces:

1. Tension T in belt at X.
2. Tension $T + \delta T$ in belt at Y.
3. Frictional reaction R at U.
4. Frictional force μR between belt and pulley.

Now again solving the forces expression in horizontal direction

$\mu R = (T + \delta T) \cos (\delta\theta / 2) - T \cos (\delta\theta / 2)$

Since $\delta\theta$ is very small

$\cos (\delta\theta / 2) = 1$

$\mu R + T = (T + \delta T)$

$\mu R = \delta T$ ...(1)

and resolving the force is vertical reaction.

$R = T \sin (\delta\theta / 2) + (T + \delta T) \sin (\delta\theta/2)$

Since, $\delta\theta$ is very small, so

$R = T\,\delta\theta/2 + (T + \delta T)\,\delta\theta/2 = T\,\delta\theta$ ...(2)

From equations (1) and (2)

$\mu(T\,\delta\theta) = \delta T$ (neglecting $\delta T.\ \delta\theta / 2$)

$\delta T / T = \mu\,\delta\theta$

On integration, we get

$^{T2}\int_{T1} \delta T/T = \int_o^{\theta} \mu d\theta$

$\log e\ T_1 / T_2 = \mu\theta$

$T_1 / T_2 = (e)^{\mu\theta}$

24.5 Power transmitted by belt drive

Power transmitted, $P = (T_1 - T_2)\ V$ (Watt)

V is the velocity of the belt = \[= \frac{{{\text{ }}{{\text{D}}_{\text{1}}}{{\text{N}}_{\text{1}}}}}{{60}}\] (m/sec)

24.6 Tension in the belts

The initial tension (T_0) ensures that the belt would not slip under designed load. Belts are mounted on the pulleys with a certain amount of tension T_0. At rest, the forces acting on the two sides of belts are equal. As the power is transmitted, the tension on the tight side increases and tension on the slack side decreases.

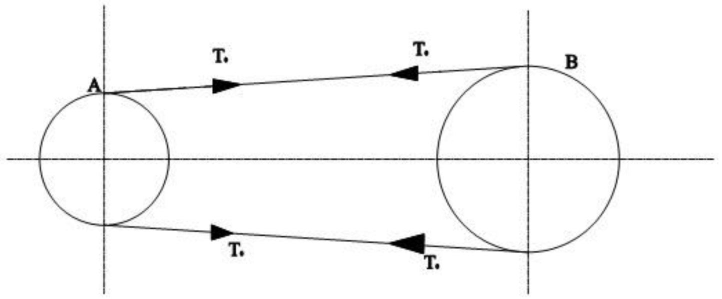


Fig 5.8a) Pulleys at rest

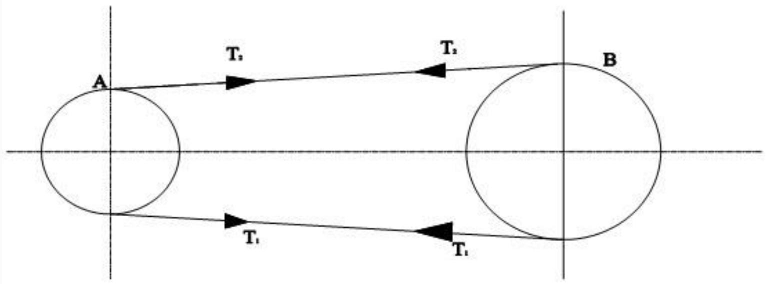


Fig 5.8 (b) Pulleys Working

Mathematically Initial tension is given by

$$T_0 = \frac{(T1+T2)}{2}$$

$$= \frac{(T1+T2+2Tc)}{2}$$

Where T_1 and T_2 are tensions on tight side and slack side and T_c is the centrifugal tension given by

$Tc=mv^2$

Where m= mass of belt per unit length and V=Velocity of belt in m/s

24.7 Condition for the transmission of Maximum power

Power transmitted by belt drive is given by

$P= (T_1-T_2)\ V$.................................(1)

Where T_1= Tension in the belt on tight side

T_2= tension in the belt on slack side

V= velocity of belts

Also $$\frac{T_1}{T_2} = {e^{\mu}}^{\theta}$$

Substituting the value of T_2, we get

$$= \left(T_1 - \frac{T_1}{e^{\mu\theta}}\right)V = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right)V = T_1.V.Z \quad \text{.................. (2)}$$

Where

$$Z = 1 - \frac{1}{e^{\mu\theta}}$$

Also maximum tension (T) in the belt considering centrifugal tension (Tc) is given by

$T=T_1+T_c$

Substituting the value T_1 in equation (2)

$P= (T-T_c)\ V.Z= (T.V-mV^3)\ Z$

Differentiating with respect to V for maximum power and equate to zero

$$\frac{\partial P}{\partial V} = 0$$

$$\frac{\partial\left(T.V.-mV^3\right).Z}{\partial V} = 0$$

$$T-3mV^2=0$$

$$\text{Or } V=\sqrt{\frac{T}{3m}}$$

Lesson 19.

25.1 VEE-BELT DRIVES

V- Belts are used where there is constraint of available space and designer wants to have compact layout of the drive system. The operation of such type of belt drives is smooth and noiseless. The life of V-belts is quite long if their contact with water and chemicals is avoided. The operating speed of V- belts is generally between 5 m/s to 50 m/s.

25.2 CHAIN DRIVES

Chain is a series of connected **links** which are typically made of metal and is used where no slippage is required. A chain may consist of two or more links. The chains on the basis of their use is classified into following three groups.

1) Hoisting and hauling chains: These chains are used for lifting, lowering, pulling or dragging operations. They are generally of oval or square shape. Figure 5.9 shows the diagrammatic representation of the same

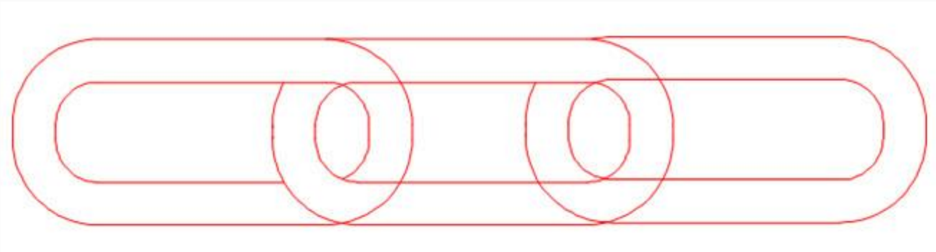


Fig 5.9 Hoisting and Hauling chains

2) Conveyor chains

These chains are made from malleable cast iron and are used for transporting material from one place to another on a conveyor system. The cross section of conveyor chain is shown in the figure 5.10

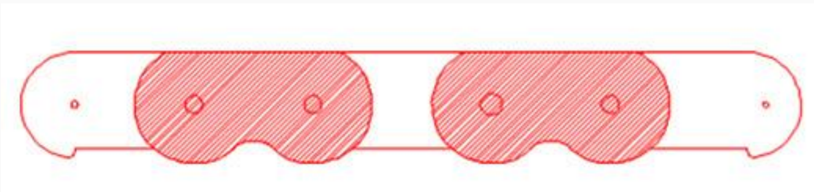


Fig 5.10 Conveyor chains

3) Power transmitting chains: As the name suggests these chains are used for power transmission. They may be of categorized as block chains, bush roller chains and inverted tooth or silent chains

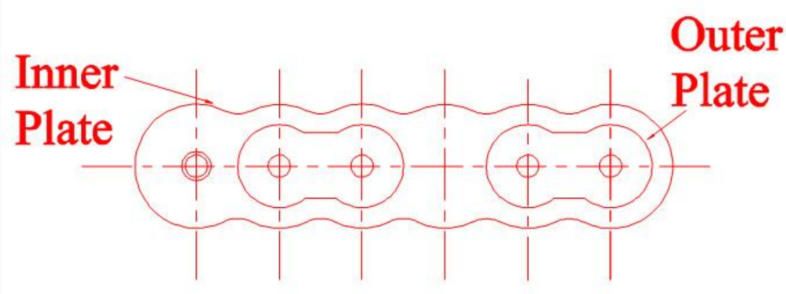


Fig 5.11 Power transmission chains

Lesson 20

Q1:- The coefficient of friction between belt and pulley is 0.39; angle of lap is 150°. The belt runs over pulley of diameter 500 mm at 150 rpm. Calculate power transmitted if maximum tension in the belt is 3000 N.

Sol:-

D = 500mm = 0.5m

N = 150rpm

μ= 0.39

$$\Theta = 150^{\circ} \times \frac{\pi}{180} = \frac{5\pi}{6}$$

We know that velocity of Belt

$$V = \frac{\pi dN}{60} = \frac{3.14 \times 0.5 \times 150}{60} = 3.925 \text{m/s.}$$

T_2 = Tension in the slack side of Belt

$$2.3 \log \left(\frac{T_1}{T_2}\right) = 0.785$$

$$\text{Log}\left(\frac{T_1}{T_2}\right) = \frac{0.785}{2.3} = 0.3413$$

$$\frac{T_1}{T_2} = 2.194 \text{ and } T_2 = 1367.3$$

Power = $(T_1 - T_2)$ V= (3000- 1367.3)3.925=6408.3 W = 6.40 KW

Q2:- The belt in 90mm wide and 10mm thick having density of1000kg/m^3. The coefficient of friction between belt and pulley is 0.3. The maximum stress in the belt is 5mPa, Calculate maximum power the belt can transmit, if angle of lap is 120°.

Sol:- We know that maximum tension in Belt

T = t.b.t. = 5 X 10^6 X 0.09 X .01

=4500N

Mass of belt per meter length

M = area X length X density

$= b.t.l.\rho$

$= \frac{90}{1000} \times \frac{10}{1000} \times 1 \times 1000$

$= 0.09 \times .01 \times 1000 = 0.9 kg/m$

Speed of Belt for greatest power

$$V = \sqrt{\frac{t}{3m}} = \sqrt{\frac{4500}{3m3 \times 0.9}} = 40.82 m/s$$

$$T_C = \frac{T}{3} = \frac{4500}{3} = 1500N$$

Tension in light side of belt

$T_1 = T - T_C = 4500 - 1500 = 3000N$

T_2 = Tension in slack side of belt

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu\theta = 0.3 \text{ X } \theta$$

$$\Theta = 120° \text{ x } \frac{\pi}{180} = 2.1 \text{ radian}$$

$$\text{Log}\left(\frac{T_1}{T_2}\right) = 0.3 \times 2.1 = 0.6.3$$

$$\left(\frac{T_1}{T_2}\right) = 1.88 \text{ or } T_2 = \frac{3000}{1.8} = 1666.6N$$

Maximum power

$P = (T_1 - T_2) v$

$= (3000-1666.6)40.82$

$= 54429.3w$ or $P = 54.4kw$

Lesson 21

Q3:- The coefficient of friction between Belt and pulley is 0.3 having angle of lap as120°. The smaller pulley has radius of 150 mm and rotates at 600rpm. Calculate power transmitted if initial tension in the belt is 1800 N.

Sol:- Velocity of Belt

$$V = \frac{\pi d_2 N_2}{60} = \frac{\pi X\, 0.3\, X\, 600}{60} = 9.42 m/s$$

Let T_1 = Tension in tight side of Belt

T_2 = tension in slack side of Belt

We know that initial tension (T_0)

$$1800 = \frac{T_1 + T_2}{2}$$

Or $T_1 + T_2 = 3600$ N……… (1)

We know

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu\theta = 0.3 \times 2.1 = 0.63$$

$\frac{T_1}{T_2}$ = 1.87……………. (2)